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STOCK MARKET PRICE SHOCKS AND MONETARY POLICY WITHIN THE US ECONOMIC CYCLE: AN EMPIRICAL ANALYSIS WITH THE VAR MODEL

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A mia nonna Maria,
sempiterna brezza estiva
nei lunghi inverni dell’esistenza.
Luce, acqua e anidride carbonica
della mia fotosintesi clorofilliana.

CONTENTS

INTRODUCTION

Over the past fifty years, the activities of the Federal Reserve have undergone major changes. The drastic policy actions taken by the FED in late 1970s, early 1980s, under the leadership of Paul Volcker, represent a break with the past, both in the conduct of monetary policy and in the control of inflation. Indeed, low inflation has emerged, if not as the primary objective of monetary policy, at least at a central focus than it was before the Great Inflation occurred. Since then, it is commonly believed that central banks have clear objectives for exerting control over interest rates, the main monetary instrument, namely, low and stable inflation and production close to the natural rate.

The Great Recession, after years of inflation targeting hegemony, has challenged the consensus that central banks should focus on stabilizing inflation and the output gap, and ignore fluctuations in asset prices, and has strengthened the viewpoint that central banks should pay attention and eventually respond to developments in asset markets. Supporters of this view argue that monetary authorities should “lean against the wind”, i.e. raise the interest rate to counteract any episode of the so-called asset price inflation, even at the cost of temporarily deviate from their inflation or output gap targets. Indeed, however low and stable inflation promotes financial stability, it also generates excess demand pressures that show up first in credit aggregates and asset prices, rather than in goods and services prices.

The purpose of this thesis is to examine, through the estimation of vector autoregression (VAR) models, the effects of stock market price and monetary policy shocks within the US economic cycle. Therefore, I will analyse how the FED reacts to a shock in the financial market and how the financial market and the US economy react to a monetary policy shock.

The variables considered in the analysis are quarterly time series: S&P 500, as a proxy for the financial market; Real GDP per capita, as a proxy for the US output; GDP Price Deflator, as a proxy for inflation; Effective Federal Funds Rate as a proxy for the interest rate.

The reference period goes from the first quarter of 1980 to the fourth quarter of 2019, thus, involving the challenges incurred by the FED to face the Great Inflation and the Great Recession.

What emerges from the analysis is that, from a financial point of view, the results are in line with the macroeconomic tenets; thus, to a positive shock in the financial index follows an expansion in the GDP, resulting on an increase in the monetary amount available in the economy, which will result in an increase in inflation; the FED would then try to contrast the increase in inflation by raising the interest rates. While, regarding the responses to monetary policy shocks, we encounter some anomalies contrary to what economic theory advocates: a positive monetary policy shock followed by an increase in the inflation rate. We may be tempted to associate, prematurely, this behaviour to the *price puzzle* phenomenon (Eichenbaum, 1992). However, there is vast literature that witnesses how this anomaly particularly involves the VAR processes (Estrella, 2015); moreover, there is not enough evidence in my analysis to prove this phenomenon.

The essay is structured as follows: the first chapter will introduce the reader to the historical and theoretical background of the thesis, concerning the evolution of US monetary policy and macroeconomic modelling; then, the second chapter will dig through the VAR methodology from a technical perspective; the third chapter is about the presentation and analysis of the variables used in the model; the fourth chapter covers the model estimation and the interpretation of the obtained results.

Thus, conclusions will be traced out.

1 THEORETICAL AND HISTORICAL OVERVIEW

1.1 THE EVOLUTION OF U.S. MONETARY POLICY

Understanding how monetary policy is designed and developed based on different historical moments, and the effects of innovations in monetary policy manoeuvres, has always been a matter of considerable importance for economists, scholars and the general public, especially, if the monetary policy into consideration refers to one of the most economically strong states in the world, which with its central bank has always influenced the conditions of the national and international economy (Judd & Rudebusch, 1998).

In this chapter my aim is to introduce and analyse the historical-economic situation of the US, with particular reference to the monetary policy undertaken by the FED at the end of the 1970s, in response to the Great Inflation and, later, to the Great Recession in 2007/2008.

To do so, I will highlight the change in monetary policy that occurred in the late '70s, early '80s, following the rise of Paul Volcker at the head of the Federal Reserve. Indeed, the drastic policy actions taken by the FED in 1979 represent a break with the past, both in the conduct of monetary policy and in the control of inflation. I will thereby illustrate the FED’s changes to the federal funds rate to provide insights into how the FED has managed both inflation and recession (Hetzel, 2017).

1.1.1 *THE US MONETARY POLICY PRE-VOLCKER*

In 1967 there was the beginning of the crisis for the whole world economy. Developed countries, suddenly found difficulties in the energy supply. During the Arab-Israeli war of 1973, the so-called Yom Kippur War, the Arab states producing oil decided to impose a quota on production to punish the supporting countries of Israel. The initiative taken by the Arab countries resulted in a steep rise in the price of crude oil which affected, in addition to the United States, the countries of western Europe and Japan. What happened was that the governments of the major oil producing countries, all members OPEC (Organization of the Petroleum Exporting Countries), decided to take control of the production and, more importantly, price setting for exports from their countries. To the sudden and unexpected interruption of the flow of the oil supply, we must consider the increasingly emphasized dependence of the U.S. from the imported oil: the crisis was just detonated. The rise in oil prices was so significant that the monetary policies of the time and the measures undertaken by the U.S. government were not enough to stem inflation, which reached 14% at the end of 1973. After a short downward movement in the years 1975-1977, due to the economic recession, the outbreak of the war between Iran and Iraq in 1980 detonates the second oil price shock. The second oil crisis will begin in 1979 and will have its full effect in 1980, triggering the Great Inflation; even if, however, Milton Friedman writes in his book, *Money Mischief, Episodes in Monetary History*: “inflation is always a monetary phenomenon” (Friedman, 1993), suggesting that the monetary policy conducted at that time, which financed massive budgets deficits and was supported by political leaders, was the main cause of inflation.

1.1.2 *The US monetary policy under the leadership of P. Volcker*

“Do we have the wit and the wisdom to restore an environment of price stability without impairing economic stability? Should we fail, I fear the distortions and uncertainty generated by inflation itself will greatly extend and exaggerate the sense of malaise and caution… Should we succeed, I believe the stage will have been set for a new long period of prosperity.”[1]

Paul Volcker

[1] Volcker, P. A. (1978). The role of monetary targets in an age of inflation. *Journal of Monetary Economics*, p. 61.

Paul Volcker became Chairman of the Federal Reserve system on August 6, 1979. Volcker, before being the head of the FED, held the position of president of the Federal Reserve Bank of New York. The new chairman, on October 6, 1979, decided to hold an extraordinary meeting of the FOMC, with the intention of deciding on the best method to control currency, credit expansion and inflation. Volcker announced that monetary policy would focus on fighting the high inflation rate and, therefore, the Federal Reserve became the leading character of the actions to reduce inflation. The drastic policy actions of the FED in 1979 represented the breaking point with the monetary policy of the past (Browne, 2001).

The revolution in the conduct of monetary policy implemented by Volcker was based on the renunciation of targeting the federal funds rate in favour of targeting the non-borrowed reserves by banks members of the federal reserve system. In this way the FED could directly control the money supply, through the non-borrowed reserves, which became the operative mechanism to control the monetary aggregate M1[2]. Associated with a greater focus on the control of money supply, there were significant increases in federal funds rates.

With higher federal funds rates, American firms sought technologies and new products which made investments more profitable to bear the effects of the higher cost of money. After the uncertainties of the first two years of the 1980s, the American economy recovered, which was then passed to the rest of the world. In 1982, the Fed returned to targeting the federal funds rate specifically. The graph below highlights the peaks in inflation in the late 1970s and early 1980s.

[2] M1 is the money supply that is composed of physical currency and coin, demand deposits, travellers' checks, other checkable deposits, and negotiable order of withdrawal (NOW) accounts. M1 includes the most liquid portion of the money supply because it contains currency and assets that either are or can be quickly converted to cash.

Fig. 1 - Inflation, consumer prices for the United States. Source: fred.stlouisfed.org

1.1.3 *The US monetary policy during the Great Recession*

The Great Recession began in December 2007 and ended in June 2009. As the financial crisis and recession deepened, measures intended to revive economic growth were implemented on a global basis. In US, the Federal Reserve’s response to the crisis evolved over time and took a number of non-traditional avenues: credit easing programs that sought to facilitate credit flows and reduce the cost of credit, and the large scale asset purchase (LSAP) programs aimed to push down longer-term public and private borrowing rates. However, for the purpose of my thesis, it is important to notice that the FED initially employed “traditional” policy actions by reducing the federal funds rate from 5.25 percent in September 2007 to a range of 0-0.25 percent in December 2008, with much of the reduction occurring in January to March 2008 and in September to December 2008. The sharp reduction in those periods reflected a marked downgrade in the economic outlook and the increased downside risks to both output and inflation. Between 2008 and 2015 FED kept the federal funds rate at zero. In 2015 the growth was stabilized so the FED began raising rates, maintaining a steady increase in rates until the beginning of 2019 (Hetzel, 2017).

Following the graph highlighting the shadow areas related to the highest and the lowest federal funds rate of all time, respectively 20% in the 80s to combat the double-digit inflation and a range that variates from 0.0% to 0.25% in 2008 in response to the financial crisis.

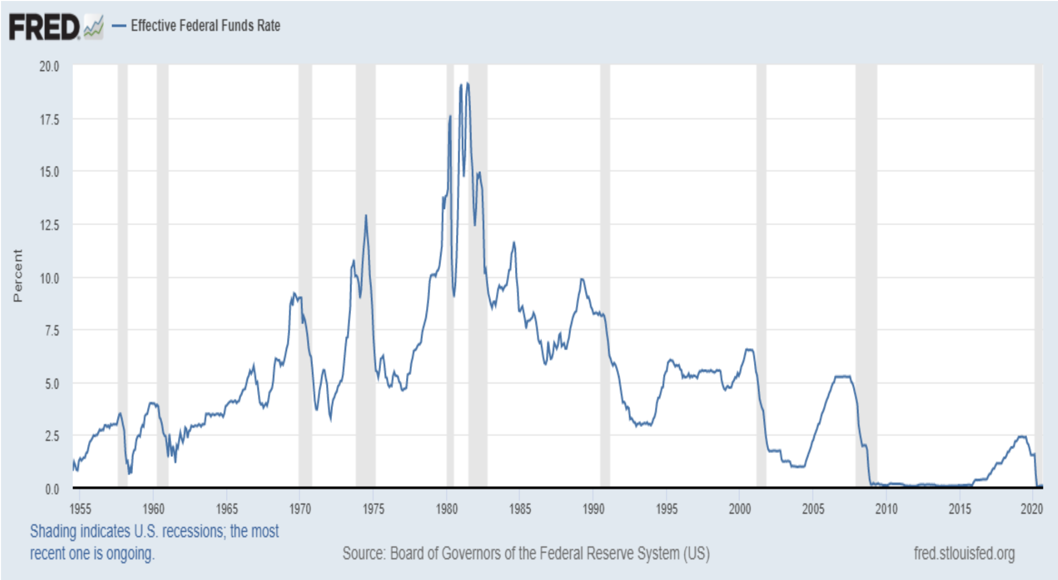


Fig. 2 - Effective Federal Funds Rate. Source: fred.stlouisfed.org

1.2 THE EVOLUTION OF MACROECONOMIC MODELLING

In a famous article, published on the Journal of Economic Perspectives, Stock and Watson (Stock & Watson, Vector Autoregressions, 2001) explain that macro-econometricians have mainly four tasks:

1) Data description: describing and summing up macroeconomic data related to a specific economic reality.
2) Forecasting: performing macroeconomic forecasts.
3) Structural Inference: quantifying what is known regarding the macroeconomic structure and investigate the unknown.
4) Policy Analysis: providing suggestions to macroeconomic policymakers.

Historically, these four objectives have been pursued through different approaches: from large models with hundreds of equations to single-equation models that focused on interactions of few variables to simple univariate time series models involving only a single variable.

However, after the macroeconomic chaos during the 70s, probably triggered by a severe oil shock in 1973, as discussed in the previous section, it was clear that the modelling used at that time was not able to explain macroeconomic interactions appropriately (Backhouse & Cherrier, 2019).

To understand why VARs soon became very popular among econometricians around the world, it is convenient to consider what models were used and what economic assumptions they used to work with. Until then, the main tool for macroeconomic analysis at disposal of central banks were the SEM models (Structural Equation Modelling), based on Keynesian macroeconomic foundations. Wren-Lewis in (Wren-Lewis, 2018) traces how the New Classical Revolution[3] in macroeconomics the 70s, as a response to the failure of Keynesian economics to explain stagflation, gained dominance, and why SEMs were displaced as policy models. The Lucas critique (Lucas, 1976) of then current large econometric policy models was the key. Because the parameters of those models were not structural, i.e. not policy-invariant, they would necessarily change whenever policy was changed. Lucas called into question the prevailing large-scale econometric models that lacked micro-foundations of optimal decision rules of economic agents. The SEMs of the day were attacked not only because they were potentially not structural, but because of the accusation that they contained "incredible restrictions".

The New Keynesian DSGE[4] models were considered better than the SEMs because they tried to solve the identification problems. The DSGE modelers have seen the Lucas Critique as a fundamental methodological prescription and not a "mere critique". To perform policy evaluation, macro-econometric models should specify aggregate relations as the result of the optimizing, forward looking individual behaviour of economic agents with respect to changes in their environment,

[3] New classical macroeconomics, sometimes simply called *new classical economics*, is a school of thought in macroeconomics that build its analysis entirely on a neoclassical framework. Specifically, it emphasizes the importance of rigorous foundations based on microeconomics, especially *rational expectations.*

[4] Dynamic stochastic general equilibrium modelling (DSGE) is a method in macroeconomics that attempts to explain economic phenomena, such as economic growth and business cycles, and the effects of economic policy, through economic models based on applied *general equilibrium theory* and microeconomic principles (*micro-foundations*).

especially with respect to changes in policy rules. Paraphrasing Lucas (Lucas, 1976): models should specify dynamic, optimal decision rules of rational individuals, in one word *micro-foundations*. When the micro-foundations are correctly specified , then the behavioural parameters ruling the model are structural or policy-invariant, i.e. such parameters have a stable value across different policy regimes.

Macroeconomists turned to DSGE models claiming them to be micro-founded in individual optimization and incorporating rational expectations (Henry & Muellbauer, 2017).

It was in this atmosphere that, in 1980, the later Nobel Prize winner Christopher Sims, in a famous article appeared on *Econometrica* (Sims, 1980), provided a new possible formalization of macroeconomic interactions: the vector autoregressions, typically called with the acronym VAR.

As Wren-Lewis notes, the proposition by (Sims, 1980) that vector autoregressive regressions (VARs) should be used as a way of modelling economic relationships without imposing incredible restrictions took hold.

In (Sims, 1980), Sims, indeed, argued that the assumptions used at the time for accomplishing econometric identification were simply "incredible". He mentioned the example that, to identify a structural system with a demand curve and a supply curve, it was standard at the time to assume that one variable shifts the demand curve but not the supply curve. The assumption that a variable could be important for one side of the market but could be excluded from affecting the other is incredible, according to Sims. He claimed that, for example, if the weather in Brazil matters for the supply of coffee, it is also expected to matter for demand; people on the demand side of the market, indeed, noticing bad weather in Brazil, would surely stockpile coffee in anticipation of an imminent rise in price. The view taken by Sims was that models based on identification assumptions such as these were not useful, thus he proposed VAR as an alternative to standard econometric models with their doubtful exclusion restrictions (Christiano, 2012).

The greatest impact of VARs has been in the construction of economic models. The great advantage proposed by Sims was the *parameter parsimony*, making the smallest assumptions. The advantage of working with a small set of assumptions is that the controversy over assumptions is thereby restricted to essentials. Indeed, because of the need to counter 'the curse of dimensionality'— the explosive increase in the numbers of parameters in a VAR as the number of variables and lags increases — in practice VARs imposed new restrictions: the minimum number of parameters or, put differently, just the right amount of predictors needed to explain the model well. These are on the number of variables and lags it is feasible to include, via the application of Bayesian priors and/or exclusion restrictions common in 'structural' VAR.

After the failure of the traditional Keynesian models, VARs have been used to select functional form and other assumptions that allow to build a fully specified stochastic general equilibrium (DSGE) models. VARs have been used in this way to guide the construction of new Keynesian models.

I now turn to an assessment of VARs in performing the four macroeconomics tasks expressed at the beginning of the paragraph.

Data description – Since VARs involve current and lagged values of multiple time series, they capture co-movements that cannot be detected in univariate or bivariate models. The standard VAR summary statistics such as impulse response functions, variance decomposition and Granger-causality tests, are useful to provide targets for theoretical macroeconomic models.

Forecasting – State-of-the-art VAR forecasting systems contain more than three variables and allow for time-varying parameters to capture important drifts in coefficients. However, adding variables to the VAR creates complications, since the number of VAR parameters increases as the square of the number of variables. The ideal model to use in forecasting is one based on a fully explicit economic theory, because fully structural models tend to have a small number of parameters, so they perform well in parsimony, furthermore, with fully structural models, it is possible to explain recent and prospective data movements in terms of intelligible economic mechanisms and shocks. As a result of the work of Sims and others (Smets & Wouters, 2003), the development of fully specified, empirically estimated DSGE models has proceeded at a rapid pace. Smets and Wouters in (Smets & Wouters, 2003) claim that DSGE models are superior to VARs in forecasting, however VARs have played and are expected to play an important role as benchmarks in evaluating the forecasting abilities of structural models.

Structural inference – The central debate is whether the assumptions made in VAR models are any more compelling than in other econometric models. Pagan and Robertson in (Pagan & Robertson, 1998) brought three criticisms to structural VAR modelling. The first criticism is about what really makes up the VAR shocks, claiming that, in large part, these shocks, like those in conventional regression, reflect factors omitted from the model and if these factors are correlated with the included variables, then the VAR estimates will contain omitted-variable bias[5]. Second, policy rules change over time, and formal statistical tests reveal widespread instability in low-dimensional VARs (Stock & Watson, 1996). Third, the timing conventions in VARs do not necessarily reflect real-time data availability, and this undercuts the common method of identifying restrictions based on timing assumptions.

Policy analysis – In one of the early papers describing the applications of vector autoregression models to economics, Thomas Sargent in (Sargent, 1979) argued that while such models were useful for forecasting, they could not be used for policy analysis. However, later this point has been reasserted, by Sargent himself in (Sargent & Hansen, 1984) and (Litterman, 1982) among others. There are two related versions of the argument of why making policy with a forecasting model is supposed to be a mistake. The first one is that such models are nothing more than summary descriptions of the historical data, usually based on sample correlations. Sargent in (Sargent & Hansen, 1984) puts forward a second version of the argument against using forecasting models for policy analysis. He observes that VARs usually incorporate policy variables into the models symmetrically with other variables, treating them all as random variables. Sargent agrees that, in principle, policy choices are random variables, with uncertainty about them as an important influence on actual behaviour. However, policymakers, choosing a policy, must make a unique choice that has a deterministic character and could not

[5] In statistics, omitted-variable bias (OVB) occurs when a statistical model leaves out one or more relevant variables.

be conducted within the confines of a model in which policy is determined by some random mechanism. It is impossible to use a statistical model to analyse policy without going behind the correlations to make an economic interpretation of them. Generating such an interpretation is what econometricians call *identification* of a model. Thus, these arguments do not constitute an objection to the use of forecasting models to guide policy choice. They only point out that when we find a way to use such a model to conduct policy choice we are, implicitly or explicitly, supplementing it with an identifying interpretation.

2 VECTOR AUTOREGRESSIVE (VAR) MODEL

2.1 THE MODEL

Economic, financial, and business variables are not only autocorrelated, but often they show also cross-correlation over many time delays (lags). There is therefore the necessity to study models that take into account the inter-temporal relationships between variables. The vector autoregressive (VAR) models are widely used to capture the evolution and the inter-dependencies between multiple time series. All the variables in a VAR are treated symmetrically by including for each variable an equation explaining its evolution based on its own lags and the lags of all the other variables in the model (Stock & Watson, Introduction to Econometrics, 3rd Edition, 2011)

Conceptually, VAR processes are the multivariate generalization of autoregressive processes (AR)[6]. A VAR is a linear model that aims to describe the evolution of a set of k endogenous variables $y_t = (y_{1t}, \dots, y_{kt}, \dots\ , y_{Kt})$ for $k = 1, \dots K$, over time (Pfaff, 2008).

The VAR process is then defined as:

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} \dots + A_P y_{t-p} + \varepsilon_t = \sum_{i=1}^{p} A_i y_{t-i} + \varepsilon_t \qquad (1.1)$$

where:

$$(\varepsilon_t | \vartheta_t) \sim N(0, \Omega)$$

$$\vartheta_t = (y_{t-1}, y_{t-2}, \dots, y_{t-p})$$

[6] An autoregressive (AR) model is a representation of a type of random process. The autoregressive model specifies that the output variable depends linearly on its own previous values and on a stochastic term. Together with the moving-average (MA) model, it is a special case and key component of the more general autoregressive-moving-average (ARMA) and autoregressive integrated moving average (ARIMA).

with A_i are $(K \times K)$ coefficient matrices for $i = 1, \ldots, p$ and ε_t is a K-dimensional process with $E(\varepsilon_t) = 0$ and time invariant positive definite covariance matrix $E(\varepsilon_t \varepsilon_t^T) = \Sigma_\varepsilon$ (white noise).

In its general matrix notation, a VAR(p) with K variables will result:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{K,t} \end{bmatrix} = \begin{bmatrix} a_{1,1}^1 & a_{1,2}^1 & \cdots & a_{1,K}^1 \\ a_{2,1}^1 & a_{2,2}^1 & \cdots & a_{2,K}^1 \\ \vdots & \vdots & \ddots & \vdots \\ a_{K,1}^1 & a_{K,2}^1 & \cdots & a_{K,K}^1 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{K,t-1} \end{bmatrix} + \cdots + \begin{bmatrix} a_{1,1}^p & a_{1,2}^p & \cdots & a_{1,K}^p \\ a_{2,1}^p & a_{2,2}^p & \cdots & a_{2,K}^p \\ \vdots & \vdots & \ddots & \vdots \\ a_{K,1}^p & a_{K,2}^p & \cdots & a_{K,K}^p \end{bmatrix} \begin{bmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{K,t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{K,t} \end{bmatrix}$$

2.2 MODEL ADEQUACY TESTS

An important step in time series modelling is conducting various diagnostic tests, thus, procedures for checking whether the VAR model represents the Data generating process (DGP) of the variables adequately. When a model is estimated, diagnostic tests can be applied to evaluate model residuals, which also serve as tests of model adequacy.

Since a reduced form is underlying every structural form, model checking usually focuses on reduced form models (Lütkepohl, 2005). If a specific reduced form model is not an adequate representation of the DGP, any structural form based on it cannot represent the DGP well. That is the reason I applied diagnostic tests, to check the accuracy of the model, on the VAR model before determining the structural form. The diagnostic tests applied to my model for residual autocorrelation, conditional heteroskedasticity and nonnormality will be discussed below in this section and the outputs will be shown in Chapter 4, Section 4.3.

2.2.1 *STABLE PROCESS TEST*

One important characteristic of a VAR(p)-process is its stability. This means that it generates stationary time series with time invariant means, variances and covariance structure, given sufficient starting values.

A VAR(p) process is defined *stable* if:

$$\det\left(I_k - A_1 z - \cdots - A_p z^p\right) \neq 0 \quad \text{for each } |z| \leq 1$$

This means that all eigenvalues of matrix A have no roots inside or on the complex unit circle (Pfaff, 2008).

If the solution of the above equation has a root for $z = 1$, then either some or all variables in the VAR(p)-process are integrated of order one, i.e., $I(1)$.

One important consequence of this property is stationarity. It can be shown the following *proposition* (Lütkepohl, 2005).

Proposition on Stationary Condition

A stable VAR(p) process yt, $t = 0, \pm 1, \pm 2, \ldots$, has time invariant means, variances and covariances structure and is, hence, stationary. If, however, $detA(z) = 0$ for $z = 1$ (i.e., the process has a *unit root*) and all others roots of the determinantal polynomial are outside the complex unit circle, then some or all of the variables are integrated, the process is, hence, nonstationary and the variables may be cointegrated. All the variables are either $I(0)$ or $I(1)$ by default.

2.2.2 *Serial Correlation Test*

The Portmanteau- and Breusch-Godfrey test is implemented for testing the lack of serial correlation in the residuals of a model: it tests whether any of a group of autocorrelations of the residuals time series are different from 0.

The null hypothesis is:

$$H_0 : B_1 = \cdots = B_h = 0$$

and correspondingly the alternative hypothesis is of the form:

$$H_1 : \exists B_i \neq 0 \; for \; i = 1, 2, \ldots, h$$

Thus, under the null hypothesis, there is no significant autocorrelation in the residuals, and they are approximately white noise.

On the other hand, under the alternative hypothesis, there is autocorrelation present, the autocorrelation values should significantly deviate from 0.

2.2.3 *CONDITIONAL HETEROSKEDASTICITY TEST*

Uncorrelated time series can still be serially dependent due to a dynamic conditional variance process. A time series exhibiting conditional heteroscedasticity is said to have autoregressive conditional heteroscedasticity (ARCH) effects. Engle's ARCH test is a Lagrange multiplier test to assess the significance of ARCH effects (Box, Jenkins, Reinsel, & Ljung, 2015).

The null hypothesis is:

$$H_0 : B_1 = B_2 = \cdots = B_q = 0$$

and the alternative hypothesis is:

$$H_1 : \; B_1 \neq 0 \cap B_2 \neq 0 \cap \ldots \cap B_q \neq 0$$

Thus, under the null hypothesis, in the absence of autoregressive conditional heteroscedasticity (ARCH) effects, we have $B_i = 0$ for all $i = 1, \ldots, q$.

The alternative hypothesis is that, in the presence of ARCH effects, at least one of the estimated B_i coefficient matrices must be significant.

2.2.4 *NONNORMALITY TESTS*

The Jarque-Bera normality tests are often used for model checking; however, normality is not a necessary condition for the validity of many of the statistical procedures related to VAR models (Box, Jenkins, Reinsel, & Ljung, 2015).

The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being three. Samples from a normal distribution have an expected skewness of 0 and an expected kurtosis of 3.

Once a VAR(p) model has been estimated, and it has passed the adequacy tests, the avenue is wide open for further analysis. Usually, the dynamic properties of a VAR(p) process are synthesized by causal inference, forecasting and/or diagnosing the empirical model's dynamic behaviour. I am referring to: impulse response functions (henceforth: IRF) and forecast error variance decomposition (henceforth: FEVD), the topics of the following sections.

2.3 IMPULSE RESPONSE FUNCTIONS

In contemporary macroeconomic modelling IRF represent an important step in econometric analysis, employing vector autoregressive models. The main purpose is to describe the evolution of a model's variables in reaction to a shock in one or more variables. This feature allows to trace the transmission of a single shock within an otherwise noisy system of equations, that makes the IRF useful tools in the assessment of economic policies (Box, Jenkins, Reinsel, & Ljung, 2015).

To illustrate the concept of IRF we must rewrite the system (1.1) in its compact notation, where L represents the *lag operator*:

$$y_t = A(L)y_t + \varepsilon_t \tag{1.2}$$

$$A(L) = A_1 L + A_2 L^2 + \cdots + A_p L^p$$

Under the hypothesis that $I - A(L)$ is invertible, with $B(L) = (I - A(L))^{-1}$, we can obtain the following moving average representation of the vector autoregressive process:

$$y_t = \varepsilon_t + B_1\varepsilon_{t-1} + B_2\varepsilon_{t-2} + \cdots + B_s\varepsilon_{t-s} \tag{1.3}$$

At this point matrix B_s may be interpreted as follows:

$$B_s = \frac{\partial y_{t+s}}{\partial \varepsilon_t}$$

Put differently, the *ij* element of B_s identifies the consequences of increasing a unit in innovations on the j^{th} VAR variable on the value of the i^{th}VAR variable at time $t+s$, keeping at 0 all other innovations at all possible dates between t and $t+s$. Such partial derivative makes sense only if it is assumed that shocks on the different variables are not correlated. Otherwise, if the variables are correlated, we would end up with a non-diagonal errors covariances and variances matrix, thus distorted results.

2.4 Forecast Error Variance Decomposition

The variance decomposition or forecast error variance decomposition (FEVD) shows, in a fitted vector autoregressive model, which proportion of variance of predictive errors on the j^{th} variable of the system at a given horizon *s*, can be attributed to innovations in the considered variables (Box, Jenkins, Reinsel, & Ljung, 2015).

The following equation allows to identify the forecast error in a VAR, *s* periods in the future:

$$(y_{t+s} - E[y_{t+s}]) = \varepsilon_{t+s} + B_1\varepsilon_{t+s-1} + B_2\varepsilon_{t+s-2} + \cdots + B_{s-1}\varepsilon_{t+1} \tag{1.4}$$

The variance of this forecast error s-periods in the future is:

$$var(y_{t+s}) = \Omega + B_1\Omega B'_1 + B_2\Omega B'_2 + \cdots + B_{s-1}\Omega B'_{s-1} \tag{1.5}$$

The variance decomposition indicates which proportion of the forecast error for a designated variable is imputable to the different variances in Ω. For the operation to be meaningful, it is necessary that the forecast error total variance would be solely function of variances and not covariances. As had happened with the IRF, the variance decomposition requires orthogonal shocks. It is crucial, thus, transforming the VAR residuals as to make them orthogonal. We then consider the structural form, addressing the problem of correlated residuals.

2.5 STRUCTURAL VAR AND IDENTIFICATION

Stock and Watson (Stock & Watson, Vector Autoregressions, 2001) claim that "a structural VAR uses economic theory to sort out the contemporaneous links among the variables. A SVAR requires identifying assumptions that allow correlations to be interpreted causally."[7]

With the aim to analyse the identification problem, we consider the equation (1.1). A VAR(p) can be interpreted as a reduced form model. A SVAR model is its structural form and is defined as:

$$y_t = \sum_{i=0}^{p} C_i y_{t-i} + B u_t \tag{1.6}$$

$$(u_t | \vartheta_t) \sim N(0,1)$$
$$\vartheta_t = (y_{t-1}, y_{t-2}, \dots, y_{t-p})$$

In this system the shocks must be orthogonal, thus it is possible to correctly interpret the IRF and the variance decomposition. The u_t are considered primitive shocks, Bernanke (Bernanke & Blinder, 1992) writes that "he thinks of the structural innovations as 'primitive' exogenous forces, not directly observed by the econometrician, which buffet the system and cause oscillations. Because these shocks are primitive, i.e., they do not have common causes, it is natural to treat

[7] Stock, J. H., & Watson, M. W. (2001). Vector Autoregressions. *Journal of Economic Perspectives*, pp. 101-115.

them as approximately uncorrelated."[8] However, the restrictions make the shocks entering one and only one equation: the matrix B may not be diagonal. This interpretation specifies a structural model with traditional characteristics; indeed, the stochastic components can be correlated to each other. Such correlation arises because different equations have one or more shock in common. The u_t are the shocks important from an economic and statistical point of view, indeed it is the response to these innovations, orthogonal to each other, which makes reasonable analysing the IRF and the variance decomposition. The main problem is that everything an econometrician has at his disposal are VAR residuals, which being correlated, are not useful to derive the IRF and the variance decomposition. However, it does exist a relation between observable VAR residuals and non-observable primitive shocks from the structural form. Indeed, if we interpret the (1.1) equation as the reduced form of (1.6), then the following relations must apply:

$$A_i = [I - C_0]^{-1} C_i$$

$$[I - C_0]\varepsilon_t = B u_t \tag{1.7}$$

The problem is to identify the parameters in B and C_i and the structural shock variations, given the variance and covariance matrix of innovations in the VAR.
We get u_t from the second equation, obtaining:

$$u_t = B^{-1}[I - C_0]\varepsilon_t$$

[8] Bernanke, B. S., & Blinder, A. S. (1992). The Federal Funds Rate and the Channels of Monetary Transmission. *The American Economic Review*, pp. 901-921.

The identification problem is therefore described considering the estimation approach of the method of moments[9]. Given the restrictions that link primitive shocks to VAR residuals, we can write:

$$u_t u_t' = B^{-1}(I - C_0)\varepsilon_t \varepsilon_t'(I - C_0)'(B^{-1})' \tag{1.8}$$

Equating sample moments with theoretical moments, as the *method of moments* suggests, we obtain:

$$I = \hat{B}^{-1}\left(I - \hat{C}_0\right)\hat{\Omega}\left(I - \hat{C}_0\right)'\left(\hat{B}^{-1}\right)' \tag{1.9}$$

that allows us to deduce only the parameters of interest where the system is identified. The necessary condition for identification is that the number of equations in (1.9) must be equal to or greater to the number of parameters to be estimated. If the VAR has length n, then the variance-covariance matrix of the VAR has $n(n + 1)/2$ free parameters. Thus, the maximum number of parameters to be estimated in the matrices B and C_i is $n(n + 1)/2$ (Box, Jenkins, Reinsel, & Ljung, 2015).

2.6 CHOLESKY DECOMPOSITION

The solution proposed by (Sims, 1980) to the identification problem was to consider $B = I$ and the lower triangular matrix $[I - C_0]^{-1}$, obtaining the exact VAR identification.

This hypothesis has strong economic and statistical implications: from an economic perspective, it is assumed that the economy has a recursive structure, from the statistical point of view the impulse response and the variance decomposition

[9] In statistics, the *method of moments* is a method of estimation of population parameters. Its starts by expressing the population moments (i.e. the expected values of powers of the random variable under consideration) as functions of the parameters of interest. Those expressions are then set equal to the sample moments.

functions depend on the order of the VAR variables (Box, Jenkins, Reinsel, & Ljung, 2015).

We further simplify the system (1.1) considering a VAR(1) with only two variables:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_{11} a_{12} \\ a_{21} a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t \end{bmatrix} \tag{1.10}$$

with:

$$\left(\begin{matrix} \varepsilon_t \\ \varepsilon_t \end{matrix} \middle| \vartheta_t \right) \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} \sigma_{12} \\ \sigma_{21} \sigma_{22} \end{pmatrix} \right]$$

$$\vartheta_t = (x_{t-1}, y_{t-1})$$

Being σ_{12} and σ_{21}different from 0, the residuals ε cannot be considered structural shocks, against which to calculate IRF and variance decomposition. A general structural form, of which (1.10) is a possible reduced form, is the following:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} c_{01} c_{02} c_{11} c_{12} \\ c_{03} c_{04} c_{21} c_{22} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \\ y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \tag{1.11}$$

with:

$$\left(\begin{matrix} u_{1t} \\ u_{2t} \end{matrix} \middle| \vartheta_t \right) \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

Thus, in this case the relationship between shocks from the structural notation and residuals from the reduced form takes the following structure:

$$[I - C_0]\varepsilon_t = Bu_t \tag{1.12}$$

$$C_0 = \begin{pmatrix} c_{01} & c_{02} \\ c_{03} & c_{04} \end{pmatrix}, B = I$$

The structural form is not identified yet. However, we can obtain the identification through triangular matrix in two alternative ways: by assuming $c_{02} = 0$ or $c_{03} = 0$.

The former method has the consequence to order the VAR placing *y* ahead and *x* next, with the economic meaning that there is no contemporaneous effect of *x* on *y*. Proceeding with the latter method, we would order *x* ahead and *y* next.

Since in my analysis I decided to perform the lower triangular matrix for the Cholesky decomposition, I then apply a lower triangular structure to C_0, the relationship between residuals from the reduced form and structural residuals results as follows:

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} = [I - C_0]^{-1} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} = \begin{bmatrix} k_{11} & 0 \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \tag{1.13}$$

with u_{1t} obtained under the hypothesis that the residuals of the VAR first equation coinciding with the structural innovations, while u_{2t} is derived as the OLS regression residual of ε_{2t} on u_{1t} and is therefore, by construction, orthogonal to u_{2t}.

To derive IRF and variance decomposition the following autoregression notation is considered:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} a_{11} a_{12} \\ a_{21} a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} k_{11} & 0 \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \tag{1.14}$$

with:

$$\left(\begin{matrix} u_{1t} \\ u_{2t} \end{matrix} \middle| \vartheta_t \right) \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

The Cholesky decomposition is a particular case of identification, but other alternative identifications are possible, based on more general specifications of the matrices *B* and $(I - C_0)$, without imposing the lower triangular structure for $(I - C_0)$ and a diagonal structure for matrix B. It is important to note that the non-diagonality of *B*, while maintaining the hypothesis of reciprocal orthogonality of structural shocks, allows to replicate a structural model with traditional properties (Gottschalk, 2001). Indeed, the stochastic components of the equations can be correlated to each other, although the variance-covariance matrix of ε is diagonal. Such correlation is resulted because different equations have one or more primitive shocks in common, therefore the matrix B is not diagonal.

3 TIME SERIES ANALYSIS

In this chapter the variables used to perform the analysis with the VAR model will be presented both from an economic perspective and a statistical one. The data used for the analysis refer to the U.S. economy and have been taken from the Federal Reserve website[10], except for S&P500 which has been extrapolated from the Yahoo finance website[11]. All the time series analysed are quarterly series and the observation sample goes from the first quarter of 1980 to the fourth quarter of 2019. The variables into account in our analysis are the following:

- the financial index S&P 500
- the GDP Implicit Price Deflator
- the Real Gross Domestic Product per capita
- the Effective Federal Funds Rate

The characteristics of each variable will be outlined, and each time series will be graphically shown. Subsequently, we will conduct the so-called Time Series Analysis (henceforth: TSA), or explorative analysis, for each single variable, to carry out the stationarity analysis.

After presenting and analyzing each individual variable, the *first difference* and the *log-transformation*, where necessary, will be applied, since the output of the stationarity analysis shows that none of time series is stationary on average.

At the end of this first exploratory phase, the graphs of the two groups of variables will be reported in order to analyze the differences, thus will be compared the graphs of the variables before and after having implemented the First Difference.

This last process will pave the way to the next chapter in which the variables will be used in our modelling process.

[10] Federal Reserve Bank of St. Louis
[11] Yahoo Finance S&P 500

3.1 STATIONARITY OF TIME SERIES

Detecting stationarity in time series data is crucial, since it has strong implications on the persistence of shocks in the economy, indeed, if a process is non-stationary, a shock will not vanish (is persistent) through time. Time series showing trend or seasonality are not stationary since the trend and seasonality will affect the value of the time series at different times.

There are several ways to ascertain whether a series is generated from a stationary process, the ones used in my analysis are:

- Visualization
- Autocorrelation Function (ACF) and partial-ACF
- Parametric tests

Visualization

The visualization approach simply determines whether a time series was generated by a stationary process by looking at its plot and check if the data show some peculiar patterns or trends that indicate a non-stationary behaviour.

However, merely by looking at the data to detect stationarity is a dubious method, indeed I proceeded applying the ACF and partial-ACF tests.

ACF and partial-ACF

Autocorrelation is the correlation of a signal with a delayed copy (or a lag) of itself as a function of the delay. When plotting the value of ACF for increasing lags (a plot called correlogram), the values tend to degrade to 0 quickly for stationary time series, while for non-stationary data the degradation will happen more slowly (Stock & Watson, 2011).

Parametric Tests

Another, more rigorous approach, to detecting stationarity in time series data is using statistical tests developed to detect specific types of stationarity: the unit root tests. In my thesis I applied to the data the augmented Dickey-Fuller test (ADF) and the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS).

They both test whether a unit root is present in a time series sample, anyway they differ for an important characteristic: the augmented Dickey-Fuller test (ADF) tests the null hypothesis that a unit root is present in a time series sample, the alternative hypothesis is stationarity or trend-stationarity; the KPSS is used for testing a null hypothesis that an observable time series is stationary around a deterministic trend (i.e. trend-stationarity) against the alternative of a unit root (Stock & Watson, 2011).

Differencing

If, after applying the tests above discussed, the time series results to be non-stationary, then we will apply the *first difference* to the data. Differencing can help stabilize the mean of a time series by removing changes in the level of a time series, and therefore eliminating (or reducing) trend and seasonality. The first difference of a time series is, indeed, the series of changes from one period to the next. If y_t denotes the value of the time series at period *t*, then the first difference of *y* at period *t* is equal to $y_t - y_{t-1}$. If the first difference is stationary and completely random (not autocorrelated), then *y* is described by a random walk model.

Log-transformation

In time series analysis, *log-transformation* is another way to transform variables and stabilize the variance of a series. Usually, for economic variables, applying the log-transformation, thus using variables in logarithms (logs), can result in forecasting improvements and can actually stabilize the variance of the underlying series (Luetkepohl & Xu, 2009).
In my analysis I applied log-transformation together with the first difference, respectively to the time series “Real GDP per capita” and “S&P 500”, as we will discuss in the following sections.

3.2 S&P 500

Created in 1957, the S&P 500 was the first U.S. market-cap-weighted stock market index, nowadays is highly regarded as the only stock market benchmark serving as an economic indicator in the Conference Board Leading Economic Index[12]. The index includes 500 leading companies and covers approximately 80% of available market capitalization. There is over USD 11.2 trillion indexed or benchmarked to the index, with indexed assets comprising approximately USD 4.6 trillion of this total. Part of this basket are shares of big companies traded on the New York Stock Exchange (Nyse), on the American Stock Exchange (Amex) and on the Nasdaq. This index is the most used to measure the trend in the U.S. stock market and is well known as a benchmark for portfolio performance. The S&P 500 index is managed by the S&P Index Committee. All Committee members are full-time professional members of S&P Dow Jones Indices' staff. The committee meets monthly in order to review pending corporate actions that may affect index constituents, statistics comparing the composition of the indices to the market, index policies covering rules for companies addition or exclusion, treatment of dividends, share counts, or other matters. The main objective of the Index Committee is to ensure that the S&P 500 remains a reliable indicator of the U.S. stock market, capable of reflecting risk and return characteristics of the wider large-cap universe, on an ongoing basis. Another mission of the Index Committee is to also monitor the liquidity component to ensure efficient portfolio trading, maintaining a minimum turnover ratio.

The following graphs show the sectoral breakdown of the basket of the 500 companies and the ten largest companies included in the index.

[12] The Conference Board Leading Economic Index is an American economic leading indicator intended to forecast future economic activity.

Sector Breakdown

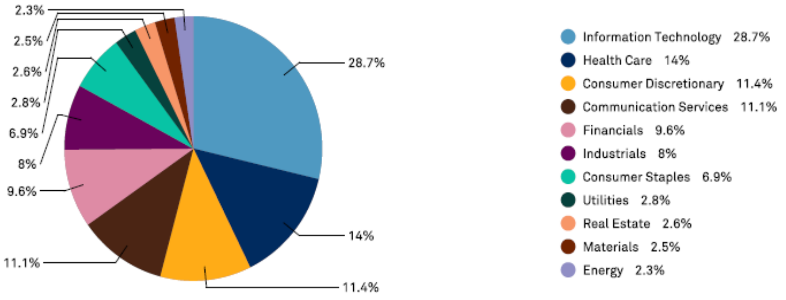


Fig. 3 - S&P 500 companies sector breakdown. Source: spglobal

Top 10 Constituents By Index Weight

CONSTITUENT	SYMBOL	SECTOR*
Apple Inc.	AAPL	Information Technology
Microsoft Corp	MSFT	Information Technology
Amazon.com Inc	AMZN	Consumer Discretionary
Facebook Inc A	FB	Communication Services
Alphabet Inc A	GOOGL	Communication Services
Alphabet Inc C	GOOG	Communication Services
Berkshire Hathaway B	BRK.B	Financials
Johnson & Johnson	JNJ	Health Care
Visa Inc A	V	Information Technology
Procter & Gamble	PG	Consumer Staples

Fig. 4 - S&P 500 ten largest companies. Source: spglobal

Methodology construction of the index

The Index Committee follows a set of guidelines to maintain the index. Complete and detailed information about these guidelines, included criteria for additions and removals, policy statements, related researches and insights, are available on the website at www.spglobal.com. These guidelines provide the transparency and equity to enable investors to replicate the index and achieve the same performance as the S&P 500. The criteria for additions and exclusion from the index will be listed below.

Criteria for additions

Universe – All constituents must be U.S. companies.

Eligibility Market Cap – In order to be included in the index, companies must have an unadjusted market cap of USD 8.2 billion or greater. This minimum is reviewed from time to time to ensure consistency with market conditions.

Public float – Companies must have a float market cap of at least USD 4.1 billion.

Adequate Liquidity and Reasonable Price – The ratio of annual dollar value traded (defined as average closing price over the period multiplied by historical volume) to float-adjusted market capitalization should be at least 1.00, and the stock should trade a minimum of 250,000 shares in each of the six months leading up to the evaluation date.

Sector Representation – Sector balance, as measured by a comparison of each GICS[13] sector's weight in an index with its weight in the S&P Total Market Index, in the relevant market capitalization range, is also considered in the selection of companies for the indices.

Company type – All eligible U.S. common equities listed on eligible U.S. exchanges can be included. REITs[14] are also eligible for inclusion. Close-end funds, ETFs, ADRs, ADS, and certain other types of securities are ineligible for inclusion.

Criteria for exclusion

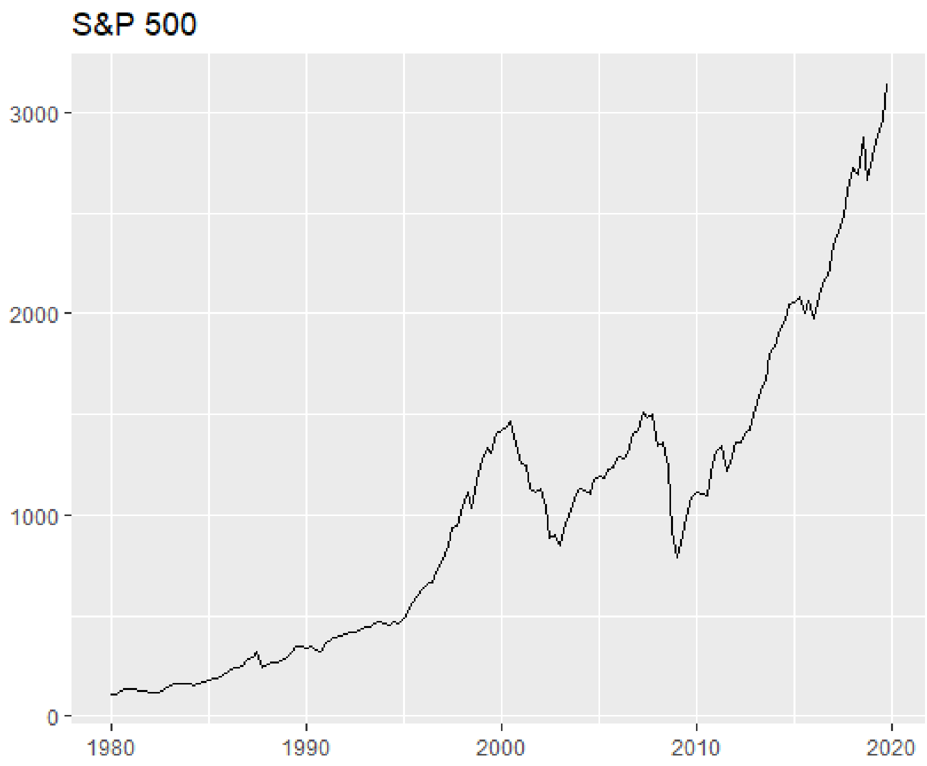
Companies that substantially breach one or more of the criteria for inclusion in the index.

Companies involved in M&A or Corporate Restructuring operations that no longer meet the inclusion criteria.

[13] The Global Industry Classification Standard (GICS) is an industry taxonomy developed in 1999 by MSCI and Standard & Poors (S&P) for use by the global financial community. The GICS structure consists of 11 sectors, 24 industry groups, 69 industries and 158 sub-industries into which S&P has categorized all major public companies.

[14] A real estate investment trust (REIT) is a company that owns, and in most cases operates, income-producing real estate.

Below is the chart and the TSA of the time series in question: after being graphically displayed, the results obtained from the stationarity tests applied to the time series will be shown and observations will be submitted. The data has been taken from the Yahoo Finance website. The data was, initially, a monthly time series; I then proceeded by cleaning the data and transforming the monthly time series into quarterly time series, taking the mean of the monthly values. The observation sample goes from the first quarter of 1980 to the fourth quarter of 2019.



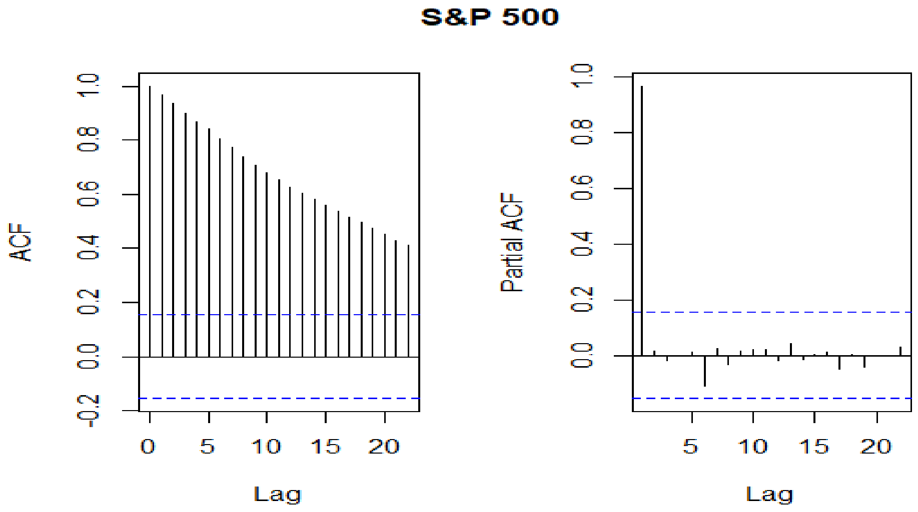
Plot 1 - S&P time series. Source: author elaboration

The time series shows a long-term trend with a mean that variates over time, thus the process does not seem to be a mean-reverting process[15].

Graphically the series does not appear to be stationary. I proceed with the estimates of the autocorrelation and partial autocorrelation function.

[15] A *mean-reverting process* is a process in which the values tend to return to the mean and fluctuations around the mean exhibit approximately similar amplitudes

The results will be graphically shown in the correlograms below.



Graph 1 - S&P 500 ACF and partial-ACF. Source: author elaboration

Looking at the correlograms of the autocorrelation and partial autocorrelation functions, they show the typical non-stationary series behavior, thus a strong persistence up to many lags ahead, meaning a sluggish decrease of the autocorrelation. Otherwise stated, the persistency is the tendency of a system to endure in the same state from one observation to the next. The ACF function shows a strong autocorrelation even up to the twentieth lag, therefore the preceding twentieth period still affects time *t*, in our case the previous twenty quarter still affect the current quarter.

After having ascertained the non-stationarity from a graphical point of view, we proceed with stationarity tests to verify stationarity from a statistical perspective. The stationarity tests been used in this analysis are:

- Augmented Dickey-Fuller test
- KPSS test

Since the functionality of the two tests above mentioned have already been discussed in the Section 3.1 on the Stationarity of time series, I will therefore limit myself to present and describe the outputs obtained after applying the tests to the time series at issue.

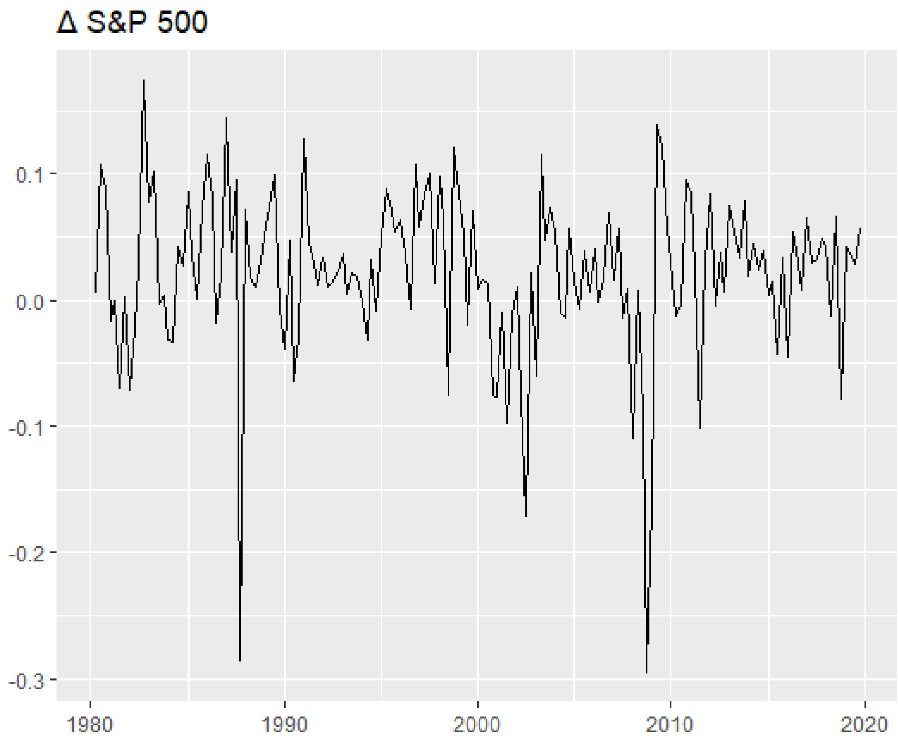
```
          Augmented Dickey-Fuller Test

data:  db$`S&P 500`
Dickey-Fuller = -0.80429, Lag order = 5, p-value =
0.9592
alternative hypothesis: stationary
```

Fig. 5 - S&P 500 Augmented Dickey-Fuller Test output. Source: author elaboration

The Augmented Dickey-Fuller test posits the stationarity of the series as alternative hypothesis; thus, the null hypothesis corresponds to non-stationarity.

Since, from the scores, the p-value (0.9592) is bigger than the standard significance level (5%) we cannot reject the null hypothesis: the time series appears to be non-stationary. With the intention to make the series stationary we will apply the *log-transformation* first and the first-order *differencing* then, methods thoroughly examined in the previous section (3.1) related to the analysis of stationarity of time series.

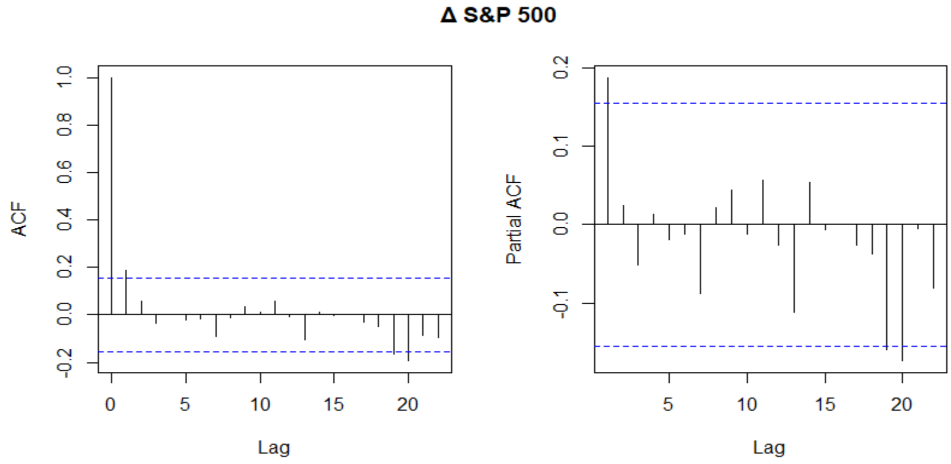
The plot and the outputs of the ACF and partial-ACF tests applied to the integrated series will be shown below.



Plot 2 - Δ S&P 500 time series. Source: author elaboration

From a graphical perspective, after applying the first difference to the data, the Δ S&P 500 shows the typical stationary time series behavior: the process appears to be mean-reverting, indeed the values of the series tend to return to the mean and fluctuations around the mean exhibit approximately similar amplitudes.

The following graph represents the output of the ACF and partial-ACF tests regarding the integrated time series at issue.



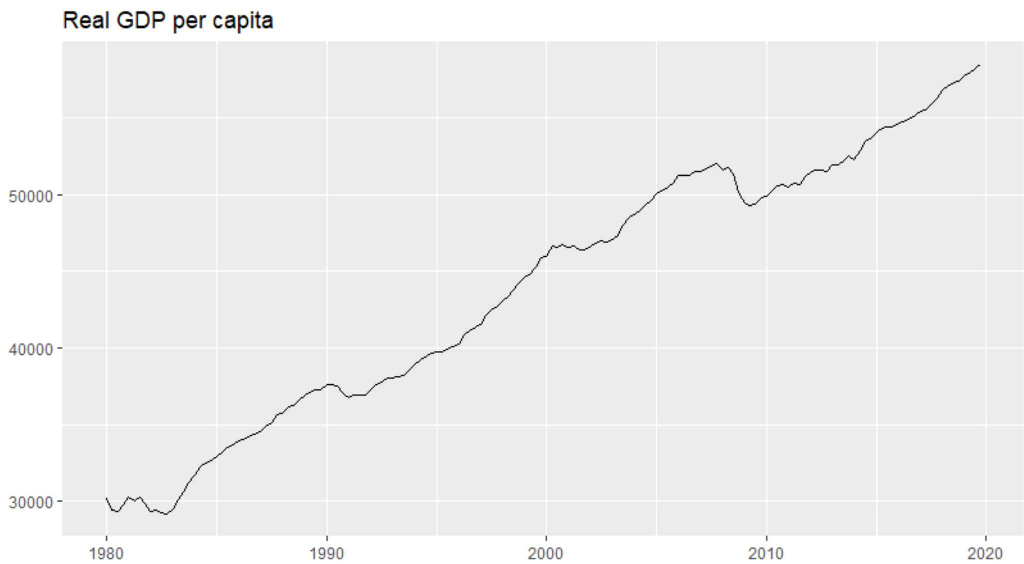
Graph 1 - Δ S&P 500 ACF and partial-ACF. Source: author elaboration

In accordance with the correlograms above, the ACF and partial-ACF show that, except the first value, which in fact is not a *lag* but it is *time 0*, the autocorrelation is very limited, none of the lags significantly excess confidence bounds: the time series appears to be stationary.

3.3 REAL GDP PER CAPITA

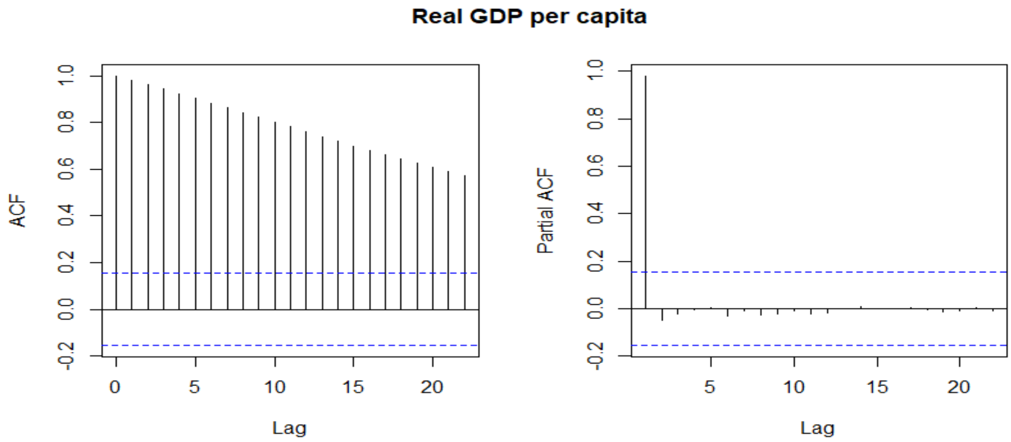
In my analysis, as a proxy for the U.S. economic output, I used the quarterly time series Real GDP per capita. To grasp the reasons behind my choice ok taking Real GDP per capita as an indicator of U.S. economy in my VAR model, an understanding of the concept of GDP and the differences between Nominal GDP and Real GDP is necessary. The Gross Domestic Product (GDP) in general is the monetary or market value of all finished goods and services made within a country's borders during a specific period. Since GDP is one of the main macroeconomic metrics used to define the stability and the growth of a nation, it is usually analyzed from two different perspectives: nominal and real. The Nominal GDP is nothing

other than GDP evaluated at current market prices of goods and services in its circulation. Nominal GDP is also referred as the current dollar GDP. The Real GDP is an inflation-adjusted measure that considers the value of all goods and services produced by an economy in a given year and is often referred to as constant-price GDP, inflation-corrected GDP or constant dollar GDP. The main difference between Nominal and Real GDP is the adjustment for inflation. The former, indeed, does nor strip out the pace of rising prices when comparing one period to another, thereby it can inflate the growth figure of a determined economy. Since inflation can make Nominal GDP higher it does not provide an accurate figure of an economic growth. As such, Real GDP offers a better ground for assessing long-term national economic performance over time as opposed to Nominal GDP.
Below is the chart and the TSA of the variable in question: after being graphically displayed, the results obtained from the stationarity tests applied to the time series will be shown and observations will be submitted. The data has been taken from the Federal Reserve website, the time series is a quarterly series, and the observation sample goes from the first quarter of 1980 to the fourth quarter of 2019.



Plot 3 - Real GDP per capita. Source: author elaboration

The Real GDP per capita time series shows a long-term trend with a mean that variates over time, thus the process does not appear to be a mean-reverting process. Graphically, we can state, the series does not seem to be stationary. I therefore proceed with the estimation of ACF and partial-ACF. The outputs will be graphically shown in the correlograms below.



Graph 2 - Real GDP per capita ACF and partial-ACF. Source: author elaboration

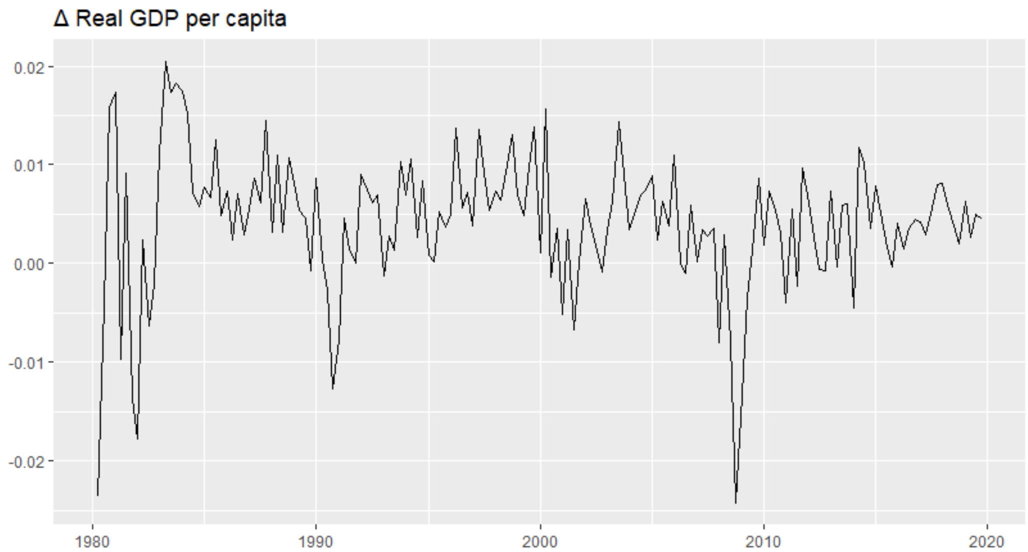
The results of the ACF and partial-ACF of the Real GDP per capita show the typical non-stationary time series behavior: strong persistence of autocorrelation that only slowly decline with increasing lags. Since both plot and ACF and partial-ACF functions show non-stationarity, we therefore proceed with the augmented Dickey-Fuller test to assess stationarity in the time series into account. Following, the output of the augmented Dickey-Fuller test for the Real GDP per capita is displayed.

```
        Augmented Dickey-Fuller Test

data:  db$`Real GDP per capita`
Dickey-Fuller = -2.1906, Lag order = 5, p-value =
0.4968
alternative hypothesis: stationary
```

Fig. 6 - Real GDP per capita Augmented Dickey-Fuller Test output. Source: author elaboration

The augmented Dickey-Fuller test, as we can observe from the figure, assumes stationarity as the alternative hypothesis. From the output, the p-value (0.4968) is less than the standard significance level (5%), thus we cannot reject the null hypothesis: the GDP Deflator time series is non-stationary as reported by the augmented Dickey-Fuller test.

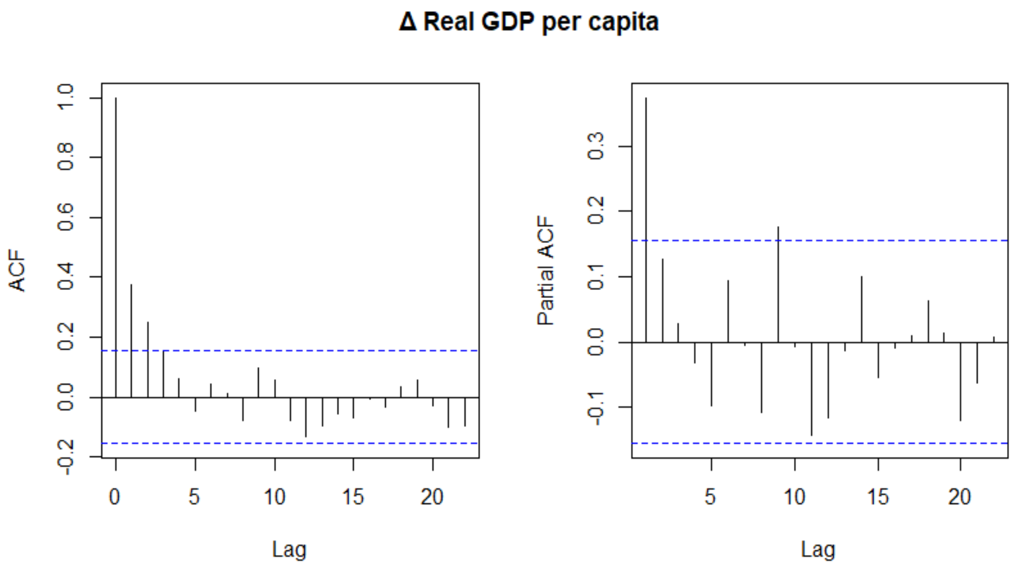
We then apply the log-transformation and then the first difference to the Real GDP per capita time series. The plot of the data and the output of the ACF and partial-ACF tests concerning the integrated Real GDP per capita time series will be shown and examined below.



Plot 4 - Δ Real GDP per capita time series. Source: author elaboration

The Δ Real GDP per capita time series, from a graphical perspective, show the typical stationary time series behavior: the process appear to be mean-reverting, indeed the values of the series tend to return to the mean and fluctuations around the mean exhibit approximately similar amplitudes.

Below is displayed the output of the ACF and partial-ACF tests performed on the Δ S&P 500 time series.



Graph 3 - Δ Real GDP per capita ACF and partial-ACF. Source: author elaboration

The ACF and partial-ACF for the Δ S&P 500 time series highlight that there are only two significant lags that excess the confidence bounds, all the rest is not significant. The process is thereby stationary with positive autocorrelation which impacts only up to the second lag, as displayed by the ACF and partial-ACF plots. Stated differently, two previous quarters still affect time *t*, a scenario in compliance with the economic and statistical doctrine (Stock & Watson, 2011).

3.4 GDP Deflator

The term inflation, from the Latin *inflatus*, generally identifies an increase in the price of goods for consumption in a predefined length of time, which generates a decrease in the purchasing power[16] of money. Just to give an example: in 1970 the

[16] The purchasing power is the amount of goods and services that can be purchased with a unit of currency.

New York Times newspaper cost 15 cents, the average price of a single-family house was 23.400,00 USD and the manufacturing average hourly pay was 3,36 USD. In 2000 the New York Times newspaper cost 75 cents, the average price of a single-family house was 166.000,00 USD and the manufacturing average hourly pay was 14,26 USD. In the U.S. as widely discussed in Chapter 1 (Section 1.1), the inflation was worrying in the 1970s, even if it never had the size and the magnitude of the extraordinarily high inflation episodes, the so-called hyperinflation[17], occasionally experimented by other countries. There are several causes that can generate inflation. Typically, inflation can result from an increase in production costs or an increase in demand for products or services.

The two different cause of inflation are also known as:

- Cost-Push Inflation: it comes from an increase in production costs, such as raw materials and wages, that causes a decline in the supply of goods while the demand remain unchanged, as a result the higher production costs are passed onto consumers in the nature of increased prices of finished goods and services;
- Demand-Pull Inflation: it derives from a strong demand for a product or service, then prices increase, and the result is the demand-pull inflation.

Other possible reasons of inflation are: in periods of economic expansion there is an increase in aggregate demand followed by an increase in prices; instead, in times of recession, there is a decrease in aggregate demand which results in a decrease in prices, also called deflation. Central banks typically implement different monetary policy actions to keep inflation under control. In general, in times of economic expansion, a restrictive monetary policy is implemented, while in times of economic recession, an expansive monetary policy is implemented.

In economics there are typically two recognized measures of inflation, similar in some respects, different in others:

- the Consumer Index Price (CPI)

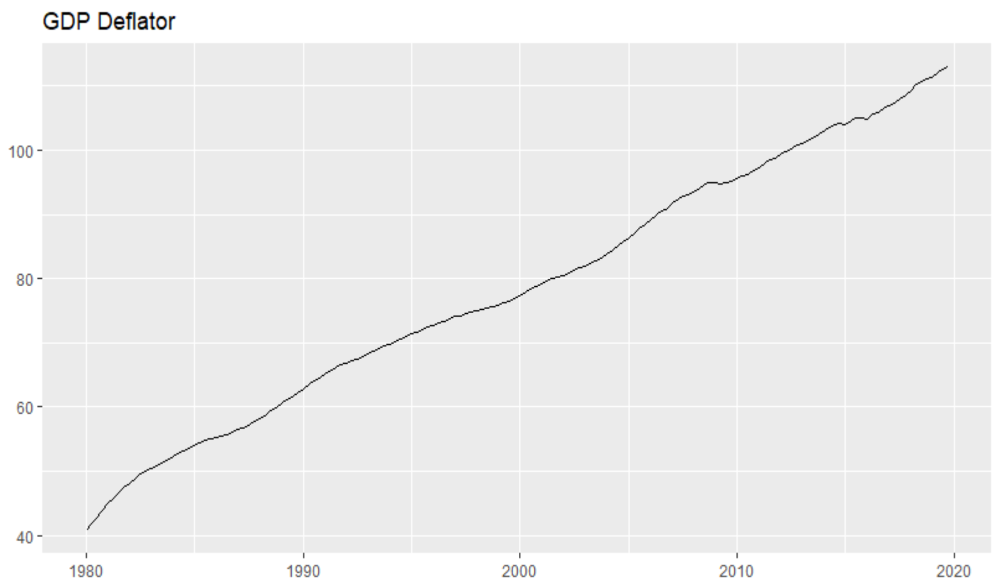
[17] Hyperinflation refers to rapid, excessive, and out-of-control general prices increases in an economy. Although hyperinflation is a rare event for developed economies, it has occurred many times throughout history in countries such as China, Germany, Russia, Hungary and Argentina.

- the GDP Price Deflator

The former measures the level of retail prices of goods and services at a specific point in time; the latter, also known as the GDP deflator or the Implicit price deflator, measures the changes in prices for all the good and services produced in an economy. Even if the trends of the GDP deflator are usually similar to the trends in the CPI, they differ in one fundamental aspect: the GDP deflator is not based on a fixed basket of goods, so it is considered a more comprehensive inflation measure than the CPI, this is also the reason because I chose the GDP deflator as the variable for my analysis. The GDP deflator formula is as follows:

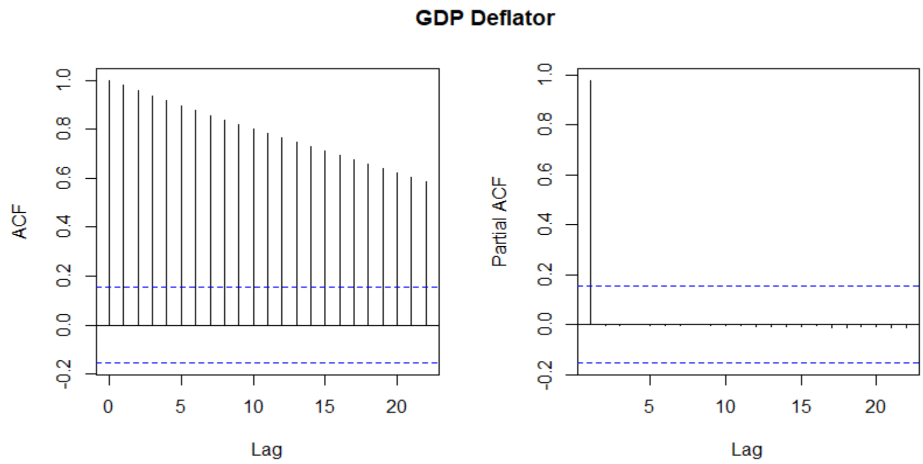
$$GDP\ Price\ Deflator = \left(\frac{Nominal\ GDP}{Real\ GDP}\right) \times 100$$

Below is the chart and the TSA of the time series in question: after being graphically displayed, the results obtained from the stationarity tests applied to the time series will be shown and observations will be submitted. The data was collected from the Federal Reserve website, the time series is a quarterly series, and the observation sample goes from the first quarter of 1980 to the fourth quarter of 2019.



Plot 5 - GDP Deflator time series. Source: author elaboration

The time series shows a long-term trend with a mean that variates over time, thus the process does not appear to be a mean-reverting process. Graphically, thus, the series does not seem to be stationary. I therefore proceed with the estimation of ACF and partial-ACF. The output will be graphically shown in the correlograms below.



Graph 4 - GDP Deflator ACF and partial-ACF. Source: author elaboration

The results of the ACF and partial-ACF show the typical non-stationary time series behavior: strong persistence of autocorrelation that only slowly decline with increasing lags. All the lags are significant, they all excess the confidence bounds: the preceding twentieth period still affects time *t*, in our case the previous twentieth quarter still affects the current quarter into account. Subsequently, having ascertained the non-stationarity of the series from a graphical point of view, we then proceed through stationarity analysis from a statistical perspective.

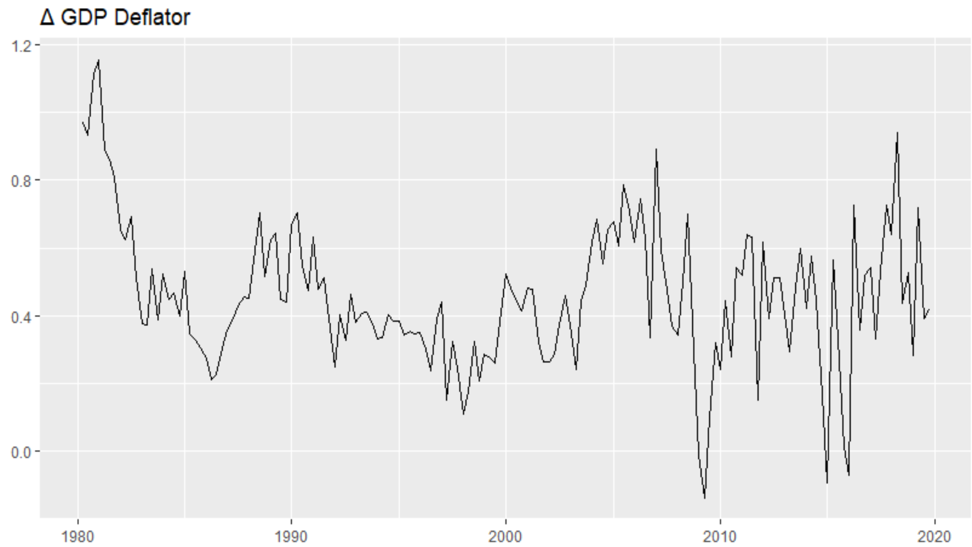
Following the output of the augmented Dickey-Fuller test for the GDP Deflator.

```
        Augmented Dickey-Fuller Test

data:  db$`GDP Deflator`
Dickey-Fuller = -2.9877, Lag order = 5, p-value =
0.1644
alternative hypothesis: stationary
```

Fig. 7 - GDP Deflator Agmented Dickey-Fuller Test output. Source: author elaboration

The augmented Dickey-Fuller test, as we can observe from the figure, assumes stationarity as the alternative hypothesis. From the output, the p-value (0.1644) is less than the standard significance level (5%), thus we cannot reject the null hypothesis: the GDP Deflator time series is non-stationary, in agreement with the augmented Dickey-Fuller test. We then apply the first difference to the GDP Deflator time series.

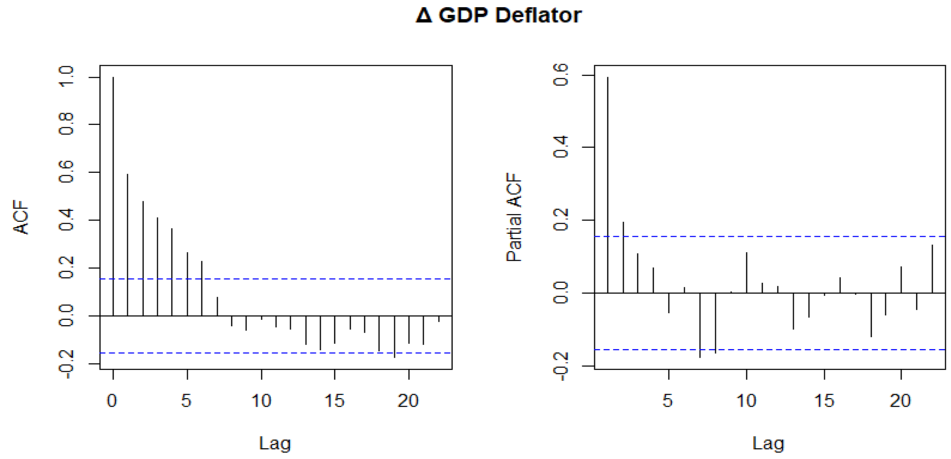
The plot and the output of the ACF and partial-ACF tests concerning the integrated time series will be shown and examined below.



Plot 6 - Δ GDP Deflator time series. Source: author elaboration

The plot for the Δ GDP Deflator time series reveals an initial trend, which typically indicates non-stationarity, but the mean quickly tends to stabilize. Therefore, from a graphical point of view, despite the initial trend the Δ GDP Deflator time series appears to be stationary.

Below the output of the ACF and partial-ACF performed on the Δ GDP Deflator is displayed.



Graph 5 - Δ GDP Deflator ACF and partial-ACF. Source: author elaboration

The ACF and the partial-ACF exhibits a feeble persistence in conformity with the initial trend observed in Plot 4, persistence that however declines quite quickly until the sixth lag. The Δ GDP Deflator is in fact the only integrated time series in the analysis that show an although moderate, positive autocorrelation; nevertheless the autocorrelation persistence is not strong and durable enough to the time series to be considered non-stationary.

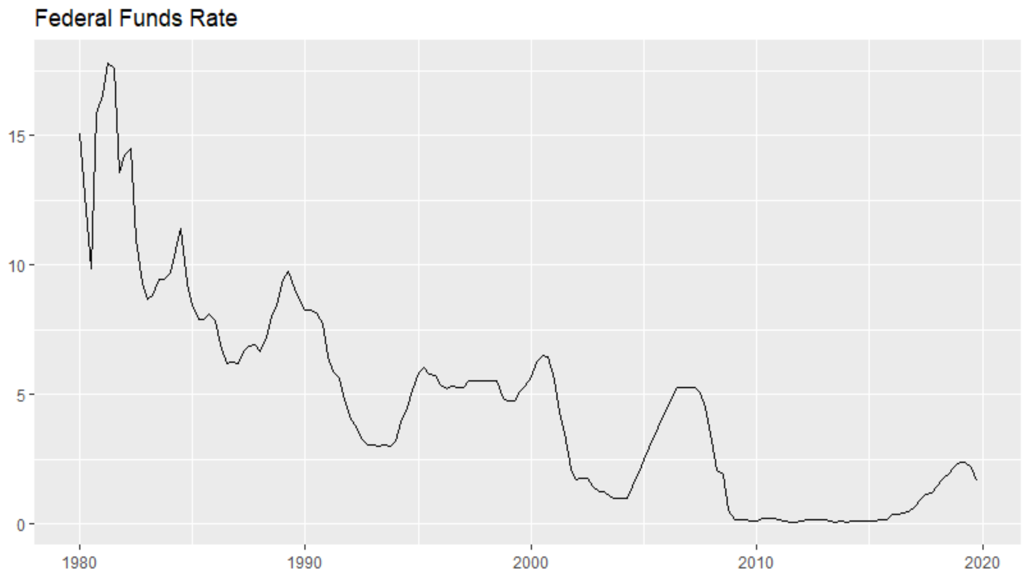
3.5 Effective Federal Funds rate

The Federal Funds rate is the target interest rate determined by the Federal Open Market Committee (FOMC) at which depositary institutions trade federal funds (balances held at Federal Reserve Banks) with each other overnight. The Effective Federal Funds rate is determined as follows.

Banks with excess liquidity will lend to other banks that need to borrow to quickly raise liquidity. The rate at which the borrowing bank will pay the loan to the lending bank is agreed between the two banks: the weighted average rate for all these sorts of transactions is called Effective Federal Funds rate. The Effective Federal Funds rate is fundamentally determined by the Federal Reserve through open market operations to reach the Federal Funds rate target. The FOMC is the Federal Reserve System’s body for monetary policymaking. The FOMC meets eight times a year to determine the federal funds target rate which will influence the effective federal funds rate in two different approaches: through open market operations or by buying and selling the government bonds (government debt). The decisions behind the actions perpetrated by the FOMC about rate adjustments are tightly related to the signals advised by specific key economic indicators that may show evidence of inflation, recession or other issues that may affect the sustainable economic growth. By way of illustration, if the FOMC considers the economy is growing too quickly and the inflation pressures are inconsistent with the Federal Reserve inflation rate, the Committee can set a higher federal funds rate target to mitigate economic activity. More specifically, the Federal Reserve may decrease liquidity by selling government bonds, as a result increasing the federal funds rate since banks have reduced liquidity to trade with other banks. Analogously the Federal Reserve can raise liquidity by buying government bonds, thereby decreasing the federal funds rate because banks have surplus liquidity for trade. With the purpose to offer a historical overview of the target for the federal funds rate, we can surely assess that it has varied a lot over the years, in response to the prevailing economic conditions at that given time. However, as also illustrated in Chapter 1 (Section 1.1) of this paper, two most noteworthy historical moments as regard the federal funds rate and the economic events to which they are linked can be listed: the highest target value at 20% in the early 1980s in response to inflation; the record low target of 0% to 0,25% in an attempt to encourage growth, with the Great Recession of 2007 to 2009.

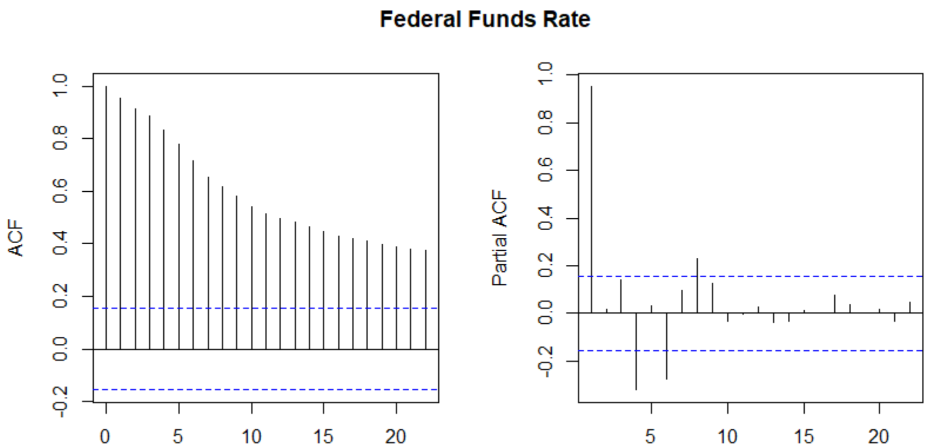
Below is the chart and the TSA of the time series in question: after being graphically displayed, the results obtained from the stationarity tests applied to the time series will be shown and observations will be submitted.

The data was collected from the Federal Reserve website, the time series is a quarterly series, and the observation sample goes from the first quarter of 1980 to the fourth quarter of 2019.



Plot 7 - Federal Funds Rate time series. Source: author elaboration

The Federal Funds Rate time series shows a long-term trend with a mean that variates over time, thus the process does not appear to be a mean-reverting process. Graphically, therefore, the series does not seem to be stationary. I hence proceed with the estimation of ACF and partial-ACF. The output will be graphically shown in the correlograms below.



Graph 6 - Federal Funds Rate ACF and partial-ACF. Source: author elaboration

The results of the ACF and partial-ACF of the Federal Funds Rate show the typical non-stationary time series behavior: strong persistence of autocorrelation that only slowly decline with increasing lags. Since both Plot 7 and ACF and partial-ACF show non-stationarity, we therefore proceed with the augmented Dickey-Fuller test to assess stationarity in the Federal Funds Rate time series.

Following the output of the augmented Dickey-Fuller test for the Federal Funds Rate time series.

```
        Augmented Dickey-Fuller Test

data:  db$`Federal Funds Rate`
Dickey-Fuller = -6.8532, Lag order = 5, p-value =
0.01
alternative hypothesis: stationary
```

Fig. 8 - Federal Funds Rate Augmented Dickey-Fuller Test output. Source: author elaboration

The augmented Dickey-Fuller test assumes *stationarity* as the alternative hypothesis; thus, the null hypothesis is undoubtedly the “non-stationarity”.

The augmented Dickey-Fuller test output for Federal Funds Rate reveals a p-value (0.01) smaller than the standard significance level (5%), thus we reject the null hypothesis: the Federal Funds Rate time series is stationary as reported by the augmented Dickey-Fuller test.

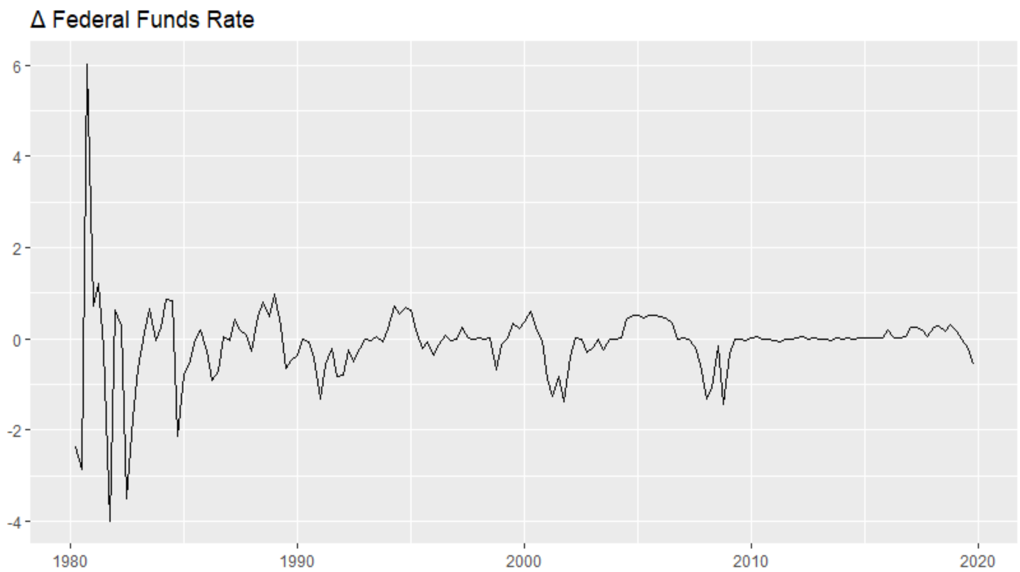
Since analyzing the time series plot and ACF and partial-ACF correlograms the Federal Funds Rate does not appear to be stationary, while the augmented Dickey-Fuller test claims otherwise, I then proceed by performing the KPSS test to detect stationarity in the Federal Funds Rate time series.

Below the output of the KPSS test for the Federal Funds Rate time series is revealed.

```
        KPSS Test for Level Stationarity

data:  db$`Federal Funds Rate`
KPSS Level = 2.518, Truncation lag parameter = 4,
p-value = 0.01
```

Fig. 9 - Federal Funds Rate KPSS Test output. Source: author elaboration

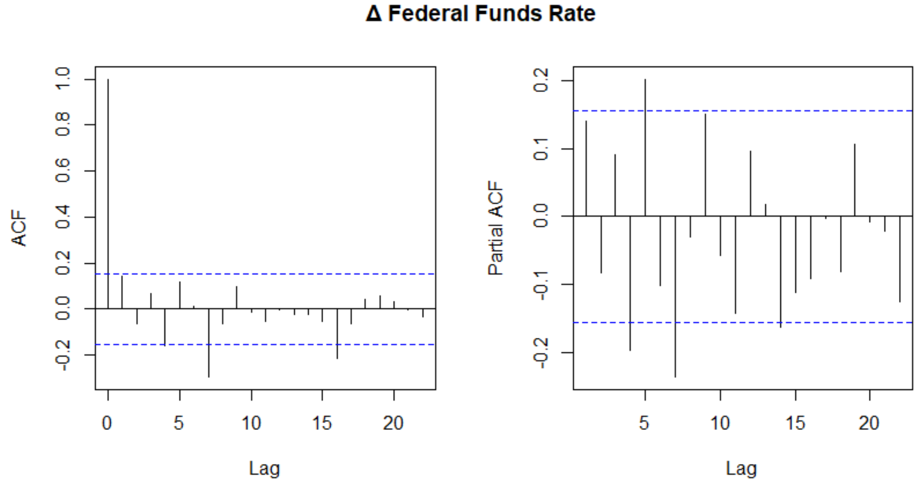
The KPSS Test, as opposed to the augmented Dickey-Fuller Test, has the *non-stationarity* as the alternative hypothesis and the *stationarity* as null hypothesis.
The results of the KPSS test for the Federal Funds Rate show a p-value (0.01) smaller than the standard significance level (5%), accordingly we reject the null hypothesis of stationarity: the Federal Funds Rate time series is non-stationary as reported by the KPSS test.
Since the augmented Dickey-Fuller and the KPSS tests result in contrasting outputs, we resolve this ambiguity by looking at the plot and the ACF and partial-ACF correlograms: we can conclude that the Federal Funds Rate time series is not stationary.
We then apply the first difference to the Federal Funds Rate time series.
The plot and the ACF and partial-ACF correlograms concerning the integrated Federal Funds Rate time series will be shown and examined below.



Plot 8 - Δ Federal Funds Rate time series. Source: author elaboration

The Δ Federal Funds Rate time series, from a graphical perspective, show the typical stationary time series behavior: the process appear to be mean-reverting, indeed the values of the series tend to return to the mean and fluctuations around the mean exhibit approximately similar amplitudes.

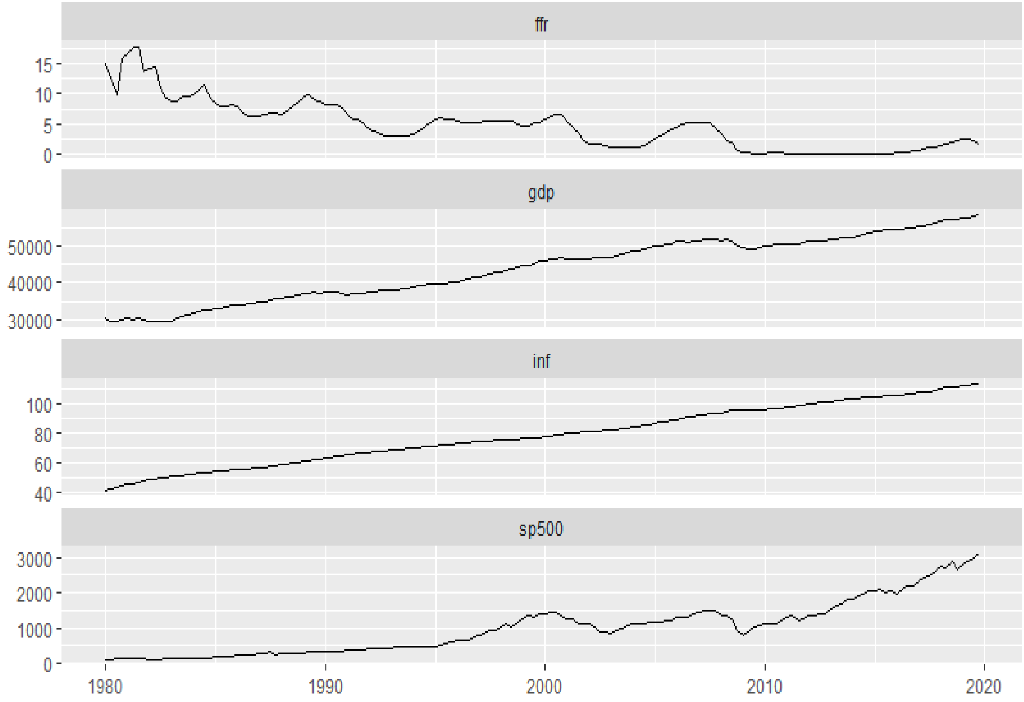
Below are displayed the ACF and partial-ACF correlograms relevant to the Δ Federal Funds Rate time series.



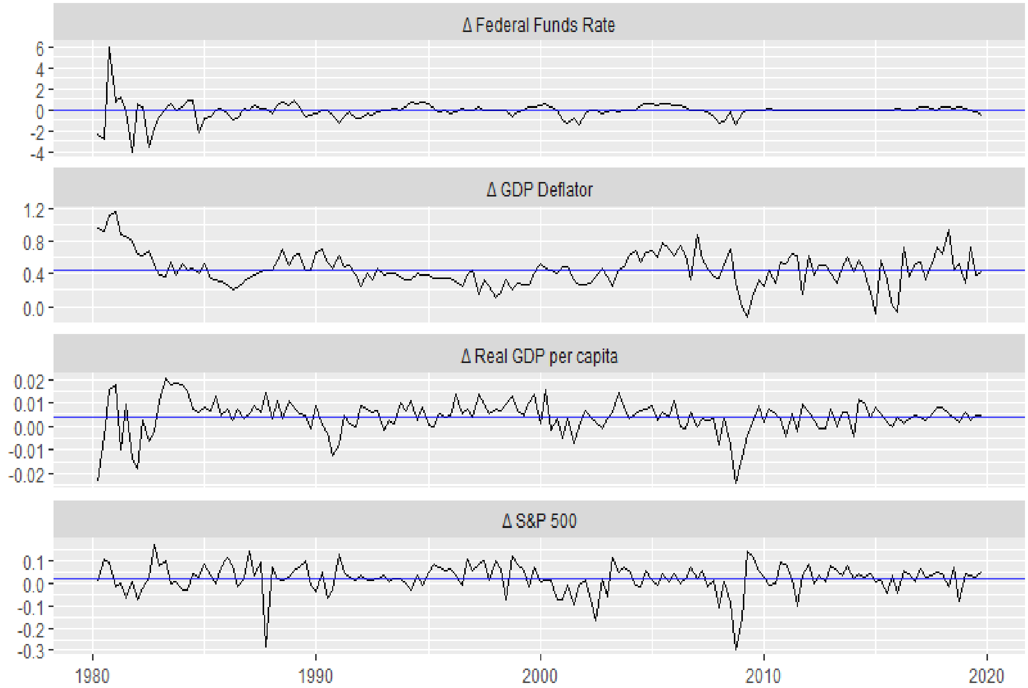
Graph 7 - Δ Federal Funds Rate ACF and partial-ACF. Source: author elaboration

In accordance with the above graph, the ACF and partial-ACF correlograms show that the autocorrelation is very limited, none of the lags significantly excess confidence bands: Δ Federal Funds Rate time series appear to be stationary.

Following, in the interest of graphically highlighting the difference between the variables before and after having transformed them, multiple times series plots will be presented.



Plot 10 – Non-stationary time series before differencing. Source: author elaboration



Plot 9 - Stationary integrated time series. Source: author elaboration

4 VAR MODEL ESTIMATION AND INTERPRETATION OF RESULTS

In this chapter the relations among the variables obtained in the previous section will be analysed using the VAR methodology, deeply discussed from a statistical perspective in Chapter 2.

To make a brief recap, through the VAR methodology each variable is regressed on *p* lags of itself and on *p* lags of the other variables. The objective of the analysis is to understand how a shock at the financial index and a monetary policy shock can impact on the economic cycle. The VAR models estimated in this chapter consider all the variables previously discussed in Chapter 3.

To better select the order of the model, thus the maximum lags number to be included in the model, I referred to the Akaike (AIC) and Bayesian Schwarz (BIC/SC) information criteria. I first implemented the models with the lag suggestions of both AIC and BIC/SC information criteria, then, after ascertaining the reliability of the models through the analysis of residuals, I chose the model that respected the parameter parsimony.

I then implemented diagnostic tests (section 4.3), to the VAR model selected, such as: The Portmanteau test for serial correlation, the Engle's ARCH test, the Jarque-Bera normality test and the stability test.

I thereby proceeded by enacting the VAR structural form, in the interest of correctly estimating the Impulse Response Functions and the Variance Decomposition.

To adopt the VAR structural form, the Cholesky decomposition has been applied, by considering $B = I$ and the lower triangular matrix $[I - C_0]^{-1}$.

Then, the impulse response functions will be displayed and discussed, from both a qualitative and quantitative perspective. Will be analysed the IRF related to shocks in the variables S&P 500 and FFR, respectively, from an economic outlook, the financial index, and the monetary policy.

Finally, the Forecast Error Variance Decomposition will be implemented, to assess what percentage of variability each variable contributes to the other variables in the autoregression.

4.1 VAR MODEL ORDER SELECTION

With the purpose of selecting the best order for my model, I carried out a test based on the *information criteria*. The outputs of the tests are displayed below.

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     3      1      1      3

$criteria
                  1            2            3
AIC(n) 1.407424e+01 1.401134e+01 1.400005e+01
HQ(n)  1.423659e+01 1.430358e+01 1.442217e+01
SC(n)  1.447388e+01 1.473070e+01 1.503912e+01
FPE(n) 1.295403e+06 1.217014e+06 1.204724e+06
                  4            5            6
AIC(n) 1.410171e+01 1.423253e+01 1.427042e+01
HQ(n)  1.465372e+01 1.491442e+01 1.508219e+01
SC(n)  1.546049e+01 1.591102e+01 1.626862e+01
FPE(n) 1.336479e+06 1.528489e+06 1.595540e+06
                  7            8
AIC(n) 1.424321e+01 1.430973e+01
HQ(n)  1.518487e+01 1.538127e+01
SC(n)  1.656112e+01 1.694735e+01
FPE(n) 1.563647e+06 1.686883e+06
```

Fig. 10 - Information criteria Test. Source: author elaboration

The lag orders suggested by the information criteria tests are respectively: VAR (1) indicated by BIC/SC information criterion; VAR (3) recommended by AIC criterion. I therefore proceed by estimating both models with 1 and 3 lags, verifying the autocorrelation of residuals of both models and, whether both models do not show autocorrelation of residuals, I will select the model that meets the criteria of the parameter parsimony, discussed in Chapter 1 (Section 1.2).

4.2 VAR MODEL ESTIMATION

In this section, I will estimate both models with 1 and 3 lags, verifying the autocorrelation of residuals of the models and, whether both models do not show autocorrelation of residuals, I will select the model that meets the criteria of the parameter parsimony.

4.2.1 *VAR (1) MODEL*

This is a VAR model of order 1, as suggested by the BIC information criteria. Written in the matrix form, our VAR (1) results to be:

$$\begin{bmatrix} sp500_t \\ gdp_t \\ inf_t \\ ffr_t \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} + A_1 \begin{bmatrix} sp500_{t-1} \\ gdp_{t-1} \\ inf_{t-1} \\ ffr_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

where the variables occur in the following order: S&P 500, GDP, GDP Deflator (inflation), Federal Funds Rate and $A_1$is the matrix (4 x 4) of AR coefficients.
In the compact notation the model will result:

$$X_t = C + A_1 X_{t-1} + \varepsilon_t$$

The output of the model is illustrated in the Appendix.

4.2.2 *VAR (3) MODEL*

This is a VAR model of order 3, as recommended by the AIC information criteria. Written in the matrix form, our VAR (3) results to be:

$$\begin{bmatrix} sp500_t \\ gdp_t \\ inf_t \\ ffr_t \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} + A_1 \begin{bmatrix} sp500_{t-1} \\ gdp_{t-1} \\ inf_{t-1} \\ ffr_{t-1} \end{bmatrix} + \cdots + A_3 \begin{bmatrix} sp500_{t-3} \\ gdp_{t-3} \\ inf_{t-3} \\ ffr_{t-3} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \\ \varepsilon_{4t} \end{bmatrix}$$

where the variables occur in the following order: S&P 500, GDP, GDP Deflator (inflation), Federal Funds Rate and $A_1, \dots, A_3$is the matrix (4 x 4) of AR coefficients.

In the compact notation the model will result:

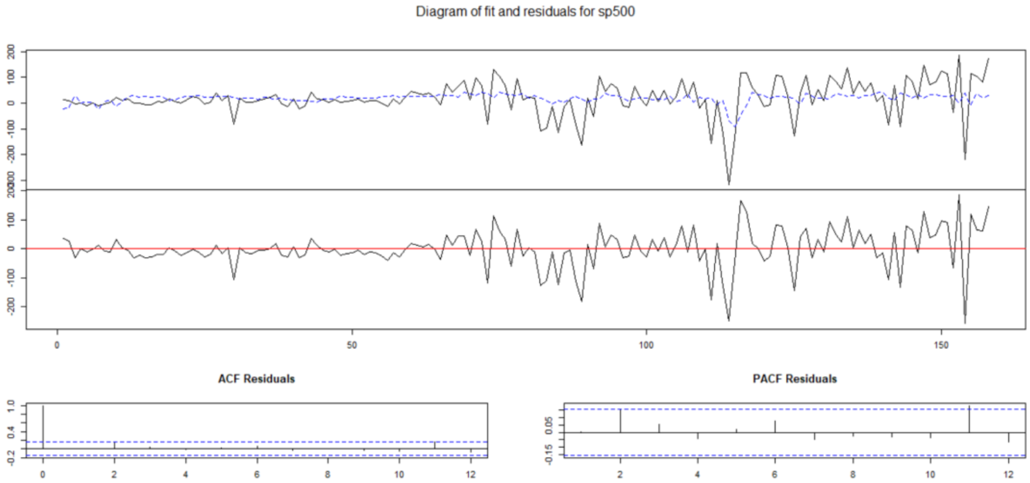
$$X_t = C + \sum_{i=1}^{3} A_i X_{t-i} + \varepsilon_t$$

The output of the model is illustrated in the Appendix.

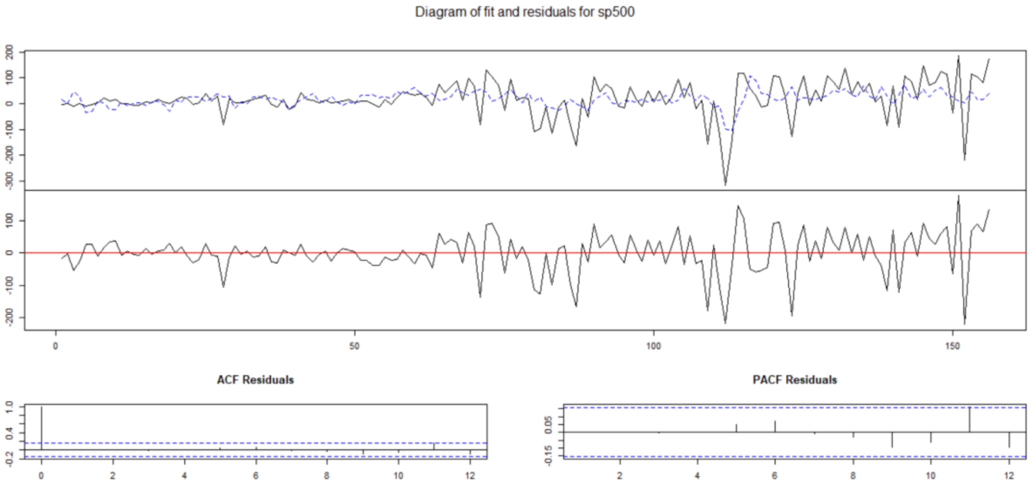
4.2.3 *ANALYSIS OF RESIDUALS FOR BOTH VAR(1) AND VAR(3) MODELS*

In this section, once extracted, the residuals of both models will be analysed. Following the plots regarding the residuals of both models and the ACF and partial-ACF are displayed. We can observe that for both models the variables relatively well fit the data. By observing the ACF and partial-ACF, none of them show autocorrelation of residuals since their functions do not excess the confidence bands. Thus, we can conclude that residuals of both models are randomly distributed with 0 mean.

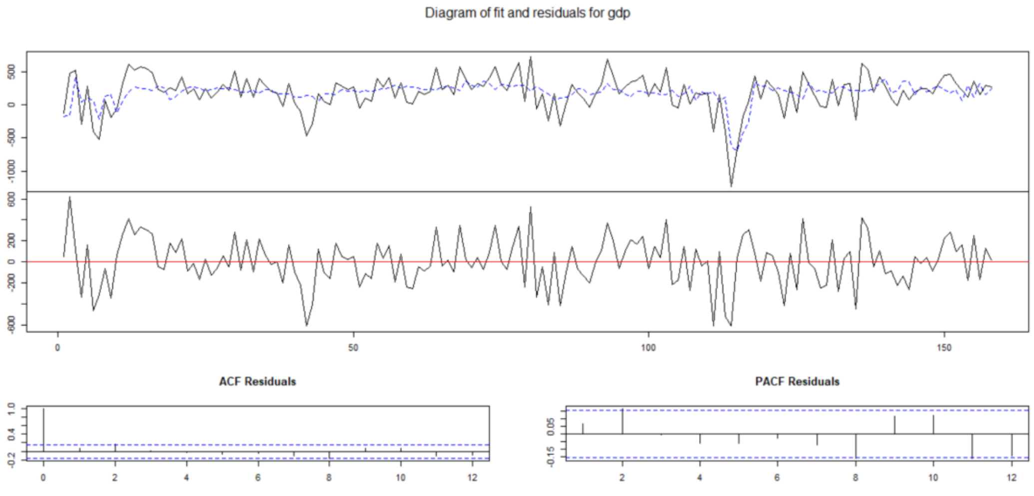
Below, the plots regarding the residuals of both models and the ACF and partial-ACF are compared.



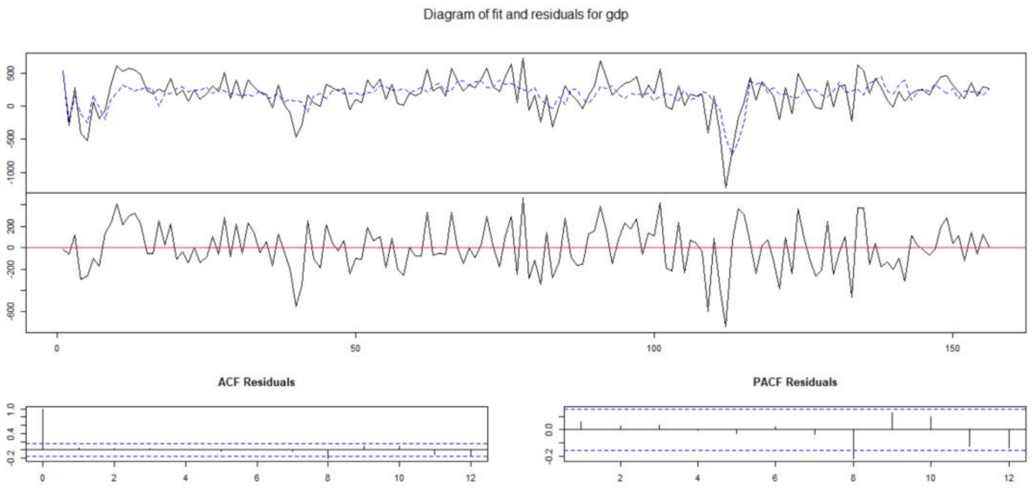
Graph 8 - S&P 500 residuals, ACF and partial-ACF, VAR(1). Source: author elaboration



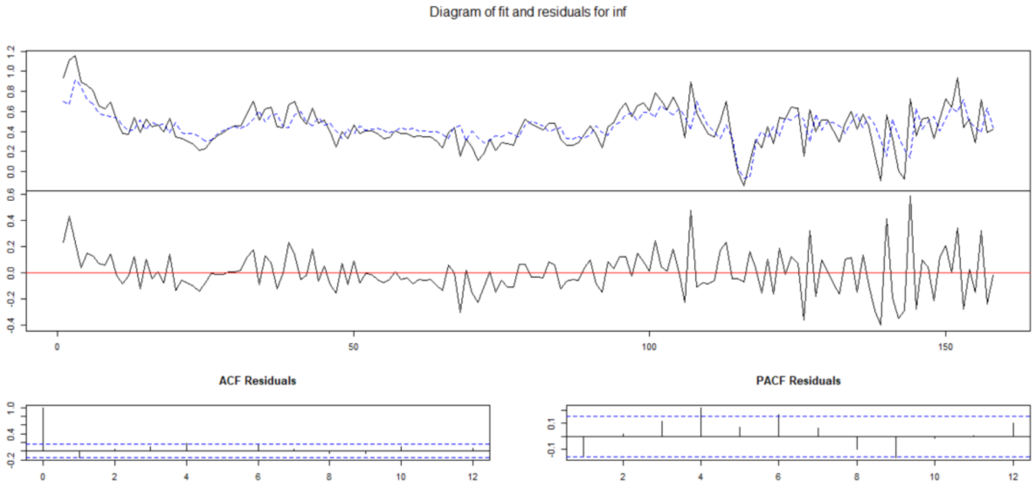
Graph 9 - S&P 500 residuals, ACF and partial-ACF, VAR(3). Source: author elaboration



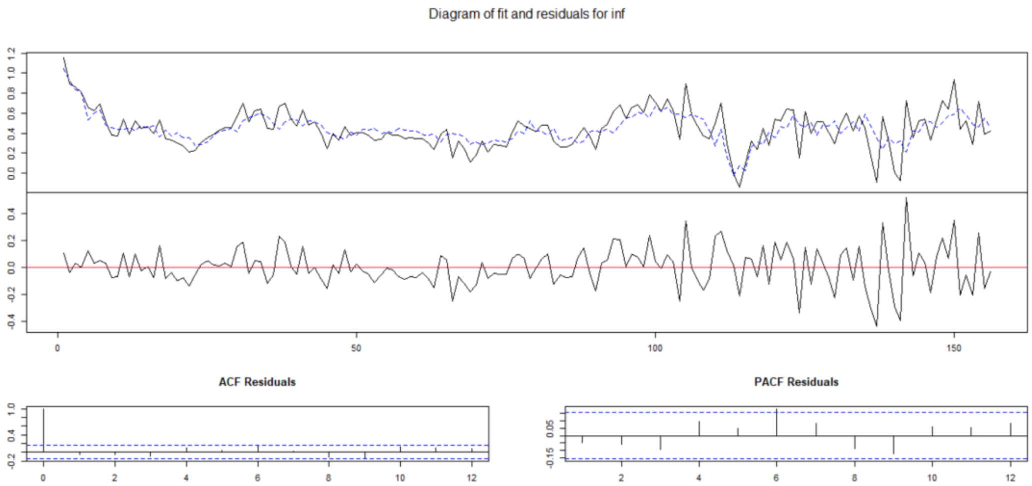
Graph 10 - GDP residuals, ACF and partial-ACF, VAR(1). Source: author elaboration



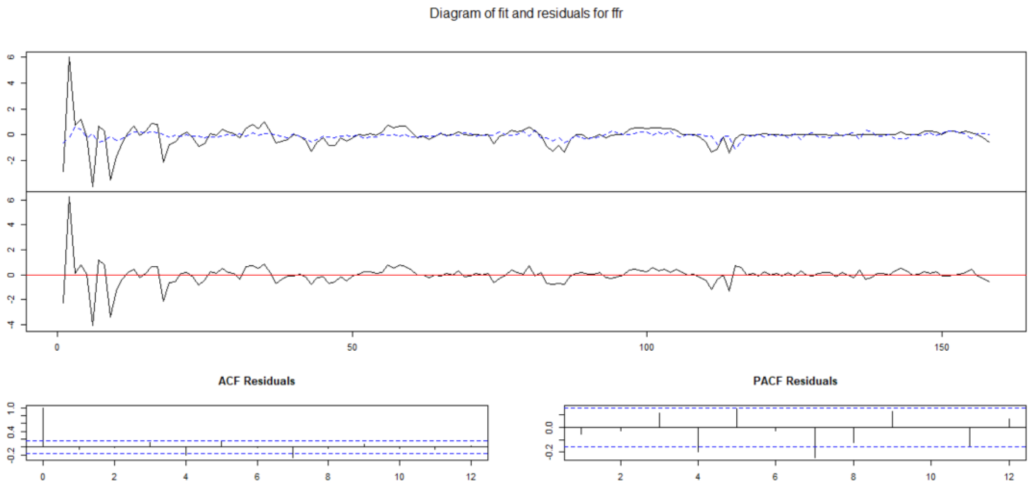
Graph 11 - GDP residuals, ACF and partial-ACF, VAR(3). Source: author elaboration



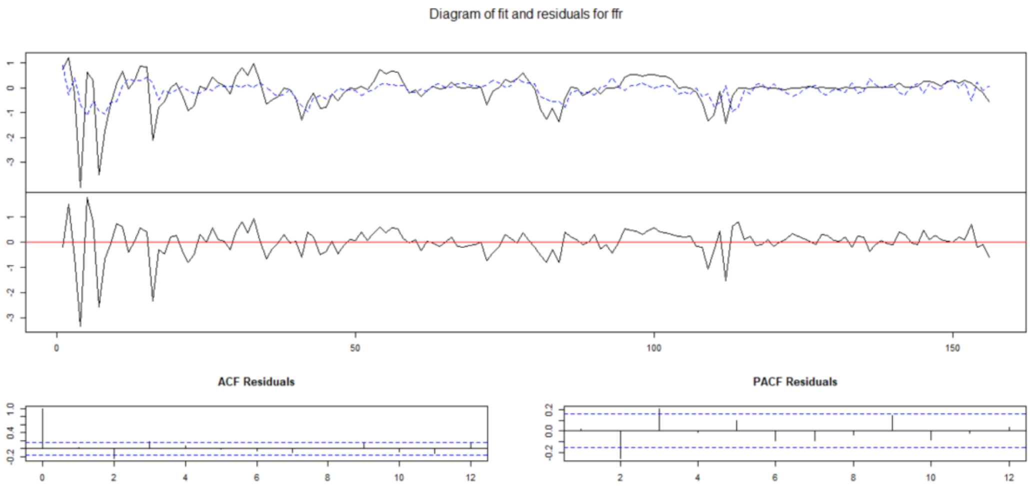
Graph 12 - INF residuals, ACF and partial-ACF, VAR(1). Source: author elaboration



Graph 13 - INF residuals, ACF and partial-ACF, VAR(3). Source: author elaboration



Graph 14 - FFR residuals, ACF and partial-ACF, VAR(1). Source: author elaboration



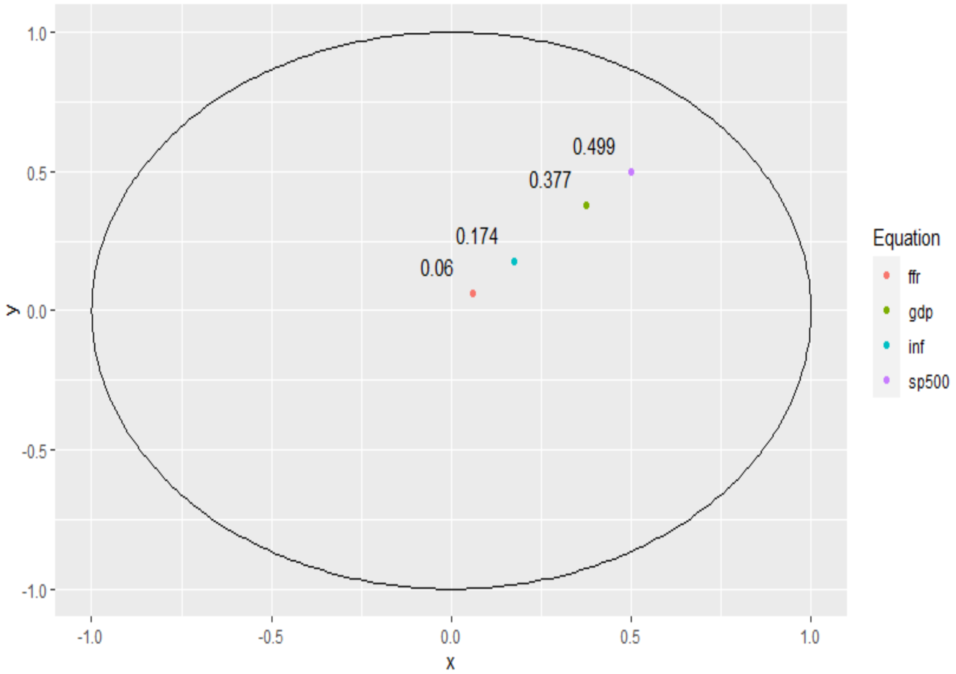
Graph 15 - FFR residuals, ACF and partial-ACF, VAR(3). Source: author elaboration

Since both model's VAR (1) and VAR (3) fit the data quite well and do not show autocorrelation in residuals, I thereby select the model in accordance with the parameter parsimony, thus the VAR (1).

4.2.4 *Stable Process*

To assess whether the system is *stable*, I then analyse if the eigenvalues of the companion coefficient matrix have no roots inside or on the complex unit circle.

The graph below shows that the model is stable since all the eigenvalues of the companion matrix have modulus less than 1.



Plot 11 - Inverse Roots of AR Characteristic Polynomial. Source: author elaboration

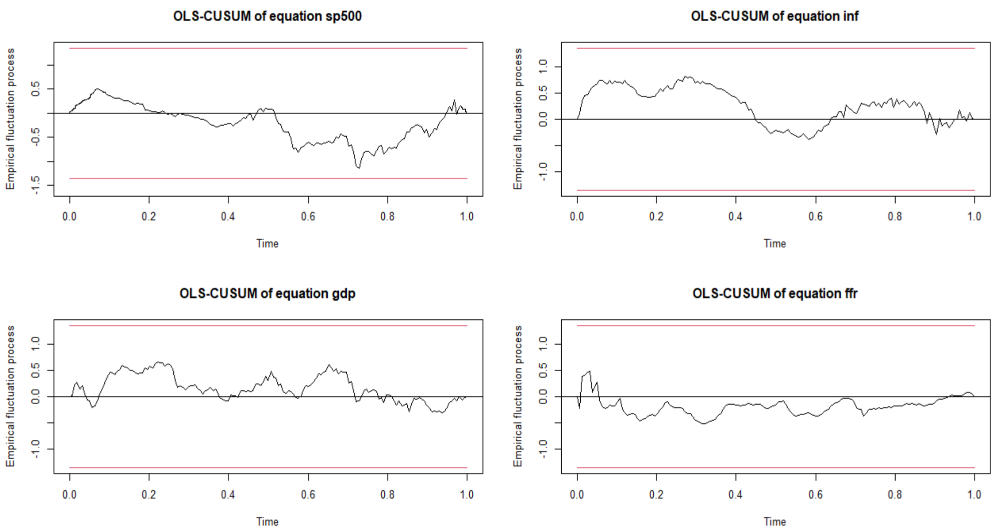
4.3 Model Adequacy Tests

In the present section I will apply diagnostic tests to the previously selected model VAR(1). The theory behind the tests has already been discussed in Chapter 2, Section 2.2.

4.3.1 *Stability Test*

When fitting a time-series model, one of the main assumptions (Chapter 2, Section 2.2) is that the coefficients are stable over time. The stability test checks this assumption: it bases the result on whether the time-series abruptly changes in ways not predicted by the model. Technically, it tests for structural breaks in the residuals. If at any point in the graph the cumulative sum (black line) excesses the red critical bounds, the structural break at that point was seen.

In the graph below the cumulative sum for all the variables is contained within the confidence bounds, thus the coefficients, and thereby the model, are deemed to be stable over time.



Plot 12 - Sum of Recursive Residuals. Source: author elaboration

4.3.2 *SERIAL CORRELATION TEST*

```
        Portmanteau Test (asymptotic)

data:  Residuals of VAR object var_bic
Chi-squared = 222.64, df = 240, p-value = 0.7829
```

Fig. 11 - Portmanteau Test output. Source: author elaboration

The output of the Portmanteau- and Breusch-Godfrey test shows a p-value (0.7829) higher than 5 % significance level, indeed, we do not reject the null hypothesis of no serial autocorrelation in the residuals.

4.3.3 *HETEROSKEDASTICITY TEST*

```
        ARCH (multivariate)

data:  Residuals of VAR object var
Chi-squared = 1268.3, df = 1200, p-value = 0.08335
```

Fig. 12 - ARCH Test output. Source: author elaboration

The output of the Engle’s ARCH test indicates a p-value (0.08335) higher than 5 % significant level, thus, we do not reject the null hypothesis that is: absence of autoregressive conditional heteroscedasticity (ARCH) effects.

4.3.4 *NONNORMALITY TESTS*

```
        JB-Test (multivariate)

data:  Residuals of VAR object var
Chi-squared = 2143.5, df = 8, p-value < 2.2e-16

        Skewness only (multivariate)

data:  Residuals of VAR object var
Chi-squared = 32.828, df = 4, p-value = 1.296e-06

        Kurtosis only (multivariate)

data:  Residuals of VAR object var
Chi-squared = 2110.7, df = 4, p-value < 2.2e-16
```

Fig. 13 - Nonnormality Test output. Source: author elaboration.

The outputs for JB-Test, skewness and kurtosis show p-values less than 5 % significance level, therefore, we must reject the null hypothesis that the data are normally distributed. However, normality in time-series analysis is a desirable but not necessary condition (Box, Jenkins, Reinsel, & Ljung, 2015).

4.4 SVAR MODEL IDENTIFICATION

To proceed and correctly interpret the Impulse Response Functions we must consider the structural form of the model that involves the simultaneous effect of the shocks involved, shocks that are, hence, orthogonal among them.

To do so, I applied the Cholesky decomposition, as anticipated in Chapter 2 (Section 2.6), thus, through a lower triangular matrix to determine the order and the impact the variables enter the equations considered.

The Cholesky decomposition imposes a recursive ordering of the identified shocks. In our case, for instance, FFR appears as the last variable, implying that FFR reacts simultaneously to all other variables shocks under consideration, while SP500, the first variable included in the model, do not react simultaneously to all other shocks, but will react with delay.

The recursive ordering of the identified shocks required by the Cholesky decomposition can be clearly observed from the SVAR estimation results displayed below.

```
SVAR Estimation Results:
========================


Estimated A matrix:
         sp500      gdp   inf ffr
sp500  1.00000  0.00000 0.000   0
gdp   -1.21414  1.00000 0.000   0
inf   -0.03089  0.02546 1.000   0
ffr    0.31683 -0.26160 5.503   1
```

Fig. 14 - SVAR matrix estimation output. Source: author elaboration

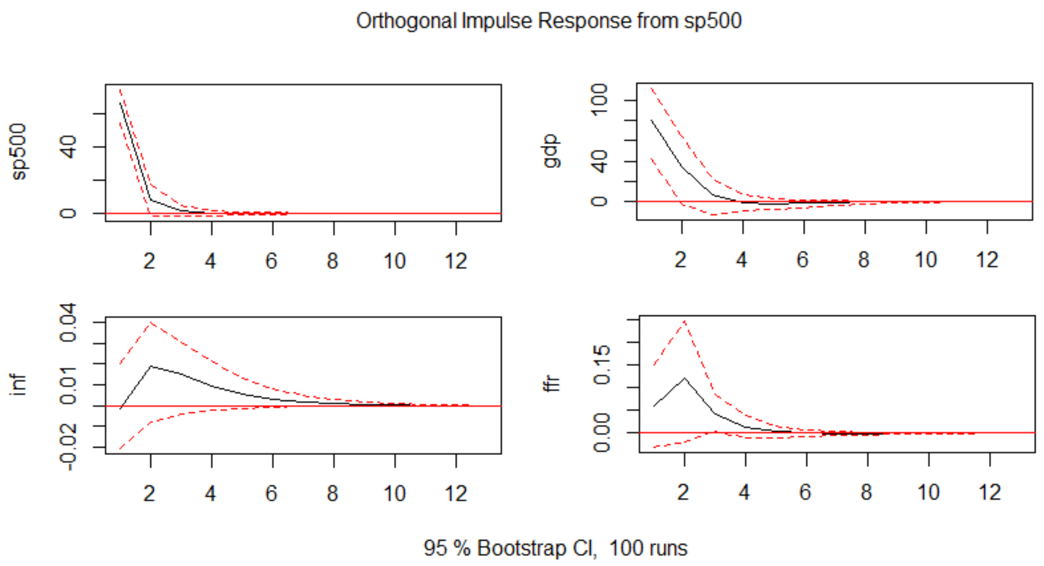
The matrix reads horizontally: the equations enter simultaneously horizontally. The aim is to identify the VAR that includes or not contemporary shocks. For instance, in our case we have a contemporary effect, on the first horizontal row, only on the equation SP500 by the variable SP500 itself, so the GDP does not have a contemporary effect on SP500, not even the INF and the FFR.

The ordering chosen to structure the VAR model is suggested by the economic theory, for instance FFR is the last variable in the ordering since it reacts to shocks in the other variables but it cannot have a contemporary effect on GDP and INF because from economic theory, if the FED raises or decreases the FFR, the effect of such policy is not immediate but it happens with a time lag (Galí, 2015).

4.5 Impulse Response Functions

The IRF are displayed in the following plots and concern the impulse responses of the variables to stock market index shocks (the first four graphs) and to monetary policy shocks (the last four graphs).

The remaining IRF related to the lower triangular matrix, as shown in the previous section, are listed in the Appendix.



Graph 16 - Impulse Response Functions to S&P 500 shock. Source: author elaboration

The IRF that shows how SP500 reacts to a shock (positive by default) in SP500 indicates that at time 0 (x-axis) the variable increases, even if already two periods later the shock has reabsorbed; except the early surge, the rest of the effect is manifested with the confidence bounds containing 0. We cannot be sure that the variable is different from 0 in that interval.

We expected this result which can be linked to the “there are no free lunches” motto. The past shocks in the financial markets here represented by the SP500 are not useful to forecast future price development in the index. The initial shock will then vanish in less than two periods following a fast trajectory.

The IRF related to a SP500 shock on GDP, highlights that when a strong shock in SP500 occurs, the GDP variable, at time 0, increases and the significance of this effect is demonstrated by the fact that the confidence bounds do not contain 0; then, already from the second period, the response starts to reabsorb. From the third period, the effect of a SP500 shock on GDP has already completely reabsorbed.

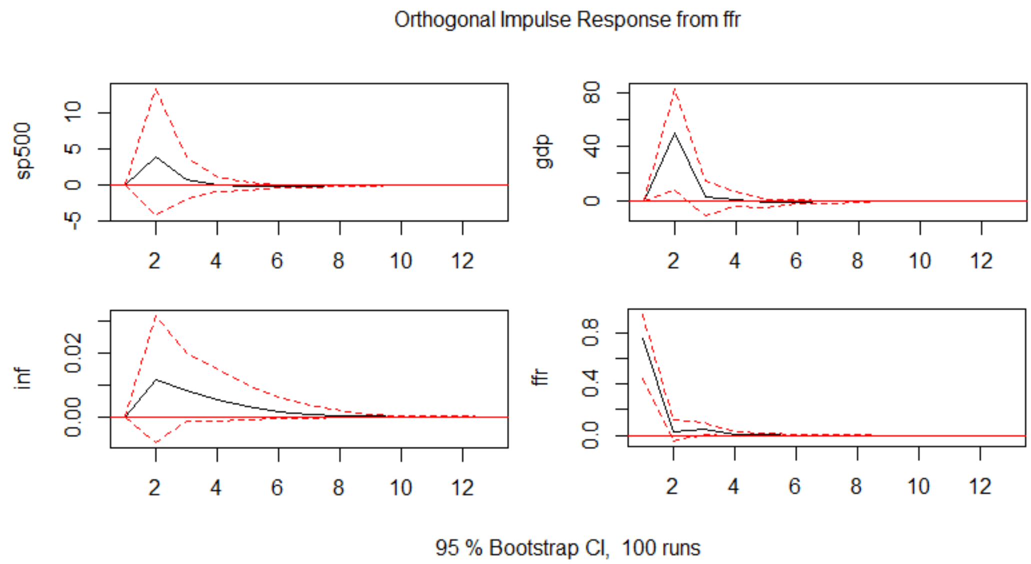
The effect here has a longer lasting impact with respect to the previous case on the SP500 itself, why is it so? We can easily understand that an increase in the SP500, that is an increase in the financial income of US investors, would lead to a positive impact on consumption and investment, that are two of the main components of GDP. The effect would then be expansionary, leading to an increase in the GDP that will last in three periods. That is, if the increase in the stock market is just a random case happening only once in a time, its effect will fade away in three periods, lacking the financial stimulus on the GDP.

The IRF plot representing how INF reacts to a shock in SP500, shows that when a positive shock occurs in SP500, INF at first does not increase, then after a period starts to rise, thereafter gradually returns to equilibrium. However, the whole reaction to the shock occurs with 0 within the confidence bounds, hence, as a matter of fact, we cannot be sure of an effect of SP500 on INF.

Linking this result with what we found out for the impact on the GDP, we can understand that an increase in the financial income would result, not immediately from our IRF, on an increase in the monetary amount available in the economy which, together with an expansion in GDP and hence in the velocity of circulation of money, will result in an increase in the inflation, with a long lasting impact of almost nine periods.

The IRF related to the reaction of FFR because of a shock in SP500, the FFR variable originally does not increase, indeed the FED rises the FFR in a later period, then progressively the effect is reabsorbed. We can be sure about the significance of the FFR increase in the second period since confidence bounds do not contain 0. Is interesting to notice that the FFR IRF and the INF IRF show a similar behaviour: both, initially, do not show any sign of reaction to an impact of a SP500 shock; then they both increase from the second period. From an economical point of view, this symmetry could be the FED reacting not directly to a shock in SP500, but rather to an increase in INF, which is in turn the result of an increase in GDP.
The FED would then try to contrast the increase in the inflation by raising the interest rates. In this sense could be even further explained the declining trend in the inflation response to the shock in the SP500. The SP500 shock would increase the inflation which will then be mitigated by the intervention of the FED policy (Galí, 2015).

Following, the IRFs related to a shock in FFR affecting the other variables and, in turn, the response of FFR because of a shock in the other variables.



Graph 17 - Impulse Response Functions to FFR shock. Source: author elaboration

The IRF of SP500 because of a shock in FFR shows that, after a slight delay, the variable increases and then, from the second period, it declines; the effect is already reabsorbed at the third period, moreover we cannot attest the significance of an increase in SP500 since the confidence bounds contain 0 during the whole process.

From a financial point of view, an increase in the interest rates impacting as a stimulus to the financial market is not so counterintuitive as it could seem at a first glance. While the average impact on financial markets could be negative, an increase in the interest rates would impact the markets by pushing investors towards more safe products, hence T-bills by the US government and SP500 stocks. Thus, even if not intense, the impact on the SP500 can be regarded as positive.

The IRF of GDP related to the shock in FFR shows that, at first, the variable decreases, then the GDP increases and next quickly declines from the third period until the effect is reabsorbed. We can certify the significance of such an increase of GDP at the second period since the confidence bounds do not contain 0.

The positive impact could be due to the increase in the SP500, retracing what we previously explained in the former case. It would be hardly explainable an increase in the GDP measure following an increase in the FED rates without pairing it to the simultaneous effect of the increase in the financial markets.

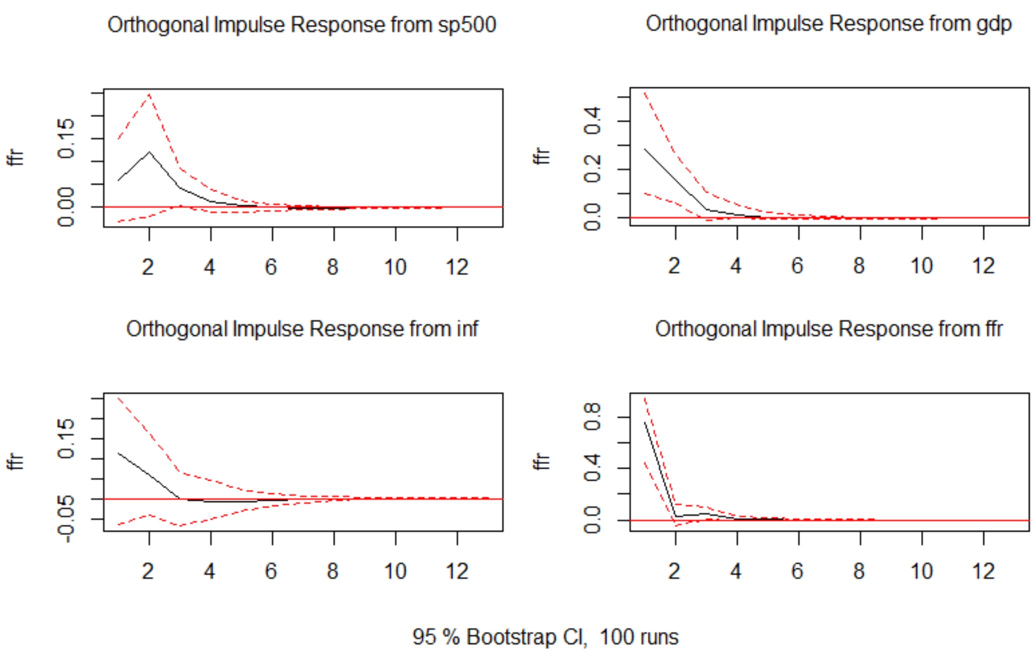
The IRF of INF reacting to the FFR shock indicates that a shock in FFR will have INF to decline at first, then rising by the second period and thereafter declining and slowly reabsorbing already by the third period, even if the reabsorption process lasts until the fifth/sixth period. However, the confidence bounds contain 0, thus we cannot be sure about the significance of the effect.

Again, it must be highlighted that from economic theory an increase in the interest rates has a decreasing impact on the inflation. But as we already noticed in the prior IRF, the increase in inflation is predominantly due to the impact on it of an increase of the SP500 and of GDP. This could be an anomaly that (Eichenbaum, 1992) first defined as *price puzzle*, an increase in the inflation rate caused by a surprise interest rate hike, contrary to the monetary policy theory (Galí, 2015), however there is not enough evidence to prove this type of phenomenon since the INF IRF shows an initial lag that should preclude the price puzzle phenomenon, which instead implies a sudden increase in inflation as a result of a positive FFR shock. Most probably, inflation reacts with a lag because its variations must be searched in the impact of the financial markets and not on the decreasing impact of the FED policy.

The IRF of FFR because of a shock in FFR shows that, at first, the variable rises and the effect is very significant, the confidence bounds do not contain 0; then, by the second period, the FFR sharply decline and, subsequently to a slight but significant increase at the third period, the effect has reabsorbed.

Thus, we can say that past policies by the FED are not significant for the future policies forecasts. It looks like the FED policies tend to follow a two-period trend, a change in the rates will be followed by a similar change in period two.

Below, the IRF related to shocks in the variables used in the model, impacting the FFR, are displayed. I will limit myself to examine only the IRF of shock in GDP and INF. From an economical perspective, this in an important analysis to perform, since contemplates the reactions of the FED to a shock in GDP and inflation.



Graph 18 - FFR Impulse Response Functions to shock in the other variables. Source: author elaboration

A positive (by default) shock in GDP makes FFR increasing at first, then the effect will progressively be reabsorbed by the third period. The significance of the rising of the variable FFR is highlighted by the fact that the confidence bounds do not contain 0 in that portion of IRF.

An increase in GDP forces the central bank to raise the rates to counterbalance the monetary expansion that the GDP increase would imply. Furthermore, from classic economic theory, an increase in the GDP would increase rates by a simple rule of offer and demand of money supply (Galí, 2015).

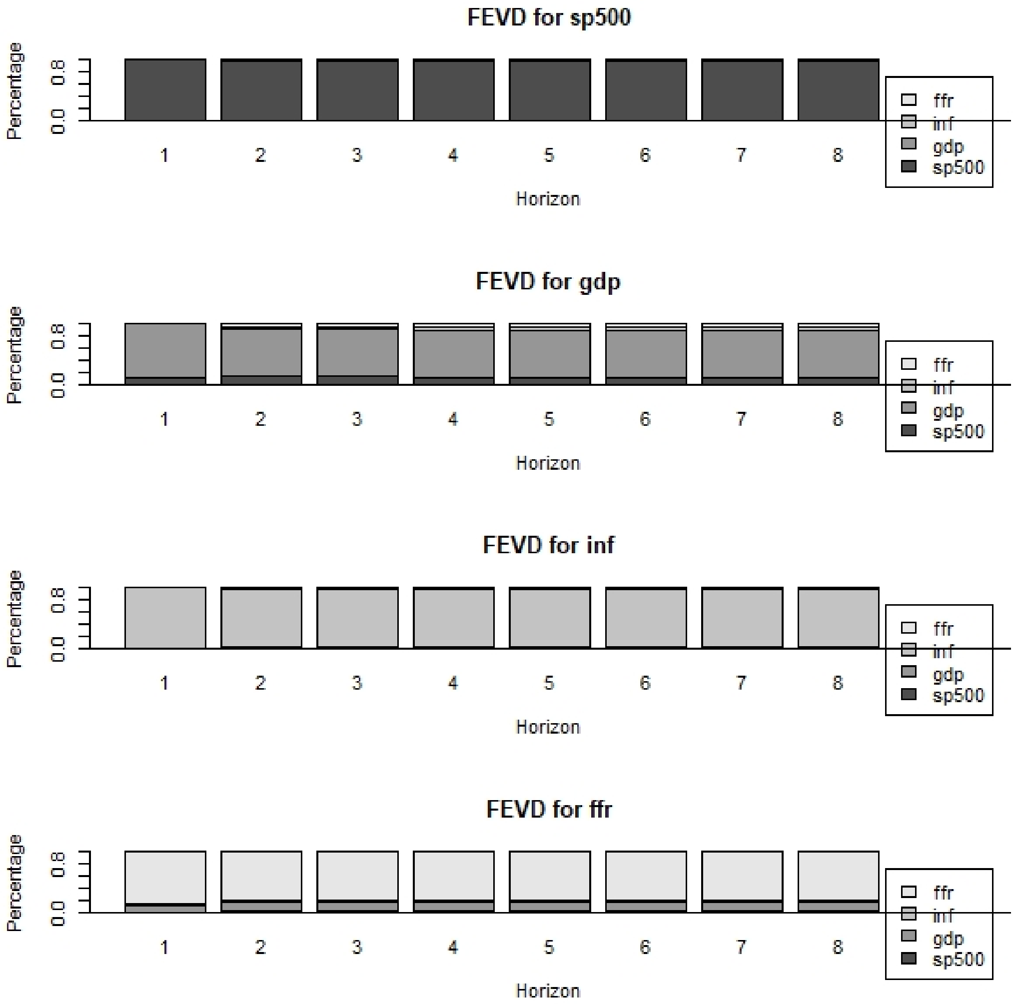
Analogously, the IRF concerning a reaction of the FED to a shock in inflation shows, originally, an increase, then the FFR quickly returns to equilibrium already by the third period. However, we cannot be sure about the significance of the shock since the confidence bounds contain 0.

Again, this is a straightforward result, an increase in the inflation leads the central bank to take a step to control it.

Thus, comparing the last two IRF (FFR reaction to shock in GDP and INF), the suggestion is that the FED is more likely to react to a GDP shock than to inflation, given that the two variables are linked by a causal relationship, since an increase in inflation is the consequence of an increase in the output of a country, however the alarm comes from the GDP because, from an economical perspective, is the GDP to drive inflation and not the other way around.

4.6 FORECAST ERROR VARIANCE DECOMPOSITION

Implementing the Forecast Error Variance Decomposition, it investigates how much of the forecast error variance of each of the variables can be explained by exogenous shocks to the other variables (Chapter 2, Section 2.4). In other terms, the FEVD helps us to determine how much, effectively, a variable shock impacts the variability of others. Typically, most of the variability of a variable will be explained by the variable itself and a minor percentage by other variables.



Graph 19 - Forecast Error Variance Decomposition for all the variables. Source: author elaboration

The FEVD for SP500 shows that 100% of the variability for the variable is explained by SP500 itself.

The FEVD for GDP indicates that, as usual, a large variability is described by the GDP variable itself, a smaller portion by SP500 and INF, then, with delay, a small portion is also explained by FFR.

The FEVD for INF, the largest part is explained by the INF variable itself and a slight portion by SP500.

The FEVD for FFR indicates that a significant portion is explained by FFR itself, a 20% of variability is described by GDP and an even smaller part by SP500.

Hence, the last FEVD related to FFR, suggests that the FED is more likely to react to a change in GDP rather than inflation, behaviour also highlighted by the FFR IRF analysed in the previous chapter, related to a shock in inflation and GDP.

CONCLUSIONS

This thesis, through the estimation of vector autoregression (VAR) models, examines the effects of stock market price and monetary policy shocks on the US economic cycle, and how the FED reacts to a shock in the financial market and how the financial market and the US economy react to a monetary policy shock.

The timeframe of the analysis goes from the first quarter of 1980 to the fourth quarter of 2019, thus, involving the challenges incurred by the FED to face the Great Inflation and the Great Recession (Hetzel, 2017).

The variables used in the model are quarterly time series: S&P 500, as a proxy for the financial market; Real GDP per capita, as a proxy for the US output; GDP Price Deflator, as a proxy for inflation; Effective Federal Funds Rate as a proxy for the interest rate.

After carrying out the time series analysis, the four variables have resulted to be non-stationary, thus, transformed to obtain stationary time series. Following, the model has been implemented and subjected to adequacy tests to assess whether the model properly represents the DGP of the variables, with positive outcome.

The results obtained are in line with the economic theory , at least for the responses to financial shocks: a positive shock in the S&P 500 stock index represents an increase in the financial income of the US investors, which will lead to a positive impact on consumption and investment, that are two of the main components of GDP. The effect would then be expansionary, leading to an increase in the GDP, boosting the monetary amount available in the economy, which, together with an expansion in GDP and hence in the velocity of circulation of money, will result in an increase in the inflation; then the FED will react, not directly to a shock in S&P 500, as claimed by the “leaning against the wind” movement, but rather to a GDP expansion causing an increase in inflation.

From a monetary policy perspective, however, I found out some anomalies: to a positive monetary policy shock follows an increase in the S&P 500 index, contrary to the economic theory (Galí, 2015). However, an increase in the interest rates impacting as a stimulus to the financial market is not so counterintuitive as it could seem at first glance, since while the average impact on financial markets could be negative, an increase in the interest rates would impact the markets by pushing investors towards more safe products, such as T-bills by US government and S&P 500 stocks. Thus, even if not intense and not so significative, the impact on the S&P 500 can be regarded as positive, in accordance with the supporters of the movement "leaning against the wind".

Analogously anomalous are the GDP and inflation responses to an increase in the interest rates. However, the positive impact, to the GDP first and to inflation then, could be due to the increase in the S&P 500, retracing what we previously said about the financial market: it would be hardly explainable an increase in the GDP followed by an increase in inflation measures, resulting from an increase in the FED rates, without pairing it to the simultaneous effect of the increase in the financial markets. Again, it must be highlighted that from economic theory, an increase in the interest rates has a decreasing impact on inflation; thus, the anomaly encountered in my thesis could lead one to believe that we are dealing with the price puzzle phenomenon. Nevertheless, it remains only a suggestion, since the analysis has not been conducted in this respect and, moreover, there is not enough empirical evidence to trace out such a phenomenon (see Chapter 4, Section 4.5).

What we can be certain about, what emerges from my analysis, is that the FED, in the last fifty years of activity, has implemented its monetary policies almost exclusively to the service of inflation targeting, with the main and clear objectives including low and stable inflation and production close to the natural rate; exerting control over real interest rates in response to expansionary scenarios of GDP, which, from economic theory, drives inflation.

APPENDIX

The outputs of the estimated VAR models are reported below.

VAR (1).

```
VAR Estimation Results:
=========================
Endogenous variables: sp500, gdp, inf, ffr
Deterministic variables: const
Sample size: 158
Log Likelihood: -2062.969
Roots of the characteristic polynomial:
0.4994 0.3766 0.1741 0.0602
Call:
VAR(y = db_ts, p = nlag$selection["SC(n)"], exogen = dum)

Estimation results for equation sp500:
======================================
sp500 = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + const + exo1

            Estimate Std. Error t value Pr(>|t|)
sp500.l1     0.10198    0.08875   1.149   0.2523
gdp.l1       0.01245    0.02537   0.491   0.6243
inf.l1     -38.07798   26.69075  -1.427   0.1557
ffr.l1       5.14183    6.92204   0.743   0.4587
const       34.29321   15.34427   2.235   0.0269 *
exo1       -58.84035   40.32500  -1.459   0.1466
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 66.7 on 152 degrees of freedom
Multiple R-Squared: 0.06752,    Adjusted R-squared: 0.03684

F-statistic: 2.201 on 5 and 152 DF,  p-value: 0.05702
```

```
Estimation results for equation gdp:
====================================
gdp = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + const + exo1

           Estimate Std. Error t value Pr(>|t|)
sp500.l1    0.37076    0.30575   1.213 0.227145
gdp.l1      0.05914    0.08742   0.676 0.499772
inf.l1   -319.17918   91.95514  -3.471 0.000675 ***
ffr.l1     66.27929   23.84786   2.779 0.006137 **
const     333.49207   52.86419   6.308 2.92e-09 ***
exo1     -655.77449  138.92796  -4.720 5.30e-06 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 229.8 on 152 degrees of freedom
Multiple R-Squared: 0.2965,     Adjusted R-squared: 0.2733
F-statistic: 12.81 on 5 and 152 DF,  p-value: 2.135e-10


Estimation results for equation inf:
====================================
inf = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + const + exo1

           Estimate Std. Error t value Pr(>|t|)
sp500.l1  2.790e-04  2.121e-04   1.316   0.1903
gdp.l1    3.804e-06  6.063e-05   0.063   0.9501
inf.l1    5.587e-01  6.378e-02   8.761 3.55e-15 ***
ffr.l1    1.554e-02  1.654e-02   0.940   0.3488
const     1.978e-01  3.666e-02   5.394 2.59e-07 ***
exo1     -2.130e-01  9.636e-02  -2.210   0.0286 *
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.1594 on 152 degrees of freedom
Multiple R-Squared: 0.4189,     Adjusted R-squared: 0.3998
F-statistic: 21.92 on 5 and 152 DF,  p-value: < 2.2e-16
```

```
Estimation results for equation ffr:
====================================
ffr = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + const + exo1

          Estimate Std. Error t value Pr(>|t|)
sp500.l1  0.0009368  0.0010932   0.857   0.3928
gdp.l1    0.0006895  0.0003126   2.206   0.0289 *
inf.l1    0.3433334  0.3287799   1.044   0.2980
ffr.l1    0.0422578  0.0852666   0.496   0.6209
const    -0.3720801  0.1890126  -1.969   0.0508 .
exo1      0.4063515  0.4967283   0.818   0.4146
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.8217 on 152 degrees of freedom
Multiple R-Squared: 0.07114,    Adjusted R-squared: 0.04058

F-statistic: 2.328 on 5 and 152 DF,  p-value: 0.04528
```

```
Covariance matrix of residuals:
           sp500      gdp      inf      ffr
sp500 4449.35538  5402.154 -0.08987  3.97607
gdp   5402.15435 52811.386 -2.14328 66.19742
inf     -0.08987    -2.143  0.02540  0.01511
ffr      3.97607    66.197  0.01511  0.67513

Correlation matrix of residuals:
          sp500     gdp       inf     ffr
sp500  1.000000  0.35242 -0.008453 0.07255
gdp    0.352415  1.00000 -0.058515 0.35058
inf   -0.008453 -0.05851  1.000000 0.11541
ffr    0.072546  0.35058  0.115412 1.00000
```

VAR (3).

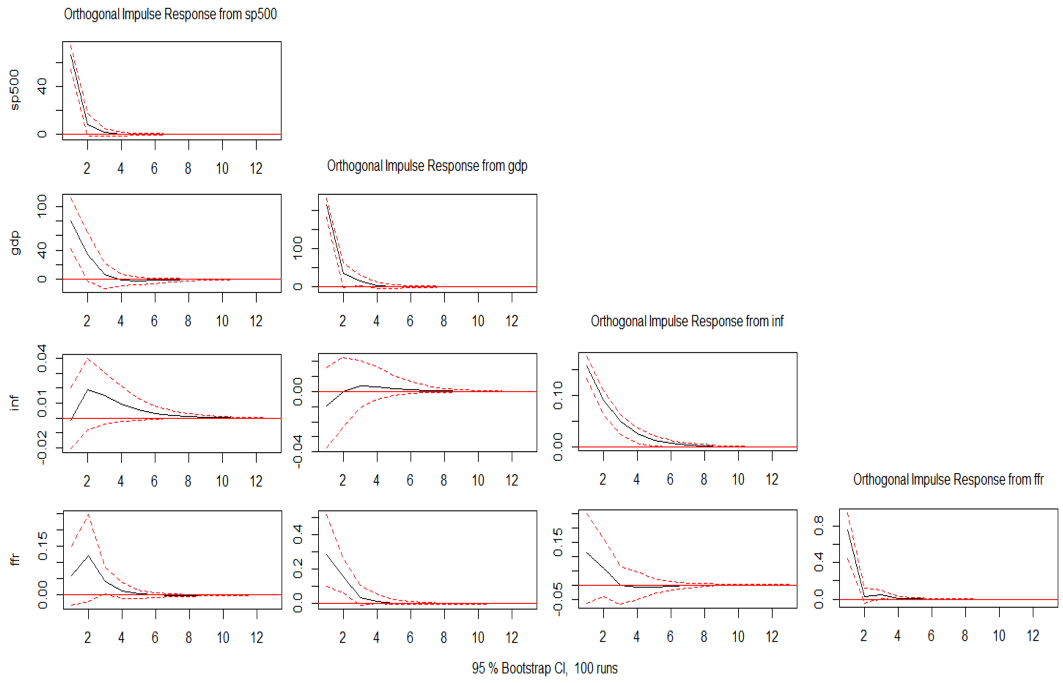
```
VAR Estimation Results:
=========================
Endogenous variables: sp500, gdp, inf, ffr
Deterministic variables: const
Sample size: 156
Log Likelihood: -1954.174
Roots of the characteristic polynomial:
0.7358 0.7358 0.547 0.547 0.5157 0.5157 0.4632 0.4632 0.434
9 0.3505 0.3505 0.05793
Call:
VAR(y = db_ts, p = nlag$selection["AIC(n)"], exogen = dum)


Estimation results for equation sp500:
======================================
sp500 = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + sp500.l2 + gd
p.l2 + inf.l2 + ffr.l2 + sp500.l3 + gdp.l3 + inf.l3 + ffr.l
3 + const + exo1

           Estimate Std. Error t value Pr(>|t|)
sp500.l1    0.04220    0.09149   0.461  0.64533
gdp.l1      0.02026    0.02748   0.737  0.46216
inf.l1    -44.59810   35.51376  -1.256  0.21125
ffr.l1      8.92786    7.28112   1.226  0.22217
sp500.l2    0.20253    0.09035   2.242  0.02653 *
gdp.l2     -0.05809    0.02569  -2.261  0.02525 *
inf.l2    -44.03387   36.99126  -1.190  0.23588
ffr.l2     11.53199    6.99280   1.649  0.10133
sp500.l3    0.10467    0.09261   1.130  0.26029
gdp.l3     -0.03397    0.02486  -1.366  0.17400
inf.l3     13.82744   34.25204   0.404  0.68704
ffr.l3     14.90836    7.13138   2.091  0.03835 *
const      63.40136   20.01267   3.168  0.00188 **
exo1      -57.08050   44.99684  -1.269  0.20668
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 65.43 on 142 degrees of freedom
Multiple R-Squared: 0.1616,     Adjusted R-squared: 0.08483

F-statistic: 2.105 on 13 and 142 DF,  p-value: 0.01714
```

```
Estimation results for equation gdp:
====================================
gdp = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + sp500.l2 + gdp.
l2 + inf.l2 + ffr.l2 + sp500.l3 + gdp.l3 + inf.l3 + ffr.l3
 + const + exo1

           Estimate Std. Error t value Pr(>|t|)
sp500.l1    0.26751    0.31204   0.857 0.392719
gdp.l1      0.06562    0.09374   0.700 0.485066
inf.l1   -390.83160  121.12710  -3.227 0.001555 **
ffr.l1     81.78139   24.83378   3.293 0.001251 **
sp500.l2    0.45847    0.30814   1.488 0.139008
gdp.l2      0.15051    0.08761   1.718 0.087988 .
inf.l2    -58.18355  126.16644  -0.461 0.645386
ffr.l2    -39.71399   23.85041  -1.665 0.098092 .
sp500.l3    0.18182    0.31586   0.576 0.565768
gdp.l3     -0.06406    0.08481  -0.755 0.451258
inf.l3     51.75612  116.82376   0.443 0.658420
ffr.l3     13.50683   24.32307   0.555 0.579557
const     333.07220   68.25742   4.880 2.82e-06 ***
exo1     -518.37718  153.47114  -3.378 0.000944 ***
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 223.2 on 142 degrees of freedom
Multiple R-Squared: 0.3706,     Adjusted R-squared: 0.3129
F-statistic:  6.43 on 13 and 142 DF,  p-value: 1.747e-09
```

```
Estimation results for equation inf:
====================================
inf = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + sp500.l2 + gdp.
l2 + inf.l2 + ffr.l2 + sp500.l3 + gdp.l3 + inf.l3 + ffr.l3
 + const + exo1

           Estimate Std. Error t value Pr(>|t|)
sp500.l1  2.595e-04  2.065e-04   1.256 0.211077
gdp.l1    7.414e-05  6.204e-05   1.195 0.234096
inf.l1    3.198e-01  8.017e-02   3.989 0.000106 ***
ffr.l1    3.499e-02  1.644e-02   2.129 0.035012 *
sp500.l2 -3.465e-05  2.039e-04  -0.170 0.865328
gdp.l2   -8.727e-05  5.799e-05  -1.505 0.134534
inf.l2    1.459e-01  8.351e-02   1.747 0.082843 .
ffr.l2    1.219e-02  1.579e-02   0.772 0.441235
sp500.l3 -5.353e-05  2.091e-04  -0.256 0.798266
gdp.l3   -3.338e-05  5.613e-05  -0.595 0.553056
inf.l3    1.580e-01  7.732e-02   2.043 0.042873 *
ffr.l3    9.909e-03  1.610e-02   0.616 0.539188
const     1.780e-01  4.518e-02   3.939 0.000128 ***
exo1     -2.751e-01  1.016e-01  -2.708 0.007604 **
---
Signif. codes:
0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.1477 on 142 degrees of freedom
Multiple R-Squared: 0.4806,     Adjusted R-squared: 0.433
F-statistic: 10.11 on 13 and 142 DF,  p-value: 8.284e-15
```

```
Estimation results for equation ffr:
====================================
ffr = sp500.l1 + gdp.l1 + inf.l1 + ffr.l1 + sp500.l2 + gdp.
l2 + inf.l2 + ffr.l2 + sp500.l3 + gdp.l3 + inf.l3 + ffr.l3
 + const + exo1

           Estimate Std. Error t value Pr(>|t|)
sp500.l1  6.938e-04  8.345e-04   0.831  0.40713
gdp.l1    4.195e-04  2.507e-04   1.673  0.09645 .
inf.l1   -8.552e-02  3.239e-01  -0.264  0.79216
ffr.l1    2.056e-01  6.641e-02   3.096  0.00236 **
sp500.l2 -4.517e-04  8.240e-04  -0.548  0.58442
gdp.l2    5.987e-04  2.343e-04   2.555  0.01166 *
inf.l2    2.379e-01  3.374e-01   0.705  0.48187
ffr.l2   -5.208e-02  6.378e-02  -0.816  0.41560
sp500.l3  1.710e-04  8.447e-04   0.202  0.83990
gdp.l3   -4.422e-05  2.268e-04  -0.195  0.84569
inf.l3   -4.376e-01  3.124e-01  -1.401  0.16351
ffr.l3    7.365e-02  6.505e-02   1.132  0.25942
const    -1.508e-01  1.825e-01  -0.826  0.41025
exo1      5.689e-01  4.104e-01   1.386  0.16785
---
Signif. codes:
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.5968 on 142 degrees of freedom
Multiple R-Squared: 0.2303,     Adjusted R-squared: 0.1598
F-statistic: 3.268 on 13 and 142 DF,  p-value: 0.0002345
```

The IRF related to the variables in the lower triangular matrix, in accordance with our structural VAR, are reported below.



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SOFTWARE

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