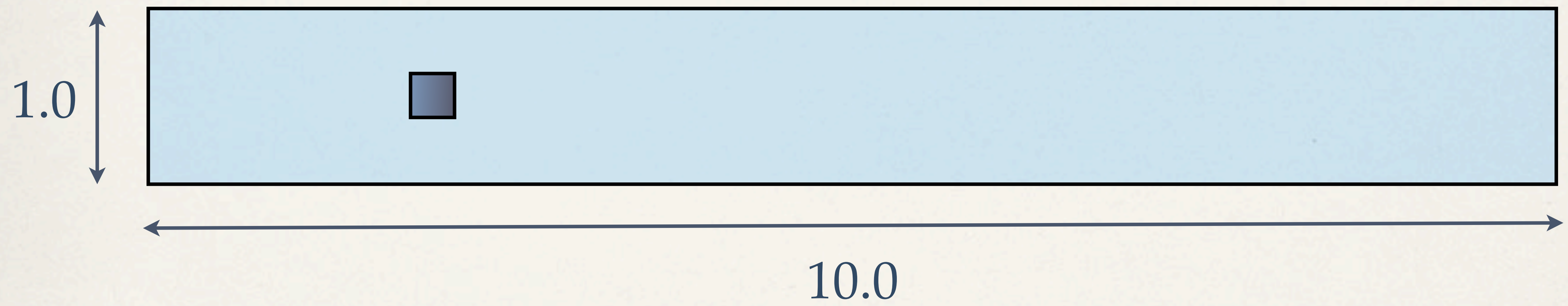
Sistema de estudio

Un fluido newtoniano incompresible que fluye a través de un canal con un obstáculo de geometría cuadrada.



Ecuaciones de gobierno

Navier-Stokes

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot (\mu \nabla \mathbf{u})$$

Continuidad

$$\nabla \cdot (\rho \mathbf{u}) = 0$$

Sistema de ecuaciones

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uu) + \frac{\partial}{\partial y}(vu) = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\Gamma\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial x}(uv) + \frac{\partial}{\partial y}(vv) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\Gamma\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma\frac{\partial v}{\partial y}\right)$$

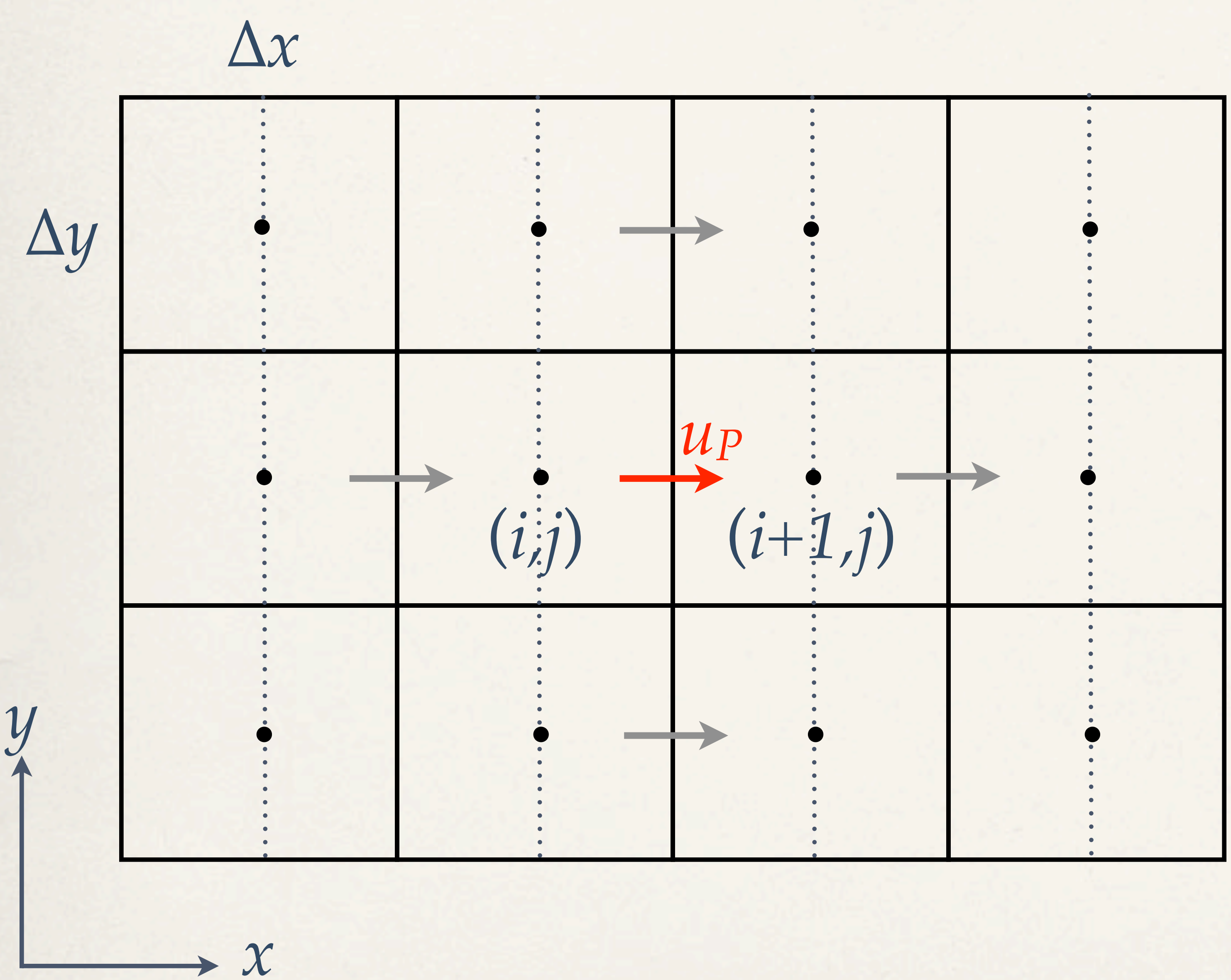
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Incógnitas:

$$(u, v, p)$$

Discretización, ecuación de u (esquema completamente implíctio)

$$a_P u_P = a_E u_E + a_W u_W + a_N u_N + a_S u_S + S_P$$



$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta V}{\Delta t}$$

Discretización, ecuación de u (esquema completamente implíctio)

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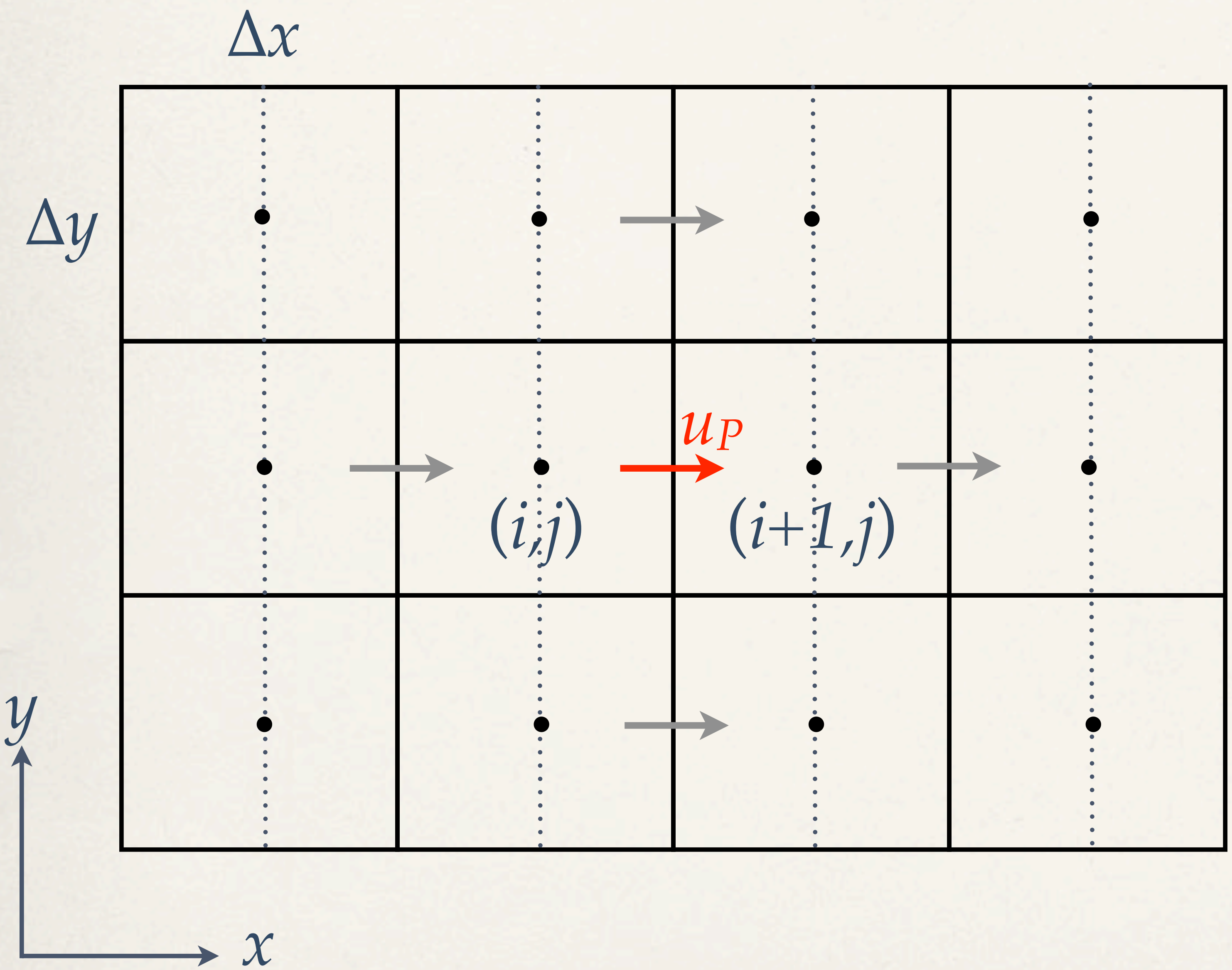
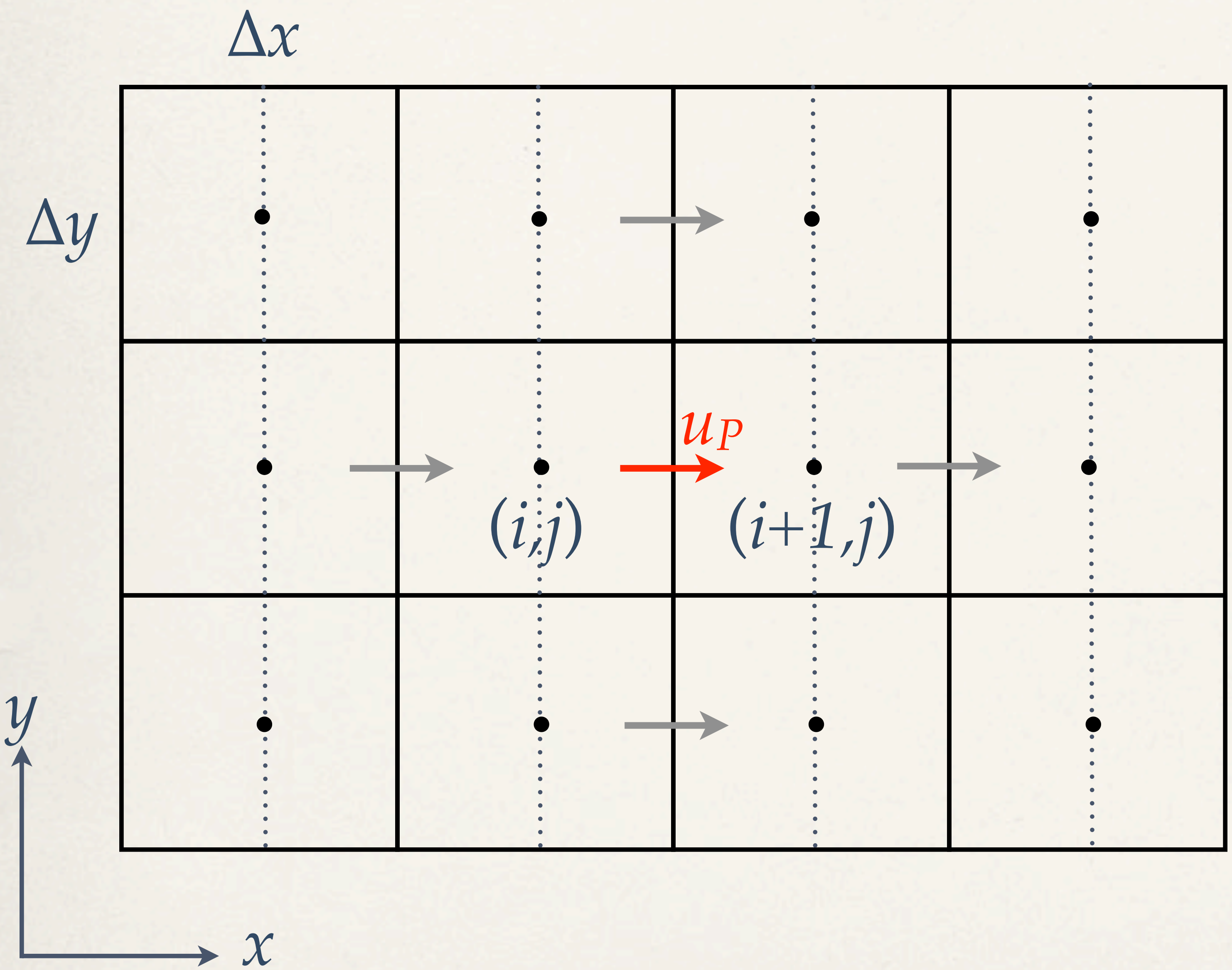


$$a_E = \Gamma_e \frac{s_e}{\Delta x_{PE}} + s_e * max[0, -u_e]$$

$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta V}{\Delta t}$$

Discretización, ecuación de u (esquema completamente implíctio)

$$a_P u_P = a_E u_E + a_W u_W + a_N u_N + a_S u_S + S_P$$



$$a_E = \Gamma_e \frac{s_e}{\Delta x_{PE}} + s_e * max[0, -u_e]$$

$$a_W = \Gamma_w \frac{s_w}{\Delta x_{WP}} + s_w * max[0, u_w]$$

$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta V}{\Delta t}$$

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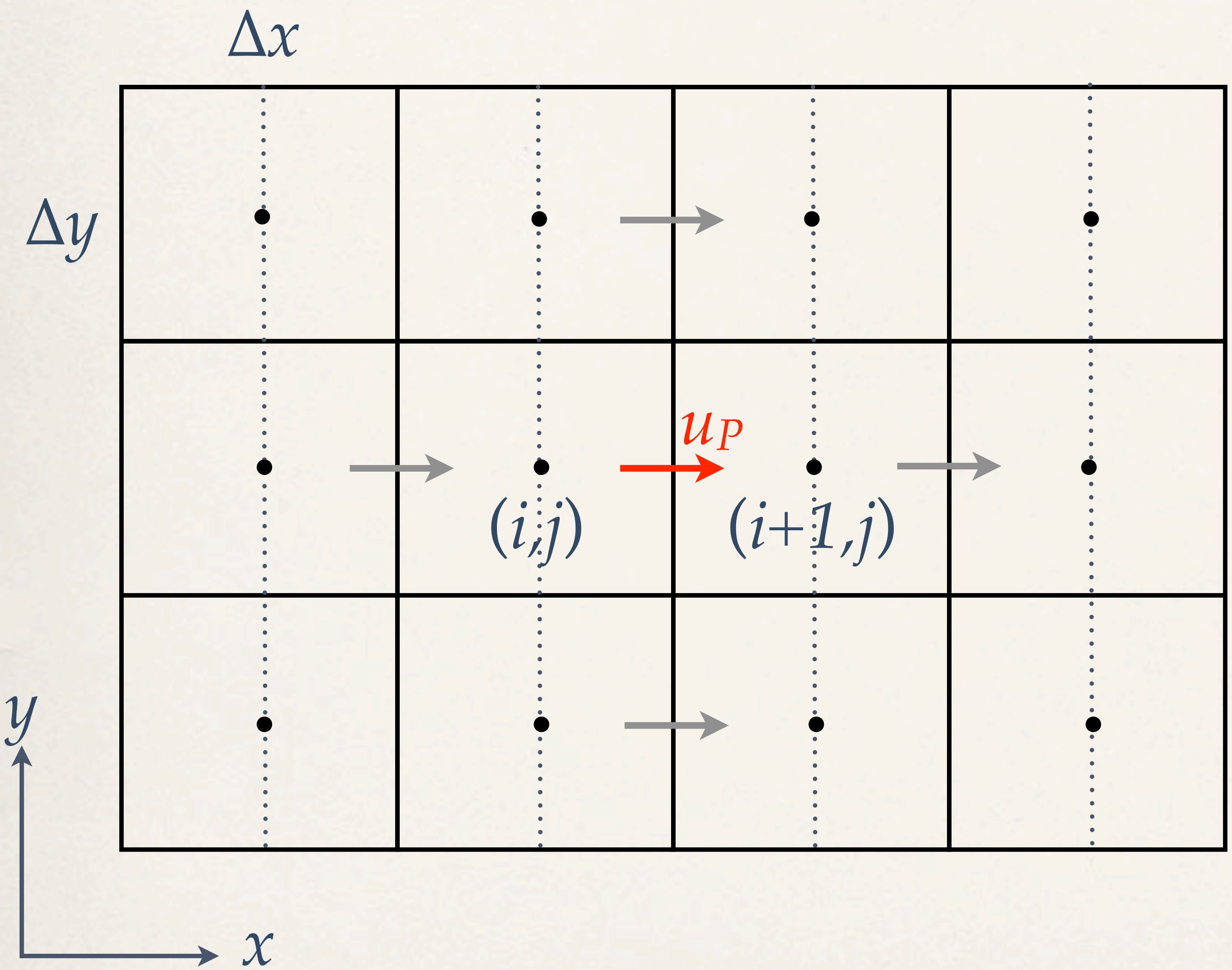


$$a_E = \Gamma_e \frac{s_e}{\Delta x_{PE}} + s_e * max[0, -u_e]$$

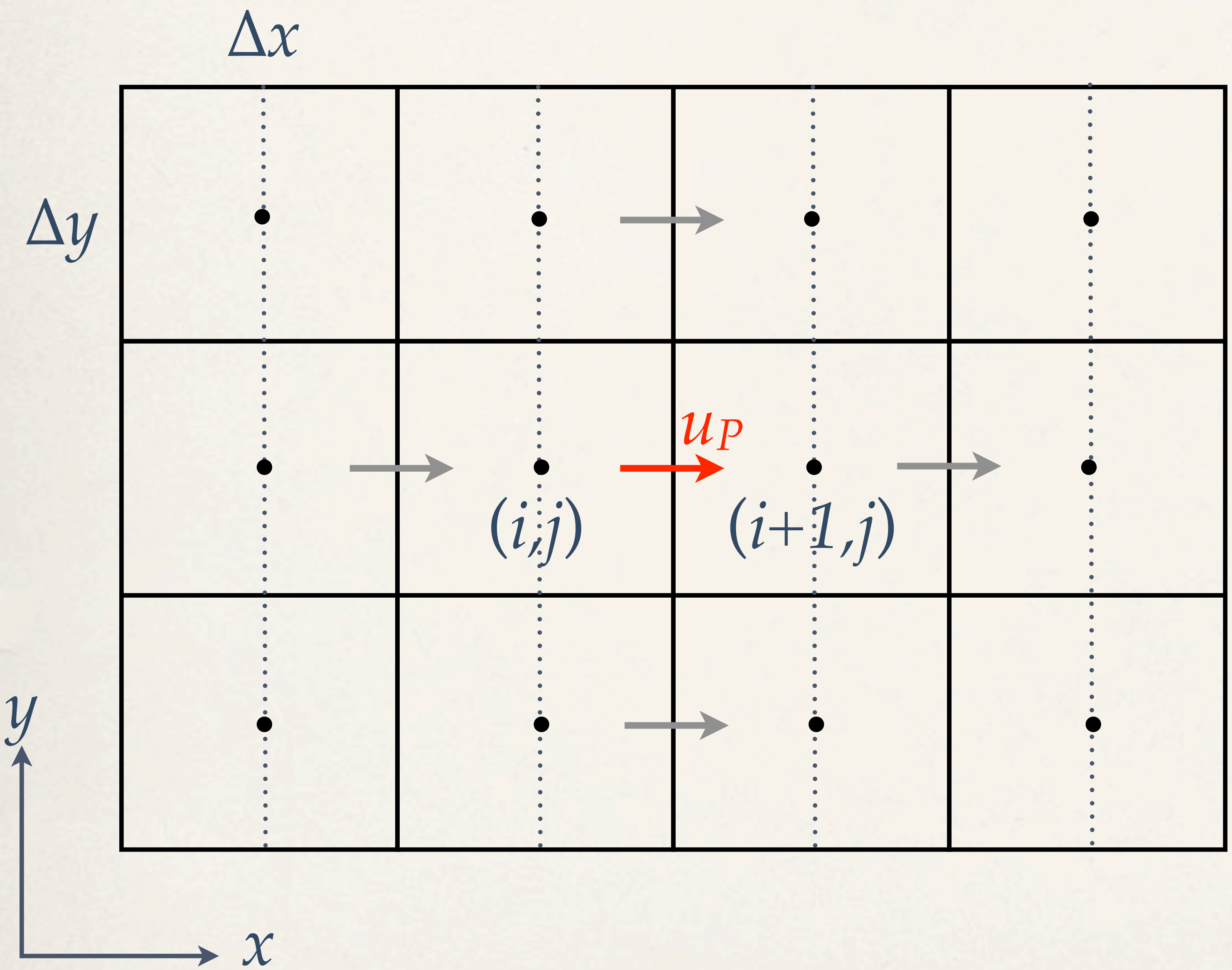
$$a_W = \Gamma_w \frac{s_w}{\Delta x_{WP}} + s_w * max[0, u_w]$$

$$a_N = \Gamma_n \frac{s_n}{\Delta y_{PN}} + s_n * max[0, -v_n]$$

$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta V}{\Delta t}$$

Discretización, ecuación de u (esquema completamente implíctio)

$$a_P u_P = a_E u_E + a_W u_W + a_N u_N + a_S u_S + S_P$$



$$a_E = \Gamma_e \frac{s_e}{\Delta x_{PE}} + s_e * max[0, -u_e]$$

$$a_W = \Gamma_w \frac{s_w}{\Delta x_{WP}} + s_w * max[0, u_w]$$

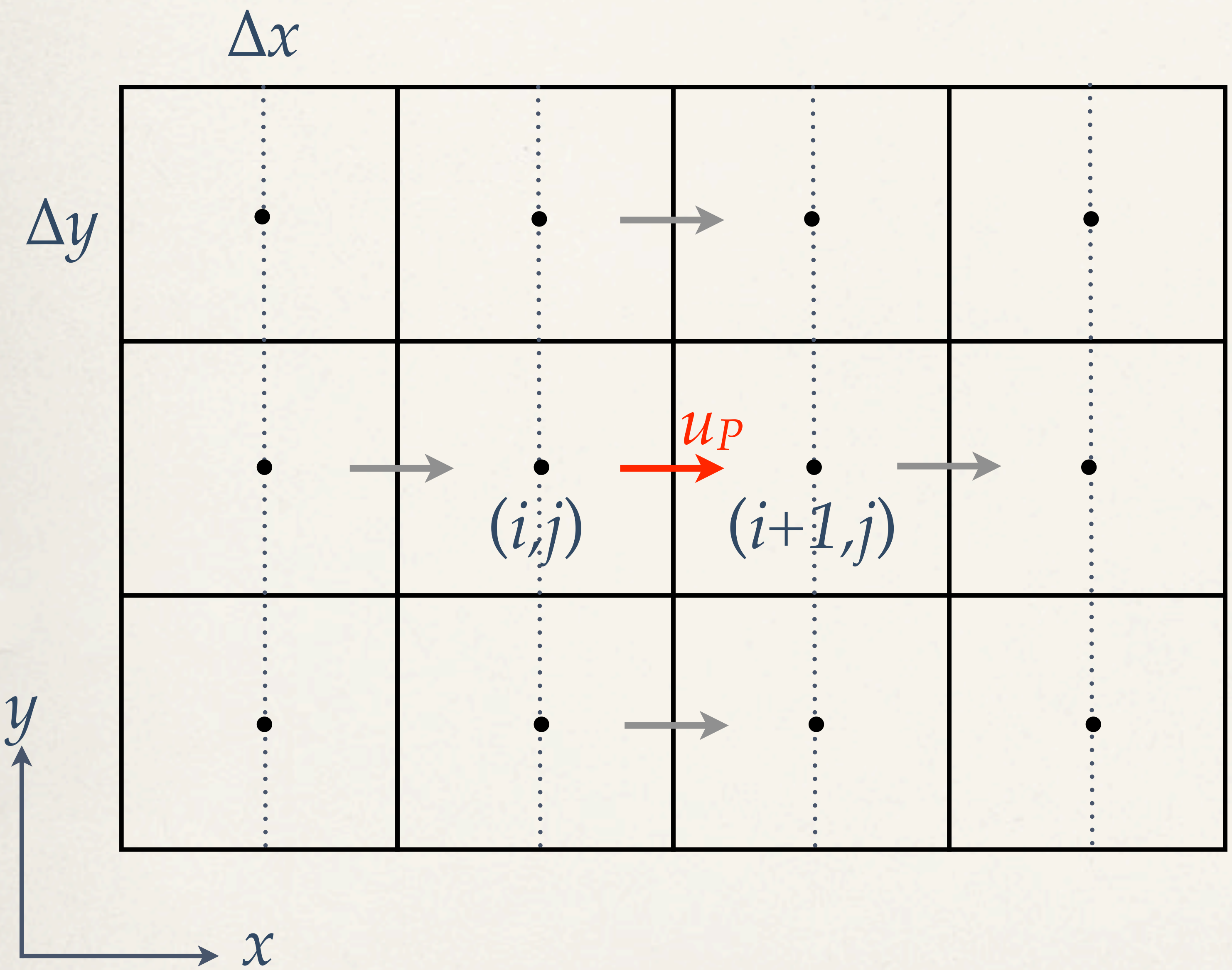
$$a_N = \Gamma_n \frac{s_n}{\Delta y_{PN}} + s_n * max[0, -v_n]$$

$$a_S = \Gamma_s \frac{s_s}{\Delta y_{SP}} + s_s * max[0, v_s]$$

$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta V}{\Delta t}$$

Discretización, ecuación de u (esquema completamente implíctio)

$$a_P u_P = a_E u_E + a_W u_W + a_N u_N + a_S u_S + S_P$$



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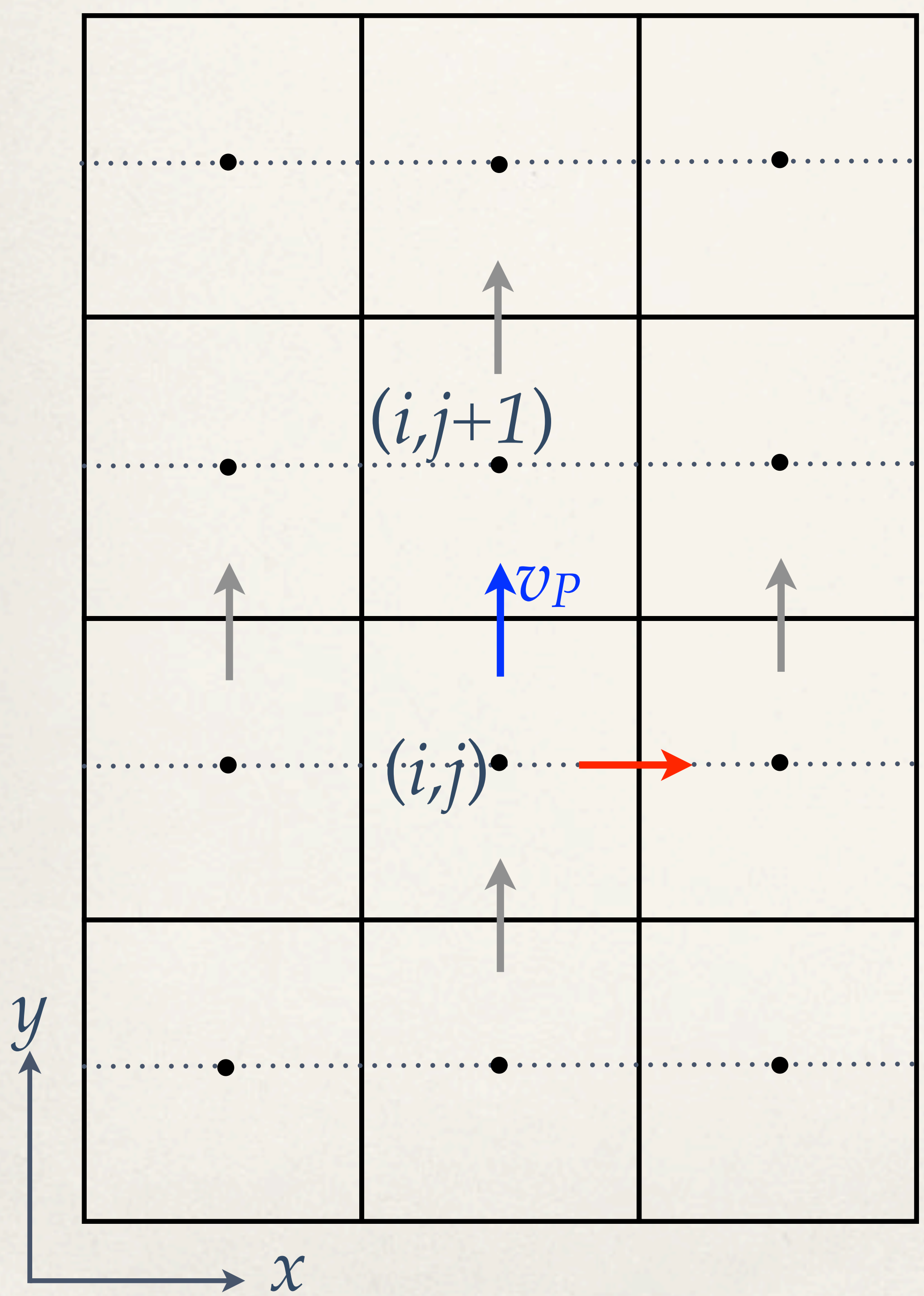
$$a_S = \Gamma_s \frac{s_s}{\Delta y_{SP}} + s_s * max[0, v_s]$$

$$S_P = u_P^0 \frac{\Delta V}{\Delta t} - \frac{(p_{i+1,j} - p_{i,j})}{\Delta x_u} \Delta V$$

$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta V}{\Delta t}$$

Discretización, ecuación de v

$$a_P v_P = a_E v_E + a_W v_W + a_N v_N + a_S v_S + S_P$$

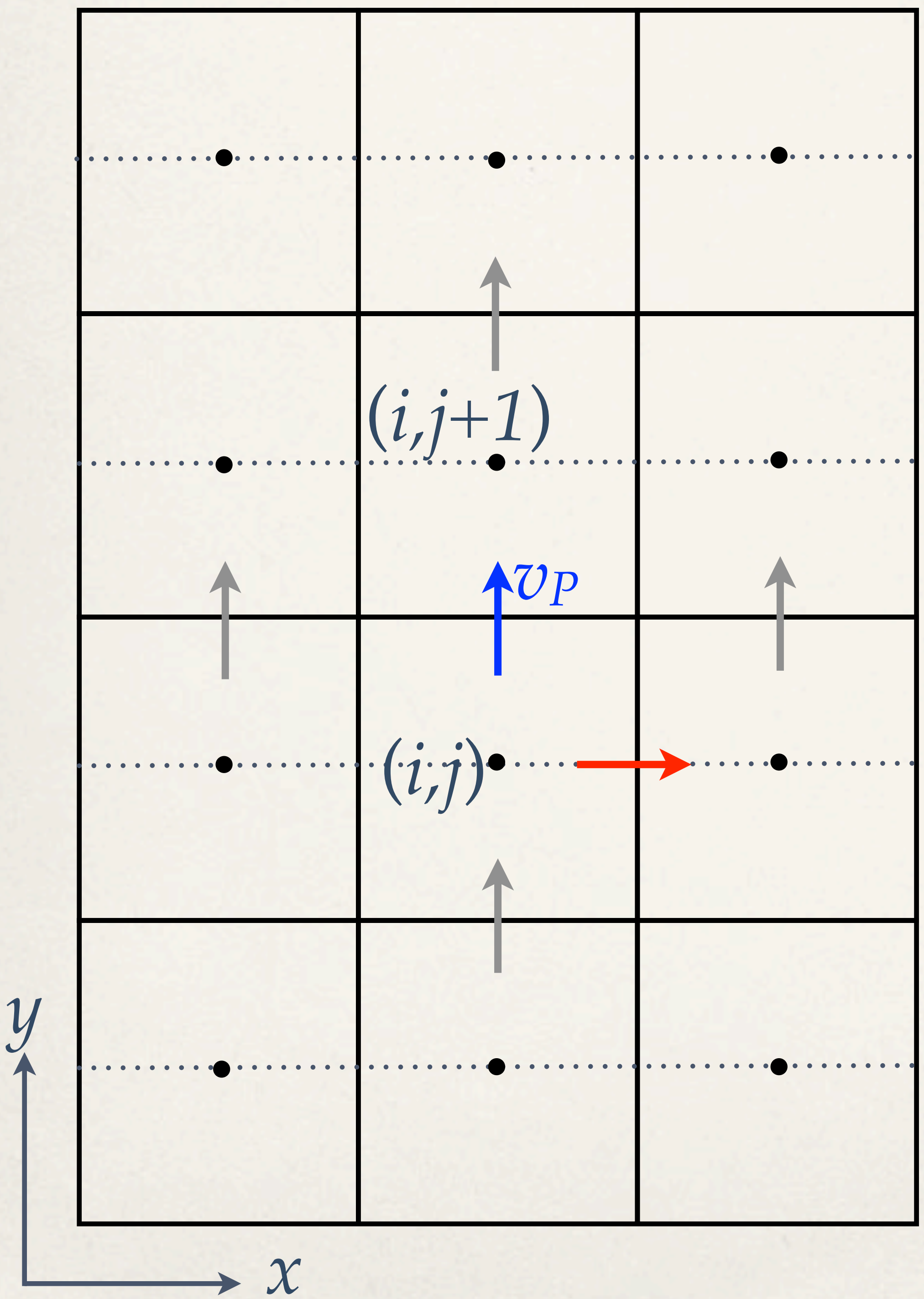


$$S_P = v_P^0 \frac{\Delta V}{\Delta t} - \frac{(p_{i,j+1} - p_{i,j})}{\Delta y_v} \Delta V$$

$$a_P = a_E + a_W + a_N + a_S + \frac{\Delta V}{\Delta t}$$

Discretización, ecuación de v

$$a_P v_P = a_E v_E + a_W v_W + a_N v_N + a_S v_S + S_P$$



$$a_E = \Gamma_e \frac{s_e}{\Delta x_{PE}} + s_e * max[0, -u_e]$$

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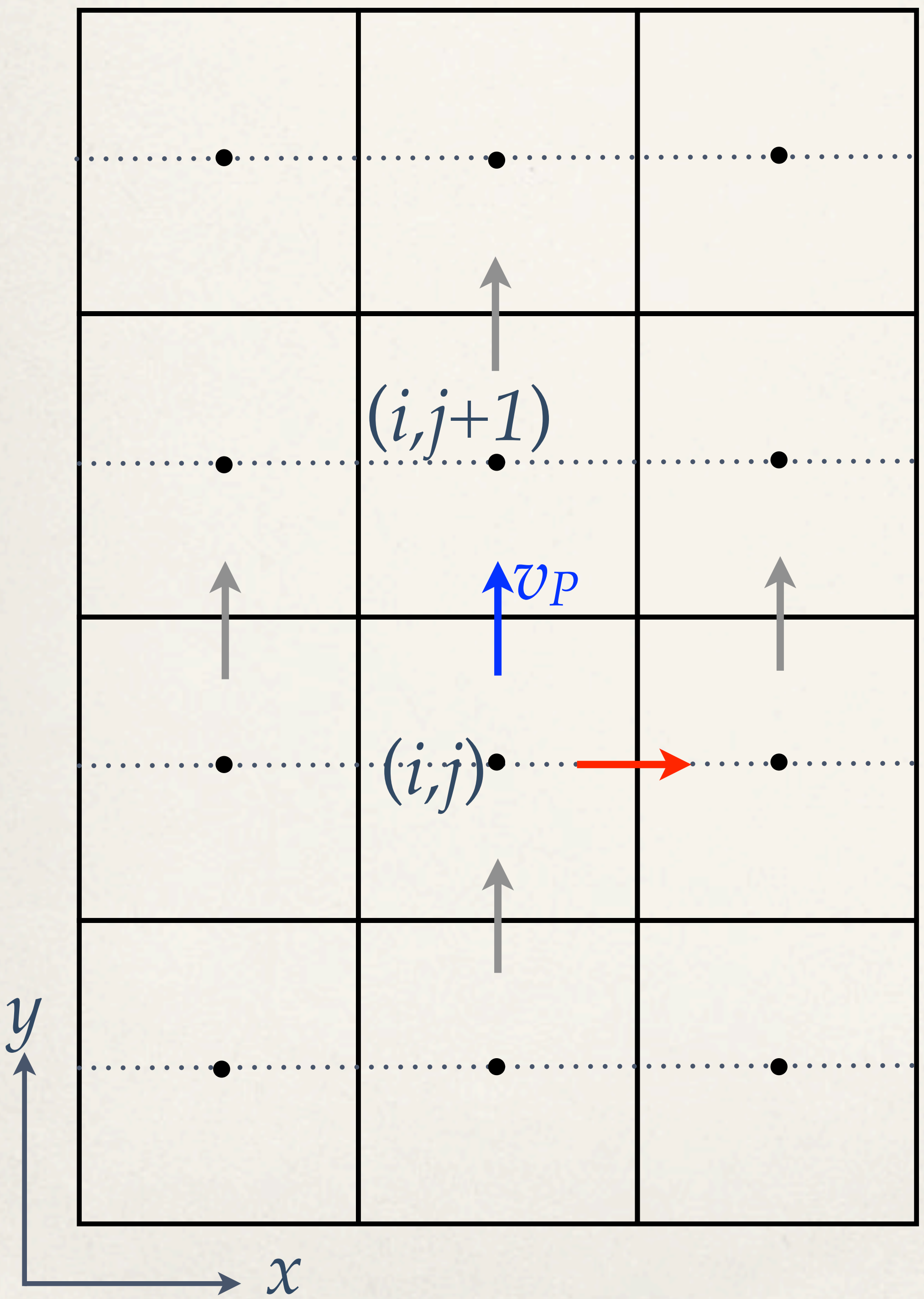
Discretización, ecuación de v

$$a_P v_P = a_E v_E + a_W v_W + a_N v_N + a_S v_S + S_P$$



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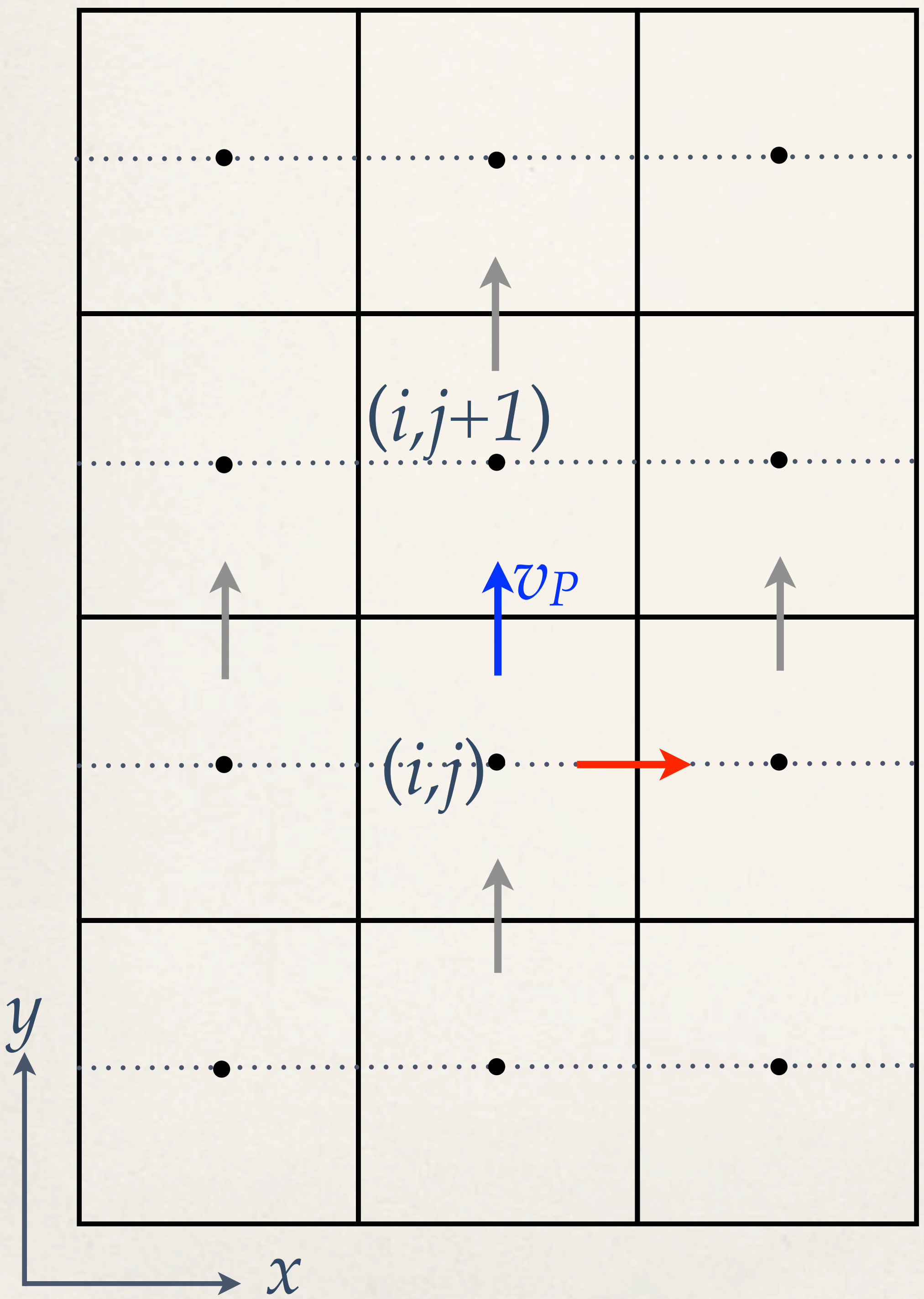
Discretización, ecuación de v

$$a_P v_P = a_E v_E + a_W v_W + a_N v_N + a_S v_S + S_P$$



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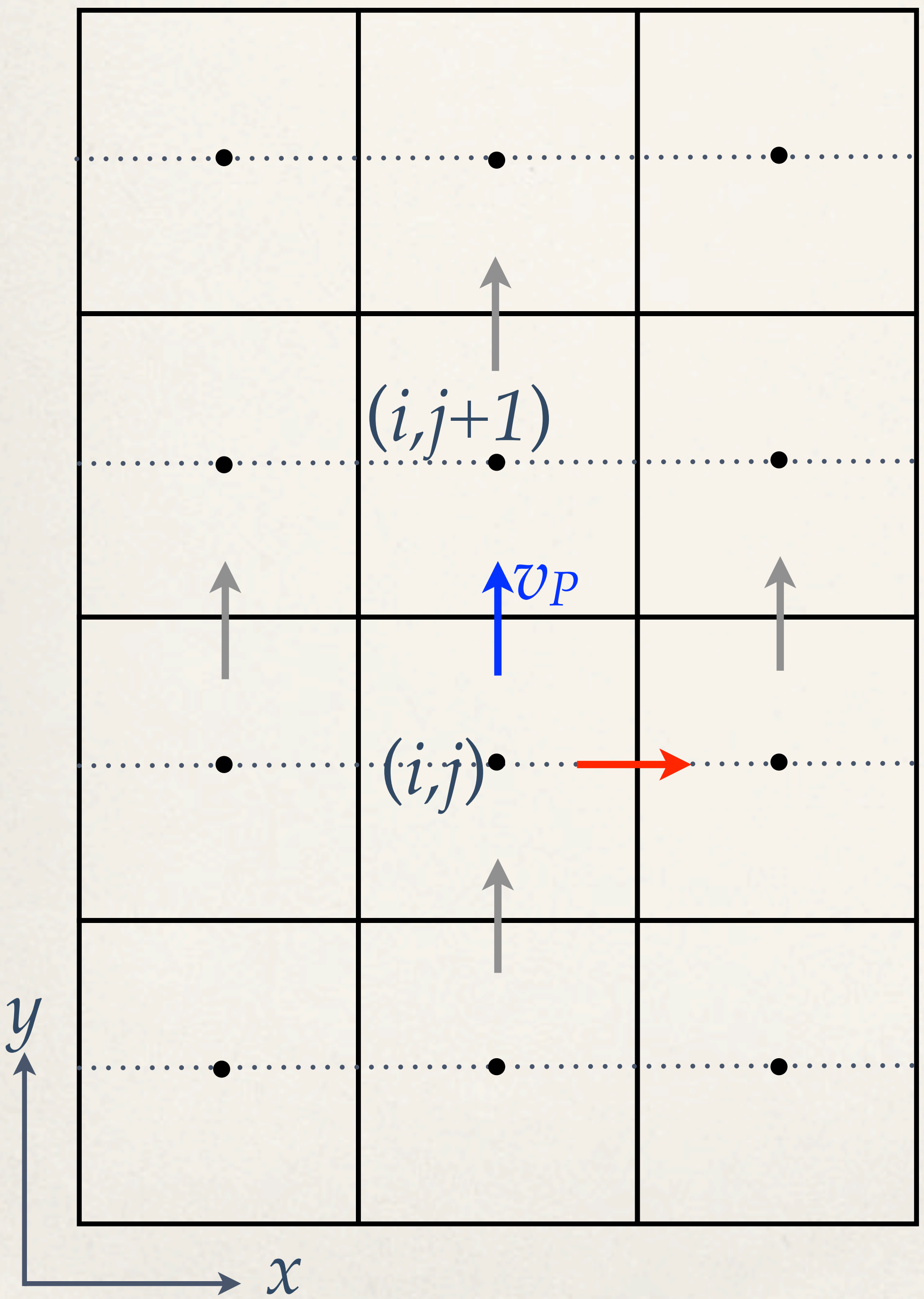
Discretización, ecuación de v

$$a_P v_P = a_E v_E + a_W v_W + a_N v_N + a_S v_S + S_P$$



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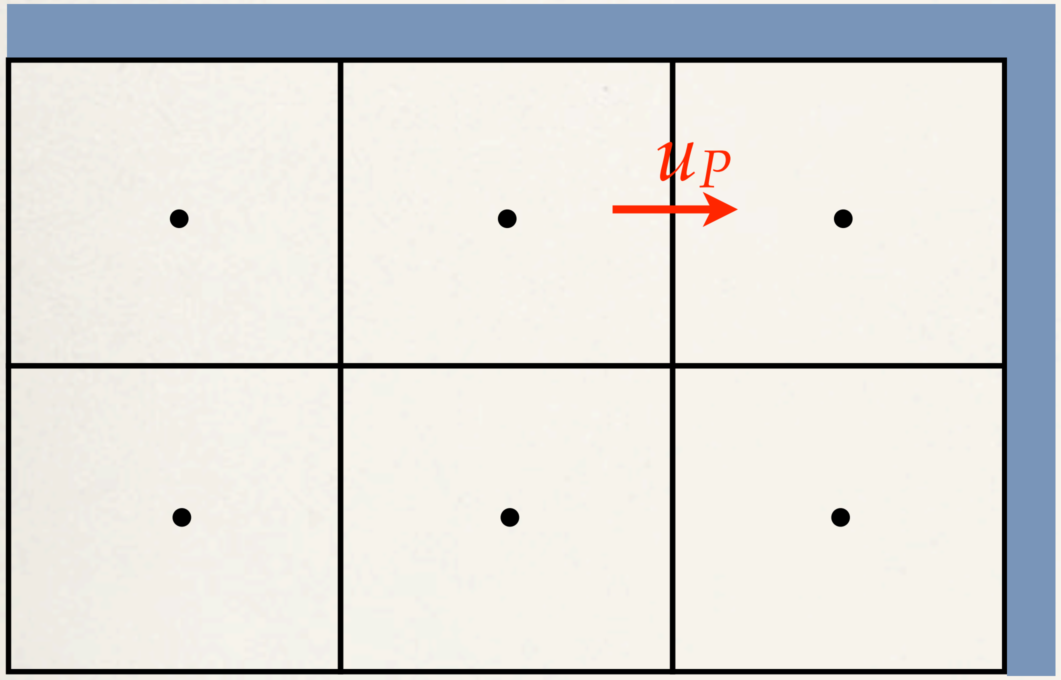
Corrección en fronteras (u)

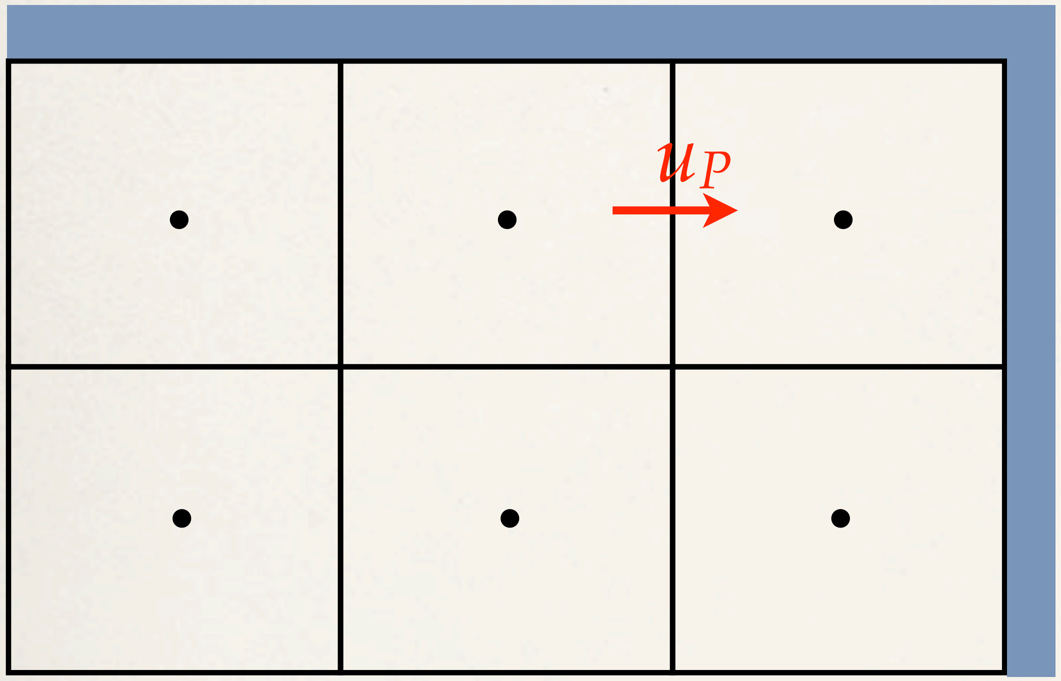


Este:

$$a_P u_P = a_W u_W + a_N u_N + a_S u_S + S'_P$$

$$S'_P = u_P^0 \frac{\Delta V}{\Delta t} - \frac{(p_{i+1,j} - p_{i,j})}{\Delta x_u} \Delta V + a_E U_0$$

Corrección en fronteras (u)



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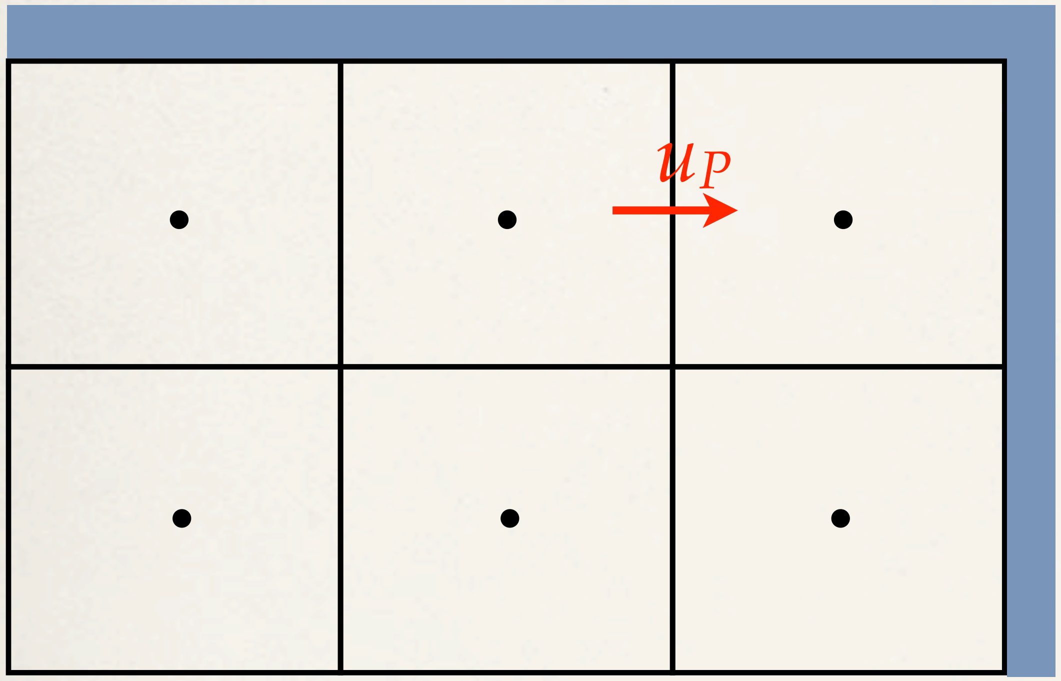

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Norte:

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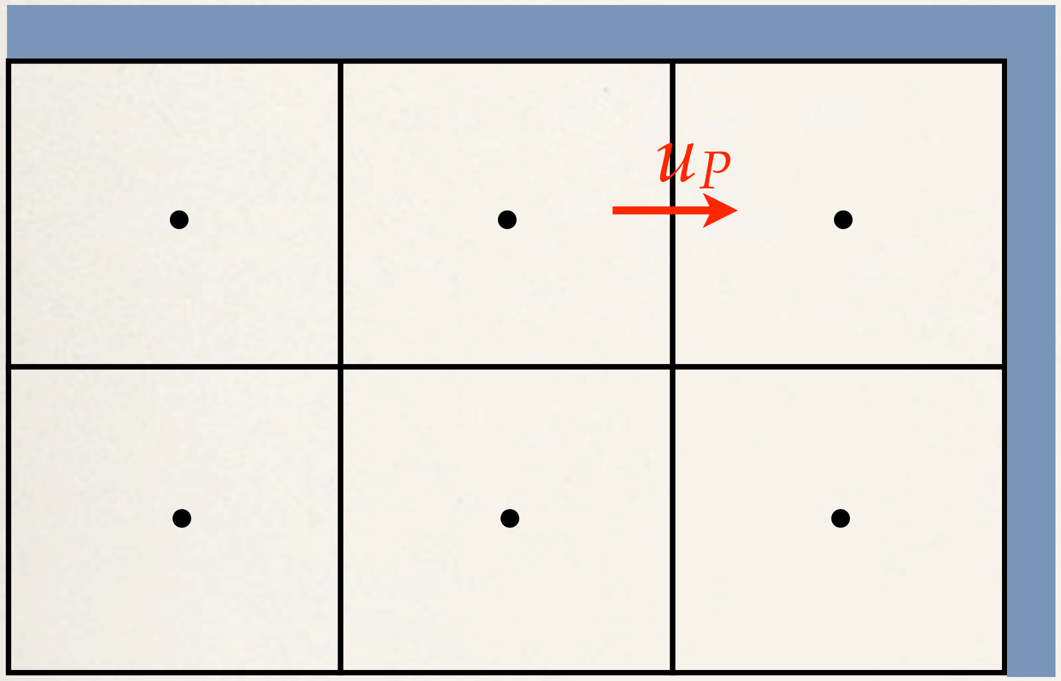

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$$a_E = \Gamma_e \frac{s_e}{\Delta x_{PE}} + s_e * max[0, -u_e]$$

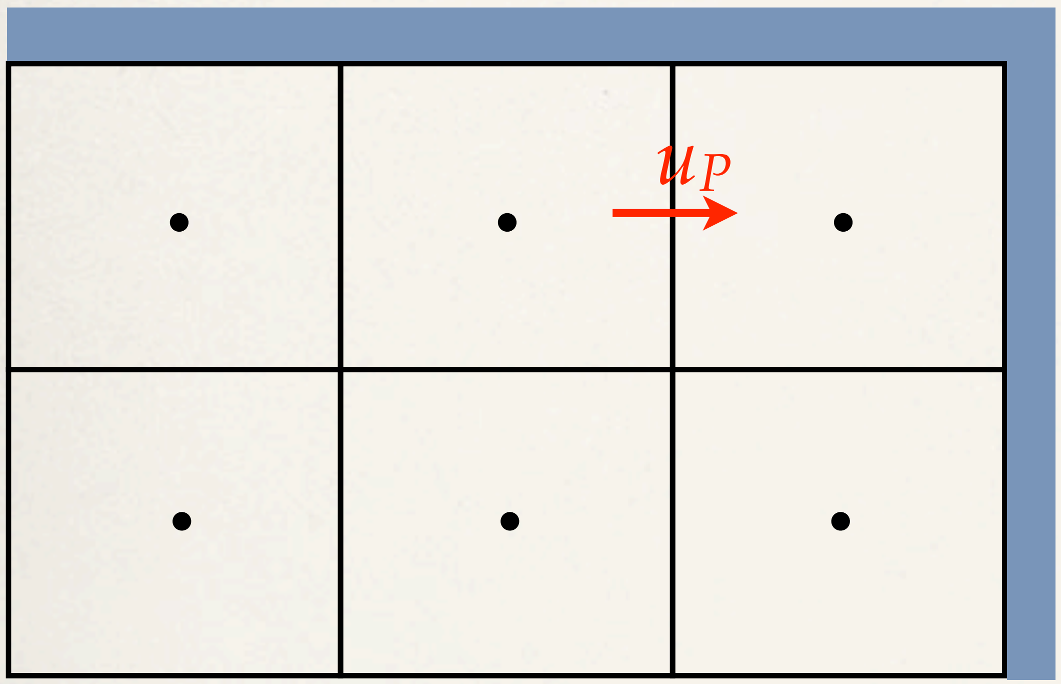
Norte:

$$a_P u_P = a_E u_E + a_W u_W + a_S u_S + S'_P$$

$$a_N = \Gamma_n \frac{s_n}{\Delta y_{PN}/2} + s_n * max[0, -v_n]$$

$$S'_P = u_P^0 \frac{\Delta V}{\Delta t} - \frac{(p_{i+1,j} - p_{i,j})}{\Delta x_u} \Delta V + a_N U_0$$

Corrección en fronteras (u)



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Norte:

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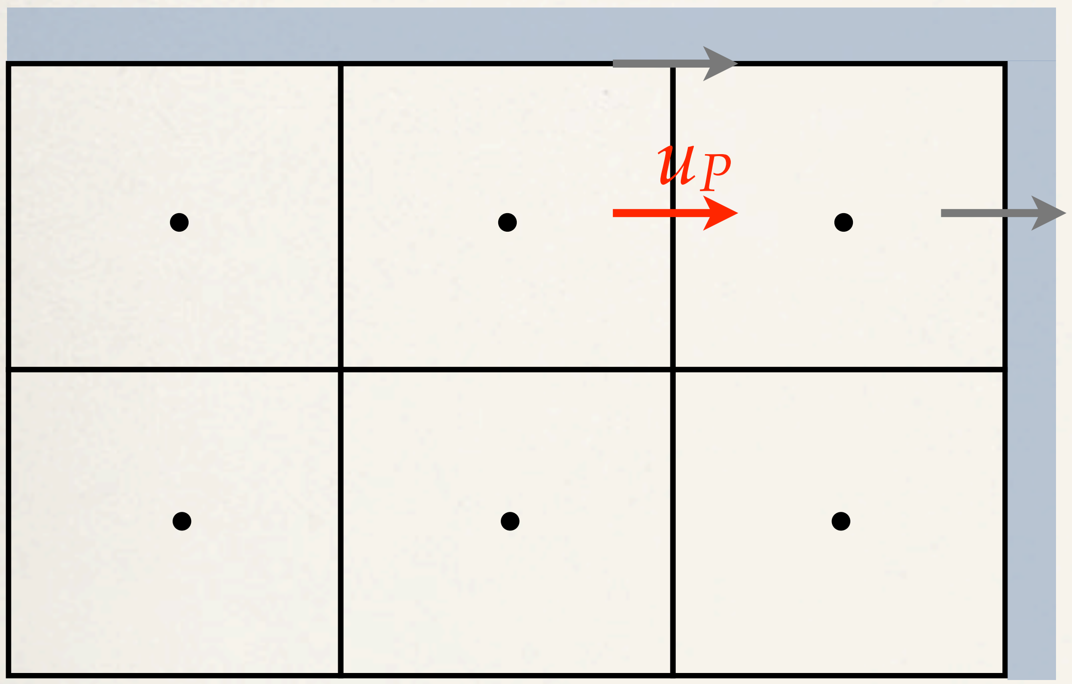
$$a_N = \Gamma_n \frac{2 s_n}{\Delta y_{PN}}$$

Corrección en fronteras (u)



$$a_N = \Gamma_n \frac{s_n}{\Delta y_{PN}/2} + s_n * max[0, -v_n]$$

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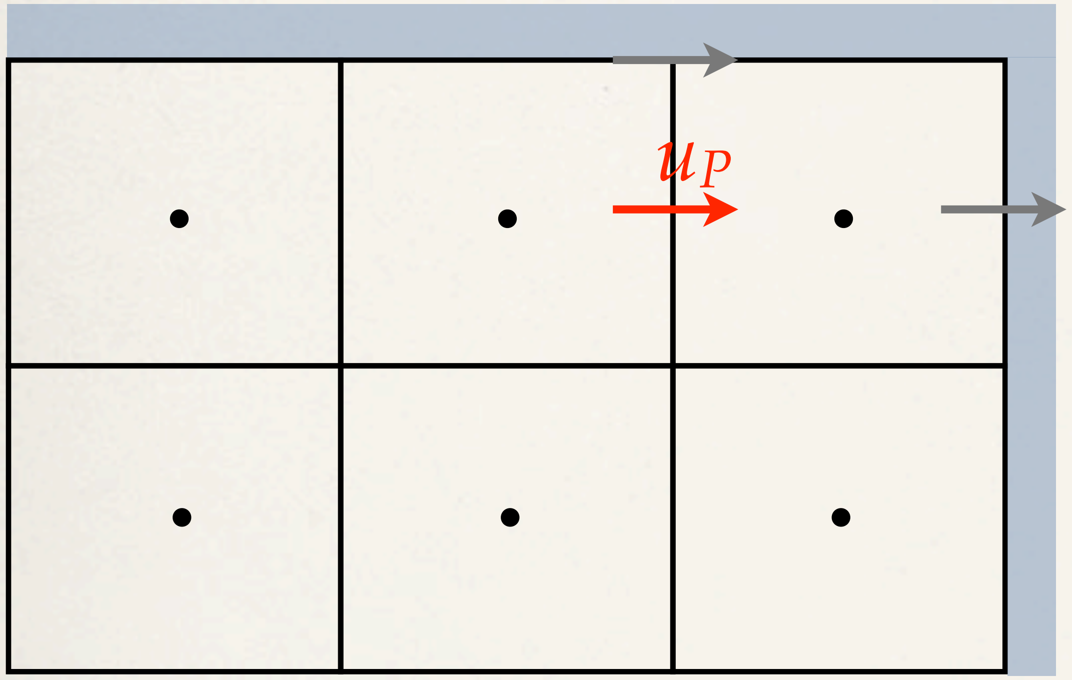

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$$a_N = \Gamma_n \frac{2s_n}{\Delta y_{PN}}$$

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Norte:

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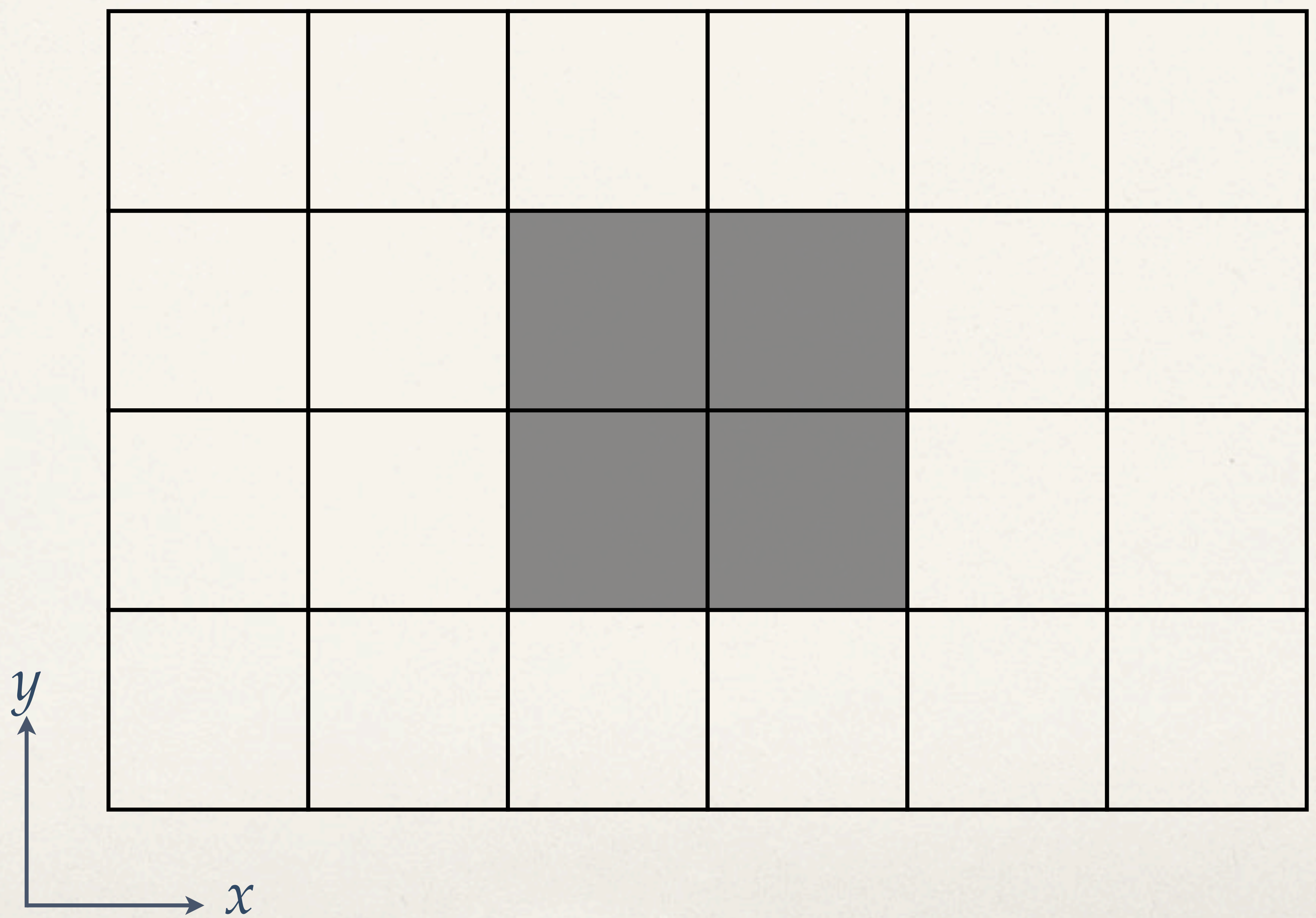
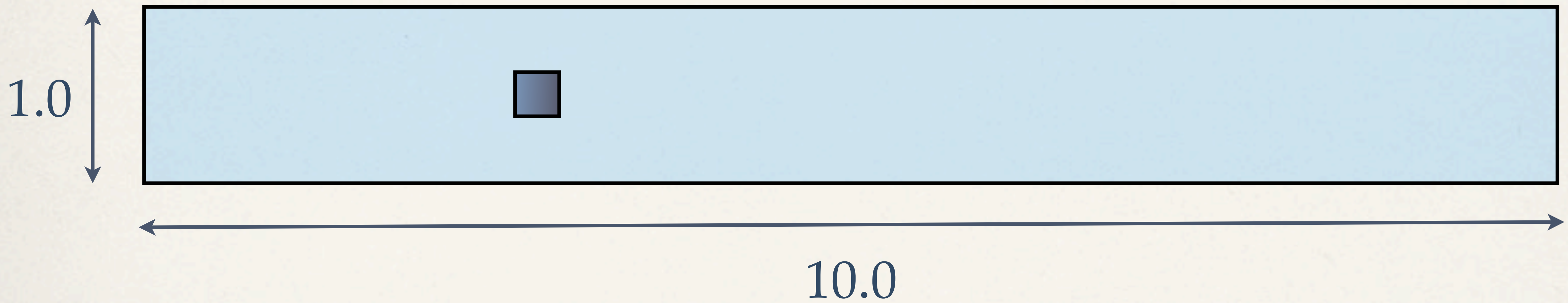
Obstáculo como frontera interna



Definición del dominio



1) Delimitamos el área de estudio

```
x0=0.0
xl=10.0
y0=-0.5
yl=0.5

nx=400
ny=40
```

Declaración de variables y arreglos

2) Variables reales para delimitar el obstáculo

```
real x0_obs, y0_obs, xl_obs, yl_obs, v_obs, u_obs
```

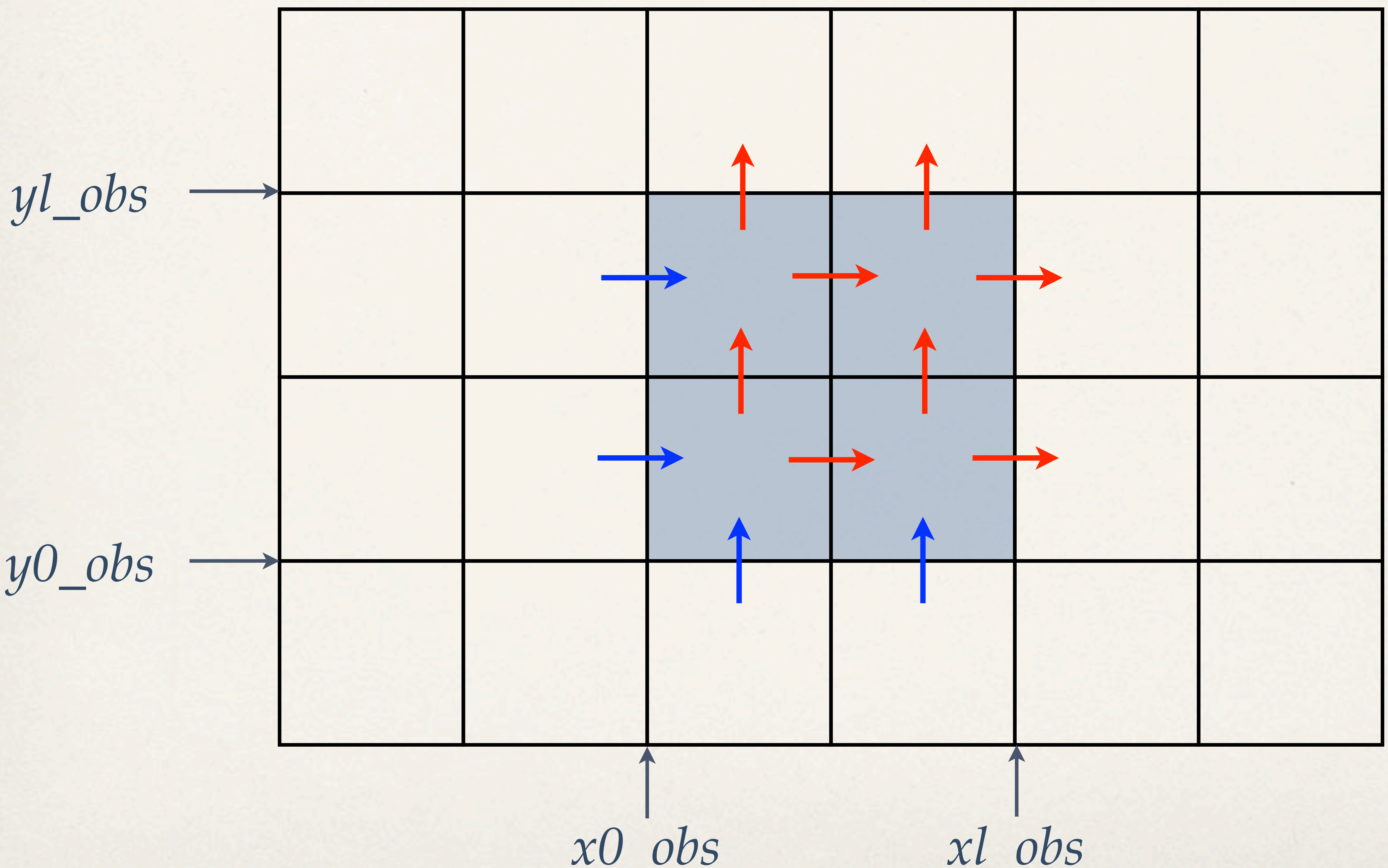
3) Arreglo de enteros para marcar el obstáculo

```
integer, allocatable, dimension(:,:) :: mark_cells
allocate (mark_cells(0:nx+1,0:ny+1))
```

4) Definimos condiciones iniciales y constantes del sistema

```
u=0.2; v=0.0; p=0.0; ap=0.0; ae=0.0; sp=0.0; du=0; dv=0;
u(0,:)=0.2; u(:,NY+1)=0.0; u(:,0)=0.0
RE=1.0    gamma=1.0/RE    itemax=10    u1=0.2; v1=0.0;
```

5) Marcamos el obstáculo con entero 1 y las velocidades en rojo y azul



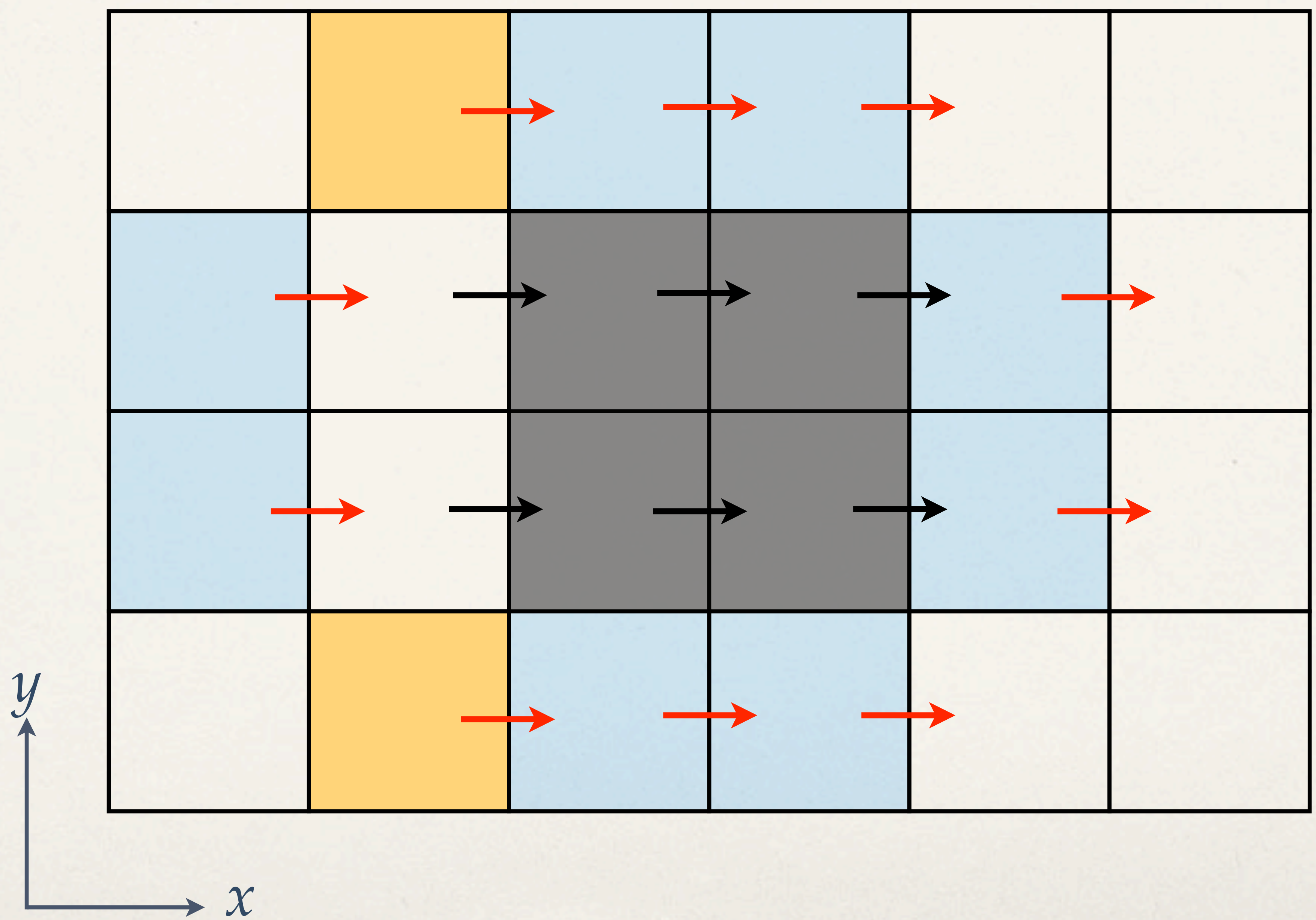
5) Marcamos el obstáculo con entero 1 y las velocidades en rojo y azul

```
x0_obs=4.0; xl_obs=4.25; y0_obs=-0.125; yl_obs=0.125
mark_cells=0
do i=1, nx
    do j=1, ny
        if((xc(i) .gt. x0_obs .and. xc(i) .lt. xl_obs) .and. (yc(j) .gt. y0_obs .and. yc(j) .lt. yl_obs))then
            mark_cells(i,j)=1; u(i,j)=0.0; v(i,j)=0.0;
        end if
    end do
end do

do i=1, nx
    do j=1, ny
        if(mark_cells(i,j) .eq. 0 .and. mark_cells(i+1,j) .eq. 1)then
        u(i,j)=0.0;
        end if
        if(mark_cells(i,j) .eq. 0 .and. mark_cells(i,j+1) .eq. 1)then
        v(i,j)=0.0;
        end if
    end do
end do
```

6) Corregimos coeficientes de ecuaciones u y v

Se corrigen velocidades en rojo por efecto de frontera



6) Corregimos coeficientes de la ecuación u

E:

```
if(mark_cells(i,j) .eq. 0 .and. mark_cells(i+2,j) .eq. 1) then
sp(i,j)=sp(i,j)+ae(i,j)*u1(i+1,j)
ae(i,j)=0.0
end if
```

W:

```
if(mark_cells(i,j) .eq. 0 .and. mark_cells(i-1,j) .eq. 1) then
sp(i,j)=sp(i,j)+aw(i,j)*u1(i-1,j)
aw(i,j)=0.0
end if
```

N:

```
if(mark_cells(i,j) .eq. 0 .and. (mark_cells(i+1,j+1) .eq. 1 .or. mark_cells(i,j+1) .eq. 1)) then
ap(i,j)=ap(i,j)+an(i,j)
sp(i,j)=sp(i,j)+2.0*an(i,j)*u1(i,j+1)
an(i,j)=0.0
end if
```

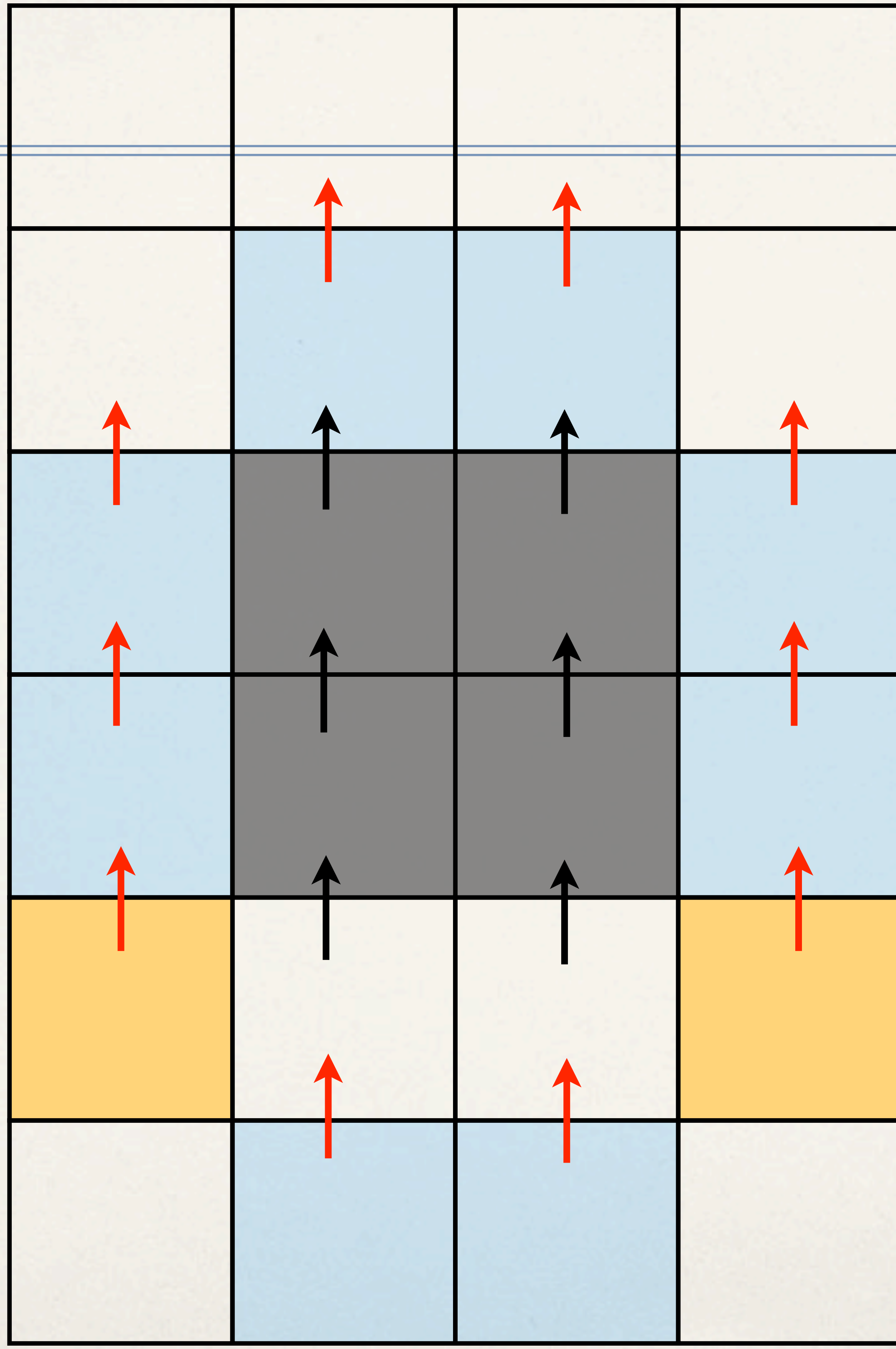
S:

```
if(mark_cells(i,j) .eq. 0 .and. (mark_cells(i+1,j-1) .eq. 1 .or. mark_cells(i,j-1) .eq. 1)) then
ap(i,j)=ap(i,j)+as(i,j)
sp(i,j)=sp(i,j)+2.0*as(i,j)*u1(i,j-1)
as(i,j)=0.0
end if
```

Obs:

```
if(mark_cells(i,j) .eq. 1 .or. (mark_cells(i,j) .eq. 0 .and. mark_cells(i+1,j) .eq. 1)) then
aw(i,j)=0.0; ae(i,j)=0.0; as(i,j)=0.0; an(i,j)=0.0; u1(i,j)=0.0
ap(i,j)=delv/dt     ! también se puede hacer    ap(i,j)=1.0 con sp(i,j)=u1(i,j)
sp(i,j)=u1(i,j)*delv/dt
end if
```

Las velocidades en rojo se corrigen por efecto de frontera



6) Corregimos coeficientes de la ecuación v

E:

```
if((mark_cells(i,j) .eq. 0) .and. (mark_cells(i+1,j+1) .eq. 1 .or. mark_cells(i+1,j) .eq. 1)) then
ap(i,j)=ap(i,j)+ae(i,j)
sp(i,j)=sp(i,j)+2.0*ae(i,j)*v1(i+1,j)
ae(i,j)=0.0
end if
```

W:

```
if((mark_cells(i,j) .eq. 0) .and. (mark_cells(i-1,j+1) .eq. 1 .or. mark_cells(i-1,j) .eq. 1)) then
ap(i,j)=ap(i,j)+aw(i,j)
sp(i,j)=sp(i,j)+2.0*aw(i,j)*v1(i-1,j)
aw(i,j)=0.0
end if
```

N:

```
if(mark_cells(i,j) .eq. 0 .and. mark_cells(i,j+2) .eq. 1) then
sp(i,j)=sp(i,j)+an(i,j)*v1(i,j+1)
an(i,j)=0.0
end if
```

S:

```
if(mark_cells(i,j) .eq. 0 .and. mark_cells(i,j-1) .eq. 1) then
sp(i,j)=sp(i,j)+as(i,j)*v1(i,j-1)
as(i,j)=0.0
end if
```

Obs:

```
if(mark_cells(i,j) .eq. 1 .or. (mark_cells(i,j) .eq. 0 .and. mark_cells(i,j+1) .eq. 1)) then
aw(i,j)=0.0; ae(i,j)=0.0; as(i,j)=0.0; an(i,j)=0.0; v1(i,j)=0.0
ap(i,j)=delv/dt
sp(i,j)=v1(i,j)*delv/dt
end if
```

7) Corrección de velocidades sólo fuera del obstáculo

```
do i=1,nx-1
do j=1,ny
   if(mark_cells(i,j) .eq. 0 .and. mark_cells(i+1,j) .ne. 1)then
      u1(i,j)=u1(i,j)+du(i,j)*(pp(i,j)-pp(i+1,j))
   end if
enddo
enddo

do i=1,nx
do j=1,ny-1
   if(mark_cells(i,j) .eq. 0 .and. mark_cells(i,j+1) .ne. 1)then
      v1(i,j)=v1(i,j)+dv(i,j)*(pp(i,j)-pp(i,j+1))
   end if
enddo
enddo
```

Una forma de comprobar resultados es verificar si las velocidades en el obstáculo son estrictamente cero.