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# Analysis of the Term Structure of Implied Volatilities

Ronald Heynen, Angelien Kemna, and Ton Vorst*

## Abstract

From various empirical work, it is well known that the volatility of asset returns changes over time. This might be one of the reasons that implied volatilities differ for options that only differ in time to maturity. We construct models for the relation between short- and long-term implied volatilities based on three different assumptions of stock return volatility behavior, i.e., mean-reverting, GARCH, and EGARCH models. We test these relations on option price data and conclude that EGARCH gives the best description of asset prices and the term structure of options' implied volatilities.

## I. Introduction

One of the basic assumptions in the derivation of the Black-Scholes formula for the valuation of stock options is that the prices of the underlying stock are lognormally distributed with constant volatility. But from several empirical studies, it is well known that the volatility of asset returns changes over time.[1] To take account of this dichotomy, alternative option valuation models have been developed.[2]

Stein (1989) used a stochastic stock return volatility model discussed by Hull and White (1987) to test the joint hypothesis of correct model specification and ex ante efficiency in the options market. He derived relations between the implied volatilities of at-the-money options with different times to maturity and tested these term structure relations of implied volatilities on stock-index options. Given that the pricing model is correct, he reports "evidence of consistent overreactive behavior in the term structure of options' implied volatilities." One way to explain

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[1]See Fama (1965), Black (1976), Merton (1980), Poterba and Summers (1986), French, Schwert, and Stambaugh (1987), and Harvey and Whaley (1991a).

[2]Cox and Ross (1976), Geske (1979), Hull and White (1987), Johnson and Shanno (1987), Scott (1987), and Wiggins (1987).



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the origin of inefficiencies is through the overreactions of traders to the arrival of new information. Alternatively, it might be possible that the stock return volatility characteristics are not correctly described by the proposed stock return volatility model. More sophisticated models could give a better description of stock return volatility.

In this paper, we test restrictions on implied volatilities for Bollerslev's (1986) Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) model and its successor, Nelson's (1991) Exponential GARCH (EGARCH) model, that have proved to be very successful in the description of stock price dynamics. Using the results of Duan (1994), we derive term structures of implied volatilities for these processes in a similar way as Stein (1989) did for a mean-reverting stochastic volatility model. We not only test the term structure on stock index options, but also on options on an individual stock. Furthermore, we use a longer lag between short-term and long-term options.[3] As in Stein (1989), we present evidence that the hypothesis that implied volatilities are formed rationally is rejected if one maintains the assumption that a GARCH or a mean-reverting stochastic volatility model gives the right description of the stock price dynamics. However, in case of the EGARCH model, the joint hypothesis of a correct model specification and ex ante efficiency is not rejected. For this model, expectations about average expected volatility might have been formed rationally.

This paper is organized as follows. In Section II, we derive relations for average expected volatilities between different times to maturity in the case of a mean-reverting stochastic volatility specification, a GARCH(1,1) stock return volatility specification, and an exponential GARCH(1,1) stock return volatility specification. In Section III, we illustrate that in all three models the implied volatility from inverting the Black-Scholes formula is a very accurate approximation of the average expected volatility. Hence, the relations from Section II for average expected volatilities also hold for implied volatilities. In Section IV, we first test whether the different model assumptions (mean-reverting, GARCH(1,1) or EGARCH(1,1)) give a good description of the stock return volatility. For this purpose, we used daily stock returns from the Amsterdam Stock Exchange (ASE) on Philips and the European Options Exchange stock index (EOE-index). Secondly, we test the restrictions for implied volatilities between different times to maturity using transactions data from the European Options Exchange on Philips and EOE-index options. Section V concludes this paper.

## II. Restrictions on the Average Expected Volatilities

This section explains how different stock return volatility models lead to different restrictions on the average expected volatility. Following the empirical results on stock return volatility behavior in recent literature, mean-reverting, GARCH(1,1), and EGARCH(1,1) models for stock return volatility are considered.

[3]Where Stein (1989) used options differing one month in time to maturity, we used options differing six months in time to maturity.

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## A. Mean-Reverting Stock Return Volatility

A mean-reverting stock return volatility process can be modeled as follows,

$$\frac{dS_t}{S_t} = \mu dt + \sigma_t dW_1, \tag{1}$$

$$d\sigma_t^2 = \alpha_0 \left(\bar{\sigma}^2 - \sigma_t^2\right) dt + \alpha_1 \sigma_t dW_2, \tag{2}$$

where $S_t$ is the stock price at $t$, $\mu$ is the mean stock price return, $dW_1$, $dW_2$ are Wiener processes; in $dW_1 \cdot dW_2 = \omega dt$, $\omega$ is the instantaneous correlation between stochastic increments $dW_1$ and $dW_2$, $\sigma_t$ is the stock return volatility, $\alpha_0$ is the coefficient of mean reversion, $\alpha_1$ is the instantaneous standard deviation of $d\sigma^2/\sigma$, and $\bar{\sigma}^2$ is the mean-reversion level.

For a stochastic volatility option model, it can be shown (Feinstein (1989)) that the value of an at-the-money option is approximately equal to the Black-Scholes value with the volatility equal to the average expected volatility of the underlying stock over the remaining lifetime of the option. This can be seen as an extension of a result of Merton (1973), which states that Black-Scholes holds with the volatility replaced by its average volatility if the volatility is a deterministic function of time. Thus, we define the average expected volatility $\sigma_{\text{Av}}(t,T)$ as

$$\sigma_{\text{Av}}^2(t,T) = \frac{1}{T}\int_t^{t+T} E_t\left[\sigma_s^2\right] ds, \tag{3}$$

where $E_t$ is the conditional expectation operator at the current time $t$, and $T$ is the time to expiration.

From the specification of the volatility process, it can be deduced that (see Stein (1989)),

$$\sigma_{\text{Av}}^2(t,T) = \bar{\sigma}^2 + \frac{1}{\alpha_0 T}\left[\sigma_t^2 - \bar{\sigma}^2\right]\left[1 - e^{-\alpha_0 T}\right]. \tag{4}$$

Equation (4) states that when the instantaneous variance $\sigma_t^2$ is above its mean level $\bar{\sigma}^2$, the average expected volatility should be decreasing in time to maturity. When the instantaneous variance is below its mean, average expected volatility should be increasing in time to maturity. The instantaneous variance cannot be observed. However, from (4), a relation can be derived between two average expected volatilities, differing in time to maturity (say $T_1$ and $T_2$), thereby eliminating the instantaneous variance $\sigma_t^2$,

$$\left[\sigma_{\text{Av}}^2(t,T_1) - \bar{\sigma}^2\right] = \frac{T_2}{T_1}\frac{\rho^{T_1} - 1}{\rho^{T_2} - 1}\left[\sigma_{\text{Av}}^2(t,T_2) - \bar{\sigma}^2\right], \tag{5}$$

where $T_1 > T_2$ and $\rho = e^{-\alpha_0}$.

Equation (5) is referred to as the term structure of average expected volatility (Stein 1989) in the case of a mean-reverting stock return volatility model. For $\rho < 1$, it can be shown that $(T_2/T_1)((\rho^{T_1} - 1)/(\rho^{T_2} - 1)) < 1$. So, given a movement in the short-term average expected volatility $\sigma_{\text{Av}}^2(t,T_2)$, there should be a smaller movement in distant average expected volatility $\sigma_{\text{Av}}^2(t,T_1)$. The constant

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of proportionality depends on the mean-reversion parameter $\rho$, as well as on the remaining time to maturities $T_1$ and $T_2$. The testability of the nonlinear relation (5) will be discussed in Section IV. In contrast to the previous process, the next two will be discrete time processes.

## B. GARCH(1,1) Stock Return Volatility

In recent studies, empirical evidence was reported that the assumption of constant conditional means and variances for returns on stock is unrealistic (Pindyck (1984) and Poterba and Summers (1986)). Furthermore, it was discovered that daily series of stock returns exhibit much higher degrees of statistical dependence than has been reported earlier. This dependence structure can be used to obtain forecasts of conditional variances. Akgiray (1989) and Pagan and Schwert (1990) showed that low-order general autoregressive conditional heteroskedastic (GARCH) models described the stock return volatility characteristics of their data very well. Akgiray (1989) compared various out-of-sample forecasts of monthly return variances and found that forecasts based on the GARCH model are superior. Further research[4] on the relation between a stock portfolio's expected return and risk, often modeled by the conditional variance, also confirmed the usefulness of GARCH models in describing the stock return data. In the case of a GARCH(1,1) specification, stock return and stock return volatility are modeled as follows,

$$\text{(6)} \qquad \mathrm{Ln}(S_t/S_{t-1}) = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \sigma_t\xi_t,$$

$$\text{(7)} \qquad \sigma_t^2 = \beta_0 + \beta_1\xi_{t-1}^2\sigma_{t-1}^2 + \beta_2\sigma_{t-1}^2,$$

where $r$ is the risk-free rate of interest, $\lambda$ is the unit risk premium, $\beta_0$, $\beta_1$, and $\beta_2$ are time independent parameters, $\beta_1 + \beta_2 < 1$ and $\xi_t \overset{\text{i.i.d.}}{\sim} N(0,1)$.

From Equation (7), one sees that the conditional stock return volatility is a function of the volatility one period earlier and the shock during that period. Thus, this model allows for clustering of periods with high volatility and periods with low volatility. In contrast to the mean-reversion model, the GARCH(1,1) model has one source of uncertainty given by $\xi_t$, while in the mean-reversion model, we have two stochastic processes.

As in the case of the mean-reverting stock return volatility model, one can derive a relation between average expected volatilities differing in times to maturity ($T_1$ and $T_2$) (see Appendix A),

$$\text{(8)} \qquad \left[\sigma_{\mathrm{Av}}^2(t,T_1) - \bar{\sigma}^2\right] = \frac{T_2}{T_1}\frac{\gamma^{T_1}-1}{\gamma^{T_2}-1}\left[\sigma_{\mathrm{Av}}^2(t,T_2) - \bar{\sigma}^2\right],$$

with $\bar{\sigma}^2 = \beta_0/(1-\beta_1-\beta_2)$ and $\gamma = \beta_1 + \beta_2$.

The constant of proportionality depends only on $T_1$ and $T_2$. Just as for the mean-reverting model, a movement in the short-term average expected volatility should coincide with a smaller movement in the long-term average expected volatility.

[4]Poterba and Summers (1986), French, Schwert, and Stambaugh (1987), Baillie and DeGennaro (1990), and Chou (1988).

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## C. EGARCH(1,1) Stock Return Volatility

Black (1976) and Christie (1982) reported that stock returns are negatively correlated with changes in return volatility. An economic explanation is given by financial and operational leverage, but Black (1976) and Christie (1982) note that leverage is probably not the sole explanation for the negative relation between stock returns and volatility. This observation led to the development of the exponential GARCH model or EGARCH model (Nelson (1991)). For an EGARCH(1,1) specification, stock return and stock return volatility are modeled as follows,

$$\text{(9)} \qquad \operatorname{Ln}\left(S_t/S_{t-1}\right) = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \sigma_t\xi_t,$$

$$\text{(10)} \qquad \operatorname{Ln}\sigma_t^2 = \beta_0 + \beta_1 \operatorname{Ln}\sigma_{t-1}^2 + \beta_2\xi_{t-1} + \beta_3\left(|\xi_{t-1}| - \sqrt{2/\pi}\right),$$

where $\beta_0, \ldots, \beta_3$ are time independent parameters, and $\xi_t$ is Gaussian white noise, $\xi_t \overset{\text{i.i.d.}}{\sim} N(0,1)$.

The asymmetric term at the right-hand side of Equation (10) accommodates the asymmetric relation between stock returns and volatility changes. If $\beta_2 = 0$ and $\beta_3 > 0$, then the innovation in $\operatorname{Ln}(\sigma_t^2)$ is positive (negative) when the magnitude of $\xi_t$ is larger (smaller) than its expected value. In case of $\beta_2 < 0$ and $\beta_3 = 0$, the innovation in the conditional variance is positive (negative) when returns innovations are negative (positive).

Pagan and Schwert (1990) generated and compared forecasts using parametric (GARCH, EGARCH) and nonparametric models. They found that Nelson's EGARCH model came closest to the explanatory power of the nonparametric models.

Defining average expected volatility for this model as,[5]

$$\text{(11)} \qquad \operatorname{Ln}\sigma_{\text{Av}}^2(t,T) = \frac{1}{T}\sum_{k=1}^{T} \operatorname{Ln} E_t\left[\sigma_{t+k}^2\right],$$

one can derive the following relation between average expected volatilities differing in times to maturity ($T_1$ and $T_2$) (see Appendix B),

$$\text{(12)} \qquad \left[\operatorname{Ln}\sigma_{\text{Av}}^2(t,T_1) - \operatorname{Ln}\bar{\sigma}^2\right] = \frac{T_2}{T_1}\frac{\left(1-\beta_1^{T_1}\right)}{\left(1-\beta_1^{T_2}\right)}\left[\operatorname{Ln}\sigma_{\text{Av}}^2(t,T_2) - \operatorname{Ln}\bar{\sigma}^2\right] + R,$$

where $\bar{\sigma}^2$ and $R$ are functions of the parameters of the model (see Appendix B).

Equation (12) differs from (5) and (8) because of the logarithmic characteristic and the appearance of the term $R$. The significance of this term is discussed in the next section.

It is also shown in the Appendix that for both the GARCH and EGARCH models there is a relation as (4) between the average volatility $\sigma_{\text{Av}}$, that is expected

[5]The use of a geometric instead of an arithmetic average has no important implications as is explained in the next section. An alternative definition is given by: $\operatorname{Ln}\sigma_{\text{Av}}^2(t,T) = \frac{1}{T}\sum_{k=1}^{T} E_t[\operatorname{Ln}(\sigma_{t+k}^2)]$. However, this definition does not fulfill a necessary condition as will also be explained in the next section.

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over a period with length $T$, contingent on the short-term volatility $\sigma_{t+1}$ and the parameters of the underlying stock return volatility process. For large values of $T$, the average expected volatility $\sigma_{\mathrm{Av}}$ tends toward the long-term mean volatility $\bar{\sigma}$. For small values of $T$, the average expected volatility $\sigma_{\mathrm{Av}}$ tends toward the short-term volatility $\sigma_{t+1}$.

The mean-reverting property for the volatility is common to stationary processes in general. For a stationary process, the volatility will always tend towards the long-term unconditional mean volatility. The previously described stock return volatility processes are examples of stationary processes; their unconditional mean variance and covariances are time independent. The constant of proportionality in Equations (5), (8), and (12) reflects the mean-reversion tendency. Furthermore, because parameters $\rho$, $\gamma$, and $\beta_1$ are smaller than 1, the value of this constant depends primarily on the times to maturity $T_1$ and $T_2$. So, the main difference in these term structure relations is determined by the specification of the level of the unconditional volatility.

## III. The Relation between Implied Volatility and Average Expected Volatility

This section discusses how the average expected volatility over the remaining time to maturity $T$, $\sigma_{\mathrm{Av}}(t,T)$, is related to implied volatility, $\sigma_{\mathrm{imp}}$, that is actually measured from inverting the Black-Scholes option pricing formula (Black and Scholes (1973)), i.e., $\sigma_{\mathrm{imp}} = C^{-1}(f)$, with $f$ as the theoretical price of the option in the case of the different stochastic processes that are considered in this paper.

If the Black-Scholes assumption of constant volatility (during the remaining time to maturity) is relaxed to the case that volatility is a completely deterministic function of time, then the Black-Scholes option pricing formula is still correct with $\sigma^2$ replaced by its average value $((1/T)\int_t^{t+T}\sigma_s^2 ds)$ during the remaining time to maturity (Merton (1973)). In order to see whether a similar result holds in the case of one of the underlying stock price processes studied in the previous section, one first has to understand what the theoretical option prices in these cases are.

Duan (1994) develops an option pricing model that reflects the changing conditional volatility of the underlying asset. For a GARCH(1,1) stock return volatility process, he shows that under the assumption that there is a representative agent who is an expected utility maximizer with certain restrictions on his utility function,[6] there exists a pricing measure $Q$ under which the option price is just the discounted expectation of the final pay-off. This pricing measure is specified by Equations (6) and (7), where one has to cancel the term $\lambda\sigma_t$ in Equation (6) and has to substitute $\xi_{t-1} - \lambda$ for $\xi_{t-1}$ in Equation (7). The theoretical option price $f$ is given by the discounted expectation of the final pay-off under this measure $Q$, i.e., $f = e^{-rT}E_t^Q[\max[S_T - K, 0]]$.

Although it is impossible to give an analytic expression for this discounted expectation, it enables Duan to approximate the theoretical option value through

[6]Duan (1994) shows that it is sufficient to assume that the utility function is of constant relative risk aversion and that changes in logarithmic aggregate consumption are normally distributed with constant mean and variance.

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Monte-Carlo simulation. From Duan's theoretical option price, the implied volatility can be calculated. To identify the parameters $\beta_0$, $\beta_1$, and $\beta_2$ in Duan's model, the values that are estimated in the next section for both the EOE-index and Philips (see Table 4) are used. The implied volatilities and the corresponding average expected volatilities are listed in Table 1. From Table 1, it can be inferred that especially for at-the-money options there is a close resemblance between the implied volatilities and average expected volatilities.

This result ensures that the relation for average expected volatilities given by Equation (8) can also be interpreted as a restriction for short- and longer-term implied volatilities of at-the-money GARCH(1,1) option prices. In the remainder of this section, a similar result will be established for the EGARCH(1,1) and mean-reverting stock return volatility cases.

If the conditional stock return volatility is modeled by Nelson's (1991) EGARCH(1,1) process, one can employ a similar argument to derive a pricing measure under which the theoretical option price is a discounted expectation of the final pay-off. In this case, the pricing measure $Q$ is specified by Equations (9) and (10) where once again one has to cancel the term $\lambda\sigma_t$ in Equation (9) and has to substitute $\xi_{t-1} - \lambda$ for $\xi_{t-1}$ in Equation (10). As in the previous case, the theoretical option price $f$ is given by the discounted expectation of the final pay-off.

Also in this case, we compare implied volatilities of the theoretical option prices (calculated via Monte-Carlo simulation using the parameter values in Table 5 of Section IV) with the corresponding (geometric) average expected volatilities (see Table 1) and come to the same conclusion.

For the EGARCH(1,1) model, the average expected volatility is defined as the geometric average of expected volatilities over the remaining time to maturity. The use of a geometric instead of an arithmetic average is justified by the close resemblance between the geometric average expected volatilities and the implied volatilities of the theoretical option prices. In fact, every definition of average expected volatility that establishes this resemblance is correct. For the alternative definition given in footnote 5 of the previous section, one could show that the average expected volatility only depends on the parameters $\beta_0$ and $\beta_1$ of the EGARCH(1,1) process. However, because implied volatilities of the theoretical EGARCH(1,1) option prices seem to depend strongly on the parameters $\beta_2$ and $\beta_3$, this particular definition is not correct.

Equation (12) of the previous section, reflecting the restriction between short- and long-term average expected volatilities in the EGARCH(1,1) case, contains at the right-hand side a term denoted by $R$. It can be shown that this term is completely negligible if this term is calculated across pairs differing six months in time to maturity, using the estimated parameters listed in the next section for Philips or the EOE stock-index.[7] Therefore, we can proceed with a simplified version of Equation (12), setting the term $R$ equal to zero.

Hull and White (1987) develop an option pricing model that incorporates a stochastically changing volatility of the underlying stock return. This stochastic volatility option pricing model is the progenitor to the forementioned models that

[7]In the case of the EOE-index, the term $R$ ranges from $1.1\ 10^{-6}$ to $2.2\ 10^{-10}$ representing cases in which the short-term time to maturity varies from, respectively, one to three months.

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TABLE 1

**Average Expected Volatility versus Theoretical Implied Volatility**

| | $S/Ke^{-rT}$ | $T$ = 3 Months: $\sigma_t = 0.1000$ | $T$ = 3 Months: $\sigma_t = 0.2000$ | $T$ = 9 Months: $\sigma_t = 0.1000$ | $T$ = 9 Months: $\sigma_t = 0.2000$ |
|---|---|---|---|---|---|
| *EOE-Index* | | | | | |
| GARCH(1,1) | | $\sigma_{Av} = 0.1466$ | $\sigma_{Av} = 0.1537$ | $\sigma_{Av} = 0.1485$ | $\sigma_{Av} = 0.1509$ |
| | 0.95 | 0.1465 | 0.1526 | 0.1481 | 0.1509 |
| | 1.00 | 0.1457 | 0.1523 | 0.1481 | 0.1508 |
| | 1.05 | 0.1463 | 0.1525 | 0.1481 | 0.1509 |
| EGARCH(1,1) | | $\sigma_{Av} = 0.1569$ | $\sigma_{Av} = 0.1637$ | $\sigma_{Av} = 0.1599$ | $\sigma_{Av} = 0.1622$ |
| | 0.95 | 0.1573 | 0.1636 | 0.1601 | 0.1621 |
| | 1.00 | 0.1569 | 0.1634 | 0.1599 | 0.1620 |
| | 1.05 | 0.1572 | 0.1636 | 0.1601 | 0.1621 |
| Mean-Reverting | | $\sigma_{Av} = 0.1463$ | $\sigma_{Av} = 0.1508$ | $\sigma_{Av} = 0.1475$ | $\sigma_{Av} = 0.1490$ |
| | 0.95 | 0.1460 | 0.1495 | 0.1476 | 0.1492 |
| | 1.00 | 0.1459 | 0.1494 | 0.1475 | 0.1491 |
| | 1.05 | 0.1460 | 0.1495 | 0.1476 | 0.1491 |
| *Philips* | | | | | |
| GARCH(1,1) | | $\sigma_{Av} = 0.1558$ | $\sigma_{Av} = 0.1714$ | $\sigma_{Av} = 0.1619$ | $\sigma_{Av} = 0.1674$ |
| | 0.95 | 0.1555 | 0.1708 | 0.1615 | 0.1672 |
| | 1.00 | 0.1554 | 0.1707 | 0.1614 | 0.1671 |
| | 1.05 | 0.1555 | 0.1708 | 0.1615 | 0.1672 |
| EGARCH(1,1) | | $\sigma_{Av} = 0.1639$ | $\sigma_{Av} = 0.1790$ | $\sigma_{Av} = 0.1717$ | $\sigma_{Av} = 0.1770$ |
| | 0.95 | 0.1639 | 0.1784 | 0.1718 | 0.1766 |
| | 1.00 | 0.1638 | 0.1783 | 0.1717 | 0.1765 |
| | 1.05 | 0.1639 | 0.1784 | 0.1718 | 0.1766 |
| Mean-Reverting | | $\sigma_{Av} = 0.1547$ | $\sigma_{Av} = 0.1704$ | $\sigma_{Av} = 0.1607$ | $\sigma_{Av} = 0.1659$ |
| | 0.95 | 0.1541 | 0.1698 | 0.1608 | 0.1661 |
| | 1.00 | 0.1540 | 0.1697 | 0.1607 | 0.1660 |
| | 1.05 | 0.1541 | 0.1698 | 0.1608 | 0.1661 |

The numbers in the table represent the implied volatilities of the simulated option prices corresponding to the different model specifications. The GARCH(1,1) and EGARCH(1,1) option prices are calculated from Duan's (1994) risk-neutral valuation measure. The mean-reverting option prices are calculated using the Hull and White (1987) stochastic volatility model. For this model assumption, it is assumed that the instantaneous correlation $\omega$ is zero. To calculate the different option model prices, the total number of simulation runs was 20,000. The fraction $S/Ke^{-rT}$ specifies the degree to which the option is in- or out-of-the-money. The implied volatilities are calculated for near-the-money option prices across three and nine months' times to maturity and different starting values for the short-term volatility $\sigma_t$. This choice corresponds to the range that is used in our empirical analysis. $\sigma_{Av}$ represents the average expected volatility over the remaining time to maturity $T$ of the option. The risk-free rate is set at 8.5 percent. The estimated parameter values of the different processes used to calculate the values in the table are listed in Tables 4, 5, and 6 of Section IV. The calculated volatilities are annualized (250 days).

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postulates an exogeneous process for stock return volatility. For the mean-reverting stock price process, Hull and White (1987) showed that under certain conditions,[8] a stochastic volatility option price can be calculated. After calculating the implied volatilities of the Hull and White stochastic volatility option price via Monte-Carlo simulation using the estimated parameter values listed in Table 6 of Section IV, these can be compared with the corresponding average expected volatilities (Table 1). Also in this case, it can be inferred from the table that the at-the-money implied volatilities are close to the corresponding average expected volatilities.

The previous results show that for all three model assumptions, there is a close resemblance (differences are less than 1 percent) between the at-the-money implied volatilities of the theoretical option prices and the average expected volatilities over the remaining time to maturity of the option. So, if market prices are correctly described by one of these three theoretical option pricing models, the implied volatilities of the market prices for at-the-money options are approximately equal to the average expected volatilities and, hence, the relations for average expected volatilities derived in the previous section should also hold for the implied volatilities of market prices. This will be our hypothesis in the empirical section on option prices.

## IV. Empirical Tests

In this section, the efficiency of the options market is tested by examining restrictions on implied volatilities. Stein (1989) reported inefficiencies in the options market, showing that longer-term implied volatilities were larger than forecasts of average volatility during the remaining time to maturity of the option based on a mean-reverting stock return volatility model. For our data set, it is tested whether this is true, comparing different model assumptions for stock return volatility behavior ((E)GARCH(1,1) and mean-reverting).

Using EOE-index data, the different model assumptions for the underlying asset return volatility process are identified and tested. The estimated parameters for these models are used to specify the term structure relations of implied volatilities derived in Section II. In Part C, these term structure relations are used to test the efficiency of the options market, comparing the results for the different model assumptions. In contrast to the cases where the term structure is described by mean-reverting or GARCH(1,1), the joint hypothesis of a correct model specification and ex ante efficiency is not rejected in the case of an EGARCH(1,1) stock return volatility model. First, a description of the data is given.

### A. The Data

The data used in this study were provided by the European Option Exchange (EOE) and the Amsterdam Stock Exchange (ASE).[9] For the estimation of the stock

[8] They assumed that the volatility is uncorrelated with aggregate consumption.

[9] The EOE started in 1978 with options on nine major Dutch stocks. The trading and clearing system is similar to the system of the CBOE (with open outcry). With an average daily turnover in 1990 of about 41,470 contracts, the EOE held its position of the largest options exchange in Europe. However, in 1991, this position was taken over by the Deutsche Termin Börse (DTB) with their screen trading system.

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return process, daily closing prices are used for Philips (stock with the largest volume in options contracts) and the EOE-index (based on the 25 largest stocks of the ASE). Options on Philips are American-style options, and on the EOE-index, we have European-style options. The period of observation was from January 21, 1988, to January 21, 1989 (250 observations). The daily returns are constructed by taking the logarithms of consecutive daily closing prices. The prices are adjusted for discrete dividends that were collected for the 25 stocks.[10]

For the estimation of the implied volatilities, near-the-money (within 5 percent) option transaction prices were taken from the expiration cycle January 23, 1989, to April 30, 1989. Short-term options with a time to maturity between zero and three months were taken to calculate the short-term implied volatility. For the long-term implied volatility, options with a time to maturity between six and nine months were taken. The long-term options always have six months longer to go than the short-term options. The two series of near-the-money option prices are collected during the period of the day that both the option and the underlying asset are traded.

In order to eliminate the nonsimultaneous price problem, as mentioned in Harvey and Whaley (1991b), option prices and stock quotes are collected in the period from one hour after the EOE opens until one hour before the EOE closes. Furthermore, it is required that the time interval between a selected short-term option price and an associated long-term option price does not exceed 15 minutes, which results, on average, in three pairs of prices during a day (and 150 pairs over the whole observation period). The time interval between these pairs varied between 1.5–2.5 hours.

The calculation of the implied volatilities is based on a binomial method where the stock-index tree is defined in terms of the index level net of the present value of the promised dividends (see Harvey and Whaley (1992) for the application of this method on an index with multiple dividends). Because proper account is taken of the discrete nature of dividends and because EOE-index options are European-style, there are no dividend and early exercise problems as described by Harvey and Whaley (1992).

Due to the fact that an index is composed of stocks that are not traded simultaneously when new information arrives, recorded index quotes may lag behind the index level used by option traders (Diz and Finucane (1991)). Since implied volatilities are calculated using recorded index levels, this lag can cause a bias. This bias for calls is reverse to the bias for puts. Therefore, implied volatilities are calculated by averaging the implied volatilities of the selected near-the-money call and the corresponding put.[11] For the estimation of the risk-free rates of interest, the end-of-the-week quotes of the Amsterdam Interbank Borrowing Rates (AIBOR) are used.

[10]For most Dutch stocks, dividend payments take place in April/May and September/October.

[11]This may also weaken the bid-ask price effect as mentioned in Harvey and Whaley (1991b).

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## B. Estimating the Underlying Stock Price Process

In this section, different model assumptions are tested for the underlying stock return volatility process. In Table 2, some descriptive statistics are reported for both Philips and EOE-index daily return series.

TABLE 2

Sample Statistics for Philips and EOE-Index Daily Stock Returns

| | Philips | EOE-Index |
|---|---|---|
| Observations | 250 | 250 |
| Mean Return | 0.0011 | 0.0012 |
| Standard Deviation | 0.0155 | 0.0102 |
| Skewness | −0.06879 | −0.03640 |
| Kurtosis | 9.2678 | 4.5560 |
| *D*-Statistic | 0.0956 | 0.0811 |

For a 1-percent significance level, the critical value of the *D*-statistic is given by 0.026. Under the assumption of normality, the asymptotic standard errors for kurtosis and skewness are respectively given by $\sqrt{24/N}$ and $\sqrt{6/N}$, where $N$ denotes the number of observations. For $N = 250$, these errors are given by 0.3098 and 0.1549, respectively.

Both series show leptokurtic behavior. The level of kurtosis differs from the level of normal kurtosis by more than five times the standard error (under the assumption of asymptotic normality), for both series. The skewness is very small (within 40 percent of the standard error) in this period, indicating that the influence of the October 1987 crash has passed. The Kolmogorov-Smirnov $D$-statistic for the null hypothesis of normality is rejected for both samples. In Table 3, the autocorrelations (AC) and partial autocorrelations (PAC) of the stock return and squared stock return series are presented for both samples.

The stock return series of both Philips and the EOE-index show low insignificant autocorrelations for lags up to five days. For the EOE-index, the squared stock return series are significantly correlated for lags up to the fifth lag. The first lag partial autocorrelation is clearly significant and the second and third lag partial autocorrelations are weakly significant, indicating that a low-order GARCH model is a likely candidate for describing stock return volatility. Philips stock displays less autocorrelation in the squared stock return series than the EOE-index, but there is significant autocorrelation for lags up to the fourth day. The first and second lag partial autocorrelations for Philips squared stock return series are also significant, indicating that, also in this case, a low order GARCH model will fit the data well. Using Ljung-Box test statistics, the null hypothesis of zero autocorrelations for lags up to the fourth day was rejected for both Philips and the EOE-index squared stock return series. Table 4 lists test statistics and parameter estimates for low-order GARCH models describing Philips and EOE-index stock returns.

The symbol $L$ represents the maximum log likelihood value. For the standardized residuals of both series the mean, variance, kurtosis, and skewness are listed. $Q_1$ and $Q_2$ give the value of the Ljung-Box test statistics for the standardized and squared standardized residuals. The symbol $SK$ represents the skewness/kurtosis test statistic. Under the null hypothesis of standard normality, this statistic is

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TABLE 3

Autocorrelations and Partial Autocorrelations for the First and Second Order Moments of EOE-Index and Philips Stock Returns

| Lag (Days) | Returns | | | Squared Returns | | |
|---|---|---|---|---|---|---|
| | AC | | PAC | AC | | PAC |
| *EOE-Index* | | | | | | |
| 1 | 0.0271 | (0.0634) | 0.0271 | 0.2146 | (0.0662) | 0.2146 |
| 2 | −0.0622 | (0.0635) | −0.0630 | 0.1785 | (0.0669) | 0.0759 |
| 3 | −0.0832 | (0.0638) | 0.0801 | 0.1641 | (0.0682) | 0.1205 |
| 4 | 0.0010 | (0.0637) | 0.0014 | 0.1649 | (0.0688) | 0.0574 |
| 5 | 0.0844 | (0.0640) | 0.0752 | 0.1010 | (0.0691) | −0.0193 |
| *Philips* | | | | | | |
| 1 | 0.0300 | (0.0600) | 0.0300 | 0.1120 | (0.0659) | 0.1120 |
| 2 | −0 0571 | (0.0661) | −0.0580 | −0.1108 | (0.0658) | −0.0208 |
| 3 | 0.0061 | (0.0659) | 0.0097 | −0.1072 | (0.0658) | −0.0872 |
| 4 | −0.0388 | (0.0659) | −0.0428 | −0.0906 | (0.0656) | −0.0009 |
| 5 | 0.0098 | (0.0657) | 0.0134 | 0.0403 | (0.0656) | 0.0388 |

The autocorrelations (AC) and partial autocorrelations (PAC) of the stock return and squared stock return series are calculated for lags up to five days. The numbers in parentheses give the standard errors for the AC. The standard error of the PAC is given by 0.0664.

asymptotically $\chi^2(2)$ distributed. In the case of EOE-index stock returns, nonsynchronous trading of the component securities causes daily portfolio returns to be autocorrelated (French, Schwert, and Stambaugh (1987)), so an MA(1) term is added in the equation for the conditional mean. The size of the standard error for the estimate of $\alpha$ and the low value of the $Q_1$ test statistic indicate that this effect is not present for the EOE-index stock return series.

The value of the $Q_2$ statistic testing the presence of first order autocorrelation in the squared residuals is only significant in the homoskedastic case. This confirms the previous results that a low order GARCH model describes the data well. Specifically, this result suggests that an ARCH(1) model specification is adequate for the description of the conditional volatility of the EOE-index stock returns. The same result holds for the Philips stock return series. However, from likelihood ratio tests,[12] the result is that the optimal model choice for both Philips and EOE-index stock returns series is the GARCH(1,1) process.[13]

In the case of the EOE-index, the *SK* test statistic rejects normality for all model assumptions at a 1-percent level, although the kurtosis for the standardized residuals is close to 3 percent. For this reason, the different models are estimated under the assumption of a standardized *t*-distribution. However, the parameter estimations and different test statistics did not change the results. For Philips stock return series, the null hypothesis of standard normality for the standardized residuals is not rejected.

[12]If $L_n$ and $L_a$ are the maximum log likelihood values under the null and the alternative hypothesis, respectively, then the test statistic $-2[L_n - L_a]$ is asymptotically $\chi^2$ distributed with degrees of freedom equal to the difference in the number of parameters under the null and the alternative.

[13]This is in line with the results of de Jong, Kemna, and Kloek (1992) who also reported a preference for the GARCH(1,1) model tested on Dutch daily closing prices.

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TABLE 4

Stock Return Test Statistics for a (G)ARCH Model Specification

| | | Constant | ARCH(1) | ARCH(2) | GARCH(1,1) | GARCH(1,2) |
|---|---|---|---|---|---|---|
| EOE-Index | $R_t = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \epsilon_t + \alpha\epsilon_{t-1}$ | | | | | |
| | $\sigma_t^2 = \beta_0 + \beta_1\epsilon_{t-1}^2 + \beta_2\sigma_{t-1}^2 + \beta_3\epsilon_{t-2}^2$ | | | | | |
| | $\lambda$ | 0.0932 | 0.1108 | 0.1198 | 0.1121 | 0.1204 |
| | | (0.0019) | (0 0014) | (0.0022) | (0.0018) | (0.0009) |
| | $\alpha$ | 0.0241 | 0.0095 | 0.0230 | 0.0253 | 0.0395 |
| | | (0.0533) | (0.0802) | (0.0697) | (0.0797) | (0.0733) |
| | $\beta_0$ | 0.0001 | 0.00008 | 0.00007 | 0.00002 | 0.00003 |
| | | (0.000008) | (0.000001) | (0.0000008) | (0.0000005) | (0.000001) |
| | $\beta_1$ | | 0.2231 | 0.1315 | 0.1623 | 0.1063 |
| | | | (0.0153) | (0.0055) | (0.0026) | (0.0052) |
| | $\beta_2$ | | | | 0.6142 | 0.4854 |
| | | | | | (0.0014) | (0.0065) |
| | $\beta_3$ | | | 0.1809 | | 0.1211 |
| | | | | (0.0113) | | (0.0149) |
| | $L$ | −340.847 | −336.205 | −333.958 | −332.947 | −332.501 |
| | Mean | 0.002 | −0.017 | −0.037 | −0.026 | −0.033 |
| | Variance | 1.001 | 1.003 | 1.001 | 1.004 | 1.004 |
| | Skewness | −0.357 | −0.412 | −0.333 | −0.437 | −0.406 |
| | Kurtosis | 4.555 | 4.254 | 4.173 | 4.243 | 4.326 |
| | $Q_1(12)$ | 13.579 | 10.068 | 10.098 | 10.226 | 10.472 |
| | $Q_2(12)$ | 34.102 | 13.464 | 9.548 | 5.188 | 5 074 |
| | $SK$ | 16.057 | 15.832 | 13.379 | 13.330 | 15.014 |
| Philips | $R_t = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \epsilon_t$ | | | | | |
| | $\sigma_t^2 = \beta_0 + \beta_1\epsilon_{t-1}^2 + \beta_2\sigma_{t-1}^2 + \beta_3\epsilon_{t-2}^2$ | | | | | |
| | $\lambda$ | −0.0220 | −0.0193 | −0.0117 | −0.0149 | −0.0202 |
| | | (0.0017) | (0.0026) | (0.0011) | (0.0020) | (0.0009) |
| | $\beta_0$ | 0.0003 | 0.0002 | 0.0002 | 0.00001 | 0.00002 |
| | | (0.00004) | (0.000001) | (0.000008) | (0.0000006) | (0.000008) |
| | $\beta_1$ | | 0.0765 | 0.0852 | 0.0566 | 0.0421 |
| | | | (0.0051) | (0.0030) | (0.0016) | (0.0068) |
| | $\beta_2$ | | | | 0.8518 | 0.7615 |
| | | | | | (0.0019) | (0.0011) |
| | $\beta_3$ | | | 0.0000 | | 0.1121 |
| | | | | (0.0029) | | (0.0032) |
| | $L$ | −394.160 | −391.360 | −390.637 | −387.978 | −387.108 |
| | Mean | 0.038 | 0.038 | 0.047 | 0.045 | 0.044 |
| | Variance | 1.033 | 1.034 | 1.071 | 1.042 | 0 823 |
| | Skewness | −0.705 | −0.811 | −0.827 | −0.830 | −0.838 |
| | Kurtosis | 3.839 | 3.853 | 3.966 | 3.824 | 3.864 |
| | $Q_1(12)$ | 16.664 | 11.240 | 11.965 | 10.607 | 11.292 |
| | $Q_2(12)$ | 35.767 | 15.113 | 15.075 | 14.639 | 14.607 |
| | $SK$ | 10.912 | 11.837 | 12.411 | 12.603 | 14.150 |

The first part of both tables lists the parameter estimates of some low order GARCH models. The numbers in parentheses give the standard errors of the parameter estimates. The risk-free rate $r$ is set to 8.5 percent. $L$ denotes the value of the maximum log likelihood. The second part lists some descriptive statistics of the standardized residuals. $Q_1(12)$ and $Q_2(12)$ are the Ljung-Box test statistics applied to the residuals and squared residuals, respectively. Under the null hypothesis that the residuals are uncorrelated, these statistics are asymptotically distributed $\chi^2$, with 12 degrees of freedom. The skewness/kurtosis ($SK$) test statistic (Bera-Jarque statistic) is asymptotically $\chi^2(2)$ distributed under the null hypothesis of normally distributed residuals.

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In Section II, some drawbacks of the GARCH model were discussed in favor of the EGARCH model. As Nelson (1991) pointed out, GARCH models do not allow for the negative correlation that is observed (Black 1976) between current returns and future returns volatility. Therefore, both samples are also tested for EGARCH characteristics. The results are listed in Table 5.

TABLE 5

Stock Return Test Statistics for an EGARCH(1,1) Specification

$$R_t = r + \lambda\sigma_t - \tfrac{1}{2}\sigma_t^2 + \sigma_t\xi_t$$

$$\mathrm{Ln}\,\sigma_t^2 = \beta_0 + \beta_1 \,\mathrm{Ln}\,\sigma_{t-1}^2 + \beta_2\xi_{t-1} + \beta_3(|\xi_{t-1}| - \sqrt{2/\pi})$$

| | |
|---|---|
| *EOE-Index* | |
| $\lambda$ = 0.1038 (0.0041) | Mean = 0.000801 |
| $\beta_0$ = −2.4 (0.001) | Variance = 1.006 |
| $\beta_1$ = 0.7396 (0.0005) | Skewness = −0.212 |
| $\beta_2$ = −0.1424 (0.002) | Kurtosis = 3.101 |
| $\beta_3$ = 0.2330 (0.0031) | $Q_1(12)$ = 11.470 |
| | $Q_2(12)$ = 7.186 |
| | $SK$ = 5.189 |
| | $L$ = −329.62 |
| *Philips* | |
| $\lambda$ = −0.0171 (0.0028) | Mean = −0.006 |
| $\beta_0$ = −1.11 (0.0001) | Variance = 0.992 |
| $\beta_1$ = 0.878 (0.0004) | Skewness = −0.411 |
| $\beta_2$ = −0.1918 (0.0024) | Kurtosis = 3.242 |
| $\beta_3$ = 0.1320 (0.001) | $Q_1(12)$ = 9.951 |
| | $Q_2(12)$ = 4.156 |
| | $SK$ = 5.541 |
| | $L$ = −385.21 |

The left-hand side of the table lists the parameter estimates of the EGARCH(1,1) Model Specification. The risk-free rate of interest $r$ is set to 8.5 percent. The numbers in parentheses give the standard errors. The right-hand side lists some descriptive statistics of the standardized residuals. $Q_1(12)$ and $Q_2(12)$ are the Ljung-Box test statistics applied to the residuals and squared residuals, respectively. Under the null hypothesis that the residuals are uncorrelated, these statistics are asymptotically distributed $\chi^2$, with 12 degrees of freedom. The skewness/kurtosis ($SK$) test statistic (Bera-Jarque statistic) is asymptotically $\chi^2(2)$ distributed under the null hypothesis of normally distributed residuals.

For both Philips and the EOE-index stock return series, the table shows significant values for the estimated parameters. For both samples, the low values of the $Q_2$ and $SK$ test statistics indicate that the EGARCH(1,1) model assumptions fit the data even better than the GARCH(1,1) process, but because the models are nonnested, it is difficult to decide which of both models performs significantly better. Akaike (1973) introduced an information criterion (AIC) for the comparison of models with different numbers of parameters estimated by the method of maximum likelihood.

The AIC is defined by AIC = $-2 \cdot$ Ln(maximum likelihood) + $2 \cdot$ (number of parameters). A smaller value of AIC means a better fit for the model. For the EOE-index, the values of AIC in the case of the GARCH(1,1) and the EGARCH(1,1) model are respectively given by 673.894 and 669.24. In the case of Philips, the

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AIC values are respectively given by 783.96 and 780.42. These results confirm the previous suggestion that the EGARCH(1,1) model can be preferred to the GARCH(1,1) model in fitting both asset return series.

In order to gain insight into the results obtained with the GARCH(1,1) and EGARCH(1,1) model specifications, the mean-reverting diffusion process is also estimated, a procedure that was carried out by Stein (1989). As Stein correctly noted, the serial correlation properties of the instantaneous volatility $\sigma_t$ cannot be observed directly. However, Poterba and Summers (1986) showed that movements in the implied volatilities cohere reasonably well with those of ex post volatilities estimated from daily returns. So assuming that implied volatility on a nearby option has the same serial correlation properties as the instantaneous volatility $\sigma_t$, an AR(1) process is fitted on a daily series of short-term (time to maturity between one and three months) implied volatilities for both Philips and EOE-index stock options in the period from January 21, 1988, until January 21, 1989. To make legitimate the use of an AR(1) model, the autocorrelations (AC) and partial autocorrelations (PAC) are calculated for both implied volatility series. The results are listed in Table 6.

To identify the estimated parameters of the AR(1) process with the parameters of the continuous time mean-reverting process, one has to discretize this diffusion process. Suppose that $\sigma_t^2$ is observed at discrete time intervals one period apart, then it can be shown (Arnold (1974)) that the observations satisfy an AR(1) process,

$$\sigma_t^2 = \theta + \rho\sigma_{t-1}^2 + \alpha_1\xi_t, \tag{13}$$

where $\xi_t$ is white noise, $\rho = e^{-\alpha_0}$, and $\theta = (1 - e^{-\alpha_0})\bar{\sigma}^2$. In Table 6, the highly significant autocorrelations and the high value for the $Q_1$ test statistic show that both daily implied volatility series are strongly correlated. For both series, only the first partial autocorrelation is strongly significant, indicating that an AR(1) process fits the data well.

## C. Term Structure Tests of Implied Volatility

In the previous section, parameter estimates were calculated for different nonnested stock return volatility model specifications for both Philips and the EOE-index, using daily stock prices ranging from January 21, 1988, until January 21, 1989. These parameter values are used to specify the ex ante relations between short- and long-term implied volatilities derived in Section II. These relations are used to perform an ex ante efficiency test on the selected (transaction) series of short- and long-term implied volatilities from January 23, 1989, until April 30, 1989. For each model assumption, the joint hypothesis of a correct model specification and rationally formed expectations of average volatility is tested. In the case of GARCH(1,1), the test is specified as follows,

$$\left[\sigma_{\mathrm{imp}}^2(t,T_1) - \bar{\sigma}^2\right] - \frac{T_2}{T_1}\frac{\gamma^{T_1}-1}{\gamma^{T_2}-1}\left[\sigma_{\mathrm{imp}}^2(t,T_2) - \bar{\sigma}^2\right] = \epsilon_t. \tag{14}$$

For the EOE-index, $\gamma = 0.7765$ (0.211) and $\bar{\sigma}^2 = 0.8948\ 10^{-4}$ ($0.03\ 10^{-4}$). For Philips stock, $\gamma = 0.9384$ (0.0187) and $\bar{\sigma}^2 = 1.6233\ 10^{-4}$ ($0.08\ 10^{-4}$). The difference between the left- and right-hand side of Equation (8) should be white noise;

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TABLE 6

**Autocorrelation and Partial Autocorrelation for Short-Term Implied Volatility Series**

| | Lag (Days) | AC | PAC | Test Statistics ($\sigma_t^2 = \theta + \rho\sigma_{t-1}^2 + \alpha_1\xi_t$) |
|---|---|---|---|---|
| *EOE-Index* | | | | |
| | 1 | 0.8871<br>(0.0561) | 0.8871<br>(0.0443) | $\rho = 0.7051$<br>(0.0201) |
| | 2 | 0.4332<br>(0.0343) | 0.0943<br>(0.0443) | $\alpha_1 = 0.154$<br>(0.0031) |
| | 3 | 0.4500<br>(0.0515) | 0.0840<br>(0.0443) | $\theta = 2.5873\ 10^{-5}$<br>($0.004\ 10^{-5}$) |
| | 4 | 0.4118<br>(0.0614) | 0.0816<br>(0.0443) | $Q_1(5) = 72.101$ |
| | 5 | 0.3163<br>(0.0516) | 0.0713<br>(0.0433) | |
| *Philips* | | | | |
| | 1 | 0.9325<br>(0.0910) | 0.9325<br>(0.0565) | $\rho = 0.911$<br>(0.0302) |
| | 2 | 0.8710<br>(0.0871) | 0.0681<br>(0.0565) | $\alpha_1 = 0.091$<br>(0.0012) |
| | 3 | 0.7991<br>(0.0898) | 0.0840<br>(0.0565) | $\theta = 9.5399\ 10^{-6}$<br>($0.007\ 10^{-6}$) |
| | 4 | 0.7812<br>(0.0852) | −0.0150<br>(0.0565) | $Q_1(5) = 65.210$ |
| | 5 | 0.7260<br>(0.0752) | −0.0312<br>(0.0565) | |

The left-hand side of the table lists the autocorrelations (AC) and partial autocorrelations (PAC) for a daily series of short-term at-the-money implied volatilities for both Philips and EOE-index stock options in the period from January 21, 1988, until January 21, 1989. The results make legitimate the use of an AR(1) process to model both time series. The corresponding parameter estimates are listed on the right-hand side of the table. $Q_1(5)$ is the Ljung-Box test statistics applied to the daily implied volatility series. Under the null hypothesis that these implied volatilities are uncorrelated, this statistic is asymptotically distributed $\chi^2$, with five degrees of freedom.

$E[\epsilon_t] = 0$ and $E[\epsilon_t\epsilon_{t-j}] = 0$ for every $j$. So, substituting the 150 pairs of associated short- and long-term annualized (250 days) implied volatilities together with the GARCH(1,1) parameter estimates, a series of 150 numbers is obtained that should be tested on zero mean and uncorrelatedness. For the other two model assumptions, the tests were carried out similarly. The results for Philips and the EOE-index across different model assumptions are listed in Table 7. To compare values of the mean of the same order of magnitude, the residuals $\epsilon_t$ are divided by $\bar{\sigma}^2$, the unconditional variance of the specific process in the case of GARCH(1,1) and mean-reverting models.

The robustness of these results depends largely on the bias in the values of the unconditional volatilities (these values are listed later in this section) resulting from measurement errors in the corresponding parameter estimates. The errors in $\bar{\sigma}$ are calculated by evaluating the range of values that emerged from the error distribution of the corresponding parameter estimates. It is found that the term structure test

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TABLE 7

Term Structure Test Statistics for Philips and EOE-Index Short- and Longer-Term At-the-Money Implied Volatilities across Different Model Assumptions

| | Mean-Reverting | GARCH(1,1) | EGARCH(1,1) |
|---|---|---|---|
| *EOE-Index* | | | |
| | Mean = 0.4719 $10^{-2}$ | Mean = 0.3817 $10^{-2}$ | Mean = 0.1001 $10^{-2}$ |
| | $T$-Value = 2.732 | $T$-Value = 2.391 | $T$-Value = 1.183 |
| | $Q_1(5)$ = 35.761 | $Q_1(5)$ = 33.772 | $Q_1(5)$ = 9.102 |
| *Philips* | | | |
| | Mean = 0.3211 $10^{-2}$ | Mean = 0.3013 $10^{-2}$ | Mean = 0.0961 $10^{-2}$ |
| | $T$-Value = 3.166 | $T$-Value = 2.841 | $T$-Value = 1.676 |
| | $Q_1(5)$ = 42.177 | $Q_1(5)$ = 31.101 | $Q_1(5)$ = 7.099 |

The table lists some descriptive statistics of the residuals $\epsilon_t$ specified in the case for a GARCH(1,1) term structure relation by Equation (14). For the other term structure relations given by Equations (5) and (12), the residuals $\epsilon_t$ can be defined in a similar way. The mean, $T$-value, and Ljung-Box statistic ($Q_1(5)$) of the residual series are calculated for each model assumption. The $Q_1(5)$ statistic is asymptotically distributed $\chi^2(5)$ under the null hypothesis of uncorrelated residuals.

results are robust within a range of two standard errors of the unconditional mean volatility estimates for all three processes.

Both the $T$-values and Ljung-Box test statistics show that, only for the EGARCH(1,1) model, the assumption of white noise is not rejected. The significant positive means in the case of mean-reverting and GARCH(1,1) would give reason to think that investors' forecasts of long-term average volatility are larger than is expected from the term structure relations. For the mean-reverting specification, it is interesting to compare the results with Stein (1989). Performing an ex ante efficiency test on the term structure of implied volatilities for S&P 100 index options, he found that the longer-term implied volatilities were larger than were expected from the average volatility forecasts based on the mean-reverting stock return volatility model. He explained these inefficiencies through the overreaction of traders to the arrival of new information. However, the alternative using an EGARCH(1,1) term structure relation shows that the joint hypothesis of a correct model specification and ex ante efficiency is not rejected for both Philips and the EOE-index. This conclusion is supported by Figures 1 and 2 where the residuals $\epsilon_t$ of the 150 sample pairs across the different model assumptions are plotted. The figures show that the absolute size of the residuals is significantly smaller in the case of the EGARCH(1,1) term structure relation.

An explanation for this result is given by the fact that the EGARCH model describes important characteristics in stock return behavior that are not captured by GARCH models. This is illustrated by the estimated level of the unconditional volatility across the different model assumptions. This estimated level could significantly vary across different model assumptions. For the EOE-index, the estimated[14] levels of the (annualized) unconditional volatility in the case of the mean-reverting, GARCH(1,1), and EGARCH(1,1) process are, respectively, 0.1481 (0.0022), 0.1496 (0.0031), and 0.1615 (0.0020). In the case of Philips, the

[14] We used the estimated parameter values of the different model assumptions listed in this section.

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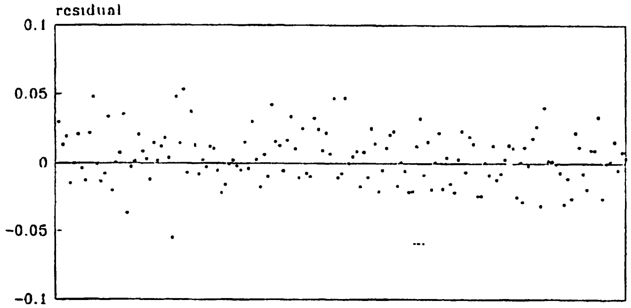
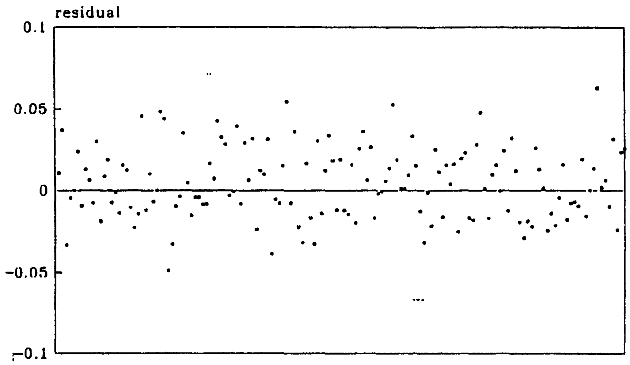
FIGURE 1

Residuals Term Structure Test for EOE-Index Options in the Period from January 23, 1989 until April 30, 1989



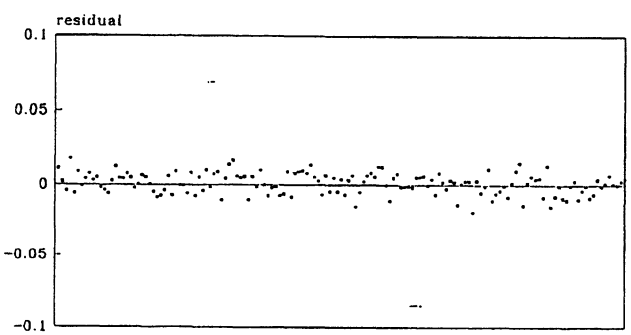

The mean−reverting case

• residuals




The GARCH(1,1) case

residuals




The EGARCH(1,1) case

residuals

Residuals are plotted for 150 intra daily sample pairs across different specified term structure models.

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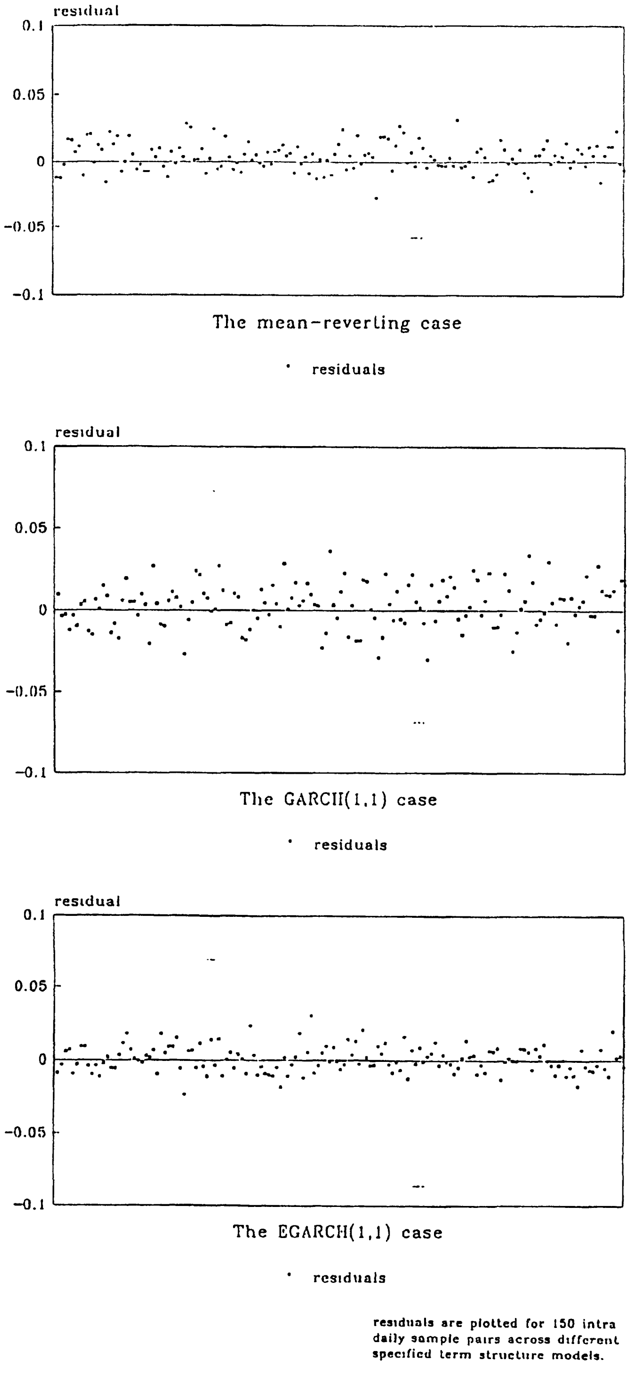
FIGURE 2

Residuals Term Structure Test for Philips Stock Options in the Period from January 23, 1989 until April 30, 1989

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estimated levels are, respectively, 0.1637 (0.0054), 0.1652 (0.0039), and 0.1758 (0.0027). Because the models are nonnested, it is difficult to decide which model correctly specifies the level of the unconditional volatility. However, for the EGARCH(1,1) process, the level of the unconditional volatility is significantly larger than in the case of the other processes.

This observation has important consequences for the interpretation of the empirical test results, which is illustrated for the EOE-index by the following example. Consider a nearby and distant implied volatility approximately satisfying the EGARCH(1,1) term structure relation. If the nearby ($T$ = 63 days) implied variance ($\sigma_{\text{imp}}(63) \approx 0.1732$) is 15 percent above the unconditional variance ($\bar{\sigma}$ = 0.1615), then the distant ($T$ = 187 days) implied variance ($\sigma_{\text{imp}}(187) \approx 0.1655$) should be approximately 5 percent above the unconditional variance. However, given the fact that the estimated levels of the unconditional variance are significantly different in the case of mean-reverting and GARCH(1,1), the same pair of implied volatilities does not satisfy the term structure relation specified for these processes. For example, in the case of GARCH(1,1), the nearby implied variance ($\sigma_{\text{imp}}(63) \approx 0.1732$) is 34 percent above the unconditional variance ($\bar{\sigma}$ = 0.1496) so the distant ($T$ = 187 days) implied variance ($\sigma_{\text{imp}}(187) \approx 0.1655$) should be approximately 11.3 percent above the unconditional variance. However, a simple calculation shows that, in this case, the distant implied variance is 22.4 percent above the unconditional mean variance, which would suggest an overreaction interpretation if the forecasts of long-term average volatility are based on the mean-reverting or GARCH(1,1) model. From this example, it can be inferred that the major determinant for the specification of the correct term structure relation is the unconditional variance, which seems for both Philips and the EOE-index correctly specified in the case of the EGARCH(1,1) process.

## V. Conclusions

In this paper, ex ante efficiency tests are constructed and performed for the relation between short- and long-term implied volatilities for the EOE-index and Philips, across different model assumptions. The joint hypothesis of a correct model specification and ex ante efficiency is rejected in the case of a mean-reverting and a GARCH(1,1) stock return volatility specification, but is not rejected in the case of the EGARCH(1,1) model. Checking robustness shows that the test results are fairly consistent. Because there is no evidence for misspecification, all model assumptions seem to give an adequate description of the underlying stock return volatility. However, a comparison of these different nonnested models based on Akaike's information criterion suggests that the EGARCH(1,1) model fits the daily stock returns better than the other two models.

This paper suggests that the major determinant for the specification of the term structure of implied volatility relations is the level of the unconditional volatility, which seems correctly specified in the case of the EGARCH(1,1) stock return volatility model. The EGARCH(1,1) process takes account of the asymmetric relation between stock returns and volatility changes. As was reported by Black (1976) and Christie (1982), the leverage effect can explain part of this relation. In contrast with the evidence of overreaction presented by Stein (1989), the results

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suggest an alternative interpretation in accordance with rational expectations theory that does not reject the efficiency of the options market, if the stock return volatility model assumptions are based on an EGARCH(1,1) process.

## Appendix A

This is the derivation of a relation for the average expected volatility in the case of a GARCH(1,1) stock return volatility process. For a GARCH(1,1) specification, stock return and stock return volatility are modeled as follows,

$$\text{(A-1)} \qquad \mathrm{Ln}\left(S_t/S_{t-1}\right) = \mu + \sigma_t \xi_t,$$

$$\text{(A-2)} \qquad \sigma_t^2 = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2,$$

where $S_t$ is the stock price at $t$, $\mu$ is the mean stock price return, $\sigma_t$ is the stock return volatility, $\beta_0$, $\beta_1$, $\beta_2$ are time independent parameters, $\beta_1 + \beta_2 < 1$, $\epsilon_t$ are the innovation variables, $\epsilon_t = \sigma_t \xi_t$, $\xi_t$ is Gaussian white noise, and $\xi_t \overset{\text{i.i.d.}}{\sim} N(0,1)$. The average expected volatility is defined by

$$\text{(A-3)} \qquad \sigma_{\mathrm{Av}}^2(t,T) = \frac{1}{T}\sum_{k=1}^{T} E_t\left[\sigma_{t+k}^2\right],$$

where $T$ is the number of periods from $t$ to the expiration date.

If one wants to compute the right-hand side of Expression (A-3), one first has to express the $\sigma_{t+k}^2$ into $\sigma_t^2$. So,

$$\text{(A-4)} \qquad \sigma_{t+k}^2 = \beta_0 + \left(\beta_1 \xi_{t+k-1}^2 + \beta_2\right)\sigma_{t+k-1}^2.$$

By induction, one derives from (A-4),

$$\text{(A-5)} \qquad \sigma_{t+k}^2 = \beta_0 + \beta_0 \sum_{m=1}^{k-1} \prod_{n=1}^{m} \left(\beta_1 \xi_{t+k-n}^2 + \beta_2\right) + \sigma_t^2 \prod_{n=1}^{k} \left(\beta_1 \xi_{t+k-n}^2 + \beta_2\right).$$

Using independence of $\xi_k$,

$$\text{(A-6)} \qquad E_t\left[\sigma_{t+k}^2\right] = \beta_0 + \beta_0 \sum_{m=1}^{k-1} (\beta_1 + \beta_2)^m + (\beta_1 + \beta_2)^{k-1}\left(\beta_1 \xi_t^2 + \beta_2\right)\sigma_t^2,$$

where $E_t$ is the conditional expectation operator at $t$. Evaluating the summations,

$$\text{(A-7)} \qquad E_t\left[\sigma_{t+k}^2\right] = \beta_0 + \beta_0 \frac{\gamma - \gamma^k}{1-\gamma} + \gamma^{k-1}\left(\beta_1 \xi_t^2 + \beta_2\right)\sigma_t^2,$$

with $\gamma = \beta_1 + \beta_2$. Now the right-hand side of (A-3) can be computed,

$$\text{(A-8)} \qquad \sigma_{\mathrm{Av}}^2(t,T) = \frac{1}{T}\sum_{k=1}^{T} E_t\left[\sigma_{t+k}^2\right] = \bar{\sigma}^2 + \left(\sigma_{t+1}^2 - \bar{\sigma}^2\right)\frac{1}{T}\frac{1-\gamma^T}{1-\gamma},$$

where $\bar{\sigma}^2 = \beta_0/(1-\beta_1-\beta_2)$, and $\sigma_{t+1}^2 = \beta_0 + \beta_1 \epsilon_t^2 + \beta_2 \sigma_t^2$.

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## Appendix B

This is the derivation of a relation for the average expected volatility in the case of an EGARCH(1,1) specification for stock return volatility. For an EGARCH(1,1) process, stock return and stock return volatility are modeled as follows,

$$\text{(A-9)} \qquad \operatorname{Ln}\left(S_t/S_{t-1}\right) = \mu + \sigma_t\xi_t,$$

$$\text{(A-10)} \qquad \operatorname{Ln}\sigma_t^2 = \beta_0 + \beta_1 \operatorname{Ln}\sigma_{t-1}^2 + \beta_2\xi_{t-1} + \beta_3\left(|\xi_{t-1}| - \sqrt{\frac{2}{\pi}}\right),$$

where $\beta_0, \ldots, \beta_3$ are time independent parameters, and $\xi_t$ is Gaussian white noise, $\xi_t \overset{\text{i.i.d.}}{\sim} N(0,1)$.

Average expected volatility is defined as follows,

$$\text{(A-11)} \qquad \sigma_{\text{Av}}^2(t,T) = \left[\prod_{k=1}^{T} E_t\left[\sigma_{t+k}^2\right]\right]^{\frac{1}{T}} \quad \text{or}$$

$$\operatorname{Ln}\sigma_{\text{Av}}^2(t,T) = \frac{1}{T}\sum_{k=1}^{T} \operatorname{Ln} E_t\left[\sigma_{t+k}^2\right],$$

where $\sigma_{\text{Av}}(t,T)$ is the geometric average expected volatility over the time span $[t, t+T]$, $E_t$ is the conditional expectation operator at $t$, and $T$ is the number of periods from $t$ to the expiration date.

To compute the average expected volatility, one first has to express $\sigma_{t+k}^2$ into $\sigma_t^2$. Define $y_t = \operatorname{Ln}\sigma_t^2$ and $A_t = \beta_2\xi_t + \beta_3(|\xi_t| - \sqrt{2/\pi})$, then by induction it follows that

$$\text{(A-12)} \qquad y_{t+k} = \beta_0\sum_{m=0}^{k-1}\beta_1^m + \beta_1^k y_t + \sum_{m=0}^{k-1}\beta_1^m A_{t+k-m-1}.$$

From this follows the expression for $\sigma_{t+k}^2$,

$$\text{(A-13)} \qquad \sigma_{t+k}^2 = \sigma_t^{2\beta_1^k}\exp\left[\beta_0\sum_{m=0}^{k-1}\beta_1^m + \sum_{m=0}^{k-1}\beta_1^m A_{t+k-m-1}\right].$$

Now the conditional expectation can be evaluated,

$$\text{(A-14)} \qquad E_t\left[\sigma_{t+k}^2\right] = \sigma_t^{2\beta_1^k} E_t\left[\exp\left[\beta_0\sum_{m=0}^{k-1}\beta_1^m + \sum_{m=0}^{k-1}\beta_1^m A_{t+k-m-1}\right]\right].$$

Factoring out the expectation operator, one obtains for $k \geq 2$,

$$\text{(A-15)} \qquad E_t\left[\sigma_{t+k}^2\right] = \sigma_t^{2\beta_1^k}\exp\left[\beta_1^{k-1}A_t\right]$$
$$\times \exp\left[\beta_0\sum_{m=0}^{k-1}\beta_1^m\right]\prod_{m=0}^{k-2} E_t\left[\exp\left[\beta_1^m A_{t+k-m-1}\right]\right].$$

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For $k = 1$ one has, of course, $E_t[\sigma_{t+1}^2] = \sigma_{t+1}^2$.

The average expected volatility can be computed if one can evaluate the factors $E_t[\exp[\beta_1^m A_{t+k-m-1}]]$. Because of the independence of the terms $A_t$ (the $\xi_t$ are independent), one only has to evaluate expressions of the form $E[\exp[\beta_1^m(\beta_2\xi_t + \beta_3(|\xi_t| - \sqrt{2/\pi}))]]$.

From straightforward calculus, one obtains,

$$\text{(A-16)} \quad E\left[\exp\left[\beta_1^m\left[\beta_2\xi_t + \beta_3\left(|\xi_t| - \sqrt{\frac{2}{\pi}}\right)\right]\right]\right]$$
$$= \exp\left[-\beta_3\beta_1^m\sqrt{\frac{2}{\pi}}\right]\left[N\left[\beta_1^m(\beta_3+\beta_2)\right]\exp\left[\frac{1}{2}\beta_1^{2m}(\beta_3+\beta_2)^2\right]\right.$$
$$\left. + N\left[\beta_1^m(\beta_3-\beta_2)\right]\exp\left[\frac{1}{2}\beta_1^{2m}(\beta_3-\beta_2)^2\right]\right],$$

with $N(a) = (1/\sqrt{2\pi})\int_{-\infty}^{a} e^{-z^2/2}dz$.

Then follows the right-hand side of Expression (A-15),

$$\text{(A-17)} \quad E_t\left[\sigma_{t+k}^2\right] = \sigma_t^{2\beta_1^k}\exp\left[\beta_1^{k-1}A_t\right]\exp\left[\left(\beta_0 - \beta_3\sqrt{\frac{2}{\pi}}\right)\frac{1-\beta_1^k}{1-\beta_1}\right.$$
$$\left. + \beta_3\beta_1^{k-1}\sqrt{\frac{2}{\pi}} + \frac{1}{2}\left(\beta_2^2+\beta_3^2\right)\frac{1-\beta_1^{2(k-1)}}{1-\beta_1^2}\right]$$
$$\times \prod_{m=0}^{k-2}\left[N\left[\beta_1^m(\beta_3+\beta_2)\right]\exp\left[\beta_1^{2m}\beta_2\beta_3\right]\right.$$
$$\left. + N\left[\beta_1^m(\beta_3-\beta_2)\right]\exp\left[-\beta_1^{2m}\beta_2\beta_3\right]\right].$$

The expression for $\sigma_{\mathrm{Av}}^2(t,T)$ follows from (A-17). For convenience, one defines $F_m(\beta_1,\beta_2,\beta_3) = N[\beta_1^m(\beta_3+\beta_2)]\exp[\beta_1^{2m}\beta_2\beta_3]$. Using expression (A-10), one identifies $\mathrm{Ln}\,\sigma_{t+1}^2$,

$$\text{(A-18)} \quad T\,\mathrm{Ln}\,\sigma_{\mathrm{Av}}^2(t,T) = \mathrm{Ln}\,\sigma_{t+1}^2 + \mathrm{Ln}\,\sigma_{t+1}^2\sum_{k=2}^{T}\beta_1^{k-1}$$
$$+ \left(\beta_0 - \beta_3\sqrt{\frac{2}{\pi}}\right)\sum_{k=2}^{T}\frac{1-\beta_1^{k-1}}{1-\beta_1} + \frac{1}{2}\left(\beta_2^2+\beta_3^2\right)\sum_{k=2}^{T}\frac{1-\beta_1^{2(k-1)}}{1-\beta_1^2}$$
$$+ \sum_{k=2}^{T}\sum_{m=0}^{k-2}\mathrm{Ln}\left[F_m(\beta_1,\beta_2,\beta_3) + F_m(\beta_1,-\beta_2,\beta_3)\right].$$

To simplify the right-hand side of Relation (A-18) one has to identify part of the expression with the unconditional variance of the EGARCH(1,1) process.

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*Lemma.* For the EGARCH(1,1) specification as given by (A-10), the unconditional variance of the process is given by,

$$\text{(A-19)} \qquad \bar{\sigma}^2 = \exp\left[\frac{\beta_0 - \beta_3\sqrt{\frac{2}{\pi}}}{1-\beta_1} + \frac{1}{2}\frac{\left(\beta_2^2+\beta_3^2\right)}{1-\beta_1^2}\right] \times \prod_{m=0}^{\infty}\left[F_m(\beta_1,\beta_2,\beta_3) + F_m(\beta_1,-\beta_2,\beta_3)\right].$$

*Proof.* From Relation (A-12), the following expression can be derived,

$$\text{(A-20)} \qquad y_t = \beta_0 \sum_{m=0}^{k-1}\beta_1^m + \beta_1^k y_{t-k} + \sum_{m=0}^{k-1}\beta_1^m A_{t-m-1},$$

$y_t = \mathrm{Ln}\,\sigma_t^2$ and $A_t = \beta_2\xi_t + \beta_3(|\xi_t| - \sqrt{2/\pi})$. The unconditional expectation of $y_t$ follows from the limit $k \to \infty$,

$$\text{(A-21)} \qquad y \overset{k\to\infty}{\to} \frac{\beta_0}{1-\beta_1} + \sum_{m=0}^{\infty}\beta_1^m\left[\beta_2\xi_{t-m-1} + \beta_3\left(|\xi_{t-m-1}| - \sqrt{\frac{2}{\pi}}\right)\right],$$

since $\sum_{m=0}^{k-1}\beta_1^m = ((1-\beta_1^k)/(1-\beta_1)) \overset{k\to\infty}{\to} (1/(1-\beta_1))$ for $|\beta_1| < 1$.

From (A-21), an expression for $E[\sigma_t^2]$ can be derived,

$$\text{(A-22)} \qquad \bar{\sigma}^2 = E\left[\sigma_t^2\right] = E\left[e^y\right] = e^{\frac{\beta_0}{1-\beta_1}}E\left[\exp\left[\sum_{m=0}^{\infty}\beta_1^m \times \left(\beta_2\xi_{t-m-1} + \beta_3\left(|\xi_{t-m-1}| - \sqrt{\frac{2}{\pi}}\right)\right)\right]\right].$$

With Relation (A-16), one can evaluate (A-22) to Expression (A-19),

$$\text{(A-23)} \qquad \bar{\sigma}^2 = \exp\left[\frac{\beta_0 - \beta_3\sqrt{\frac{2}{\pi}}}{1-\beta_1} + \frac{1}{2}\frac{\left(\beta_2^2+\beta_3^2\right)}{1-\beta_1^2}\right] \times \prod_{m=0}^{\infty}\left[F_m(\beta_1,\beta_2,\beta_3) + F_m(\beta_1,-\beta_2,\beta_3)\right]. \quad \square$$

Combining Expression (A-19) with Relationship (A-18) for $\bar{\sigma}^2$, one obtains,

$$\text{(A-24)} \qquad T\left[\mathrm{Ln}\,\sigma_{\mathrm{Av}}^2(t,T) - \mathrm{Ln}\,\bar{\sigma}^2\right] = \left(\frac{\mathrm{Ln}\,\sigma_{t+1}^2 + \left(\beta_3\sqrt{\frac{2}{\pi}} - \beta_0\right)}{1-\beta_1}\right.$$

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$$\times \sum_{k=1}^{T} \beta_1^{k-1} - \frac{\frac{1}{2}\left(\beta_2^2+\beta_3^2\right)}{1-\beta_1^2} \sum_{k=1}^{T} \beta_1^{2(k-1)}$$
$$+ \sum_{k=1}^{T-1} \left[ \sum_{m=0}^{k-1} \mathrm{Ln}\left[F_m\left(\beta_1,\beta_2,\beta_3\right)+F_m\left(\beta_1,-\beta_2,\beta_3\right)\right] \right.$$
$$\left. - C\left(\beta_1,\beta_2,\beta_3\right) \right] - C\left(\beta_1,\beta_2,\beta_3\right),$$

where $C(\beta_1,\beta_2,\beta_3) = \sum_{m=0}^{\infty} \mathrm{Ln}\left[F_m(\beta_1,\beta_2,\beta_3)+F_m(\beta_1,-\beta_2,\beta_3)\right]$, which is a convergent sequence if $|\beta_1| < 1$. Evaluating the sums over the index-variable $k$ one obtains,

$$\text{(A-25)} \qquad T\left[\mathrm{Ln}\,\sigma_{\mathrm{Av}}^2(t,T) - \mathrm{Ln}\,\bar{\sigma}^2\right] = \left(\mathrm{Ln}\,\sigma_{t+1}^2 + \frac{\left(\beta_3\sqrt{\frac{2}{\pi}}-\beta_0\right)}{1-\beta_1}\right)$$
$$\times \frac{1-\beta_1^T}{1-\beta_1} - \frac{\frac{1}{2}\left(\beta_2^2+\beta_3^2\right)}{1-\beta_1^2}\frac{1-\beta_1^{2T}}{1-\beta_1^2}$$
$$+ \sum_{k=1}^{T-1} \left[ \sum_{m=0}^{k-1} \mathrm{Ln}\left[F_m\left(\beta_1,\beta_2,\beta_3\right)+F_m\left(\beta_1,-\beta_2,\beta_3\right)\right] \right.$$
$$\left. - C\left(\beta_1,\beta_2,\beta_3\right) \right] - C\left(\beta_1,\beta_2,\beta_3\right).$$

This expression can be written in the following form,

$$\text{(A-26)} \qquad \mathrm{Ln}\,\sigma_{\mathrm{Av}}^2(t,T) = \mathrm{Ln}\,\bar{\sigma}^2 + \frac{1}{T}\left[\left(1-\beta_1^T\right)f(t)+g(T)\right],$$

$$\text{(A-27)} \qquad f(t) = \left(\mathrm{Ln}\,\sigma_{t+1}^2 + \frac{\left(\beta_3\sqrt{\frac{2}{\pi}}-\beta_0\right)}{1-\beta_1}\right)\frac{1}{1-\beta_1},$$

and

$$\text{(A-28)} \qquad g(T) = -\frac{\frac{1}{2}\left(\beta_2^2+\beta_3^2\right)}{1-\beta_1^2}\frac{1-\beta_1^{2T}}{1-\beta_1^2}$$
$$+ \sum_{k=1}^{T-1} \left[ \sum_{m=0}^{k-1} \mathrm{Ln}\left[F_m\left(\beta_1,\beta_2,\beta_3\right)+F_m\left(\beta_1,-\beta_2,\beta_3\right)\right] \right.$$
$$\left. - C\left(\beta_1,\beta_2,\beta_3\right) \right] - C\left(\beta_1,\beta_2,\beta_3\right).$$

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