**PROJECT 2**

Project 2: Graph Algorithms and Related Data Structures

Singles-source shortest path algorithm, Minimum Spanning Tree (MST), and Strongly Connected Components (SSCs)

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**Project Overview**

This project includes Implementation and analysis of below algorithms

1. Single source shortest path Algorithm (Dijkstra’s)
2. Minimum Spanning Tree Algorithm (Prim’s)
3. Strongly Connected Components.

Along with implementation and analysis, Pseudocode, Data structures used and examples have also been included.

**Dijkstra’s Single source shortest path algorithm**

This algorithm is used to compute shortest distance of all the vertices from the given vertex. At every step, we add the vertex in the set/cloud with minimum distance from the given node.

**Pseudo code:**

**Algorithm** Dijkstra’s:

G🡨 graph

S🡨 source

Let PQ be the Min-heap

Let dist be the dictionary for tracking costs

Let pred be the dictionary for tracking predecessor

PQ.push(0,S)

For all v in G:

dist[v]= infinity

pred[v]=null

dist[S]=0

While PQ is not empty:

node= heappop(pq) //return vertex having minimum cost

for each neighbour v for node:

if dist[v]>dist[u]+weight[u,v]

dist[v]=dist[u]+weight[u,v]

pred[v]=u

heappush(pq,dist[v],v)

**Data Structures used:**

* **Dictionary**:

1. For storing graph in adjacency list format, dictionary is used where key is node /sources and it will contain pairs of destination and edge weights as values.

For example, If there is an edge from source A to destination B and has weight 2 and edge to destination C with weight 3 then in the dictionary it would be stored as {‘A’: [‘B’,2], [‘C’,3]}.

1. For storing costs/distance for each node
2. For storing predecessor for each node

* **Min-Heap**

1. Min heap is used to store node and its cost. This is used to get the node having minimum distance from the source.

**Detailed analysis of runtime:**

1. For initialization of the graph the required complexity is O(V) where V is the number of vertices. Here we set the distance of each vertex to infinity and predecessor of each vertex to Null.
2. We have used min heap based priority queue and time complexity to build(add element to heap and perform heapify operation)the min heap is O(log V).
3. The heappop operation removes the minimum element from the heap. Each extraction takes log V and this would be performed until all the vertices are removed from the heap and hence its complexity would be O(V log V).
4. The relax operation (distance operation) is performed for all the adjacent nodes i.e. E times (number of edges). Each operation takes O(log V) and we do it for each adjacent edge hence its complexity is O(E log V)
5. Therefore, total time complexity is O (V log V) + O (E log V). Hence total time complexity is **O((V+E) log V)** where V is number of vertices and E is number of edges. **If all the vertices are reachable from source the time complexity would be O(E log V)**

**Minimum Spanning Tree algorithm using Prim’s**

We have to find a spanning tree with minimized total weight from given weighted undirected graph.

**Pseudo code:**

**Algorithm** Prims:

G🡨 graph

S🡨 source

Let PQ be the Min-heap

Let keys be the dictionary for tracking weight

Let pred be the dictionary for tracking predecessor

Let visited be the dictionary for tracking if node is visited.

PQ.push(0,S)

For all v in G:

keys[v]= infinity

pred[v]=null

visited[v]=false

keys[S]=0

While PQ is not empty:

node= heappop(pq) //return vertex having minimum cost

visited[node]=true

for each neighbour v for node:

if visited[v]=false AND keys[v]>weight(u,v):

keys[v]=weight(u,v)

heappush(pq,(weight(u,v),v))

pred[v]=u

**Data Structures used:**

* **Dictionary**:

1. For storing graph in adjacency list format, dictionary is used where key is node /sources and it will contain pairs of destination and edge weights as values.

For example, If there is an edge from source A to destination B and has weight 2 and edge to destination C with weight 3 then in the dictionary it would be stored as {‘A’: [‘B’,2], [‘C’,3]}

1. For storing weights for each node
2. For storing predecessor for each node
3. To check if node is visited or not

* **Min-Heap**

1. Min heap is used to store node and its cost. This is used to get the node having minimum distance from the source.

**Run time analysis:**

1. For initialization of the graph the required complexity is O(V) where V is the number of vertices. Here we set the distance of each vertex to infinity and predecessor of each vertex to Null.
2. We have used min heap based priority queue and time complexity to build (add element to heap and perform heapify operation the min heap is O(log V).
3. The heappop operation removes the minimum element from the heap. Each extraction takes log V and this would be performed until all the vertices are removed from the heap and hence its complexity would be O(V log V).
4. The key/weight update operation is performed for all the adjacent nodes i.e. E times (number of edges). Each operation takes O(log V) and we do it for each adjacent edge hence its complexity is O(E log V)
5. Therefore, total time complexity is O (V log V) + O (E log V). Hence total time complexity is **O((V+E) log V)** where V is number of vertices and E is number of edges.

Overall complexity will be **O (E log v)**

**Strongly Connected Components**

In Strongly connected components, we find maximal subgraphs such a way that each vertex reaches all other vertices. We use Depth First search to get the result. Then Transpose on the graph is taken and DFS is applied on Transposed graph in the order of finished times found in first DFS.

**Pseudo Code**

**Algorithm** Strongly\_connected\_components:

Let G🡨 graph

Let visited be the dictionary to check if node is visited

Let S be the stack to arrange nodes according to finish time

Dfs\_main(graph) //function call for DFS on G

Let G’ be the graph with reversed edges

Let res be the nodes connected to given node

Let final be the list to store strongly connected components

Let reverse\_visited be the dictionary to check if node in G’ is visited

While S not empty:

vertex=S.pop()

If reverse\_visited[vertex ]!=1:

scc\_dfs(vertex) //function call for DFS on G’

final.append(res)

res=[]

return final

Function scc\_dfs(Vertex v):

reverse\_visited[v]=1

res.append(v)

for all neighbours node in G’[v]:

if(reverse\_visited[node]!=1)

scc\_dfs(node)

Function DFS\_main(Graph graph):

For v in graph:

If visited[v]!=1:

Dfs\_visit(v)

Function DFS\_visit(Vetex v):

Visited[v]=1

For each neighbour node in v:

If(visited[node] ! =1):

DFS\_visit(node)

S.push(node)

**Data Structures used:**

* Dictionary:

1. For storing graph in adjacency list format, dictionary is used where key is node /sources and it will contain pairs of destination and edge weights as values.

For example, If there is an edge from source A to destination B and has weight 2 and edge to destination C with weight 3 then in the dictionary it would be stored as {‘A’: [‘B’,2], [‘C’,3]}

1. To check if nodes are visited

* Stack: Stack is used to store the nodes according to their finished times.
* List: List is used to store the strongly connected nodes.

**Run-Time Analysis:**

* Depth First search’s time complexity is O(E+V) where V is number of vertices and E is number of Edges. Transpose of the graph will also take O(E+V). Stack’s push and pop operation takes O (1). As Strongly connected Component involves DFS and transpose and again DFS, ultimately time complexity will be O(E+V) where E are the edges and V is vertices.

**Steps to Execute the code:**

1. Go to the folder where the files are located which have the code and graph inputs text file.
2. Run files (dijkstra.py, prims.py and strongly\_connected\_components.py) individually using python3 filename.py.
3. After running each of the file, it will display the menu to select which graph to run the algorithm on. The graphs are pre-defined in the files.
4. Files undirectedGraph1.txt, undirectedGraph2.txt, undirectedGraph3.txt, undirectedGraph4 has undirected graph inputs. Files directedGraph1, directedGraph2 has directed graph inputs. scc\_1.txt, scc\_2.txt, scc\_3.txt and scc\_2 has files for strongly connected components.
5. In graph files first line has the graph information where first number is number of vertices, second number is number of edges and third is ‘u‘or ‘d’ which specifies if the graph is directed or undirected. The last line has the source node. The other lines specify source, destination and weight between the nodes.
6. Menu’s have been given as per the requirements for example strongly connected components only directed graphs is used.

**Sample Input and Outputs**

1. **Dijkstra’s Single Source shortest path algorithm**

**Input**:

10 20 d

1 2 5

2 5 6

5 8 2

8 10 4

10 9 7

9 6 8

6 3 1

3 1 4

1 4 3

2 3 15

4 6 6

4 5 7

4 7 1

7 6 8

7 9 7

7 10 9

7 8 12

7 5 11

2 8 9

6 10 5

1

**Output**

Path (1 —> 2): Minimum cost = 5, Route = ['1', '2']

Path (1 —> 5): Minimum cost = 10, Route = ['1', '4', '5']

Path (1 —> 8): Minimum cost = 12, Route = ['1', '4', '5', '8']

Path (1 —> 10): Minimum cost = 13, Route = ['1', '4', '7', '10']

Path (1 —> 9): Minimum cost = 11, Route = ['1', '4', '7', '9']

Path (1 —> 6): Minimum cost = 9, Route = ['1', '4', '6']

Path (1 —> 3): Minimum cost = 10, Route = ['1', '4', '6', '3']

Path (1 —> 4): Minimum cost = 3, Route = ['1', '4']

Path (1 —> 7): Minimum cost = 4, Route = ['1', '4', '7']

2) **Minimum Spanning Tree Algorithm (Prim’s)**

**Input**

10 14 U

0 5 343

1 6 954

1 5 879

2 5 1054

2 4 1364

3 6 433

4 5 1106

8 0 464

0 7 1435

1 7 811

6 7 837

4 9 766

3 9 1053

1 9 524

1

**Output**

Edge : ( 5 , 0 ) Weight: 343

Edge : ( 1 , 5 ) Weight: 879

Edge : ( 7 , 6 ) Weight: 837

Edge : ( 5 , 2 ) Weight: 1054

Edge : ( 9 , 4 ) Weight: 766

Edge : ( 6 , 3 ) Weight: 433

Edge : ( 0 , 8 ) Weight: 464

Edge : ( 1 , 7 ) Weight: 811

Edge : ( 1 , 9 ) Weight: 524

Total Cost of minimum spanning tree is: 6111

3) **Strongly Connected Components**

**Input**:

12 17 D

A B 1

B C 2

B E 4

B D 3

C F 2

E F 6

E B 8

E G 3

F C 1

F H 7

G H 3

G J 2

H K 4

I G 3

J I 1

K L 3

L J 2

L

**Output**:

The strong connected components are: [['A'], ['B', 'E'], ['D'], ['C', 'F'], ['H', 'G', 'I', 'J', 'L', 'K']]

**Output Screens**

1. Sample output screens are available in output screens folder.