1. A robot singularity is a configuration in which the robot end-effector becomes blocked in certain directions.

The three types of singularities are defined by which joint alignment cause the problem:

1. Wrist singularities - These happen when two of the robot wrist axes line up with each other. This can cause these joints to try and spin 180 degrees instantaneously.
2. Shoulder singularities - These happen when the centre of the robot's wrist aligns with the axis of 1st joint. The first and last joint of robot line up with each other.
3. Elbow singularities - These happen when the centre of the robot's wrist lies in the same plane as joints 2 and 3. It looks like the robot has "reached too far", causing the elbow to lock in position.

$$v = J(\theta)\dot{\theta}, \quad \tau = J^T(\theta)F$$

$J(\theta) \in R^{6 \times n}$, where n = no. of joints.

$$\text{rank } J(\theta) \leq \min(6, n)$$

It is full rank if rank of $J(\theta) = \min(6, n)$

It is singular at θ^* if rank $J(\theta^*) < \max_{\theta}$ rank $J(\theta)$.

If $n < 6$: tall, kinematically deficient Jacobian

$n = 6$: square

$n > 6$: fat, redundant Jacobian

5c

link	a_i	α_i	d_i	θ_i
1	0	$90°$	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	$-90°$	0	θ_4
5	0	0	0	θ_5
6	0	0	d_6	θ_6

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 1 & d_6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} C_1[C_5C_6C_{234} - S_6S_{234}] - S_1S_5C_6 & -C_1[C_5S_6C_{234} + C_6S_{234}] + S_1S_5S_6 & C_1S_5C_{234} + S_1C_5 & d_x \\ \cancel{C_1S_5C_{234}+} \; C_1S_5S_6 + S_1C_5C_6C_{234} - S_1S_6S_{234} & -C_1S_5S_6 - S_1C_5S_6C_{234} & -C_1C_5 + S_1S_5C_{234} & d_y \\ S_6C_{234} + C_5S_6S_{234} & C_6S_{234} - C_5S_6S_{234} & S_5S_{234} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

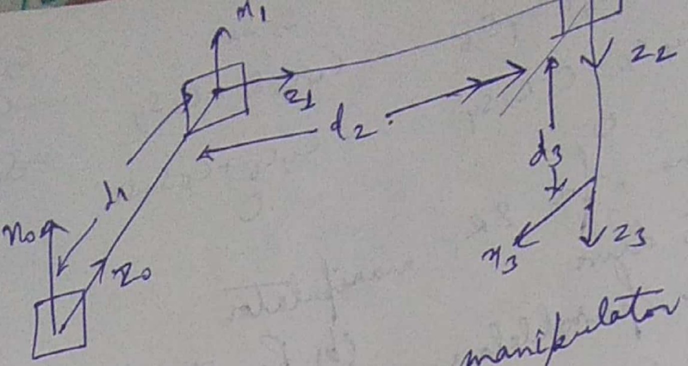
$$d_x = a_2C_1C_2 + a_3C_1C_{23} + d_6[C_1S_5C_{234} + S_1C_5]$$

$$d_y = a_2S_1C_2 + a_3S_1C_{23} - d_6[C_1C_5 + S_1S_5C_{234}]$$

$$d_z = a_2S_2 + a_3C_2S_{23} + d_6S_5S_{234}$$

$$\therefore T_0^6 = \begin{bmatrix} -C_6S_5 & S_5S_6 & C_5 & d_3 + d_6C_5 \\ -C_4C_5C_6 + S_4S_6 & C_4C_5S_6 + C_6S_4 & -C_4S_5 & d_2 - d_6C_4S_5 \\ -C_4S_6 - C_5C_6S_4 & -C_4C_6 + C_5S_4S_6 & -S_4S_5 & d_2 - d_6S_4S_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6.



DH parameters for Cartesian manipulator

link	a_i	α_i	d_i	θ_i
1	0	$-90°$	d_1	0
2	0	$90°$	d_2	$90°$
3	0	0	d_3	$-90°$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1 A_2 A_3 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Attaching a spherical wrist to the robot gives

$$T_0^5 = T_0^3 \; T_3^6$$

The matrix T_0^3 is given above.

$$T_6^3 = A_4 A_5 A_6 = \begin{bmatrix} R_6^3 & O_6^3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_4 C_5 C_6 - S_4 S_6 & -C_4 C_5 S_6 - S_4 C_6 & C_4 S_5 & C_4 S_5 d_6 \\ S_4 C_5 C_6 + C_4 S_6 & -S_4 C_5 S_6 + C_4 C_6 & S_4 S_5 & S_4 S_5 d_6 \\ -S_5 C_6 & S_5 S_6 & C_5 & C_5 d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} -C_6 S_5 & S_5 S_6 & C_5 & d_3 + d_6 C_5 \\ -C_4 C_5 C_6 + S_4 S_6 & C_4 C_5 S_6 + C_6 S_4 & -C_4 S_5 & d_2 - d_6 C_4 S_5 \\ -C_4 S_6 - C_5 C_6 S_4 & -C_4 C_6 + C_5 S_4 S_6 & -S_4 S_5 & d_1 - d_6 S_4 S_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

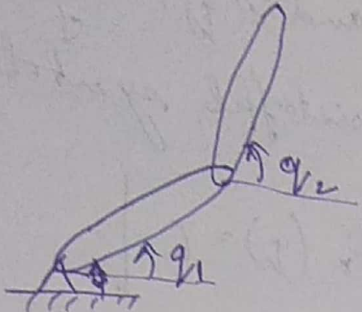
7. Configuration for 2R manipulator
(a) Direct drive (b) Remotely driven
(c) 5-bar parallelogram arrangement.

(a) Direct drive robots configuration eliminate the amplification of inertia and mechanical backlash associated with other drives. Joint angle position sensors, required for real-time servo-level control, are generally considered an important part of the drive train. Less often, velocity feedback sensors are provided.

(b) Manipulators with remotely actuated joints are used. Suited for medical robotic systems for which it is desirable to have all active components away from the area a procedure is performed. Small size industrial motor driven robots is another familiar example. It is convenient to have the motors located at the base of the robot for compactness reasons.

(c) For some special purpose applications, parallel-link robots are more suitable than serial link robot. These robots generally have three or six links in parallel, each link attached to a fix base and to a moving working platform. With proper design, a five link parallelogram manipulator can have five degrees of freedom motion of the working platform.

8.



$$v_{c_1} = \begin{bmatrix} -\frac{l_1}{2} sq_1 & 0 \\ \frac{l_2}{2} sq_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix} \qquad \omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot q_1 \end{bmatrix}$$

$$v_{c_2} = \begin{bmatrix} -l_1 sq_1 & -\frac{l_2}{2} sq_2 \\ l_1 cq_1 & \frac{l_2}{2} cq_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix} \qquad \omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot q_2 \end{bmatrix}$$

$$K = \frac{1}{2} \sum_{i=1}^{2} m_i v_{c_i}^T v_{c_i} + \frac{1}{2} \sum_{j=1}^{2} \omega_i^T I_i \omega_i = \frac{1}{2} \dot q^T D \dot q$$

$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_2^2 + I_1 & m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) \\ m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix}$$

Potential energy, $V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$

$$\phi_k(q) = \left[\frac{\partial V}{\partial q_1}\right] = m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1$$

$$\left[\frac{\partial V}{\partial q_2}\right] = m_2 g \frac{l_2}{2} \cos q_2$$

$$\left[\frac{\partial V}{\partial q}\right] = \begin{bmatrix} m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1 \\ m_2 g \frac{l_2}{2} \cos q_2 \end{bmatrix}$$

c_{112} & $c_{221} \neq 0$

Rest all 6 c_{ij} are zero.

10. When ~~you~~ we are already provided $D(q)$ and $V(q)$, we just need to calculate C_{ijk} i.e. Christoffel symbol of first kind.

$$\boxed{\sum_j d_{kj}\ddot{q}_j + \sum_{i,j} C_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau}$$

Equation of motion

$$\phi_k = \frac{\partial V}{\partial q_k}$$

$$\boxed{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau} \quad \text{—— (1)}$$

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right].$$

where k is the no. of joints.

For 2 joints, $i \to (1,2)$, $j \to (1,2)$ and $k\,(1,2)$

There are 8 components of C_{ijk}

$$\sum_{i,j,k=1}^{2} C_{ijk}$$

Hence all the values in eq^n (1) are obtained.

12.

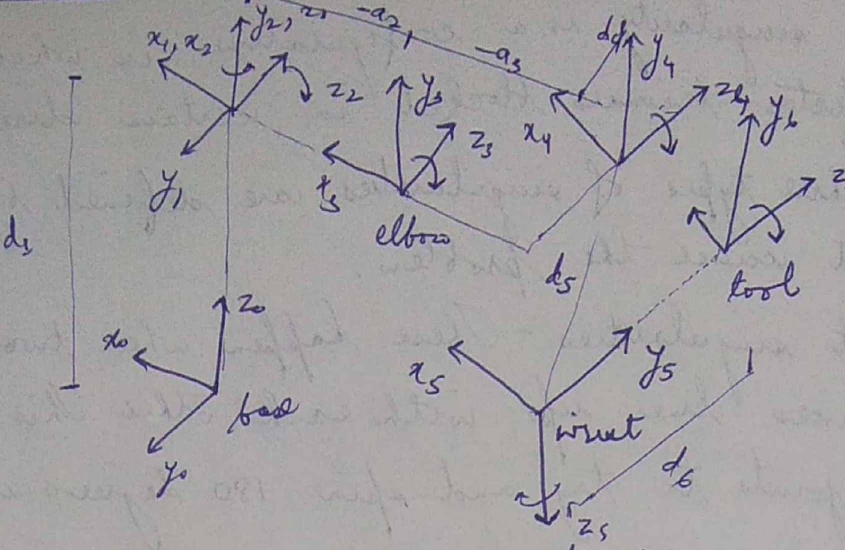


Fig: The UR5 robot in zero position:

The DH parameters are specified as:-

a_i = distance from z_i to z_{i+1} measured along X_i

α_i = angle from z_i to z_{i+1} measured along X_i.

d_i = distance from X_{i-1} to X_i measured along z_i

θ_i = angle from X_{i-1} to X_i measured about z_i

The DH parameters are:

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	d_1	θ_1
2	90°	0	0	θ_2
3	0	a_2	0	θ_3
4	0	a_3	d_4	θ_4
5	90°	0	d_5	θ_5
6	-90°	0	d_6	θ_6.