Printed Pages:04 **Sub Code: BAS-103**

Paper Id: | 233418 | **Roll No.** | | | | | | | | | | |

B. TECH.
(SEM I) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS I

Time: 3 Hours ***Total Marks: 70***

समयः 03 घण्टे पूर्णांकः 70

Note:

1. Attempt all Sections. If require any missing data; then choose suitably.

2. The question paper may be answered in Hindi Language, English Language or in the mixed language of Hindi and English, as per convenience.

नोटः 1. सभी प्रश्नो का उत्तर दीजिए। किसी प्रश्न में, आवश्यक डेटा का उल्लेख न होने की स्थिति में उपयुक्त डेटा स्वतः मानकर प्रश्न को हल करें।

2. प्रश्नों का उत्तर देने हेतु सुविधानुसार हिन्दी भाषा, अंग्रेजी भाषा अथवा हिंदी एवं अंग्रेजी की मिश्रित भाषा का प्रयोग किया जा सकता है।

SECTION A

1. **Attempt *all* questions in brief.** **2 x 7 = 14**

निम्न सभी प्रश्नों का संक्षेप में उत्तर दीजिए।

a. If A is a Hermitian matrix, then show that iA is Skew-Hermitian matrix.
यदि A एक हर्मिटियन (Hermitian) मैट्रिक्स है, तो दिखाएँ कि iA यह स्क्यू-हर्मिटियन (Skew-Hermitian) मैट्रिक्स है।

b. Find the eigen value of the matrix $A=\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix}51\\ 51\end{bmatrix}$.
आइजेन मान $\begin{bmatrix}51\\ 51\end{bmatrix}$ के संगत मैट्रिक्स $A=\begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix}$ का आइजेन वेक्टर ज्ञात करें।

c. If $y=\cos^{-1}x$, prove that $(1-x^2)y_2-xy_1=0$.
यदि, $y=\cos^{-1}x$ तो सिद्ध कीजिए कि $(1-x^2)y_2-xy_1=0$.

d. If $u=\sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{5}{2}\tan u$.
यदि, $u=\sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$ तो सिद्ध कीजिए कि $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{5}{2}\tan u$.

e. Find the percentage error in measuring the volume of a rectangular box when the error of 1% is made in measuring each side.
यदि प्रत्येक भुजा को मापने में 1% की त्रुटि होती है तो एक आयताकार बॉक्स के आयतन को मापने में कितनी प्रतिशत त्रुटि होगी?

f. Evaluate $\iint y\,dxdy$ over the part of the plane bounded by the line $y=x$ and the parabola $y=4x-x^2$.
रेखा $y=x$ और परवलय $y=4x-x^2$ से घिरे क्षेत्र के भाग के लिए $\iint y\,dxdy$ की गणना कीजिए।

g. Find curl of a vector field given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$.

$\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$ द्वारा परिभाषित वेक्टर फ़ील्ड $\vec{F}$ का कर्ल (curl) ज्ञात करें।

SECTION B

2. Attempt any *three* of the following: **7 x 3 = 21**

निम्न में से किसी तीन प्रश्नों का उत्तर दीजिए।

a. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find its inverse.

मैट्रिक्स $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ के लिए केली-हैमिल्टन प्रमेय को सत्यापित करें और इसका व्युत्क्रम ज्ञात करें।

b. If $y\sqrt{x^2 - 1} = \log_e(x + \sqrt{x^2 - 1})$, prove that
$(x^2 - 1)y_{n+1} + (2n+1)xy_n + n^2 y_{n-1} = 0$.

यदि $y\sqrt{x^2 - 1} = \log_e(x + \sqrt{x^2 - 1})$, तो सिद्ध कीजिए कि
$(x^2 - 1)y_{n+1} + (2n+1)xy_n + n^2 y_{n-1} = 0$.

c. Expand $f(x, y) = y^x$ about $(1,1)$ up to second degree terms and hence evaluate $(1.02)^{1.03}$.

$(1,1)$ के सापेक्ष $f(x, y) = y^x$ का द्वितीय डिग्री के पदों तक विस्तार करें और तदोपरान्त $(1.02)^{1.03}$ की गणना कीजिए।

d. Evaluate the double integral $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{(y^4 - a^2x^2)}}\,dxdy$ by changing the order of integration.

समाकलन के क्रम को बदलकर डबल इंटीग्रल $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{(y^4 - a^2x^2)}}\,dxdy$ का मान ज्ञात कीजिए।

e. Find the directional derivative of scalar function $f(x, y, z) = xyz$ at point $P(1,1,3)$ in the direction of the outward drawn normal to the sphere $x^2 + y^2 + z^2 = 11$ through the point P.

गोले $x^2 + y^2 + z^2 = 11$ पर बिंदु P से गुजरते हुए बाहर की ओर खीचें गये नार्मल की दिशा में अदिश फलन $f(x, y, z) = xyz$ का बिन्दु $P(1,1,3)$ पर दिशात्मक अवकलज (directional derivative) ज्ञात कीजिए।

SECTION C

3. Attempt any *one* part of the following: **7 x 1 = 7**

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Test the consistency for the following system of equations and if system is consistent, solve them:

समीकरणों की निम्नलिखित निकाय के लिए संगतता (consistency) का परीक्षण करें और यदि निकाय सुसंगत है, तो उन्हें हल करें:

$$x+y+z=6,$$
$$x+2y+3z=14,$$
$$x+4y+7z=30.$$

(b) Find the eigen values and corresponding eigen vectors of the matrix ***A*.**

मैट्रिक्स A के आइजेन मान और संगत आइजेन वेक्टर ज्ञात कीजिए।

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}.$$

4. Attempt any *one* part of the following: 7 x 1 = 7

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Trace the curve $x^2y^2=(a^2+y^2)(a^2-y^2)$ in xy-plane, where a is constant.

xy-तल में वक्र $x^2y^2=(a^2+y^2)(a^2-y^2)$ जहां a एक नियतांक है, का अनुरेखण करें।

(b) If $u=\dfrac{x^2y^2}{x^2+y^2}+\cos\left(\dfrac{xy}{x^2+y^2}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\frac{x^2y^2}{x^2+y^2}.$$

यदि $u=\dfrac{x^2y^2}{x^2+y^2}+\cos\left(\dfrac{xy}{x^2+y^2}\right)$ तो सिद्ध कीजिए कि:

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\frac{x^2y^2}{x^2+y^2}$$

5. Attempt any *one* part of the following: 7 x 1 = 7

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Find the Jacobian of the functions $y_1=(x_1-x_2)(x_2+x_3)$, $y_2=(x_1+x_2)(x_2-x_3)$, $y_3=x_2(x_1-x_3)$, hence show that the functions are not independent. Find the relation between them.

फलन $y_1=(x_1-x_2)(x_2+x_3)$, फलन $y_2=(x_1+x_2)(x_2-x_3)$, फलन $y_3=x_2(x_1-x_3)$, का जेकोबियन (Jacobian) ज्ञात कीजिए। दिखाएं कि फलन स्वतंत्र नहीं हैं। उनके बीच संबंध ज्ञात कीजिए।

(b) A rectangular box, which is open at the top, has a capacity of 32 cubic feet. Determine, using Lagrange's method of multipliers, the dimensions of the box such that the least material is required for the construction of the box.

एक आयताकार बॉक्स, जो शीर्ष पर खुला है, की क्षमता 32 घन फीट है। लाग्रेंज की मल्टीप्लायर विधि (Lagrange's method of multipliers) का उपयोग करते हुए, बॉक्स के आयामों का इसप्रकार निर्धारण करें कि बॉक्स के निर्माण के लिए कम से कम सामग्री की आवश्यकता हो।

6. Attempt any *one* part of the following: 7 x 1 = 7

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Evaluate $\iiint_R (x-2y+z)\,dz\,dy\,dx,$ where R is the region determined by $0 \le x \le 1, 0 \le y \le x^2, 0 \le z \le x+y$.
$\iiint_R (x-2y+z)\,dz\,dy\,dx,$ को ज्ञात कीजिए, जहां R क्षेत्र $0 \le x \le 1, 0 \le y \le x^2, 0 \le z \le x+y$ द्वारा निर्धारित है।

(b) Use Dirichlet's integral to evaluate $\iiint xyz\,dx\,dy\,dz$ throughout the volume bounded by $x=0, y=0, z=0$ and $x+y+z=1$.
Dirichlet's integral की सहायता से $x=0, y=0, z=0$ और $x+y+z=1$ से घिरे हुए आयतन के लिए $\iiint xyz\,dx\,dy\,dz$ को ज्ञात कीजिए।

7. Attempt any *one* part of the following: **7 x 1 = 7**
निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Apply Gauss divergence theorem to evaluate $\iint_S \vec{F}.\hat{n}\,ds$, where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface of the region bounded by the cylinder $x^2+y^2=4, z=0, z=3$.
गॉस डाइवर्जेंस प्रमेय (Gauss divergence theorem) का प्रयोग करते हुए $\iint_S \vec{F}.\hat{n}\,ds$ आकलन कीजिए, जहां $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ और S, बेलन $x^2+y^2=4, z=0$ $z=3$. से घिरे क्षेत्र, की सतह है ।

(b) Evaluate $\oint_C \vec{F}.\vec{dr}$ by Stoke's theorem, where $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ and C is the boundary of the triangle with vertices at $(0,0,0), (1,0,0)$ and $(1,1,0)$.
स्टोक के प्रमेय द्वारा $\oint_C \vec{F}.\vec{dr}$ का आकलन कीजिए, जहां $\vec{F} = y^2\hat{i} + x^2\hat{j} - (x+z)\hat{k}$ और C ऐसे त्रिभुज की सीमा है जिसके शीर्ष $(0,0,0), (1,0,0)$ और $(1,1,0)$ है।

Subject Code: KAS103T

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Roll No:

BTECH
(SEM I) THEORY EXAMINATION 2021-22
ENGINEERING MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Notes:

- *Attempt all Sections and Assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief	Marks**(10X2=20)**
Q1(a)	If the matrix A= $\begin{bmatrix}-1 & 2 & 3\\ 0 & 3 & 5\\ 0 & 0 & -2\end{bmatrix}$, then find the eigen value of $A^3+5A+8I$.	1
Q1(b)	Reduce the matrix $\begin{bmatrix}1 & 1 & 1\\ 3 & 1 & 1\end{bmatrix}$ into normal form and find its rank.	1
Q1(c)	Find the envelope of the family of straight line $y=mx+\frac{a}{m}$, where m is a parameter.	2
Q1(d)	Can mean value theorem be applied to $f(x)=\tan x$ in $[0,\pi]$.	2
Q1(e)	State Euler's Theorem on homogeneous function.	3
Q1(f)	Find the critical points of the function $f(x,y)=x^3+y^3-3axy$.	3
Q1(g)	Find the area bounded by curve $y^2= \ x \ and \ x^2=y$.	4
Q1(h)	Find the value of $\int_0^1\int_0^x\int_0^{x+y} dx\,dy\,dz$.	4
Q1(i)	Find a unit normal vector to the surface $z^2=x^2+y^2$ at the point $(1,0,-1)$.	5
Q1(j)	State Stoke's Theorem.	5

SECTION-B	Attempt **ANY THREE**of the following Questions	Marks**(3X10=30)**
Q2(a)	Find the characteristic equation of the matrix A= $\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$, compute A^{-1} and prove that $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I=\begin{bmatrix}8 & 5 & 5\\ 0 & 3 & 0\\ 5 & 5 & 8\end{bmatrix}$.	1
Q2(b)	State Rolle's theorem and verify Rolle's theorem for the function $f(x)=\frac{sinx}{e^x} \ in \ [0,\pi]$.	2
Q2(c)	If u, v and w are the roots of $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$, cubic in λ, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	3
Q2(d)	Find the volume bounded by the cylinder $x^2+y^2=4$ and the plane $y+z=4$ and z = 0.	4
Q2(e)	Apply Green's theorem to evaluate $\int_C [(2x^2-y^2)dx + (x^2+y^2)dy]$,where C is the boundary of the area enclosed by the x-axis and the upper half of the circle $x^2+y^2=a^2$.	5

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q3(a)	Find the value of k for which the system of eqations $(3k-8)x + 3y+3z = 0$, $3x + (3k-8)y+3z = 0$, $3x+3y+(3k-8)z=0$ has a non-trivial solution.	1
Q3(b)	Find the eigen values and eigen vectors of matrix A= $\begin{bmatrix}2 & 1 & 1\\ 2 & 3 & 2\\ 3 & 3 & 4\end{bmatrix}$.	1

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ENGINEERING MATHEMATICS-I

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	
Q4(a)	If $f(x) = \frac{x}{1+e^{\frac{1}{x}}}$; $x \neq 0$ and $f(0) = 0$, then show that the function is continuous but not differentiable at $x = 0$.		2
Q4(b)	If $y = \left(x + \sqrt{1+x^2}\right)^m$, find $y_n(0)$.		2

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	
Q5(a)	Expand x^y in powers of $(x-1)$ and $(y-1)$ up to the third-degree terms and hence evaluate $(1.1)^{1.02}$.		3
Q5(b)	A rectangular box which is open at the top having capacity 32c.c. Find the dimension of the box such that the least material is required for its constructions.		3

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	
Q6(a)	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.		4
Q6(b)	Find the position of the C.G. of a semicircular lamina of radius, a if its density varies as the square of the distance from the diameter.		4

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**	
Q7(a)	Find the directional derivative of $\nabla(\nabla f)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$ where $f = 2x^3y^2z^4$.		5
Q7(b)	Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational and hence find function $\emptyset$ such that $\vec{F} = \nabla\emptyset$.		5

Subject Code: KAS103T

PAPER ID-311288

Roll No:

B.TECH
(SEM I) THEORY EXAMINATION 2020-21
ENGINEERING MATHEMATICS-I

Time: 3 Hours *Total Marks: 100*

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

Qno.	Question	Marks	CO
a.	Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.	2	1
b.	State Rank-Nullity Theorem.	2	1
c.	State Rolle's Theorem.	2	2
d.	Discuss all the symmetry of the curve $x^2y^2 = x^2 - a^2$	2	2
e.	If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	2	3
f.	If $x = e^v \sec u, y = e^v \tan u$, then evaluate $\frac{\partial(x,y)}{\partial(u,v)}$.	2	3
g.	Evaluate $\int_0^1 \int_0^{x^2} e^{y/x}\, dydx.$	2	4
h.	Calculate the volume of the solid bounded by the surface x = 0, y =0, x+y+z=1 and z=0.	2	4
i.	Show that the vector $\vec{V} = (x+3y)\hat{\imath} + (y-3z)\hat{\jmath} + (x-2z)\hat{k}$ is solenoidal.	2	5
j.	State Green's theorem.	2	5

SECTION B

2. Attempt any *three* of the following:

Qno.	Question	Marks	CO
a.	Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$	10	1
b.	If $y = e^{tan^{-1}x}$, prove that. $(1 + x^2)y_{n+2} + [(2n+2)x-1) y_{n+1} + n(n+1) y_n = 0$.	10	2
c.	If $u^3+v+w = x+y^2+z^2$, $u+v^3+w = x^2+y+z^2$, $u+v+w^3 = x^2+y^2+z$,Show that: $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1-4xy(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}$	10	3
d.	Evaluate by changing the variables, $\iint_R (x+y)^2\, dx\, dy$ where R is the region bounded by the parallelogram x+y=0, x+y =2, 3x-2y=0 and 3x-2y = 3.	10	4
e.	Use divergence theorem to evaluate the surface integral $\iint_S (xdydz + ydzdx + zdxdy)$ where S is the portion of the plane x+2y+3z=6 which lies in the first octant.	10	5

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SECTION C

3. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Find non-singular matrices P and Q such that PAQ is normal form. $\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 3\\ 0 & 1 & 1\end{bmatrix}$	10	1
b.	Find the eigen values and the corresponding eigen vectors of the following matrix. $A = \begin{bmatrix}2 & 0 & 1\\ 0 & 3 & 0\\ 1 & 0 & 2\end{bmatrix}$.	10	1

4. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Find the envelope of the family of lines $\frac{x}{a}+\frac{y}{b}=1$, where a and b are connected by the relation $a^n+b^n=c^n$	10	2
b.	If y = sin (m $\sin^{-1}$x), find the value of y_n at x= 0.	10	2

5. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Divide 24 into three parts such that continued product of first,square of second and cube of third is a maximum.	10	3
b.	If $u = sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$,prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\cot u$. Also evaluate $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$.	10	3

6. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Evaluate the following integral by changing the order of integration $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dydx$.	10	4
b.	A triangular thin plate with vertices (0,0),(2,0) and (2,4) has density $\rho = 1+x+y$. Then find: (i) The mass of the plate. (ii) The position of its centre of gravity G.	10	4

7. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	A fluid motion is given by $\vec{v}=(ysin\,z-sinx)\hat{\imath}+(x\,sin\,z+2yz)\hat{\jmath}+(xycos\,z+y^2)\hat{k}$.Is the motion irrotational? If so, find the velocity potential.	10	5
b.	Verify Stoke's theorem for the function $\vec{F}=x^2\hat{\imath}+xy\hat{\jmath}$ integrated round the square whose sides are x=0,y=0,x=a,y=a in the plane z=0.	10	5

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Paper Id: 199103 Roll No:

B. TECH.
(SEM I) THEORY EXAMINATION 2019-20
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions.

Q. No.	Question	Marks	CO
a.	Show that vectors (1, 6, 4), (0, 2, 3) and (0, 1, 2) are linearly independent.	2	1
b.	Define Lagrange's mean value theorem.	2	2
c.	If $u = x(1-y), v = xy$, find $\frac{\partial(u,v)}{\partial(x,y)}$.	2	3
d.	Show that vector $\vec{V} = (x+3y)\hat{\imath} + (y-3z)\hat{\jmath} + (x-2z)\hat{k}$ is solenoidal.	2	5
e.	Find the value of 'b' so that rank of $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & b \end{bmatrix}$ is 2.	2	1
f.	Evaluate $\int_0^2 \int_0^1 (x^2 + 3y^2) dy dx$.	2	4
g.	Find grad $\emptyset$ at the point (2, 1, 3) where $\emptyset = x^2 + yz$	2	5
h.	If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3
i.	Find $\frac{du}{dt}$ if $u = x^3 + y^3, x = a\cos t, y = b\sin t$.	2	3
j.	Find the area lying between the parabola $y = 4x - x^2$ and above the line $y = x$.	2	4

SECTION B

2. Attempt any *three* of the following:

Q. No.	Question	Marks	CO
a.	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ and hence find A^{-1}.	10	1
b.	If $y = e^{m\cos^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. Hence find y_n when $x = 0$.	10	2
c.	If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$.	10	3
d.	Evaluate the integral by changing the order of integration: $I = \int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$.	10	4
e.	Verify Stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)\hat{\imath} + 2xy\hat{\jmath}$ integrated round the rectangle in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a, y = b$.	10	5

SECTION C

Paper Id: 199103 Roll No:

3. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	For what values of λ and μ the system of linear equations: $x+y+z=6$, $x+2y+5z=10$, $2x+3y+\lambda z=\mu$ has (i) a unique solution (ii) no solution (iii) infinite solution. Also find the solution for $\lambda=2$ and $\mu=8$.	10	1
b.	Find the rank of the matrix $A=\begin{bmatrix}1 & 3 & 4 & 2\\ 2 & -1 & 3 & 2\\ 3 & -5 & 2 & 2\\ 6 & -3 & 8 & 6\end{bmatrix}$ by reducing it to normal form.	10	1

4. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	Verify the Cauchy's mean value theorem for the function e^x and e^{-x} in the interval$[a,b]$. Also show that 'c' is the arithmetic mean between a and b.	10	2
b.	Trace the curve $r^2=a^2\cos 2\theta$.	10	2

5. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	If $u=f(2x-3y,3y-4z,\ 4z-2x)$, prove that $\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0$.	10	3
b.	Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.	10	3

6. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	Evaluate $\iint (x+y)^2 dx\,dy$, where R is the parallelogram in the xy-plane with vertices $(1,0),(3,1),(2,2),(0,1)$ using the transformation $u=x+y,\ v=x-2y$. https://www.aktuonline.com	10	4
b.	Find the volume of the region bounded by the surface $y=x^2,\ x=y^2$ and the planes $z=0,\ z=3$.	10	4

7. Attempt any *one* part of the following:

Q. No.	Question	Marks	CO
a.	Verify the divergence theorem for $\vec{F}=4xz\hat{\imath}-y^2\hat{\jmath}+yz\hat{K}$ taken over the rectangular parallelepiped $0\le x\le 1,\ 0\le y\le 1,\ 0\le z\le 1$.	10	5
b.	Find the directional derivative of $\emptyset(x,y,z)=x^2yz+4xz^2$ at (1, -2, 1) in the direction of $2\hat{\imath}-\hat{\jmath}-2\hat{k}$. Find also the greatest rate of increase of $\emptyset$.	10	5

Printed Pages: 03 **Sub Code: KAS103**

Paper Id: 199103 **Roll No.**

B.Tech.
(SEM-I) THEORY EXAMINATION 2018-19
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions.

Q no.	Question	Marks	CO
a.	Find the rank of the matrix $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$.	2	1
b.	Find the stationary point of $f(x, y) = x^3 + y^3 + 3axy, a > 0$	2	3
c.	If $x = r\cos\theta,\ y = r\sin\theta, z = z$ then find $\frac{\partial(r,\theta,z)}{\partial(x,y,z)}$.	2	3
d.	Define del ∇ operator and gradient.	2	5
e.	If $\phi = 3x^2y - y^3z^2$, find grad $\boldsymbol{\phi}$ at point (2, 0, -2).	2	5
f.	Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dxdy.$	2	4
g.	If the eigen values of matrix A are 1, 1, 1, then find the eigen values of $A^2 + 2A + 3I.$	2	1
h.	Define Rolle's Theorem	2	2
i.	If u = $x^3y^2\sin^{-1}$(y/x), then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3
j.	In RI = E and possible error in E and I are 20 % and 10 % respectively, then find the error in R.	2	3
k.	State the Taylor's Theorem for two variables.	2	3

SECTION B

2. Attempt any *three* of the following:

Q no.	Question	Marks	CO
a.	Using Cayley- Hamilton theorem find the inverse of the matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Also express the polynomial B= $A^8-11A^7-4A^6+A^5+A^4-11A^3-3A^2+2A+I$ as a quadratic polynomial in A and hence find B.	10	1

Q no.	Question	Marks	CO
b.	If y = Sin(m $\sin^{-1}$x), prove that : $(1 - x^2)\, y_{n+2} - (2n+1)x\, y_{n+1} - (n^2 - m^2)y_n = 0$ and find y_n at x = 0.	10	2
c.	If *u, v, w* are the roots of the equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$, then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.	10	3
d.	Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates. Hence show that $\int_0^{\infty} e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$.	10	4
e.	Verify the divergence theorem for $\vec{F} = (x^3 - yz)\hat{i} + (y^3 - zx)\,j + (z^3 - xy)\hat{k}$, taken over the cube bounded by planes x = 0, y = 0, z = 0, x = 1, y = 1, z = 1.	10	5

SECTION C

3. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Find inverse employing elementary transformation A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$	10	1
b.	Reduce the matrix A to its normal form when A = $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$. Hence find the rank of A.	10	1

4. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	If $\sin^{-1} y = 2\log(x+1)$ show that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$	10	2
b.	Verify Lagrange's Mean value Theorem for the function $f(x) = x^3$ in $[-2,2]$	10	2

5. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Find the maximum or minimum distance of the point (1, 2, -1) from the sphere $x^2 + y^2 + z^2 = 24$.	10	3
b.	If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$	10	3

6. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Change the order of integration and then evaluate: $\int_0^2 \int_{\frac{x^2}{4}}^{3-x} x\,y \; dy\,dx$.	10	4
b.	Calculate the volume of the solid bounded by the surface x=0, y=0, x+y+z=1 & z=0.	10	4

7. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Prove that $\left(y^2 - z^2 + 3yz - 2x\right)\hat{\mathrm{i}} + \left(3\mathrm{xz} + 2\mathrm{xy}\right)\hat{\mathrm{j}} + \left(3\mathrm{xy} - 2\mathrm{xz} + 2\mathrm{z}\right)\hat{\mathrm{k}}$ is both Solenoidal and Irrotational.	10	5
b.	Find the directional derivative of $\Phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point P(1, 1, 1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.	10	5

KAS103 CORRECTION M 11.12.18

Q NO 1 : DO ANY TEN QUESTIONS