Unit-1 Sets

Define Sets?

A well defined collection of distinct objects are called sets

- objects in a set are called its member or elements
- sets are denoted by capital letters like A, B, C
- Elements of sets are represented by a, b, c

ex- if a is element of set A

Then we write $a \in A$ i.e. a belongs to A

a is the element of A.

Collection of all vowels in English alphabet $A = \{a, e, i, o, u\}$

* Some special sets:-

Z = set of all integers
N = set of all natural no.
R = set of all real no.
Q = set of all rational no.

* Representation of sets:-

Two methods to represents sets :-

i) Roaster or Tabular method
ii) set builder or rule method.

i) Roaster method-

in this all elements of a set are listed with braces { } and are separated by comma (,).

example :

i) Set of all factors of 12

$A = \{1, 2, 3, 4, 6, 12\}$

ii) Set of all letters in mathematics

$B = \{m, a, t, h, e, m, a, t, i, c, s\}$

2) Rule method - In this we list the property satisfied by all the elements of sets.

Ex: 1 $A = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{x : x \in N \text{ and } x < 8\}$

Ex: 2

$B = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}\}$

$B = \{x : x \in \frac{N}{N+1} \text{ and } x \in N \text{ and } 1 \leq N\}$

Types of Sets :-

→ Empty Sets - A set containing no element at all is called empty null, void set. eg: $\{x : x \in N \text{ \& } 2 < x < 3\}$

$\{x : x^2 = 9 \text{ \& } x \text{ is an even Integer}\}$

→ Singleton Set: A set containing exactly one element is called singleton set.

eg: $\{x : x \in N \text{ and } x^2 - 9\} = \{3\}$

Finite Set: A set is said to be finite if it consist of only finite no. of elements.

$A = \{a, e, i, o, u\}$

$= \{\text{a set of vowels}\}$

infinite set - A set those are not finite and they are called infinite sets.

Equal sets - Two sets a and b are equal if they have exactly the same element

ex- $a = b$

$A = \{1, 2, 3\}$ $B = \{3, 2, 1\}$

→ orders of elements in the sets have no effect.

eg: $A = \{1, 2, 3\}$ $B = \{3, 2, 1\}$

Repetition of this element in the set has no meaning.

Cardinal No. of Sets -

The no. of distinct elements containing in a finite set A is called cardinal of set A and denoted by $n(A)$ or $\text{card}(A)$

* Equivalent set :-

Two finite set A and B are said to be Equivalent if $n(A) = n(B)$

$n(A) = n(B)$ Hence A and B are called Equivalent sets.

* Sub-Sets:

- A set(A) is said to be subset of (B) if every element of A is also an element of B.

if x elements belongs A then x elements belongs B.

1. Every set A is subset of itself

   $A \subseteq A$ $\subseteq$ denoted subset

2. Null set is considered subset of any set

   $\phi \subseteq A$

3. if A is the subset of B and B is the subset of C then A is subset of C.

No. of Subset of a Set:

if a set contains "n" elements then the no. of sub sets is 2^n

Superset:

if A is the subset of B then B is called superset of A and written as

$A \subset B$ Superset denoted by $\supseteq$

$B \supseteq A$

Proper Subset:-

Any sub set A is said to be proper subset of another set B. if A is a subset of B and there is atleast one element of B, which doesn't belongs to A, then in proper subset of B.

Universal Subset -

Universal set is which contains all the elements of object a including its own elements.

denoted by "U" ex: $A: \{1, 3, 5, 7\}$

$B = \{2, 4, 6, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

* Operation of Sets :-

1. Intersection -

denoted by on two sets A and B.

and $\{x : x \in A \text{ and } x \in B\}$

$\therefore A \cap B = \phi$ (disjoint sets) = null / empty.

if $A \cap B$ is equal to the ϕ then they are called null/empty or disjoint sets.

2. Complement -

if U is the universal set and A be any subset of U, then the absolute complement of A denoted by A^c A' are the sets of elements which belongs to U but does not belongs to A.

$A' = \{x : x \in U \text{ and } x \notin A\}$

3. Relative Complement - (Difference)

Relative complement of B with respect to A is

$A - B = \{x : x \in A \text{ and } x \notin B\}$

$B' = A - (A \cap B)$

ex: $A = \{5, 3, 17, 12, 19\}$

$B = \{17, 19, 6\}$

i) Find the relative compliment of B in A

$A - B = \{5, 3, 12\}$

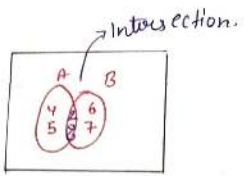
ii) Find the relative complement of A in B.

$B - A = \{6\}$

* Symmetric difference :-

The symmetric difference of set A with respect to set B is the set of elements which are either of set A and B but not in the intersection and denoted by

$A \Delta B \quad A \oplus B$



$A \oplus B = (A \cup B) - (A \cap B)$
$= (A-B) \cup (B-A)$

$A \oplus B = \{4,5,6,7\}$

$A = \{a,b,c,d,e\}$
$B = \{a,b,h,i,j\}$ $\quad A \oplus B = \{c,d,e,h,i,j\}$

Proper Subset :-

if B is the subset of A and B is not equal to A

ex: $A = \{1,2,3,4,5,6,8,10\}$

$B = \{2,6,8\} \quad C = \{1,2,3,4,5,6,8\}$

$B \subseteq A$ (Proper Subset)

$C \subseteq A$ Subset

$A \subseteq C$

* Set properties -

1) Empty sets :-

The unique sets with no elements called Empty sets and denoted by ϕ (NULL)

i) $\phi \subseteq A = \phi$

ii) $A \cup \phi = A$

iii) $A \cap \phi = \phi$

iv) $A \cap A^c = \phi$

2) Disjoint Sets :

A and B are called disjoint set if $A \cap B = \phi$

Set $A_1, A_2 ---- A_n$ are called mutually disjoint.

eg: $A = \{1,2\}$

$B = \{3,4\}$ are disjoint set.

→ The set of even and odd integer are called disjoint sets.

3) Power Sets :

→ A set A denoted by P(A) is the subset of A

Power set A is the set of all the subset of A.

ex: $A = \{1,2,3\} = 2^n, 2^3 = 8$

$P(A) = \{\phi\}, \{1\}, \{2\}, \{3\}, \{4\}, \{2,3\}, \{1,2\}, \{2,1,3\}$

- if $A \subseteq B$ then the powerset of A $P(A) \subseteq P(B)$
- if A has n elements then P(A) has 2^n elements.

* Multisets -

it is an unordered collection of elements in which multiplicity of an element maybe one or more than one or zero (1,or,0)

* Multiplicity : no of times elements repeated again and again

ex (i) $A = \{l,l,l,m,n,o,o,l,m,n,n\}$

$l = 4, m = 2, n = 3, o = 2$

ii) $B = \{a,a,a,a,a,c\}$

$a = 5, c = 1$

Operations of multisets:-

i) Union of multisets -

Union of multiset is a set such that maximum multiplicity of an element in a and b.

ex $A = \{1,1,2,2,2,3,3,3,3\}$

$B = \{1,2,2,2,2,2,2,3,3\}$

$A \cup B = \{2\cdot1, 6\cdot2, 4\cdot3\}$

ii) Intersection of multisets:

The intersection of two multisets A and B is a multiset that multiplicity of one element is equal to the minimum of multiplicity in A and B.

denoted by $A \cap B$

ex: $A = \{1,1,2,3,5,6,6,6\}$

$B = \{1,2,5,4,3,6\}$

$A \cap B = \{1,2,6\}$

Difference of multisets:-

It is a multiset such that multiplicity of an Element is Equal to the multiplicity of element A minus the multiplicity of element B.

ex: $A = \{1,1,1,2,2,3,2,3,5,5\}$

$B = \{1,1,2,2,2,4,4,3,3,3,3,3,5,5,5\}$

i) $A - B = \{1\}$

ii) $B - A = \{4,4,3,3,3,5\}$

ii) Sum of multisets:-

It is the set in the multiplicity of the element is equal to the sum of multiplication of the element in A and B.

ex $A = \{1,3,4,4,5,5\}$

$B = \{1,1,3,3,4,5,6,7\}$

$A+B = \{1,1,1,3,3,3,4,4,4,5,5,5,6,7\}$

$= \{3\cdot1, 3\cdot3, 3\cdot4, 3\cdot5, 1\cdot6, 1\cdot7\}$

* Cardianality of multiplieset:-

It is the no. of distinct element of multiset without considering the multiplicity of an element.

$A = \{\overset{1}{1,1,1}, \overset{2}{2,2}, \overset{3}{3}, \overset{4}{5}, \overset{5}{9,9}\}$

$n(A) = \{5\}$

$B = \{\overset{1}{x}, \overset{2}{y}, \overset{3}{z}, \overset{4}{i}, i\}$

$n(B) = \{4\}$

Ques Let P and Q be two multiset

$P = \{3\cdot a, 2\cdot b, 1\cdot c\}$ $Q = \{4\cdot a, 3\cdot b, 2\cdot d\}$

a) $P \cup Q = \{4\cdot a, 3\cdot b, 1\cdot c, 2\cdot d\}$

b) $P \cap Q = \{3\cdot a, 2\cdot b\}$

c) $P - Q = \{1\cdot c\}$

d) $Q - P = \{1\cdot a, 1\cdot b, 2\cdot d\}$

e) $P + Q = \{7\cdot a, 5\cdot b, 1\cdot c, 2\cdot d\}$

Ordered pair and Cartesian product:

Pair of object whose component occur in a special order it is written by listing two components in the specific order separating them by (,) and synthesizing in ()

$$\overset{A \quad B}{(a, b)}$$

first component → a, second component → b

Cartesian product:- Let A and B be two sets then the cartesian product of A and B is defined as

$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$

→ Cartesian product is the set of all ordered pair whose first components come from A and second come from B.

ex: $A = \{a, b, c\}$ $B = \{1, 2\}$

i) $A \times B = \{(a,1)\ (a,2)\ (b,1), (b,2).\ (c,1)\ (c,2)\}$

ii) $B \times A = \{(1,a)\ (1,b)\ (1,c)\ (2,a)\ (2,b)\ (2,c)\}$

$[A \times B \neq B \times A]$ → Cartesian of $A \times B$ is not equal to cartesian of $B \times A$.

Properties of Cartesian product:

for the four sets A, B, C, D

i) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

ii) $(A - B) \times C = (A \times C) - (B \times C)$

iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

iv) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Prove:-

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

$= A \times (B \cap C)$

then

$(x, y) \in A \times (B \cap C)$

$= x \in A$ and $y \in (B \cap C)$

$= x \in A$ and $(y \in B \cap y \in C)$

$= (x, y) \in A \times B$ and $(x, y) \in (A \times C)$

on RHS

$(A \times B) \cap (A \times C)$

conversely $(x, y) \in A \times B$ and $(x, y) \in (A \times C)$

$(x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

Hence $x \in A$ and $y \in (B \cap C)$

$\boxed{(x, y) \in A \times B \cap C}$ proved

RELATIONS –

Let A and B be two sets then the relation from A to B is the subset of cartesian product $A \times B$

R is the relation from A to B then R is the set of ordered pair A, B where $a \in A$ and $b \in B$

→ Total number of distinct relation from set A to set B is 2^{mn} when $n(A) = m$ and $n(B) = n$

a is related to b by relation R

R → called binary relation from A to B

eg $A = \{1, 2, 5\}$, $B = \{2, 4\}$

$A \times B = \{(1,2)\ (1,4), (2,2)\ (2,4)\ (5,2)\ (5,4)\}$

relation $R: x < y$ $R = \{(1,2)\ (1,4)\ (2,4)\}$

• **Domain** - The set a belongs to A belongs to R for some b belongs to B is called domain of R

$\{a \in A : (a,b) \in R$ for some $b \in B\}$ is called domain of R.

• **Range** - The set $b \in B$ $(a,b) \in R$ for some $a \in A$ is called range of R denoted by $Ran(R)$

Operations on relation -

1) **Union** - Given two relation R_1 and R_2 we will define the union of these relation by $R_1 \cup R_2 = \{(a,b) | (a,b) \in R_1, \forall (a,b) \in R_2\}$

eg $A = \{1,2\}$ $B = \{3,4\}$

$A \times B = \{(1,3), (1,4), (2,3) (2,4)\}$

$R_1 \subset A \times B$

$R_1 = (1,3) (2,3) (2,4)$

$R_2 = (1,4) (2,5) (2,4)$

$R_1 \cup R_2 = (1,3) (2,3) (1,4) (2,5)$ * common will not be considered

2) **Intersection** - Given two relation R_1 & R_2 we will define the intersection of these two relation denoted by $R_1 \cap R_2$

$R_1 \cap R_2 = \{(a,b) | (a,b) \in R_1 \cap (a,b) \in R_2\}$

eg. $R_1 = \{(1,3) (2,4)\}$

$R_2 = \{(1,3) (2,5)\}$

$R_1 \cap R_2 = \{(1,3)\}$

only common will taken.

3) **Difference** - Give two relation R_1 and R_2 within we will define the difference of these two relations denoted by $R_1 - R_2$

$R_1 - R_2 = \{(a,b) | (a,b) \in R_1 \cap (a,b) \notin R_2)$

eg. $R_1 = \{(1,3) (2,4)\}$

$R_2 = \{(2,4) (2,5)\}$

$R_1 - R_2 = (1,3)$

4) **Composition** - Given two relation R_1 and R_2 the composition of relation is define as follows

$R_1 O R_2 = \{(a,c) | (a,b) \in R_1 \cap (b,c) \in R_2)$

eg $R_1 = (1,2) (2,4)$, $R_2 = (2,5), (4,6)$

$R_1 O R_2 = aRc$

$R_1 = (1,2) (2,4)$ $R_2 = (2,5) (4,6)$

$(a,b)(a,b)$ $(b,c) (b,c)$

$= (1,5) (2,6)$

5) **Inverse** - Given a relation R we will define its inverse denoted by R_1^{-1}

$R_1^{-1} = \{(b,a) | (a,b) \in R\}$

eg $R_1 = (1,2) (2,4)$

$R_1^{-1} = (2,1) (4,2)$

Q1 $R_1 = (1,2)\ (2,3)\ (3,4)$

$R_2 = (1,1)\ (1,2)\ (2,1)\ (2,2)\ (2,3)\ (3,1)\ (3,2)\ (3,3)$

Solve $R_1 \cup R_2$

$= (1,1)\ (2,1)\ (2,2)\ (3,1)\ (3,2)\ (3,3)\ (3,4)$

ii) $R_1 \cap R_2$

$= (1,2)\ (2,3)$

iii) $R_1 - R_2$

$= (3,4)$

iv) R_1^{-1}

$= (2,1)\ (3,2)\ (4,3)$

v) $R_1 \circ R_2$

$= (1,1)\ (1,2)\ (1,3)\ (2,1)\ (2,2)\ (2,3)$

vi) $R_2 \circ R_1$

$= \{(1,2)\ (2,2)\ (3,2)\ (1,3)\ (2,3)\ (3,3)\ (2,4)\ (3,4)\}$

Reflexive relation -

Let R be a relation in set A then R be the relation then R called the reflexive relation.

if $(a,a) \in R$ for all $\in A$

Let $A = \{1,2,3,4\}$ $(a,a) = (1,1)\ (2,2)\ (3,3)\ (4,4)$

$R_1 = \{(1,1)\ (2,4)\ (3,3)\ (4,1)\ (4,4)\}$

$R_2 = \{(1,1)\ (2,2)\ (2,3)\ (3,3)\ (4,4)\}$

R_1 is not reflexive But R_2 is reflexive

R_3 is not reflexive But R_4 is reflexive.

Irreflexive Relation -

Let R be a relation in set A then R be the relation then $a \in A$, $(a,a) \in R$ is called irreflexive and it is also called anti-reflexive relation.

let $A = \{1,2,3,4\}$

$R_1 = \{(1,2)\ (2,1)\ (3,3)\ (4,4)\}$

$R_2 = \{(1,2)\ (1,3)\ (2,4)\}$

Symmetric relations :-

Let R be the relation in set A then R is said to be the symmetric relation.

$(a,b) \in R \Rightarrow (b,a) \in R$

$xRy = yRx$

$A = \{1,2,3,4\}$

$R_1 = \{(1,2)\ (2,1)\ (3,4)\ (4,3)$ Symmetric relation$\}$

$R_2 = \{(1,3)\ (3,1)\ (3,4)\}$

Anti symmetric relation -

a relation R on set A is said to be anti-symmetric if $(a,b) \in R$ and $(b,a) \notin R$ then $a = b$

$A = \{1,2,3,4\}$

$R_1 = \{(1,2)\ (3,3)\ (4,3)\}$ anti symmetric.

Q. Can a relation be symmetric and anti symmetric both of together then prove that ?

$R = \{(a,b)\ (b,b)\ (c,c)\}$

$(a,b) \in A$, $(b,a) \in R$

$R = \{(a,a)\ (b,b)\ (c,c)\}$

Transitive relation:-

A relation R on set A is said to be transitive if
$(a,b) \in R,\ (b,c) \in R \Rightarrow (a,c) \in R$

Asymmetric:-

A relation R on set A is said to be asymmetric if
$(a,b) \in R \Rightarrow (b,a) \notin R$

$(a,b) \in R \Rightarrow (b,a) \notin R$

$A = \{1,2,3,4\}$

$R_1 = \{(1,2)(2,3)(3,1)(4,2)\}$

Inverse relation :- (R^{-1})

Let R be a relation from set A to B then inverse relation of A is defined as the

eg: $\{(b,a) : (a,b) \in R\}$

$A = \{1,2,3\}$ $B = \{a,b\}$

$R = \{(1,a)(1,b)(3,a)(2,b)\}$ be relation from A to B

$R^{-1} = (a,1)(b,1)(a,3)(b,2)$

* Identity Relation:

A relation R in set A is said to be the identity relation if $(I_a) = \{(x,x) : x \in A\}$

$A = \{2,4,6\}$

$I_a = \{(2,2)(4,4)(6,6)\}$

Complement of relations $(\bar{R}, R^c)$ -

$\bar{R} = \{(a,b) \mid (a,b) \in A \times B \text{ and } (a,b) \in R\}$

$\bar{R} = \{(A \times B) - R\}$

$A = \{a,b\}$ and $B = \{1,2,3\}$

$R = \{(a,2)(b,1)(b,3)\}$

Find $\bar{R} = ?$

$A \times B = \{(a,1)(a,2)(a,3)(b,1)(b,2)(b,3)\}$

$R = \{(a,2)(b,1)(b,3)\}$

$\bar{R} = A \times B - R$
$= \{(a,1)(a,3)(b,2)\}$

Composition of relation -

Let a, b, c the three sets

Let R is the relation from A to B that is R is a subset $R \subseteq A \times B$

→ S is the relation from B and C i.e $S \subseteq B \times C$

The composite of R and is denoted by $R \circ S$ when

$R \circ S = \{(a,c) \in A \times C :$ for some $b \in B$, $(a,b) \in R$ and $(b,c) \in S$

ex: let $A = \{1,2,3\}$ and $B = \{P,Q,R\}$. $C = \{x,y,z\}$

$R = \{(1,P)(1,Q)(2,P)(2,Q)(1,Q)(2,x)(3,P)(3,Q)(3,x)\}$ $A \times B$

$S = \{(P,y)(Q,x),(Q,y)(R,x)(R,z)(Q,z)(R,y)(R,z)\}$

$R \circ S = (a,c)$

$\{(1,y)(1,z)$ $(2,y)$
$(1,x)(1,y)(2,x)(2,z)$
$(3,x)(3,y)(3,z)\}$

$A \times B$ $B \times C$
(a,b) (b,c)
R S
(a,c)

Q. $R = \{(1,2)(3,4)(2,2)\}$

$S = \{(4,2), (2,5), (3,1), (1,3)\}$

Find $R \circ S$, $S \circ R$, $R \circ (S \circ R)$, $R \circ R$, $S \circ S$.

Equivalence relation – or Equality (R, S, T)

A relation R said to be equivalent relation in set A if and only if (i) R is reflexive i.e. for all $a \in R$, $(a,a) \in R$

ii) R is symmetric i.e. $(a,b) \in R$, then $(b,a) \in R$

iii) R is transitive i.e. $(a,b) \in R$, and $(b,c) \in R$ then $(a,c) \in R$

Q. $A = \{a, b, c\}$

$R_1 = \{\ \}$ not equivalent

$R_2 = \{a,a\}$ equivalent

$R_3 = \{(a,a)(b,b)(c,c)(b,a)\}$ not equivalent

$R_4 = \{(a,a)(a,c),(b,a)(c,a)\}$ not equivalent

$R_5 = \{(a,a),(b,b)(c,c)(a,b)(a,c)(b,a)\}$ equivalent

$R_6 = A \times A$ equivalent

Partial order relation – (R, A, T) POSET

a relation R on set A is said to be partial order relation if i) R is reflexive i.e. for all $a \in R$, $(a,a) \in R$

ii) R is anti-symmetric i.e. $(a,b) \in R$, then $(b,a) \notin R$

iii) R is transitive i.e. $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

$R_1 = \{\ \}$ It is POSET

$R_2 = \{(1,1)(2,2)(3,3)\}$ yes, It is POSET

$R_3 = \{(1,1)(2,2)(3,3)(1,2)(2,1)\}$ It is not Poset

$R_4 = \{(1,1)(2,2)(3,3)(1,3)(2,3)\}$ It is POSET

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Some proof of Identity law –

A) IDEMPOTENT LAW –

i) $A \cup A = A$

ii) $A \cap A = A$

Proof

$A \cup A = A$

$A \cup A = \{x : x \in A \text{ or } x \in A\}$

$= \{x : x \in A\}$

$= A$ Hence proved.

$A \cap A = A$

$A \cap A = \{x : x \in A \text{ and } x \in A\}$

$= \{x : x \in A\}$

$= A$

B) IDENTITY LAW – (imp.)

i) $A \cup \phi = A$

proof

$A \cup \phi = \{x : x \in A \text{ or } x \in \phi\}$

$= \{x : x \in A\}$

$= A$

ii) $A \cap U = A$

$A \cap U = \{x : x \in A \text{ and } x \in U\}$

$= \{x : x \in A\}$

$= A$

C) COMMUTATIVE LAW

i) $A \cup B = B \cup A$

Let $x \in A \cup B$

$x \in A$ or $x \in B$

$x \in B$ or $x \in A$

$x \in B \cup A$

$A \cup B \subseteq B \cup A$ — (1)

Let $y \in B$ or $y \in A$

$y \in A$ or $y \in B$

$y \in (A \cup B)$

$B \cup A \subseteq A \cup B$

Comparing (1) and (2) eq.

$\boxed{A \cup B = B \cup A}$

* Note

$A = B$

then $A \subseteq B$ and $B \subseteq A$

II) $A \cap B = B \cap A$

Let $x \in A \cap B$

$x \in A$ and $x \in B$

$x \in B$ and $x \in A$

$x \in B \cap A$

$A \cap B \subseteq B \cap A$ — (1)

Let $y \in B$ and $y \in A$

$y \in A$ and $y \in B$

$y \in (A \cap B)$

$B \cap A \subseteq A \cap B$

comparing eqn (1) and (2)

$\boxed{A \cap B = B \cap A}$

D) Associative Law-

I) $(A \cup B) \cup C = A \cup (B \cup C)$

Let $x \in (A \cup B) \cup C$

$x \in (A \cup B)$ or $x \in C$

$x \in A$ or $x \in (B \cup C)$

$x \in A \cup (B \cup C)$

$(A \cup B) \cup C \subseteq A \cup (B \cup C)$ — (1)

RHS

Let $y \in A \cup (B \cup C)$

$y \in A$ or $y \in B$ or $y \in C$

$y \in (A \cup B)$ or $y \in C$

$y \in (A \cup B) \cup C$

$y \in (A \cup B) \cup C$

$A \cup (B \cup C) \subseteq (A \cup B) \cup C$ — (2)

Comparing eqn (1) and (2)

$(A \cup B) \cup C = A \cup (B \cup C)$ Hence proved

E) DISTRIBUTIVE LAW-

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

LHS

Let $x \in A \cup (B \cap C)$

$x \in A$ or $x \in (B \cap C)$

$x \in A$ or $(x \in B$ and $x \in C)$

$(x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$x \in A \cup B$ and $x \in A \cup C$

$(x \in A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ — (1)

Let $y \in (A \cup B) \cap (A \cup C)$

$(y \in A$ or $y \in B)$ and $(y \in A$ or $y \in C)$

$y \in A$ or $(y \in B$ and $y \in C)$

$y \in A$ or $y \in (B \cap C)$

$y \in A \cup (B \cap C)$

$A \cup B \cap (A \cup C) \subseteq A \cup (B \cap C)$ — (2)

comparing eqn (1) and (2)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De' Morgan law

1) $(A \cup B)' = A' \cup B'$

Let $x \in (A \cap B)'$ then

$x \notin (A \cap B)$

$x \notin A$ or $x \notin B$

$x \in A'$ or $x \in B'$

$x \in (A' \cup B')$

$(A \cap B)' \subseteq A' \cup B'$

let $y \in A' \cup B'$

$y \in A'$ and $y \in B'$

$y \notin A$ and $y \notin B$

$y \notin$ Both A and B

$y \notin (A \cap B)$

$y \in (A \cap B)'$

2) $(A \cup B)' = A' \cap B'$

Compliment law-

1(a) $A \cup A' = U$ b) $A \cap A' = \phi$

2(a) $U' = \phi$ 2(b) $\phi' = U$

Involution law-

i) $(A')' = A$

$A = \{1, 2, 3, 4\}$

$A' = \phi$

$(A')' = \{1, 2, 3, 4\}$

Q. write the Sets in Builder form -

A) The Set A consisting of element a, e, i, o, u.

$A = \{x : x$ is a vowel in english language$\}$

B) Set $B = \{1, 4, 9, 16, 25, 36\}$

$B = \{x : x^2$ where $x \le 6\}$

c) Set $C = \{2, 4, 6, 8\}$

$C = \{x : x$ is an even integer between 2 to 8 or $1 < x \le 8\}$

D) $D = \{3, 6, 9, 12, 15\}$

$D = \{x : x$ is a multiple of 3 where $x < 6$

E) $E = \{-4, -3, -2, -1, 0, 1, 2, 3\}$

$E = \{x : x$ is an integer $-4 \le x \le 3\}$

Q. A computer company must hire 20 programmer to handle System program Job & 30 programmer for application program. of those hired, 5 are expected to perform Jobs of both, How much must be hired.

no of programmer in Set A, $n(A) = 20$

$n(B) = 30$

$n(A \cap B) = 5$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 20 + 30 - 5$

So the company hire = 45 programmer.

Equivalence class (Reflexive, Transitive, Symmetric)

$[x] = \{y / y \in A$ and $(x, y) \in R\}$

Ex- $A = \{1, 2, 3, 4, 5\}$

$R = \{(1,1)(2,2)(3,3)(4,4)(5,5)(1,2)(2,1)(4,5)(5,4)\}$

$[1] = \{1, 2\}$

$[2] = \{2, 1\}$

$[3] = \{3\}$

$[4] = \{4, 5\}$

$[5] = \{5, 4\}$

Equivalence class

[1]	[3]	[4]
$\{1, 2\}$	$\{3\}$	$\{4, 5\}$
↓	↓	↓
Partition 1	P_2	P_3

$P_1 \cup P_2 \cup P_3$ $\{1, 2, 3, 4, 5\}$

if intersect $P_1 \cap P_2 \cap P_3 = \phi$

Q Let $A = \{1, 2, 3, 4, 5\}$

$R = \{a, b\} / a+b$ is even Relate to R is defined as Set A. Find the equivalence class & partition.

- Reflexive $a, b = a, a$

  $a + a = 2a = a + a$
- Symmetric $a+b = b+a$ is even
- Transitive $a+b$ is even

  $b+c$ is even

  So, $a+c$ is even Hence, relation is equivalence

$[1] = (1, 3, 5)$

$[2] = (2, 4)$

$[3] = (1, 3, 5)$

$[4] = (2, 4)$

$[5] = (1, 3, 5)$

Equivalence class of element $\{1, 3, 5\}$ are same and E.C. of $\{2, 4\}$ are same

$\{1, 3, 5\}$ $\{2, 4\}$ Equivale class

P_1 P_2

any element 1, 3, 5 can be choosen as representation of E.C. 1, 3, 5 and same with the element.

Quotient Set

The collection of all distinct equivalent classes of set A induced by an equivalence relation is called Quotient set of A by R; it is denoted by A/R.

$\frac{A}{R} = \{[A], a \in A\}$

$R = \{(1,3)\ (1,1)\ (2,2)\ (2,4)\ (3,1)\ (3,3)\ (4,2)\ (4,4)\}$

$[1] = \{1,3\}$ $\quad [3] = \{1,3\}$

$[2] = \{2,4\}$ $\quad [4] = \{2,4\}$

$\frac{A}{R} = [1]\ \ [2]$

$\{1,3\}\ \{2,4\}$

Partition

The partition of set A is a collection of mutally disjoint non empty subset of A whose union is A so P is called partition of A if.

i) P contain non empty subset of A

i.e., $A_1, A_2 --- A_n$

where $A_i \subseteq A$ for $\{i = 1,2,3 --- n\}$

Partition set A का subset होना चाहिए

ii) union of all subset of A under partition give A.

i.e. $A_1 \cup A_2 \cup A_3 -- A_n = A$

or $\cup A_i = A$, for $i = 1,2,3 --- n$

iii) The element of partition P are particular disjoint

$A_i = A_j = \phi$

check the validity of partition -

Q. $A = \{1,2,3,4,5,6,7,8\}$

1) $P_1 = \{\{1,2,3,4\}\ \{5,6\}\ \{8\}\}$

property,

$\{1,2,3,4\} \subseteq A$ $\{5,6\} \subseteq A$ $\{8\} \subseteq A$ true

$P_1 \cup P_2 \cup P_3 = \{1,2,3,4\} \cup \{5,6\} \cup \{8\}$

$= \{1,2,3,4,5,6,8\}$ false, not complete union.

It is not a valid partition

Q $A = \{1,2,3,4,5,6,7,8,9\}$

are relation defined on $A \times A$ such that.

$A \times B$ (a,b) R (c,d) is $a+d = b+c$

1) Prove that R is equivalent relation.

for reflexive

$(a,b)\ R\ (a,b)$ $(a,b) \in A$

$a+b = b+a$ which is have for all A

$\therefore$ R is reflexive.

Given $(a,b)\ R\ (c,d)$ such that

$a+d = b+c$

then $(c,d)\ R\ (a,b)$

$c+b = d+a$

which satisfy given condition

$\therefore$ R is symmetric

if $(a,b)\ R\ (c,d)$ and $(c,d)\ R\ (e,f)$

then $(a,b)\ (c,d)\ (e,f) \in A \times A$

$a+d = b+c$ — (1)

$e+f = c+d$ — (2)

adding the eq.

$c+d+e+f = b+c$

$(a,b)\ R\ (e,f)$

then R is transitive.

2) Find equivalence class $[(2,5)]_R$

let $(a,b)\ R\ (c,d)$ on $A \times A$

let $(2,5)\ R\ (c,d)$ on $A \times A$

$a+d = b+c$

$2+d = 5+c$

$d-c = 3$

$d = 3+c$

for

$c=1$	$d=4$	$(1,4)$
$c=2$	$d=5$	$(2,5)$
$c=3$	$d=6$	$(3,6)$
$c=4$	$d=7$	$(4,7)$
$c=5$	$d=8$	$(5,8)$
$c=6$	$d=9$	$(6,9)$
$c=7$	$d=10$	$(7,10)$
$c=8$	$d=11$	$(8,11)$
$c=9$	$d=12$	$(9,12)$

equivalence class of $[2,5]$

$\{(1,4)\ (2,5)\ (3,6)\ (4,7)\ (5,8)\ (6,9)\}$

Closure of relation

closure of relation R with respect to property P is the relation obtain by adding minimum of ordered pair to R to obtain property

1) Reflexive closure - if relation R defined on set A then the $R \cup I_a$ is reflexive closure of relation 'R' and it is denoted by R_f, where I_a is the Identity relation defined on set A.

2) Symmetric closure - if R is a relation defined on set A $R \cup R^{-1}$ is the symmetric closure of relation 'R' and it is denoted by R_s where R^{-1} is the inverse relation

3) Transitive closure - if R is a relation defined on set A then transitive closure of relation 'R' is defined as $R \cup R^2 \cup R^3 \cup \cdots R^n$ it is denoted by R^* or R^+

Q $A = \{1,2,3,4\}$

$R = \{(1,1)\ (1,3)\ (2,4)\ (3,1)\ (3,3)\ (4,3)\}$

Reflexive closure

$R_f = R \cup I_a$

$R_f = \{—\} \cup \{(2,2)\ (4,4)\}$

$R_f = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,3)\ (2,4)\ (3,1)\ (4,3)\}$

* The closure of relation R w.r.t property P is the relation obtain by adding minimum no. of ordered pair to R to obtain property

Symmetric closure -

$R_s = R \cup R^{-1}$

$R_s = R \cup \{(4,2)\ (3,4)\}$

$R_s = \{(1,1)\ (1,3)\ (2,4)\ (3,1)\ (3,3)\ (4,3)\ (4,2)\ (3,4)\}$

Transitive closure -

$A = \{1,2,3\}$

$R = \{(1,2)\ (2,3)\ (3,3)\}$

$R^2 = R.R = (1,2)\ (2,3)\ (3,3)$
$(1,2)\ (2,3)\ (3,3)$

$R^2 = (1,3)\ (2,3)\ (3,3)$

$R^3 = R^2 R = (1,3)\ (2,3)\ (3,3)$
$(1,2)\ (2,3)\ (3,3)$
$(1,3)\ (2,3)\ (3,3)$

$R^+ = R \cup R_2 \cup R_3$

$(1,2)\ (2,3)\ (3,3) \cup (1,3)\ (2,3)\ (3,3) \cup (1,3)\ (2,3)\ (3,3)$

$R^+ = \{(1,2)\ (2,3)\ (3,3)\ (1,3)\}$

* Countability of Set-

i) if the set contain specific or finite element it is called as finite set.

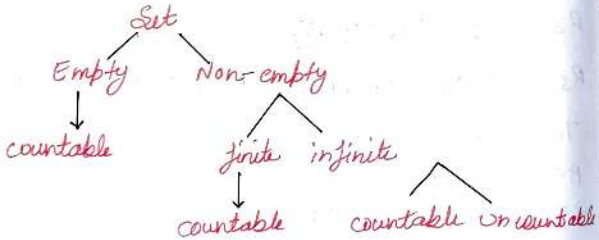
ii) if set contain infinite no. of element then it is infinite set.

iii) countable set (denumerable)

if there is one to one mapping b/w the elements of set and natural no. then it is called as countable set.

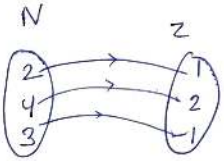
Uncountable set – if there cannot be one to one mapping between elements of sets and natural no. then it is called as uncountable set.

* Any set which is finite or denumerable is called countable set



Q. $F: Z \rightarrow N$

$$f_n = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even positive integer.} \\ \frac{1-n}{2} & \text{if } n \text{ is odd positive integer.} \end{cases}$$



WARSHALL'S ALGORITHM –

2019

Q. $R = \{(1,4)\ (2,1)\ (2,3)\ (3,1)\ (3,4)\ (4,3)\}$

$A = \{1, 2, 3, 4\}$

$$P_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

for P_1

considering 1st column 1st row

(2,3) (4)

(2,4) (3,4)

$$P_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Placing 1 at above location

for P_2 2nd column 2Row

$\{\phi\}$ $\{1,3,4\}$

$$P_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Column Row

(2,4) (1,4)

(2,1) (2,4) (4,1) (4,4)

$$P_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 1 & 0 & 1 & 1 \end{array}$$

P_4 4th column 4th row

(1, 2, 3, 4) (1, 3, 4)

(1,1) (1,3) (1,4) (2,1) (2,3) (2,4)
(3,1) (3,3) (3,4) (4,1) (4,3) (4,4)

$$P_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

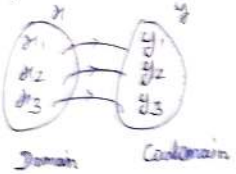
Transitive closure

R^* or R^+

(1,1) (1,3) (1,4) (2,1) (2,3) (2,4)
(3,1) (3,3) (3,4) (4,1) (4,3) (4,4)

Function -

Let x and y be two non empty sets, A function x to y that assign to each element x belong to x $x \in x$ and $y \in y$



Domain = (x_1, x_2, x_3)

Codomain = (y_1, y_2, y_3)

e.g. let $A = \{1, 4, 5, 4\}$

$B = \{a, b, c, d\}$

$F \to A \to B$

$F = \{(1,a)(2,c)(3,d)(4,d)\}$

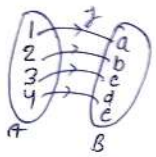
Find domain, codomain, range

domain = $\{1, 2, 5, 4\}$

Codomain = $\{a, b, c, d\}$

range = $\{a, c, d\}$

* One to one function (Injective)



$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \; \forall \; x_1 x_2 \in x$

e.g. $f: z \to z$

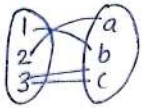
$f(x) = x+1$

let $f(x_1) = f(x_2)$

$(x_1+1) = (x_2+1)$

$x_1 = x_2 \; \forall \; x_1 x_2 \in z$

* onto function (Surjective)



$F: R \to R$

$f(x) = 2x+1$

$F(x) = y$

$2x+1 = y$

$x = \frac{y-1}{2}$

Bijective function -

A function which is one to one and onto function is called bijective function

(1) $f(x) = 2x+1$

$F \to R \to R$

one to one $f(x_1) = f(x_2)$

$2x_1+1 = 2x_2+1$

$2x_1 = 2x_2$

$x_1 = x_2$

onto function

$f(x) = 2x+1$

$f(x) = y$

$2x+1 = y$

$x = \frac{y-1}{2}$

It is bijective function.

(2) $f(x) = \sqrt{x}$

$F \to R^2 \Rightarrow R^+$

one to one $f(x_1) = f(x_2)$

$\sqrt{x_1} = \sqrt{x_2}$

$(\sqrt{x_1})^2 = (\sqrt{x_2})^2$

$x_1 = x_2$

onto function

$f(x) = \sqrt{x}$

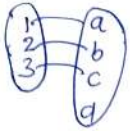
$f(x) = y$

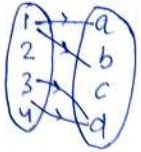
$\sqrt{x} = y$

$x = y^2$

It is bijective

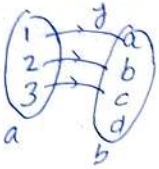
Into function - A function which is not onto function is called Into fn.



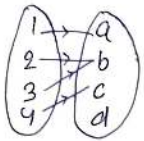


One to one into function

A function which is one to one and into both

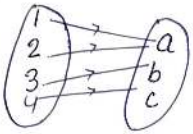


Many to one function - It is a function where element of a range corresponds to more than one element in domain.



if two or more element have same image.

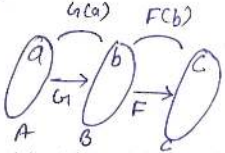
Many to one onto function -



* Composition function -

G is function from set A to B and F is a function from set B to C the composition of function F & G denoted by FOG or FOG = F(G(a))

F o (G(a))



Q. G is a function from set a,b,c itself such that G(a) = b, {a,b,c} to set {1,2,3} F(a)=1, F(b)=2, F(c)=3 find composition of F & G Fog.

G(a) = b f(a) = 1
G(b) = c f(b) = 2
G(c) = a f(c) = 3

Q. F(x) = 3x+2
g(x) = 4x+8

Fog(x) = F(g(x)) = 3(4x+8)+2
gof(x) = 4x+8 = 4(3x+2)+8 = 12x+16
FoF = 3x+8 = 3(8x+2)+2 = 9x+8
gog(x) = 4x+8 = 4(4x+8) = 16x+40

21/9/22

* Composition of function is associative -

(hog)of = ho(gof)

let F→x→y = F(x)=y, x ∈ x, y ∈ y
g→y→z g(y)=z, y ∈ y, z ∈ z
h→z→w h(z)=w, z ∈ z, w ∈ w

(hog) of	ho(gof)
(hog)of(x)	ho(g of(x)
hog(y)	(g(y)
h(z) = w	ho(z) = w

* Composition of function is not comutative only associative.

Q. f, g and h function are defined on the set of natural no. such that f(x) = x+1, g(x) = 2x

$h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$

Find.

i) FoF	ii) gof	iii) Fog	iv) goh
FoF(x)	2(x+1)	x+1	goh(x)
x+1	2x+2	2x+1	2(h(x)
(x+1)+1			when x is odd
= x+2			2.1 = 2
			when x is even
			2.0 = 0

(fog)oh	go(fog)oh
fog(x) oh	go(2x+1)oh
x+1	2(2x+1)oh
(2x+1) oh	4x+2 oh(x)
when x is even,	if x is even = 2
when x is odd 3	if x is odd = 6

Permutation -

Let x be a finite set, the set of all bijective function defined on set X is called the set of Permutation function defined on x. $X = \{1,2\}$ $n = 2$

$2! = 2$

$P_1 = \begin{pmatrix} 1, 2, 3 \\ 1, 2, 3 \end{pmatrix}$ $P_4 = \begin{pmatrix} 1, 2, 3 \\ 2, 1, 3 \end{pmatrix}$

$P_2 = \begin{pmatrix} 1, 2, 3 \\ 1, 3, 2 \end{pmatrix}$ $P_5 = \begin{pmatrix} 1, 2, 3 \\ 3, 2, 1 \end{pmatrix}$

$P_3 = \begin{pmatrix} 1, 2, 3 \\ 2, 3, 1 \end{pmatrix}$ $P_6 = \begin{pmatrix} 1, 2, 3 \\ 3, 1, 2 \end{pmatrix}$

Inverse of permutation -

Inverse of permutation can be obtained by exchanging the two rows and rearranging the column, so the the rows are in order.

$x = \{1,2,3,4\}$

$P = \begin{pmatrix} 1, 2, 3, 4 \\ 4 \; 3 \; 1 \; 2 \end{pmatrix}$ $P^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{bmatrix}$

$P_1^{-1} = \begin{bmatrix} 4 & 3 & 1 & 2 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

Composition or product of permutation -

Let P_1 and P_2 be two permutation on set X then product and composition of P_1 and P_2 is denoted by $P_1 o P_2 (n)$
$= P_1 (P_2 (n))$

$X = \{a, b, c\}$

$P_1 = \begin{Bmatrix} a & b & c \\ c & a & b \end{Bmatrix}$ $P_2 = \begin{Bmatrix} a, b, c \\ b \; c \; a \end{Bmatrix}$

Find $P_1 o P_2$, $P_2 o P_1$

$P_1 o P_2 (a) = P_1 (b) = a$
$P_1 (o) P_2 (b) = P_1 (c) = b$
$P_1 o P_2 (c) = P_1 (a) = c$

$P_1 o P_2 = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}$

$P_2 o P_1 (a) = P_2 (c) = a$
$P_2 o P_1 (b) = P_2 (a) = b$
$P_2 o P_1 (c) = P_2 (b) = c$

$P_2 o P_1 = \begin{pmatrix} a & b & c \\ a & b & c \end{pmatrix}$

Q. $\{5, 6, 7\}$

$P_1 = \begin{Bmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{Bmatrix}$ $P_2 = \begin{Bmatrix} 5 & 6 & 7 \\ 7 & 5 & 6 \end{Bmatrix}$

Find $P_1 o P_2$, P_1^{-1}, P_2^{-1}, $P_2^{-1} o P_1^{-1}$

$P_1 o P_2 (5) = P_1 (7) = 7$
$P_1 o P_2 (6) = P_1 (5) = 6$
$P_1 o P_2 (7) = P_1 (6) = 5$

$P_1 o P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 7 & 6 & 5 \end{pmatrix}$

$P_1 = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$

$P_1^{-1} = \begin{pmatrix} 6 & 5 & 7 \\ 5 & 6 & 7 \end{pmatrix}$

$P_1^{-1} = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$

Unit - 2 Group Theory

N = Set of Natural Number
Z = Set of Integers
R = Set of Real number
Q = Set of Rational number
C = Set of complement no.

$F : X \to X$ Unary operation
$F : X^2 \to X$ Binary operation
$F : X^3 \to X$ Ternary operation
$F : X^n \to X$ n-array operation

Binary operation -

Let G be a non-Empty set then

$G \times G = \{(a,b)\ a \in G, b \in G\}$

if $F : G \times G \to G$ then F is said to the binary operation on G

It is denoted by $(+, \cdot, 0, *)$

Algebraic structure -

A non-empty set together with one or more than one binary operation is called algebraic operation. $(Z, +, \cdot)$

Property of Algebraic structure -

closure → Distributive
Associative → Cancellation
Commutative → Indempotent

Element
→ Identity
→ Inverse

closure property -

The * will be on binary operation if and only if $a*b$ belongs to G $\forall a \in G$ and $a*b$ is unique this is said to the closure property of the binary operation and set G is said to be close with respect to binary operation

eg $Z = \{\infty, -3, -2, -1, 0, 1, 2, 3 --- \infty)$

$(-3) + (-2) = (-5)$

$-3 - 2 \in Z$

$-5 \in Z$

2. Commutative property -

$a*b = b*a$

3. Associative property

$(a+b)*c = a*(b+c)$

4. Distributive property

→ $a*(b+c) = (a*b) + (a*c)$

right distributive

→ $(b+c)*a = (b*a) + (c*a)$

left distributive.

Cancellation law

$a*b = a*c$

$b = c$ left cancellation

$b*a = c*a$

$b = c$ Right cancellation

Idempotent property-

The element $a \in S$ is called idempotent element w.r.t operation if $\boxed{a * a = a}$

Identity element

$a * e = a$

$e * a = a$

eg $Z = \{-\infty, -3, -2, -1, 0, 1, 2, 3, \ldots \infty\}$

$(Z +)$	$(Z *)$
$-5 + 0 = -5$	$a \times 1 = a$
$0 + (-5) = -5$	$1 \times a = a$

An element in a set S is called an Identity element w.r.t binary operation * if of any element in S.

Inverse element

$a * a^{-1} = e$ (Identity)

$\downarrow$ Inverse element

$Z = \{-\infty, -3, -2, -1, 0, 1, 2, 3, \ldots \infty\}$

$(10) + (-10) = 0$

closure property $\longrightarrow$ Algebraic Structure

$\downarrow$

Associative property $\longrightarrow$ Semi-group

$\downarrow$

Identity element $\longrightarrow$ monoid

$\downarrow$

Inverse Element $\longrightarrow$ Group

$\downarrow$

Commutative property $\longrightarrow$ Abelian Group

Semi group - Let S be a non-Empty set and * be a binary operation on set then $(S, *)$ is called Semi-group if it follows 1) closure property
2) Associative property

Monoid - Let S be a non-empty set and * be a binary operation on set then $(S, *)$ is called monoid if it follows.

1) closure property, 2) Associative property.
3) Identity element.

Group - Let S be a non-empty set and * be a binary operation on set $(S, *)$ is called group if it follows

1) closure property 2) Associative property
3) Existence of identity element 4) Existence of Inverse element

Abelian group - Let S be a non-empty set and * be a binary operation on set then $(S, *)$ is called Abelian group if it follows -

1) closure property 2) Associative property
3) Existence of Identity element 4) Existence of Inverse element
5) commutative property.

All abelian groups are group but all groups are not Abelian group.

Q. Prepare the composition table for multiplication on the element in the set $A = \{1, \omega, \omega^2\}$, where ω is the cube root of unity. Show $(A, \times)$ is an abelian group.

	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

1) closure

$a \times b \in A$

It is closure

Associative

$a \times (b \times c) = (a \times b) \times c$

let $a = \omega, b = 1, c = \omega^2$

$\omega \times (1 \times \omega^2)$

$\omega \times \omega^2$

$= \omega^3$

$= 1$

$(\omega \times 1) \times \omega^2$

$\omega \times \omega^2$

$= \omega^3$

$= 1$

Identity

$a \times e = a$

$e \times a = a$

$e = 1$

Inverse

$a \times a^{-1} = e$

$e = 1$

$1 = 1$

$\omega^{-1} = \omega^2$

$\omega^{2-1} = \omega$

Commutative

$a \times b = b \times a$

let $a = 1 \& b = \omega$

$1 \times \omega = \omega \times 1$

$\omega = \omega$

Q. Prove that $G = \{1, -1, i, -i\}$ is an abelian group with respect to multiplication, where $i = \sqrt{-1}$

	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

closure

$a * b \in G$

Commutative

$(a \times b) = b \times a$

$i \times (-i) = (-i) \times i$

$-i^2 = -i^2$

$1 = 1$

associative

$(a \times b) \times c = a \times (b \times c)$

$a = 1, b = i, c = -i$

$(1 \times i) \times -i = 1 \times (i \times -i)$

$i \times (-i) = -i^2$

$-i^2 = 1$

$1 = 1$

Identity

$a \times e = a$

$a \times a = a$

$e = 1$

Inverse

$a \times a^{-1} = e$

$e = 1$

$1 = 1$

$\omega^{-1} = \omega^2$

$\omega^{2-1} = \omega$

Q. Prove that $G = \{1, -1, i, -i\}$ is an abelian group with respect to multiplication, where $i = \sqrt{-1}$

	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

closure property

$a * b \in G$

Associative

$(a \times b) \times c = a \times (b \times c)$

$a = 1, b = i, c = i$

$(1 \times i) \times (-i) = 1 \times (i \times -i)$

$i(-i) = -i^2$

$-i^2 = 1$

$1 = 1$

Identity

$a \times e = a$

$e = 1$

Inverse

$a \times a^{-1} = e$

$e = 1$

$1^{-1} = 1$

$-1^{-1} = -1$

$i^{-1} = -i$

$-i^{-1} = i$

Commutative

$a \times b = b \times a$

$i \times (-i) = (-i) \times i$

$-i^2 = -i^2$

$1 = 1$ LHS = RHS

Q. The binary operation $*$ on A is $a * b = a + a + 2ab$ $\forall a, b \in A$, determine whether the binary operation is associative and commutative. Also find identity and inverse element if exist.

Associative

$(a * b) * c = a * (b * c)$

let $a, b, c \in A$

LHS $\underbrace{(a * b)}_{a} * \underbrace{c}_{c}$

$= \underbrace{(a + b + 2ab)}_{a} * \underbrace{c}_{c}$

[illegible]

[illegible]

[illegible]

Part of Unit - 1

Mathematical Induction -

Principal of mathematical Induction

A prove by mathematical induction then $P(x)$ is true for every positive integer and consist of following step:

1. Inductive Base
2. Inductive Hypothesis
3. Inductive Step

1. Inductive Base -

It show that $P(1)$ is true for $n=1$

or

$P(x)$ is true for $n = x$ (a)

2. Inductive Hypothesis -

It assume that $P(K)$ is true for some arbitrary K.

3. Inductive Step -

It show that $P(K+1)$ is true by using Hypothesis $P(K)$

Q. Show that sum of square of n the integer $\frac{n(n+1)(2n+1)}{6}$

Let $P(n) = 1^2+2^2+3^2+ - - - + n^2 = \frac{n(n+1)(2n+1)}{6}$

i) Inductive Base

for $n=1$

$P(1) = \frac{1(1+1)(2+1)}{6} = \frac{6}{6} = 1$

$1^2 = 1$

$1 = 1$

ii) Inductive hypothesis

Let $P(K)$ is true

i.e. $1^2+2^2+3^2+ - - - + K^2 = \frac{K(K+1)(2K+1)}{6}$

iii) Inductive Step

for $n = K+1$

$P(K+1) = 1^2+2^2+3^2+ - - - + K^2+(K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$

now, LHS

$1^2+2^2+3^2+ - - - + K^2+(K+1)^2 = \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$= \frac{K(K+1)(2K+1)+6(K+1)^2}{6}$

$= \frac{K(K+1)(2K+1)+6(K+1)^2}{6}$

$= \frac{(K+1)}{6}[K(2K+1)+6(K+1)]$

$= \frac{(K+1)}{6}[2K^2+K+6K+6]$

$= \frac{(K+1)}{6}[2K^2+4K+3K+6)$

$= \frac{(K+1)}{6}[2K(K+2)+3(K+2)]$

$= \frac{(K+1)(2K+3)(K+2)}{6}$ RHS

Q. prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133

i) Inductive base $P(n) = 1$

$$11^{n+2} + 12^{2n+1} = 11^3 + 12^3 = 3059$$

$$P(1) = 133\,(23)$$

$P(1)$ is divisible by 133

Hence, $P(1)$ is true.

ii) Inductive Hypothesis.

$P(K)$ is true

$P(K) = 11^{K+2} + 12^{2K+1}$ is divisible by 133

assume $P(K)$ is true

iii) Inductive step

$P(K+1) = 11^{K+3} + 12^{2K+3}$ is divisible by 133

$$= 11^{K+2} \cdot 11 + 12^{2K+1} \cdot 12^2$$

$$= 11^K \cdot 11^{K+2} + 144 \cdot 12^{2K+1}$$

$$= 11 \cdot 11^{K+2} + (133+11)\, 12^{2K+1}$$

$$= 11 \cdot 11^{K+2} + 133 \cdot 12^{2K+1} + 11 \cdot 12^{2K+1}$$

$$= 11 \cdot 133m + 133 \cdot 12^{2K+1}$$

$$= 133\,(11m + 12^{2K+1})$$

$$= 133\,N$$

$P(K+1)$ is divisible by 133

Q. use PMI to show that $2^n < n!$ for $n \geq 4$.

Assume that $P(n)$ for $n > 4$

$$2^n < n!$$

i) Inductive Base

$$P(4) = 2^4 = 16$$

$$4! = 4 \times 3 \times 2 = 24$$

$$2^4 < 4!$$

Hence, $P(4)$ is true.

ii) Inductive Hypothesis

$P(K)$ is true

$P(K) = 2^K$ is less than $K!$

Assume, $P(K)$ is true $\quad 2^K < K! \quad$ — (1)

iii) Inductive Step

$P(K+1) = 2^{K+1}$ is less than $(K+1)!$

$$= 2^K \cdot 2$$

from eqn (1)

$$2^K < K!$$

multiply by both sides by 2

$$2 \cdot 2^K < 2\,(K!)$$

$$2 \cdot 2^K < 2\,(K+1)\,K!$$

$$2 \cdot 2^K < (1+1)\,K!$$

$(1+1)\,K\,(K-1)$

$(K+K)\,(K-1)$

$2K\,(K-1)$

$2K^2 - 2K$

$2K\,(K-1)$

Q. Show that

$$3+33+333+\cdots+333\cdots 3.3 = \frac{10^{n+1}-9n-10}{27}$$

Ackerman Function -

It is a function with two arrangment each of which can be assigned only non negative integer value.

It is a non primitive recursion function.

A(m,n)

1) if $m=0, n\neq 0$ then $A(0,n) = n+1$

2) if $m\neq 0, n=0$ then $A(m,0) = A(m-1), 1$

3) if $m\neq 0, n\neq 0$ then $A(m,n) = A(m-1), A(m,n-1)$

Q. Calculate A(1,1) and A(1,2) using Ackerman function.

A(1,1)

$m=1$

$n=1$

using 3rd property, A(m-1), A(m, n-1)

A(1-1), A(1, 1-1)

~~using 2nd~~ A(0, A(1,0))

~~A(1-1),1~~ A(1,0)+1

~~A(0,1)~~ = A(1-1), 1+1

~~A(1,1)~~ = A(0,1) + 1

(1+1) + 1

= 3

$m=1$

$n=2$

A(1-1), A(1, 2-1)

A(0, A(1,1))

~~A(1,1)~~ A(

Inverse function –

Let $f: x \to y$ be the bijective function f is the function that assign every element $y \in Y$ and unique element $x \in X$ such that $f(x) = y$ inverse of f is denoted by G_0

$$f: X \to Y$$
$$f^{-1}: Y \to X \quad \text{if } f(x) = y \Rightarrow f^{-1}(y) = x$$

eg. $X = \{1, 2, 3\}$

$Y = \{K, L, M\}$

Theorem of Inverse –

If a function $f \to X \to Y$, $f^{-1} = y \to X$ or in

the function G is called inverse of f if $fog = I_y$ and $gof = I_n$

$gof = x \to y$

$g(y) = x$

$f(x) = y$

now $fog(y) = f(g(y))$

$= f(x)$

$$\boxed{fog = I_y}$$

$$gof(x) = y(y)$$

$$\boxed{gof = I_n}$$

Q. Let $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(1,1)(1,5)(2,2)(2,3)(2,6)(3,2)(3,3)(5,1)(5,5)(4,4)(5,1)(5,5)(6,2)(6,6)\}$

Find the equivalence class

Reflexive = $(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)$

Symmetric $(1,5)(5,1)(2,3)(3,2)(2,6)(6,2)(3,6)(6,3)$

transitive $(1,5)(5,1)$ $(1,1)$

$(2,2)$ $(2,3)$ $(2,3)$

transitive

$[1] = \{1, 5\}$

$[2] = \{2, 3, 6\}$

$[3] = \{2, 3, 6\}$

$[4] = \{4\}$

$[5] = \{1, 5\}$

$[6] = \{2, 3, 6\}$

$[1] \to \{1, 5\} \to P_1$

$[2] \to \{2, 3, 6\} \to P_2$

$[4] \to \{4\} \to P_3$

$P_1 \cup P_2 \cup P_3 = \{1, 2, 3, 4, 5, 6\}$

if intersect $P_1 \cap P_2 \cap P_3 = \phi$

Q. How many symmetric and reflexive relation are possible for containing n elements.

Reflexive Relation $= 2^{(n)^2 - n} = 2^{n(n-1)}$

Symmetric Relation $= 2^n \times 2^{\frac{n(n-1)}{2}}$

Q. Find $A = (2, 4, 5, 7, 8) = B$

aRb if and only if $a + b \leq 12$ find relation matrix.

$A = 2, 4, 5, 7, 8$ $B = 2, 4, 5, 7, 8$

$R = (2,2)(2,4)(2,5)(2,7)(2,8)$

$(4,2)(4,4)(4,5)(4,7)(4,8)$

$(5,2)(5,5)(5,7)$

$(7,2)(7,4)(7,5)$

$(8,2)(8,4)$

$$\begin{array}{c|ccccc} & 2 & 4 & 5 & 7 & 8 \\ \hline 2 & 1 & 1 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 1 \\ 5 & 1 & 1 & 1 & 1 & 0 \\ 7 & 1 & 1 & 1 & 0 & 0 \\ 8 & 1 & 1 & 0 & 0 & 0 \end{array}$$

Q. Let z be the group of Integers with binary operation $*$ define by $a*b = a+b-2$ for all $a,b \in z$ find the identity element.

$a*e = a$ $\qquad$ $a*b = a+b-2$

$a+e-2 = a$ $\qquad$ $a*e = a+e-2$

$\boxed{e = 2}$

Q. classify countable and uncountable set. prove that z of all integers is countable finite set

z $0, +1, -1, +2, -2, +3, -3$

N 1 2 3 4 5 6 7

$$f_n = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{1-n}{2} & \text{if } n \text{ is odd} \end{cases}$$

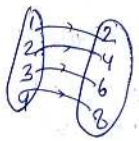
Q. Show that $f: N \to N$ given by $f = 2x$ is one to one but not onto

one to one $\qquad$ not onto

let $f(x) = \lambda$

$2x = \lambda$

$x = \frac{\lambda}{2}$



for each unique value of λ there will be a unique element form hence

Q. Justify R is equivalence relation $(a,b) R (c,d)$

$(x,y) R (a,b)$

$(a,b) R (c,d)$

Reflexive

$(x,y) R (a,b) \longrightarrow x^2+y^2 = a^2+b^2$

Symmetric $\quad (a,b) \in R \Rightarrow (b,a) \in R \; \forall a,b \in A$

$x^2+y^2 = a^2+b^2 \quad (x,y) R (a,b)$

$a^2+b^2 = x^2+y^2 \quad (a,b) R (x,y)$

Transitive

$x^2+y^2 = a^2+b^2$ — (1)

$a^2+b^2 = c^2+d^2$ — (2)

from eq (1) & (2)

$x^2+y^2 = c^2+d^2$

$(x,y) R (c,d)$

Q Justify that for any set A, B.

$A - A \cap B = A - B$ $\qquad$ $\boxed{A - B = (A \cap B)'}$

LHS

$\underbrace{A}_{A} - \underbrace{A \cap B}_{B}$

$(A \cap (A \cap B))'$ $\qquad$ apply de morgan's law

$A \cap A' \cup B'$ $\qquad$ Distributive law

$A \cap A' \cup A \cap B'$

ϕ

$(A-B) \cup (B-A) = A \cup B - (A \cap B)$

LHS

Let $x \in (A-B) \cup (B-A)$

$x \in A$ and $x \notin B$ or $x \in B$ and $x \notin A$

$x \in A$ or $x \in B$ and $x \notin A$ or $\notin B$

$x \in (A \cup B)$ and $\notin (A \cap B)$

$x \in (A \cup B)$ and $x \notin (A \cap B)$

$x \in (A \cup B - A \cap B)$

$x \in (A-B) \cup (B-A) \subseteq A \cup B - (A \cap B)$ — (1)

RHS

$A \cup B - A \cap B$

Let $x \in A \cup B - x \notin (A \cap B)$

$x \in A$ or $x \in B$ and $x \notin A$ or $x \notin B$

$x \in A$ and $x \notin B$ or $x \notin B$ and $x \notin A$

$x \in (A-B)$ or $x \in (B-A)$

$x \in (A-B) \cup (B-A)$

$A \cup B - (A \cap B) \subseteq (A-B) \cup (B-A)$ — (2)

$(A \cup B) - (A \cap B) = (A-B) \cup (B-A)$

Q. Find the numbers between 1 to 500 that are not divisible by any of the integer 2, 3, 5 or 7.

Rule

two digits

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

for three digit -

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

For four digit

$n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) - n(A \cap B) - n(A \cap C) - n(A \cap D)$
$- n(B \cap C) - n(B \cap D) - n(C \cap D) + n(A \cap B \cap C) + n(A \cap C \cap D) + n(B \cap C \cap D)$
$- n(A \cap B \cap C \cap D)$

Total no. = 500

Let $A = 2$, $B = 3$, $C = 5$, $D = 7$

$\frac{500}{2} + \frac{500}{3} + \frac{500}{5} + \frac{500}{7} - \frac{500}{6} - \frac{500}{10} - \frac{500}{14} - \frac{500}{15} - \frac{500}{21} - \frac{500}{35}$

$+ \frac{500}{30} + \frac{500}{70} + \frac{500}{105} + \frac{500}{210}$

(Ignore values (digits) after decimal)

$\Rightarrow 250 + 166 + 100 + 71 - 83 - 50 - 35 - 33 - 23 - 14 + 16 + 7 + 4 + 2$

$= 500 -$

Q. Find the numbers between 100 to 1000 that are divisible by any of the integer 3, 5 or 7. $A = 3$, $B = 5$, $C = 7$

1 to 100

$= \frac{100}{3} + \frac{100}{5} + \frac{100}{7} - \frac{100}{15} - \frac{100}{21} - \frac{100}{35} + \frac{100}{105}$

$= 33 + 20 - 14 - 6 - 4 - 2 + 0$

$= 55$

$\frac{1000}{3} + \frac{1000}{5} + \frac{1000}{7} - \frac{1000}{15} - \frac{1000}{21} - \frac{1000}{35} + \frac{1000}{105}$

$333 + 200 + 142 - 66 - 47 - 28 + 9$

$= 547$

$543 - 55 + 1$

$= 489$

COSET-

Let $(Z,+)$ integer be an additive group

if $(H,*)$ is the subgroup of $(G,*)$ and $a \in G$ then $\{ha : h \in H\} = H*a$ is called right coset generated by H and a. and also $a*H = \{ah : h \in H\}$ is called left coset generated by a and H.

Properties of Coset-

1) $a \in aH, \forall a \in G$
2) $aH = H$ if $a \in H$
3) $aH = bH$ or $aH \cap bH = \phi$
4) aH is subgroup of G where $a \in H$
5) Union of distinct cosets provide the group

* Index of coset
if H is the subgroup of group G then no. of distinct left or right coset of H in G is called Index of H in group G. It is denoted by $i_G H$ or $[G:H]$ and according to language of theory $\frac{O(G)}{O(H)}$ = total no. of distinct coset of H

Let $G = \{1,-1,i,-i\}$, $H = \{1,-1\}, *\}$ Find the left coset.

Left coset = No of element in a subgroup.

$1 \in G \quad 1 \times H = 1 \times \{1,-1\} = \{1,-1\}$
$-1 \in G \quad (-1) \times H = -1 \times \{1,-1\} = \{-1,1\}$ } H
$i \in G \quad (i) \times H = i \times \{1,-1\} = \{i,-i\}$
$-i \in G \quad (-i) \times H = -i \times \{1,-1\} = \{-i,+i\}$ } H

Right Coset =

$1 \in G \quad H \times 1 = \{1,-1\} \times 1 = \{1,-1\}$
$-1 \in G \quad H \times (-1) = \{1,-1\} \times -1 = \{-1,1\}$
$i \in G \quad H \times (i) = \{1,-1\} \times i = \{i,-i\}$
$-i \in G \quad H \times (-i) = \{1,-1\} \times -i = \{-i,i\}$

* if Left coset = Right coset then it is a Abelian group.

Let $(Z,+)$ be an additive group and $H \subset Z$ find the left coset where H = multiple of 3

$Z = \{-5,-4,-3,-2,-1,0,1,2,3,4,5,6,\ldots\}$

$H = \{\ldots -9,-6,-3,0,9,12,15 \ldots \infty\}$

Right coset
$H+0 = 0+\{-6,-3,0,3,6\} = \{-6,-3,0,3,6\}$
$H+1 = 1+\{-6,-3,0,3,6\} = \{-5,-2,1,4,7\}$
$H+2 = 2+\{-6,-3,0,3,6\} = \{-4,-1,2,5,8\}$
$H+3 = 3+\{-6,-3,0,3,6\} = \{-3,0,3,6,9\}$

Left coset
$0+H = \{-6,-3,0,3,6\}+0 = \{-6,-3,0,3,6\}$
$\phi+H = \{-6,-3,0,3,6\}+1 = \{-5,-2,1,4,7\}$
$2+H = \{-6,-3,0,3,6\}+2 = \{-4,-1,2,5,8\}$
$3+H = \{-6,3,0,3,6\}+3 = \{-3,0,3,6,9\}$

Find all the left coset of $H = \{1,11\}$ group $\{U_{30}, X_{30}\}$

$U = \{1,7,11,13,17,19,23,29\}$
$H = \{1,11\}$

X_{30} = Mod

$77 = \frac{77}{30} = 17$ remainder is Mod

$1 X_{30} H = 1 X_{30} \{1,11\} = \{1,11\} = \{1,11\}$ — H_1
$7 X_{30} H = 7 X_{30} \{1,11\} = \{7,77\} = \{7,17\}$ — H_2
$11 X_{30} H = 11 X_{30} \{1,11\} = \{11,121\} = \{11,1\}$ — H_1
$13 X_{30} H = 11 X_{30} \{1,11\} = \{13,143\} = \{13,23\}$ — H_3
$17 X_{30} H = 17 X_{30} \{1,11\} = \{17,187\} = \{17,7\}$ — H_2
$19 X_{30} H = 19 X_{30} \{1,11\} = \{19,209\} = \{19,29\}$ — H_4
$23 X_{30} H = 23 X_{30} \{1,11\} = \{23,253\} = \{23,13\}$ — H_3
$29 X_{30} H = 29 X_{30} \{1,11\} = \{29,319\} = \{29,19\}$ — H_4

Lagranges theorem

Statement 1:

order of each subgroup of each finite group is divisor of order of group.

$$\boxed{\frac{O(G)}{O(H)} = K}$$

Statement 2:

No. of distinct left or right subgroup in group G is subgroup

Proof -

Let $(G, *)$ be a group of order n

i.e. $O(G) = n$

$G = \{a_1, a_2, a_3 ---- a_n\}$

and $(H, *)$ be subgroup of order m

$O(H) = m$

Let $H = h_1, h_2, h_3 --- h_m\}$

So left coset

$a_1 * H = \{a_1 * h_1, a_2 * h_2, a_n * h_m\}$

$a_1 H, \quad a_2 H, a_3 H --- a_K H$

$G = |a_1 H| \cup |a_2 H| --- \cup |a_K H|$

$n = m \cup m \cup \quad --- K$

$n = m \times K$

$\frac{n}{m} = K$

$$\boxed{\frac{O(G)}{O(H)} = K}$$

SHIVAM

Let if $(Z, +)$ is the group where Z is set of integer and H is a subgroup of G. Find the index of H in Z. (contains multiple of six (6)).

$G(Z, +)$

$H = \{-\infty, -12, -6, 0, 6, 12, 18 --- \infty\}$

Left coset

$0 + H = 0 + \{-12, -6, 0, 6, 12\} = \{-12, -6, 0, 6, 12\}$

$1 + H = 1 + \{-12, -6, 0, 6, 12\} = \{-11, -5, 1, 7, 13\}$

$2 + H = 2 + \{-12, -6, 0, 6, 12\} = \{-10, -4, 2, 8, 14\}$

$3 + H = 3 + \{-12, -6, 0, 6, 12\} = \{-9, -3, 3, 9, 15\}$

$4 + H = 4 + \{-12, -6, 0, 6, 12\} = \{-8, -2, 4, 10, 16\}$

$5 + H = 5 + \{-12, -6, 0, 6, 12\} = \{-7, -1, 5, 11, 17\}$

$6 + H = 6 + \{-12, -6, 0, 6, 12\} = \{-5, 0, 6, 12, 18\}$

Lagrange's theorem

Statement 1:

order of each subgroup of each finite group is divisor of order of group.

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$G = \{a_1, a_2, a_3 ---- a_n\}$

and $(H, *)$ be subgroup of order m

$O(H) = m$

Let $H = h_1, h_2, h_3 --- h_m\}$

So left coset

$a_1 * H = \{a_1 * h_1, a_2 * h_2, a_n * h_m\}$

$a_1 H, \quad a_2 H, a_3 H --- a_K H$

$G = |a_1 H| \cup |a_2 H| --- \cup |a_K H|$

$n = m \cup m \cup \quad --- K$

$n = m \times K$

$\frac{n}{m} = K$

$$\boxed{\frac{O(G)}{O(H)} = K}$$

Let if $(Z, +)$ is the group where Z is set of integer and H is a subgroup of G. Find the index of H in Z. (contains multiple of six (6)).

$G(Z, +)$

$H = \{-\infty, -12, -6, 0, 6, 12, 18 --- \infty\}$

Left coset

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$5 + H = 5 + \{-12, -6, 0, 6, 12\} = \{-7, -1, 5, 11, 17\}$

$6 + H = 6 + \{-12, -6, 0, 6, 12\} = \{-5, 0, 6, 12, 18\}$

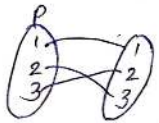
Permutation -

Let P be a finite set having N distinct element then 1:1 onto mapping from P to itself is called Permutation of degree n.

$P = (1, 2, 3)$

$F : P \to P$

$(\sigma) \cdot P_1 > \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$



Let P be set consisting of n distinct element then the element of P can be Permutated

$P = (1, 2, 3)$

$3! = 3.2.1 = 6$

$P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ $P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ $P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

$P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ $P_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ $P_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

Types of Permutation -

Identity Permutation -

It is defined on set x denoted by I_x. if image of each element is element itself then is called Identity Permutation.

Equal Permutation :-

Let F and G be two permutation on a set S then we defined

$F = G$ if $F(x) = g(x)$

eg. $F = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix}$ $g = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 4 & 2 & 1 & 3 \end{pmatrix}$

$F(1) = 2$
$F(2) = 4$
$F(3) = 3$
$F(4) = 1$

$g(1) = 2$
$g(2) = 4$
$g(3) = 3$
$g(4) = 1$

Inverse Permutation -

Let F is a permutation of degree n defined on set S

$F : S \xrightarrow[\text{onto}]{\text{one-one}}$

$F = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & \cdots & a_n \\ b_1 & b_2 & b_3 & b_4 & \cdots & b_n \end{pmatrix}$

$F^{-1} = \begin{pmatrix} b_1 & b_2 & b_3 & b_4 & \cdots & b_n \\ a_1 & a_2 & a_3 & a_4 & \cdots & b_n \end{pmatrix}$

$F = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}$

$F^{-1} = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$

imp

Cyclic Permutation -

A Permutation which replaces an object cyclicly is called Cyclic Permutation of degree n.

$P = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

Identity element is not considered in cyclic permutation

Cyclic Per $(1\ 2\ 3\ 4)$ length = 4

$t = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{pmatrix}$

$(1, 2)$ $(3, 4, 6)$ (5) ×

$l = 2$ $l = 3$ $l = 1$

* It is also a disjoint cycle

Disjoint cycle

Two cycles are said to be disjoint if they have no symbol in common

Transposition -

A cycle of length two is called transposition

Eg. the cycle $(2,3)$ $l=2$

Multiplication or Product of cycle -

If the cycle $\underset{1}{(1,2,3)}$ and $\underset{2}{(5,6,4,1)}$ represent the permutation of degree 6.

$$P_1 \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 4 & 5 & 6 \end{pmatrix} \quad P_2 \begin{pmatrix} 2 & 5 & 6 & 4 & 1 & 3 \\ 2 & 6 & 4 & 1 & 5 & 3 \end{pmatrix}$$

$$P_1 o P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 4 & 5 & 6 \\ 2 & 3 & 5 & 1 & 6 & 4 \end{pmatrix}$$

$$P_1 o P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 5 & 1 & 6 & 4 \end{pmatrix}$$

Even and odd Permutation -

$$t = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{pmatrix}$$

~~(1,2) (2,1) (3,4) (4,6) (6,3)~~

$(1,2)$ $(3,4,6)$ $(5)_x$

$(1,2)$ $(3,4)$ $(3,6)$

odd.

Examine whether the following permutations - is even or odd -

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$$

$(1,2,5,6)$ $(3,4)$ $(7)_x$ $(8,9)$

$\underset{1}{(1,2)}$ $\underset{2}{(1,5)}$ $\underset{3}{(1,6)}$ $\underset{4}{(3,4)}$ $\underset{5}{(8,9)}$ odd (5)

Properties -

i) A Permutation cannot be even and odd

ii) A cycle of length N can be expressed as the product of N-1

iii) Identity permutation is always an even permutation.

iv) The product of two odd permutation is odd permutation.

v) The product of two even permutation is even permutation

vi) The product of an even and odd permutation is odd permutation

$A = \{1,2,3,4,5\}$

Find $(1,3) o (2,4,5) o (2,3)$

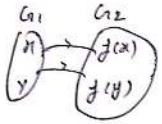
$$\underset{P_1}{\begin{pmatrix} 1 & 3 & 2 & 4 & 5 \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}} o \underset{P_2}{\begin{pmatrix} 2 & 4 & 5 & 1 & 3 \\ 4 & 5 & 2 & 1 & 3 \end{pmatrix}} o \underset{P_3}{\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 4 & 5 \end{pmatrix}}$$

$$P_2 o P_3 \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix} \qquad P_1 o P_3 \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 5 & 2 \end{pmatrix}$$

Homomorphism -

If $(G_1, \cdot)$ and $(G_2, *)$ are the two groups then a function $f: G_1 \to G_2$ is called Homomorphism.

if $(x\ y) = f(x) * f(y)$, $\forall\ x, y \in G_1$



Types of Homomorphism

Monomorphism -

if the map f is injective then this Homomorphism is called Monomorphism.

Epimorphism -

if the map f is surjective (onto) then this Homomorphism is called Epimorphism.

Isomorphism -

If the G_1 and G_2 are two groups then a function $f: G_1 \to G_2$ is called Isomorphism if

i) f is Homomorphism $(x \cdot y) = f(x) * f(y)$, $\forall\ x, y \in G$

ii) f is bijective function

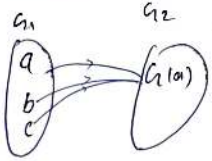
Automorphism -

An Isomorphism of group G onto itself is called Automorphism that $f: G \to G$

condition (i) it should be Homomorphism

ii) It should be bijective

Trivial Homomorphism -



many to one.

Q. If $G_1 = (Z\ +)$ and $G_2 = G_1$ and mapping $f: G_1 \to G_2$ is defined $f(x) = 2x\ \forall\ x \in Z$ Show that f is Homomorphism.

$f(x+y) = f(x) + f(y)$, $\forall\ x \in G_1$

LHS

$f(x+y)$

$f(x)$

$2(x+y)$

$2x + 2y$

$f(x) + f(y)$

So f is Homomorphism.

* imp

Theorem of Homomorphism -

if f such that $G_1 \to G_2$ is a homomorphism than show that

i) $f(e) = e'$ where e and e' are the identity element of G_1 & G_2 respectively

ii) $f(x^{-1}) = [f(x)]^{-1}\ \forall\ x \in G_1$

Proof!

Let $x \in G_1$ $\quad f(x) \in G_2$

$x^{-1} \in G_1$ $\quad f(x)^{-1} \in G_2$

(1) $e' = f(e)$

Let $x \in G \Rightarrow f(x) = G'$

$\Rightarrow e' \circ f(x) = f(x)$ $\qquad f(x \circ y) = f(x) \circ f(y)$

$\Rightarrow e' \circ f(x) = f(e \circ x)$

$e' \circ' f(x) = f(e) \circ f(x)$ (Apply left cancellation law)

$e' = f(e)$

Proof 2

$[f(x)]^{-1} = f(x^{-1})$

$f: G \to G_1$

Let $x \in G$ $\quad f(x) \in G'$

$x^{-1} \in G$ $\quad f(x)^{-1} \in G'$

$f(x \circ y) = f(x) * f(y) \; \forall x, y \in$

$f(x) \circ' f(x)^{-1} = f(x \circ x^{-1})$

$f(x) \circ' f(x^{-1}) = f(e)$

from Proof 1 $f(e) = e'$

$f(x) \circ' f(x^{-1}) = e'$

$f(x^{-1}) = [f(x)]^{-1}$

Hence proved.

Show that any infinite cyclic group is isomorphic group $(Z, +)$

Let us suppose $C_\infty = \{\langle g \rangle, g, g^2, g^3 --- g^n\}$

$(Z, +) = \{n, n \in Z\}$

$\theta : C_\infty \to Z$

$\theta(g^n) = n$ — (1)

Isomorphism

One to one

Let $\theta(g^{n_1}) = \theta(g^{n_2})$

$\boxed{n_1 = n_2}$

So, one to one mapping.

Homomorphism

Let $g^{n_1}, g^{n_2} --- \in C_\infty$

$\theta(g^{n_1}) = n_1, \quad \theta(g^{n_2}) = n_2$

Now

$\theta(g^{n_1} \cdot g^{n_2}) = \theta(g^{n_1 + n_2})$

$n_1 + n_2$ by eq (1)

$\theta(g^{n_1}) + \theta(g^{n_2})$

Onto

Let $n \in Z$ then there exist

$g^n \in C_\infty$

$\theta(g^n) = n$

θ is onto function.

Hence infinite cyclic group is isomorphic.

Find all subgroups of $(Z_5, +_5)$

$Z_5 = \{0, 1, 2, 3, 4\}$

Ring $(R, +)$

i) $(R, +)$ is an abelian group

ii) $(R \cdot)$ is a semi group

$(\cdot)$ is a distributive over $+$

$a \cdot (b+c) = (ab) + (a \cdot c)$

Ring are commutative if

i) $a \cdot b = b \cdot a$ under multiplication

ii) R with unity

$a \cdot e = a = e \cdot a$

$e = 1$

If $1 \in R$ such that $a \cdot 1 = a \; \forall a \in R$

the $1 \in R$ is called unit or identity element

element of ring R and rings is called Ring with unity.

Integral Domain-

There must be atleast two element holding the properties

1) Commutative $(a \cdot b = b \cdot a)$

2) Unit element $(a \cdot e = a, \; e \cdot a = a)$

3) Ring without zero divisor.

Ring with zero divisor-

i) $a \neq 0, b \neq 0$

$a \cdot b = 0$

ex

$a = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \quad b = \begin{bmatrix} 0 & 0 \\ 6 & 3 \end{bmatrix}$

$a \cdot b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

ii) $5 \times_3 6 = \frac{30}{3} = 0$

Mod

Ring without zero divisor-

Show that if $a \cdot b \neq 0$

if $a \neq 0$

$b \neq 0$

Show that set of Integer Z for addition and multiplication of Integer is a commutative ring with unity.

Abelian group $(+)$

i) closure

$a, b \in Z$

$a+b \in Z$

ii) Associative

$\forall a, b, c \in Z$

$a = 1, b = 2, c = 3$

$a + (b+c) = (a+b) + c$

$1 + (2+3) = (1+2) + 3$

$1 + 5 = 3 + 3$

$6 = 6$

iii) Identity

$a + e = a$

$2 + 0 = 2$

$e = 0$

$-2 + 0 = -2$

iv) Inverse

$a + (a^{-1}) = e \quad \forall a \in Z$

$1 + (-1) = 0$

$2 + (-2) = 0$

v) Commutative

$a + b = b + a \quad \forall a, b \in Z$

$1 + 2 = 2 + 1$

$3 = 3$

$-1 + 3 = 3 + (-1)$

$2 = 2$ satisfied, abelian group

II) Check $(Z, \times$ is a Semigroup)

a) Closure

$a, b \in Z$

$a \cdot b \in Z$

b) associative.

$a \times (b \times c) = (a \times b) \times c$

$1 \times (2 \times 3) = (1 \times 2) \times 3$

$1 \times 6 = 2 \times 3$

$6 = 6$

III) Check left and right distributive.

LD

$a \times (b+c) = a \times b + a \times c$

$2 \times (3+4) = (2 \times 3) + (2 \times 4)$

$14 = 14$

RD

$(a+b) \times c = (a \times c) + (a \times b)$

$(2+3) \times 4 = (2 \times 4) + (3 \times 4)$

$20 = 20$

To check commutative Ring with unity.

<u>Commutative</u>

$a \times b = b \times a$

let $a = 2$ $b = 3$

$2 \times 3 = 3 \times 2$

$6 = 6$

$-2 \times 3 = 3 \times -2$

$-6 = -6$

Identity.

$a \times e = a$

$2 \times 1 = 2$

$e = 1$

Hence 1 is an identity element and $1 \in Z$

Hence $(Z, +,)$ algebraic structure is a commutative ring with unity.

Q. Show that system $(E+)$ is a commutative ring without unity with usual multiplication and where E is the set of even Integers.

Show that $(Z_5 +_5 \times_5)$ is a commutative Ring with unity.

$Z_5 = \{0, 1, 2, 3, 4\}$

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

1) closure hold property

2) commutative

$a +_5 b = b +_5 a$

$(1 +_5 2) = (2 +_5 1)$

$3 = 3$

3) Associative

$a +_5 (b +_5 c) = (a +_5 b) +_5 c$

$1 +_5 (2 +_5 3) = (1 +_5 2) +_5 3$

$1 +_5 (0) = 3 +_5 3$

$= 1$

4) Identity

$0 +_5 0 = 0$

$1 +_5 0 = 1$

$2 +_5 0 = 2$

$3 +_5 0 = 3$

$4 +_5 0 = 4$

0 is Identity element.

5) Inverse

$0 +_5 0 = 0$

$1 +_5 4 = 0$

$2 +_5 3 = 0$

$3 +_5 2 = 0$

$4 +_5 1 = 0$

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

INTEGRAL DOMAIN-

1. Commutative 2. unit element 3 without zero divisors

Let I be any non-empty set with addition and multiplication operation are two binary operation $(+ \cdot)$ on field then the system is called field if it has following condition satisfy

i) $F+$ is an abelian group

ii) $F, \cdot$ is an abelian group.

when $F' = \{x \in F / x \neq 0\}$

iii) The set is distributive under binary operation of multiplication over addition.

iv) $a \cdot (b+c) = (a \cdot b) + (a \cdot c) \ \forall \ a, b, c \in F$

Example of field

- Set of real no.
- Set of rational no.
- Set of complex no

Q. $S = \{a + b\sqrt{2}, a, b \in Z\}$ the given set is integral domain ? check whether the given set is field or not.

$x = a + b\sqrt{2} \quad y = c + d\sqrt{2} \quad z = e + f\sqrt{2}$

i) R_1 closure $x+y \in S$, $x, y \in S$

ii) R_2 commutative $x+y = y+x$

$a + b\sqrt{2} + c + d\sqrt{2} = c + d\sqrt{2} + a + b\sqrt{2}$

iii) R_3 associative $(x+y)+z = x + (y+z)$

$(a + b\sqrt{2} + c + d\sqrt{2}) + (e + f\sqrt{2}) =$

$a + b\sqrt{2} + (c + d\sqrt{2} + e + f\sqrt{2})$

$R_4 = x + e = x$

$a + b\sqrt{2} + e = a + b\sqrt{2}$

$\therefore 0 + 0\sqrt{2} \in S$

$R_5 =$ inverse

$x + m = e$

$a + b\sqrt{2} + m = 0$

$m = -(a + b\sqrt{2})$

Integral domain

commutative

$xy = yx$

$(a + b\sqrt{2})(c + d\sqrt{2}) = (c + d\sqrt{2})(a + b\sqrt{2})$

$ac + ad\sqrt{2} + bc\sqrt{2} = ac + bc\sqrt{2} + ad\sqrt{2} + 2bd$
$+ 2bd$

unit element

$x \cdot e = x$

$(a + b\sqrt{2}) \cdot e = (a + b\sqrt{2})$

$e = 1$

without zero divisor

$xy = 0 \qquad x = 0$ or $y = 0$

$(a + b\sqrt{2})(c + d\sqrt{2}) = 0$

$ac + bc\sqrt{2} + ad\sqrt{2} + 2bd = 0$

$(ac + 2bd) + (bc + ad)\sqrt{2} = 0$

$ac + 2db = 0$ or $\sqrt{2}(ad + bc) = 0$

$a = 0, \ b = 0, \ c = 0, \ d = 0$

field $x, (a+b) + (a+c) = a(b+c)$

left distributive $= x(y+z)$
$= xy + xz$

Right distributive -

$(x+y)\cdot z = x\cdot z + y\cdot z$

right derivative $= (a+b\sqrt{2}+c+d\sqrt{2})\cdot(e+f\sqrt{2})$

$[(a+b\sqrt{2})\times(e+f\sqrt{2})] + [(c+d\sqrt{2})(e+f\sqrt{2})]$

$ae + af\sqrt{2} + be\sqrt{2} + 2bf + ce + cf\sqrt{2} + de\sqrt{2} + df2$

multiplicative inverse -

$x = a+b\sqrt{2}$

$x^{-1} = \dfrac{1}{a+b\sqrt{2}}$

$= \dfrac{1}{a+b\sqrt{2}} \; \dfrac{a-b\sqrt{2}}{a-b\sqrt{2}}$

$= \dfrac{a-b\sqrt{2}}{a^2-2b^2}$

$= \dfrac{a}{a^2-2b^2} + \dfrac{-b\sqrt{2}}{a^2-2b^2}$

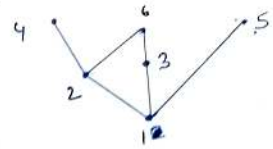
It is an satisfying integral but not field

Unit-3

Hasse diagram-

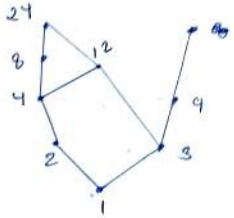
1) $x = \{1,2,3,4,5,6\}$ $(x, /)$

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,4),\ (2,6)\ (3,6)$



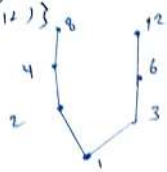
2) $x = \{1, 2, 3, 4, 8, 9, 12, 18, 24\}$

$\{(1,1) \ldots (1,6)$
$(2,4) \ldots (2,24)$
$(3,9), (3,12)\ (3,18)(3,24)$
$(4,8)\ (4,12)\ (4,24)$
$(12,24)\ (18,12) \ldots$

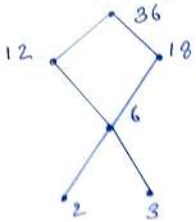


3) Construct a Hasse diagram representing the partial ordering $\{(a,b)$ a divides $b\}$ on $\{(1,2,3,4,6,8,12)\}$

$\{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6),\ (1,8)\ (1,12)$
$(2,4)\ (2,6)\ (2,8)\ (2,12)$
$(3,6),\ (3,12)$
$(4,8)\ (4,12)\ (6,12)\ \}$



Q. $(2,3,6,12,18,36)$



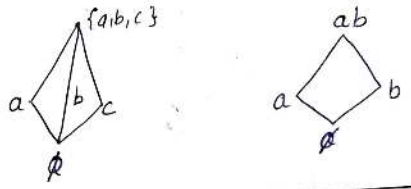
$\{(2,6)\ (2,12)\ (2,18)\ (2,36)$
$(3,6)\ (3,12)\ (3,18)\ (3,36)$
$(6,12)\ (6,18)\ (6,36)$
$(12,36)$

Poset & lattice -

$X = \{1, 2, 3, 4, 8, 9, 12, 18, 24\}$ $(X, /)$

$S = (a, b)$ Make the Hasse diagram of Power set.

$P_S = \{\phi, a, b, ab\}$



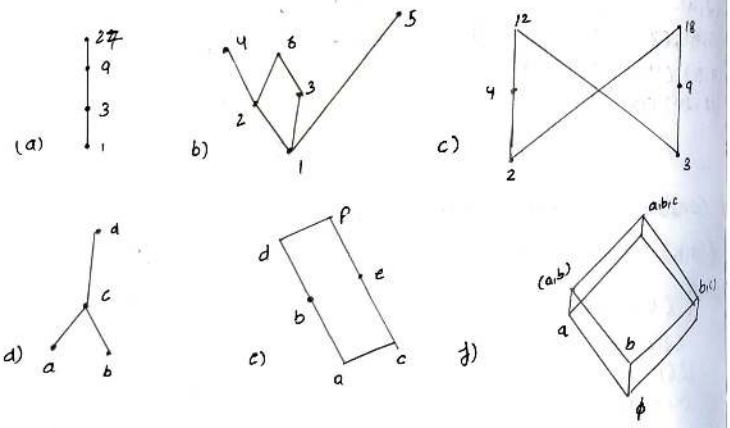


fig	Greatest	least	Maximal	Minimal
a	27	1	27	1
b	—	1	4,5,6	1
c	—	—	(12,18)	(2,3)
d	d	—	d	a,b
e	f	a	f	a
f	{a,b,c}	ϕ	{a,b,c}	ϕ

Upper bound -

Let S be an non-empty subset of P. an element a belongs to P, $a \in P$ is called upper bound of S if $x \le a$ for all $x \in S$

Least upper bound - (LUB) Super mom

If a is any upper bound of S such that $a \le b$ for all other upper bound of B.

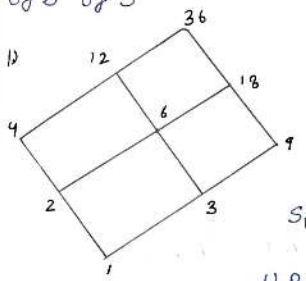
Lower bound -

an element $a \in P$ is called lower bound of S if $a \le x, \forall x \in S$

Greatest lower bound -

If a is lower bound such that $b \le a$ for all other lower bound of 'b' of S

1)

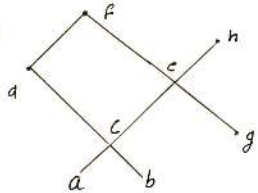


$S_1 = \{4, 6\}$

$S_2 = \{2, 3, 6\}$

$S_1 = 4, 6$	$S_2 = \{2, 3, 6\}$
$U.B = \{12, 36\}$	$U.B = \{6, 12, 18, 36\}$
$LUB = \{12\}$	$LUB = 6$
$LB = \{2, 1\}$	$LB = 1$
$GLB = \{2\}$	$GLB = 1$

2.)



$S_1 = \{c, g\}$

$S_2 = \{d, e, h\}$

$S_1 = \{c, g\}$	$S_2 = \{d, e, h\}$
U.B. $= \{f, e, h\}$	U.B. $= \phi$
L.U.B $=$ e	LUB $= \phi$
L.B $= \phi$	LB $= \{a, b, c\}$
GLB $= \phi$	GLB $= c$

~~LAT~~ $A = \{1, 2, 3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$

$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\ (1,3)\}$

Check POSET (R, A, T)

$R_1 =$ Ref., Anti-sy, Trans

$R_2 =$ Ref., Anti-sy, Trans.

Hence, POSET (A, R_1) (A, R_2)

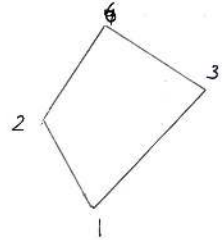
LATTICE (Imp) -

~~A Poset~~ A lattice is a Partially order set (Poset) in which every pair of element such as $a, b \in L$ has greatest lower bound and ~~lower bound~~ Least upper bound.

LUB (Super mum) of a $\{a, b\} \subseteq L$ and denoted by $a \vee b$ or $a \oplus b$ and is called Join or Sum of a and b

GLB (Inf mum) of a Subset is denoted by $a \wedge b$ or $a * b$ is called meet or product of a and b

Q.1



LUB table

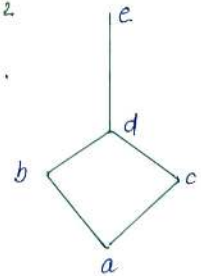
$\vee$	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

GLB table

$\wedge$	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

For every pair of elements in a given Poset both LUB & GLB exists hence we can say that a given Poset S lattice.

Q.2



LUB table

$\vee$	a	b	c	d	e
a	a	b	c	d	e
b	b	b	d	d	e
c	c	d	c	d	e
d	d	d	d	d	e
e	e	e	e	e	e

GLB table

$\wedge$	a	b	c	d	e
a	a	a	a	a	a
b	a	b	a	b	b
c	a	a	c	c	c
d	d	b	c	d	d
e	a	b	c	d	e

It is lattice

Part of unit-2

Order & Generator of element -

order of group = Total no. of elements

order of element G^n , n is order of element

Q. Find the order & generator of element of order 6 under $(+_6)$

Sol. $\{0,1,2,3,4,5,+_6\}$

$O(G) = 6$

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4						
5	5	0	1	2	3	4

order of element -

$O(0) = 0 = 0^1$

$O(1) = 0 = 1^6$

$O(2) = 0 = 2^3 = 2 +_6 2 +_6 2$

$O(3) = 0 = 3^2 = 3 +_6 3$

$O(4) = 4^3 = 4 +_6 4 +_6 4$

$O(5) = 5^6 = 5 +_6 5 +_6 5 +_6 5 +_6 5 +_6 5$

$O(0) = 1$

$\boxed{O(1) = 6}$

$O(2) = 3$

$O(3) = 2$

$O(4) = 3$

$\boxed{O(5) = 6}$

Trick

Generator of element

0	1	2	3	4	5
$0^0 = 0$	$1^6_{+_6} = 0$	$2^3 = 0$	3^2	4^3	$5^6 = 0$
$0^1 =$	$1^1 = 1$	$1 \times$			$5^5 = 1$
$0^2 =$	$1^{+_2} = 2$				$5^4 = 2$
	$1^{+_3} = 3$				$5^3 = 3$
	$1^{+_4} = 4$				$5^2 = 4$
	$1^{+_5} = 5$				$5^1 = 5$

Since 1 & 5 are generator all ~~Just of~~ elements so 1, 5 are "

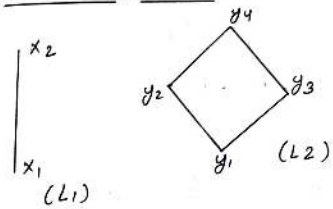
Normal Subgroup

if H is a subgroup of G and if $Ha = aH$ $\forall a \in G$ then every subgroup H is called Normal Subgroup.

Unit-3 continue---

Multiplication of lattice -

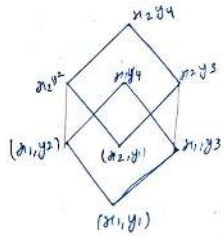
1)



$L_1 = \{x_1, x_2\}$

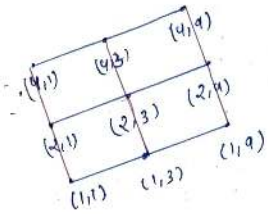
$L_2 = \{y_1, y_2, y_3, y_4\}$

$L_1 \times L_2$ $\{(x_1,y_1)\ (x_1,y_2)\ (x_1,y_3)\ (x_1,y_4)$
$(x_2,y_1)\ (x_2,y_2)\ (x_2,y_3)\ (x_2,y_4)\}$



2) $L_1 = \{1, 2, 4\}$, $L_2 = \{1, 3, 9\}$

$L_1 \times L_2 = \{(1,1)\ (1,3)\ (1,9)\ (2,1)\ (2,3)\ (2,9)\ (4,1)\ (4,3)\ (4,9)\}$



Types of lattice -

a) Complete Lattice

A lattice P is called complete if every non-empty subset glb & lub

ex $\{Z, \leq\}$

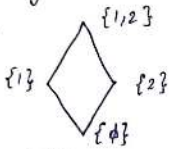
$\{S = x \in Z \mid x > 0\}$

It is a infinite set so it is not complete lattice

Every finite set is a complete lattice as it contain glb and lub

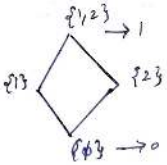
Bounded lattice -

If a lattice has both greatest and least element then it is called as a Bounded lattice



Properties of Bounded lattice -

1) $a \vee 1 = 1$ join (V) - LUB
2) $a \wedge 1 = a$ meet (∧) - GLB
3) $a \vee 0 = a$
4) $a \wedge 0 = 0$



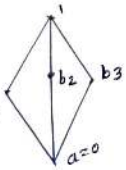
c) Modular lattice -

Lattice with join and meet operation proves the following property

$a \vee (b \wedge c) = (a \vee b) \wedge c$, where $a \leq c$

$\leq$ a precedes c
a is related to c

ex



Let $a = 0 = b_1$ $c = b_2$

on LHS

$a \vee (b \wedge c) = 0 \vee (b_1 \wedge b_2)$

$0 \vee 0$

$= 0$

on RHS

$(a \vee b) \wedge c = (0 \vee b_1) \wedge b_2$

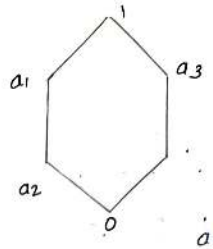
$= b_1 \wedge b_2$

$= 0$

LHS = RHS

Hence it is Modular lattice

Q.)



Let $a = a_2$ $b = a_3$ $c = a_1$

on LHS

$a \vee (b \wedge c) = a_2 \vee (a_3 \wedge a_1)$

$= a_2 \vee 0$

$= a_2$

On RHS

$a \wedge (b \vee c) = a_2 \wedge (a_3 \vee a_1)$

$= a_2 \wedge 1$

$= 0$

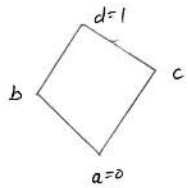
LHS ≠ RHS

It is not modular lattice

Compliment lattice -

It is said to be complement $a \vee x = 1$ (UB) $a \wedge x = 0$ (LB)

Ex

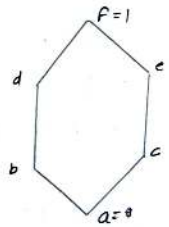


1) $a \vee d = 1$ (d)
2) $a \wedge d = 0$ (a)

$\boxed{a^c = d \quad d^c = a}$

$b \vee c = 1$ (d)
$b \wedge c = 0$ (a)

$\boxed{b^c = c \quad c^c = b}$



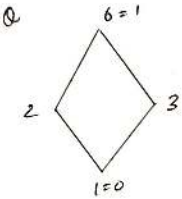
$a \vee f = 1$ (f) $a^c = f$
$a \wedge c = 0$ (a) $f^c = a$

$d \vee e = f$ $d^c = e$
$d \wedge e = e$ $e^c = d$

$b^c = c$
$c^c = b$

a Bounded lattice P is said to be compliment lattice if every element of P has atleast 1 complement

Q $(D_6/) = 1, 2, 3, 6$



$\vee$	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

$\wedge$	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

$1^c = 6$ $\quad 2^c = 3$

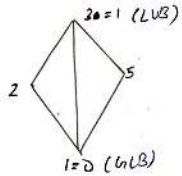
$6^c = 1$ $\quad 3^c = 2$

Distributive lattice-

A lattice is called distributive if it is satisfied both the properties 1) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Q



let $a=1$, $b=2$, $c=3$

2) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

on LHS

$a \vee (b \wedge c) = 1 \vee (2 \wedge 3)$

$= 1 \vee 1$

$= 1$

on RHS

$(a \vee b) \wedge (a \vee c)$

$(1 \vee 2) \wedge (1 \vee 3)$

$= 2 \wedge 3$

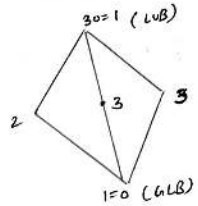
$= 1$

If In distributive lattice, both the properties are satisfy then the given lattice is a distributive lattice

Distributive lattice theorem -

If every element of P has atleast most one compliment then it is called Distributive lattice.

1)



$1^c = 30$

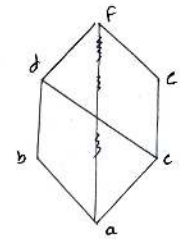
$30^c = 1$

$2^c = 5$

$5^c = 2$

It is not distributive because it has more than one compliment

2)



$a^c = f$

$f^c = a$

$b^c = e$

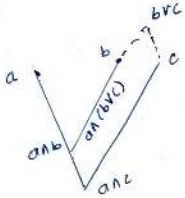
$e^c = b$

It is not distributive

Distributive inequalities-

1) $a \vee b \wedge (a \vee c) \leq a \vee (b \wedge c)$

2) $(a \wedge b) \vee (a \wedge c) \leq a \wedge (b \vee c)$



Proof

$(a \wedge b) \vee (a \wedge c) \leq a \wedge (b \vee c)$

$a \wedge b \leq a$, $a \wedge b \leq$ | $a \wedge c \leq c$

$a \wedge b \leq a$, $a \wedge b \leq$ | $a \wedge c \leq a$, $a \wedge c \leq b \vee c$

$\Rightarrow a \wedge c \leq a$

$a \wedge (b \vee c)$ is UB of $a \wedge b$, $a \wedge c$

$a \wedge b \vee (a \wedge c) \leq a \wedge (b \vee c)$

LUB $\quad$ UB

Uniqueness Theorem -

Let L be bounded distributive lattice such that complement of Every element (if exist) is unique.

Proof

Let a is a subset of L, $a \subset L$ and compliment are a_1 and a_2

$a \vee a_1 = 1 (UB) = a \vee a_2$

$a \wedge a_1 = 0 (LB) = a \wedge a_2$

Let a_1, a_2 are equal to c

$a_1 \vee 0$

$a_1 \vee (a \wedge a_2)$

$(a_1 \vee a) \wedge (a_1 \vee a_2)$

$1 \wedge (a_1 \vee a_2)$

$= a_1 \vee a_2$

$a_2 = a_2 \vee 0$

$a_2 \vee (a \wedge a_1)$

$(a_2 \vee a) \wedge (a_2 \wedge a_1)$

$1 \wedge (a_2 \wedge a_1)$

$a_2 \vee a_1$

$a_1 = a_2 = c$

SHIVAM

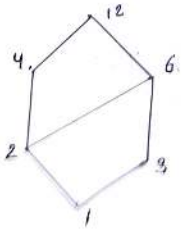
Sublattice -

A non-empty subset A' of a lattice L is said to be sublattice if all $a \forall a, b \in L' \Rightarrow a \vee b, a \wedge b \in L$

LUB GLB

Ex $(D_{12} /)$

$\{1, 2, 3, 4, 6, 12\}$



$(1, 4, 6)$

LUB = 4

GLB = 1

$(1, 6)$

LUB = 6

GLB = 1

Principle of Duality - (2 marks)

It states if it relates to then if it converted into inverse

So in such a case LUB is converted GLB and GLB is converted into LUB

Homomorphism -

$f: L \to M$ (if Function mapping in b/w two lattice)

1) Join Homomorphism. $f(x \vee y) = f(x) \vee f(y)$

2) meet Homomorphism. $f(x \wedge y) = f(x) \wedge f(y)$

3) order Homomorphism. if $x \leq y \to f(x) \leq f(y)$

Lattice Homomorphism -

if Join & meet Homomorphism both exist then it is called as. lattice Homomorphism

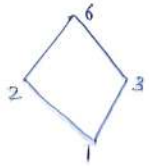
Isomorphism - one one, onto $(\vee \wedge)$

Auto morphism - Isomorphism exist from $L \to L$

Isomorphism into itself.

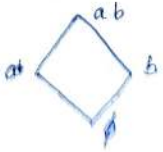
Justify that $(D_6 /)$ and Power set S are lattice given Isomorphism Prove that it is Isomorphism.

$D_6 = \{1, 2, 3, 6\}$



$S = \{a, b\}$

$P(S) = \{\phi, a, b, a, b\}$



L_1	L_2
1	ϕ
2	a
3	b
6	ab

$f(x \vee y) = f(x) \vee f(y)$

$f(2 \vee 3) = f(6)$

$f(x) \vee f(y)$

$(a) \vee (b)$

$\{a, b\}$

$\{6\}$

$f(x \wedge y) = f(x) \wedge f(y)$

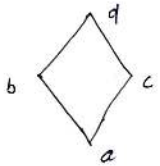
$f(2 \wedge 3) = f(2) \wedge f(3)$

$f(1) = \phi \quad a \wedge b$

$\phi = \phi$

Hence, it is a Isomorphism

Lattice is called Boolean algebra if it is distributed & complimented both.



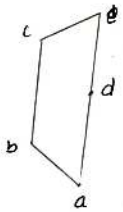
$a^c = d \qquad b^c = c$

$d^c = a \qquad c^c = b$

each element has one compliment so it is Boolean algebra

* Lattice is boolean algebra if for each element has one compliment.

Q. Proof This lattice is boolean algebra or not



$a^c = e \qquad c^c = d$

$e^c = a \qquad d^c = d$

$b^c = c \qquad d^c = c$

$e^c \; b \qquad e^c = d$

So, the given lattice is not boolean algebra

because d has more than 1 compliment

Boolean is a distributive complemented lattice.

To prove

Lattice $\{B, +, \cdot, 0, 1\}$

1) commutative $\quad a+b = b+a \;,\; a \cdot b = b \cdot a$

2) distributive $\quad a+(b \cdot c) = (a+b) \cdot (a+c)$

3) Identity $\quad a+0 = a$

$a \cdot 1 = a \qquad 0, 1 \in B$

4) complement law

$a + a' = 0$

$a \cdot a' = 1$

Example

$D_{35} = \{1, 5, 7, 35\}$

$a+b = \text{lcm}(a,b) \qquad a \cdot b = \text{gcd}(a,b)$

+	1	5	7	35
1	1	5	7	35
5	5	5	35	35
7	7	35	7	35
35	35	35	35	35

·	1	5	7	35
1	1	1	1	1
5	1	5	1	5
7	1	1	7	7
35	1	5	7	35

1) commutative held

2) Distributive let $(a=7, b=5, c=1)$

$a+(b \cdot c) = (a+b) \cdot (a+c)$

$7+(5 \cdot 1) = (7+5) \cdot (7 \cdot 1)$

$7+1 = 35 \cdot 7$

$7 = 7$

$a \cdot (b+c) = (a \cdot b) + (a \cdot c)$

$7 \cdot (5+1) = (7 \cdot 5) + (7 \cdot 1)$

$1 = 1$

3) Identity element

$a+1 = a$

↓ zero element

$a \cdot 35 = a$

↓ unit element

4) Inverse element

$a+a' = 35$

a	$a' = 1$
1	
5	
7	
35	

$a \cdot a' = 0$

a	a'
1	35
5	
7	
35	

LAWS

1) Idempotent law

a) $a+a = a$

b) $a \cdot a = a$

2) Boundedness law

a) $a+1 = 1$

b) $a \cdot 0 = 0$

3) Absorption law

a) $a+(a \cdot b) = a$

b) $a \cdot (a+b) = a$

4) Associative law

a) $(a+b)+c = a+(b+c)$

b) $(a \cdot b) \cdot c = a(b \cdot c)$

5) Uniqueness of complement

$a+x = 1$

and $a \cdot x = 0$ Hence $x = a'$

6) Involution law

$(a')' = a$

7) Complement law

$0' = 1$

$1' = 0$

8) De Morgan's law

a) $(a+b)' = a' \cdot b'$

b) $(a \cdot b)' = a' + b'$

De Morgan law - (imp.)

1) $(a+b)' = a' b'$

let $a = a+b$

$x = a' \cdot b'$

$a+x = 1$ / $a \cdot x = 0$ / $x = a'$

$(a+b) + (a' \cdot b')$

$[(a+b)+a'] \cdot [(a+b)+b'] =$ (left distributive law)

$[(a+a')+b] \cdot [(b+b')+a]$ (Associative law)

$[1+b] \cdot [a+1]$ (Complement law)

$1 \cdot 1$ (Boundedness law)

1

$a \cdot x = 0$

$(a+b) \cdot (a' \cdot b') = 0$

$(a+b) \cdot (a' \cdot b')$

~~a(a+b)~~ $(a \cdot a' b') + b(a' \cdot b')$ distributive law

$(a \cdot a') b' + (b \cdot b') a'$ associative law

$(0 \cdot b' + (0 \cdot a')$ complement law

$0+0$ Boundedness law

$= 0$

$x = a'$

$a' \cdot b' = (a+b)'$

$(a \cdot b)' = a' + b'$

let $x = a \cdot b$

$a = a' + b'$

$a + x = 1 \qquad a \cdot x = 0 \qquad x = a'$ {uniqueness of compliment

$(a' + b') + (a \cdot b) = 1$

$(a' + b') + (a \cdot b)$

$[(a' + b') + a] \cdot [(a' + b') + b]$

$[(a' + a) + b'] [a' + (b' + b)]$

$[1 + b'] \cdot [a' + 1]$

$1 \cdot 1$

$= 1$

now $a \cdot x = 0$

$(a' + b')(a \cdot b) = 0$

$(a' + b')(a \cdot b)$

$(a' \cdot a \cdot b) + (b' \cdot a \cdot b)$

K-MAP

Normal form

1) Minterm | Sum of product (SOP) | Disjunction Normal form (DNF)
2) Maxterm | Product of Sum (POS | Conjuctive Normal form (CNF)

A) Minterm |SOP| DNF

$xyz + x'y'z$

X	Y	Z	F
0	0	0	1 ✓
0	0	1	1 ✓
0	1	0	1 ✓
0	1	1	0
1	0	0	1 ✓
1	0	1	1 ✓
1	1	0	0
1	1	1	0

minterm value of Truth table (1)

$x'y'z' + x'y'z + x'yz' + xy'z' + xy'z$

Max term of Truth table (0)

$(x + y' + z') \cdot (x' + y' + z) \cdot (x' + y' + z')$

Convert DNF into CNF -

1) Denote given DNF function F
2) Find F' which be sum of those term
3) Now find F''
4) Simplify F'' of RHS using Demorgan's law.
5) F'' will be required CNF.

$DNF = x'(xy) + xy + xy + xy$

$CNF = 1$

1) $f(xy) = xy + xy + xy$

2) $f(xy) = xy$

3) $f'(xy) = (xy)$

4) Boolean $CNF = x \cdot y$

CONVERSION OF CNF to DNF

$f(xyz) = (x+y+z)(x+y+z)+(x+y+z)$

$f'(xyz) = (x+y+z)(x+y+z)(x+y+z)(x+y+z)(x'+y'+z)$

$f'(xyz) = [(x+y+z)(x+y+z)(x+y+z)(x+y+z)(x'+y'+z)]$

$f'(xyz) = [(x+y+z)+(x+y+z)+(x+y+z)+(x+y+z)+(x'+y'+z)]$

Simplify using Boolean Expression -

1) $f(xy) = xy + xy + x'y$

$x(y+y') + x'y$

$x(1) + x'y$

$x + x'y$

$(x+x')(x+y)$

$1 \cdot (x+y)$

$x+y$

2) $A \oplus B \oplus AB = A+B$

$AB + \oplus B = C$

$C \oplus A B$

$C(AB) + ABC'$

$C[A'+B'] + ABC'$

$A \oplus B[A'+B'] + ABC'$

$(AB+A'B)(A+B') + ABC'$

$(AB'A + AB'B' + A'BA' + A'BB' + AB(AB' + A'B))'$

$\boxed{A \cdot A' = 0, B \cdot B' = 0}$

$AB' + A'B + AB[(A'+B) \cdot (A+B')]$

$AB' + A'B + AB[A' \cdot A + A' \cdot B' + BA + BB']$

$AB' + A'B + AB[A'B + AB]$

$AB' + A'B + ABA'B + ABAB$

$AB + A'B + AB$

$AB[B+B'] + A'B$

$AB + A'B$

$AB + A'B$

$A'B + AB$

$(A+A')(A+B)$

$= 1(A+B)$

$= A+B$

$A+B = B+A$

K-MAP

K-map is used to simplify the Boolean expression. The K-map is given using Boolean logic expression.

Formula $2^n = n$ is the no of variable

	n
	$n = 2$

1) $2^2 = 4$ 2) $2^3 = 8$ 3) $2^4 = 16$

$A \backslash B$		
0	0	1
1	3	2

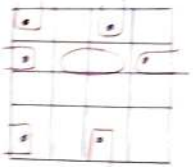
$A \backslash BC$	00	01	11	10
0	0	1	3	2
1	4	5	7	6

$AB \backslash CD$	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

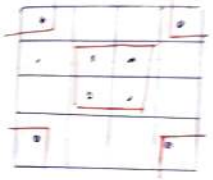
Rules for creating groups in K-map

- NO zero(0) allowed
- Groups can be created horizontal but cannot be diagonal
- Overlapping is allowed
- Groups should be as large as possible
- Groups must contain 2^n cell

Pair group 2 cell



Pair group 4 cell



Pair group 2 cell

1) $F(A,B) = \Sigma(0,2,3)$

A \ B	B'	B
A'	1	
A	1	1

$A + B'$

$F(A,B,C) = \Sigma(0,1,2,3,5)$

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}C$	1	1	1	1
A'	0	1	3	2
	4	5	7	6

$A' + \bar{B}C$

$F(A,B,C,D) = \Sigma(0,1,4,5,8,9,12,13)$

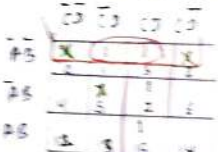
AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 0	1 1	3	2
$\bar{A}B$	4	5	7	6
AB	1 12	1 13	15	14
$A\bar{B}$	1 8	1 9	11	10

C'

K-map with don't care condition -

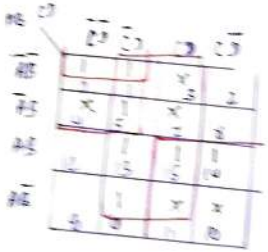
Sometimes the X or 0 may be assumed 0 or 1 as per the requirement to simplify the term

(2) $F(A,B,C,D) = \sum m(1,3,2,7,15) + d(0,4,5)$



$F = A'B' + CD$

(3) $Y = (A,B,C,D) = \sum(0,1,5,9,3,14,15) + d(2,4,7,12,13)$

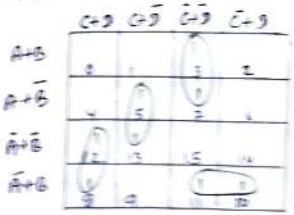


$Y = \bar{A}\bar{C} + D + A\bar{C}$

(5) $F(A,B,C) = \sum m(2,3,4,5) + \sum d(6,7)$

$F = A + B$

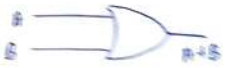
(6) $F(A,B,C,D) = \prod(3,5,7,8,10,12,13)$



$F = (\bar{A}+C+D)\cdot(\bar{B}+C+\bar{D})\cdot(A+\bar{C}+\bar{D})\cdot(\bar{A}+B+\bar{C}$

Circuit diagram -

1) OR Gate



2) AND Gate

3) NOT Gate

4) NAND Gate

5) NOR Gate

6) XOR Gate

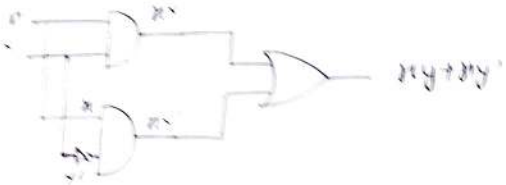
7) XNOR Gate

AND (A) (B)

OR (A) (B)

c) $f(x,y) = (x \wedge y) \vee (x \wedge y')$

$(xy) + (xy')$



MAK HIS

UNIT-4 Mathematical Logic

Study of Reasoning Techniques

i) Proposition calculus – statement (True or False)

ii) Predicate calculus – Predicates (Proposition containing variables)

Mathematical logic
- Proposition calculus → Sentences
  - Declarative (True, false)
  - Non-declarative (Question, explanation, command)
- Predicate calculus

Proposition variable – Represents simple statement by letter

p, q, r are known as proposition variable

Proposition constant

p: 2+2=0 (f)

i.e. "p is false"

Proposition
- ATOMIC (Primary / Primitive) → Single
- COMPOUND (Molecular / composite) → Two or more

Connective –

The word and phrase or (symbols) used to form compound proposition is called connective.

Name	Symbol	Connection word
Negation	$\neg, \sim$	NOT
Conjuction	$\wedge$	AND
Disjunction	$\vee$	OR
Implication or conditional	$\Rightarrow (\sim P \vee Q)$	If then
Equivalence or Biconditional	$\Leftrightarrow$ $(P \rightarrow Q) \wedge (Q \rightarrow P)$	If and only if.

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Conjuction – (AND hates false)

If P and Q are the two statement then conjuction of P and Q is the compound statement denoted by and read as P and Q

The compound statement P and Q is true when both P and Q are true otherwise it is false.

Eg. P: Ram is healthy (T)

$\boxed{P \wedge Q}$ Q: He has blue eyes (T)

Ram is healthy and has blue eyes (T)

P	Q	$P \wedge Q$
$\boxed{T}$	$\boxed{T}$	$\boxed{T}$ → Ans
T	F	F
F	T	F
F	F	F

Disjunction –

If P and Q are two statement then disjunction of P and Q is the compound statement denoted by $(P \vee Q)$ and read as P or Q.

The statement P or Q is true if atleast one of P or Q is true.

eg P: He will go to delhi

Q: He will go to Kolkata

$P \vee Q$: He will go to delhi or Kolkata (T)

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation –

If P is any proposition then negation of P is denoted by $\neg P$

if P is true then negation is false and vice-versa.

P	$\sim P$
T	F
F	T

1) $P \wedge (\sim Q \vee Q)$

P	Q	$\sim Q$	$\sim Q \vee Q$	$P \wedge (\sim Q \vee Q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

2) $\sim (P \vee Q) \vee (\sim P \wedge \sim Q)$

P	Q	$\sim P$	$\sim Q$	$P \vee Q$	$\sim (P \vee Q)$	$\sim P \wedge \sim Q$	$\sim (P \vee Q) \vee (\sim P \wedge \sim Q)$
T	T	F	F	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	T	F	F	F
F	F	T	T	F	T	T	T

2) $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r)$

r	q	p	$\sim q$	$\sim r$	$\sim p$	$(\sim q \wedge r)$	$(q \wedge r)$	$p \wedge r$	$\sim p \wedge (\sim q \wedge r)$
T	T	T	F	F	F	F	T	T	F
T	T	F	F	F	T	F	T	T	F
T	F	T	F	T	F	T	F	F	F
T	F	F	F	T	T	T	F	T	F
F	T	T	T	F	F	F	F	F	F
F	T	F	T	F	T	T	F	F	T
F	F	T	T	T	F	T	F	F	F
F	F	F	T	T	T	T	F	F	T

3. if Tomorrow is Sunday then Today is Saturday.

$\underbrace{\quad}_{p}$ $\underbrace{\quad}_{q}$

Final ans T, F alternate

if then

$p \rightarrow q = \sim p \vee q$

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

✓ Trick

p	q	$\sim p$	$\sim p \vee q$
F	F	T	T
F	T	T	T
T	F	F	F
T	T	F	T

if 1st state true & 2nd statement false then it is false

if and only if

$p \Leftrightarrow q \Rightarrow ((p \rightarrow q) \wedge (q \rightarrow p))$

p	q	$p \Leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

if both statement false or both statement true it is True

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NAND $\rightarrow p \uparrow q \Rightarrow \sim(p \wedge q)$

NOR $\rightarrow p \downarrow q \Rightarrow \sim(p \vee q)$

XOR $\rightarrow p \oplus q \Rightarrow pq' + p'q$

$(p \wedge q') \vee (p' \wedge q)$

---

1) converse $q \rightarrow p$

2) Contrapositive $\sim q \rightarrow \sim p$

3) inverse $\sim p \rightarrow \sim q$

Q If it rains, then I will stay at home [2020-21]

find converse, contrapositive & inverse for statement

p: if it rain

q: I will stay at home

Implication

$p \rightarrow q$ Given, it is representation of this statement.

1) converse $q \rightarrow p$

If I will stay at home then it will rain

2) contrapositive $\sim q \rightarrow \sim p$

If I will not stay at home then it will not rain.

3) Inverse $\sim p \rightarrow \sim q$

if it does not rain then I will not stay at home

Q. P: Tomorrow is Sunday

Q: Today is Saturday.

Find the implication statement → Truth table.

Implication

$P \rightarrow Q$ if tomorrow is Sunday then today is Saturday.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Q. P: if it Rains

Q: The Crop will grow.

Find out the implication, Converse, Inverse, Contrapositive

1) Implication

$P \rightarrow Q$ if it Rains then Crop will grow.

2) Converse

$Q \rightarrow P$

if the crop grow then it will rain.

3) Inverse $\sim P \rightarrow \sim Q$

If it is not rain the crop will not grow.

4) Contrapositive $\sim Q \rightarrow \sim P$

The crop will not grow if there is no rain.

1) Tautology -

when the output is only true

2) Contradiction -

when the output is only false

3) contigency -

At least one value should be true

Q. $(P \rightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)$

Find whether it is Tautology or not.

2) $P \wedge (Q \wedge \sim P)$

Prove contradiction.

3) $\sim (P \wedge Q)$

prove contigency.

1)

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$	$(P \rightarrow Q) \wedge (Q \rightarrow R)$	$(P \rightarrow Q) \wedge (Q \rightarrow R) \rightarrow (P \rightarrow R)$
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

Here it is Tautology

2)

p	q	$\sim p$	$q \wedge \sim p$	$p \wedge (q \wedge \sim p)$
F	F	T	F	F
F	T	T	T	F
T	F	F	F	F
T	T	F	F	F

Hence, It is contradiction, all values are false

3)

p	q	$(p \wedge q)$	$\sim(p \wedge q)$
F	F	F	T
F	T	F	T
T	F	F	T
T	T	T	F

Hence, it is contingency more than one value is true.

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Algebra of proposition -

1) Idempotent laws

a) $p \vee p = p$

b) $p \wedge p = p$

2) ASSOCIATIVE LAWS

a) $(p \vee q) \vee r = p \vee (q \vee r)$

b) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

3) Commutative law

a) $p \vee q = q \vee p$

$p \wedge q = q \wedge p$

4) Distributive law

a) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

b) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

5) Identity law

a) $p \vee F^{(1)} = p$

(b) $p \vee T^{(1)} = T(1)$

(c) $p \wedge T^{(1)} = p$

d) $p \wedge F^{(1)} = F(0)$

6) Compliment law

a) $p \vee \sim p = T$

b) $p \wedge \sim p = F$

c) $\sim T = F$

d) $\sim F = T$

7) Involution law

a) $\sim(\sim p) = p$

8) De morgan's law

a) $\sim(p \vee q) = \sim p \wedge \sim q$

b) $\sim(p \wedge q) = \sim p \vee \sim q$

a) Prove the given statement is equivalence $p \vee (p \wedge q) = p$

on LHS $p \vee (p \wedge q)$

$= (p \wedge T) \vee (p \wedge q)$ [Identity]

$= p \wedge (T \vee q)$ [Distributive]

$= p \wedge T$ [Identity]

$= p$ [Identity]

RHS

Hence Given statement is equivalence

2) Solve the following without using truth table

$p \to (q \to r) \equiv (p \wedge q) \to r$

LHS $p \to (q \to r)$

$= \sim p \vee (q \to r)$ [Implication]

$= \sim p \vee (\sim q \vee r)$ "

$= (\sim p \vee \sim q) \vee r$ [Associative]

$= \sim(p \wedge q) \vee r$ De morgans

$= (p \wedge q) \to r$ Implication.

3) $(p \wedge q) \vee (p \wedge \sim q)$

$= p \wedge (q \vee \sim q)$ [Distributive]

$= p \wedge T$ [compliment]

$= p$ [Identity]

4) $(P \vee Q) \to R \equiv (P \to R) \wedge (Q \to R)$

LHS $(P \vee Q) \to R$

$= \sim(P \vee Q) \vee R$ [Implication]

$= (\sim P \wedge \sim Q) \vee R$ [De morgans]

$= (\sim P \vee R) \wedge (\sim Q \vee R)$ [Distributive]

$= (P \to R) \wedge (Q \to R)$ [Implication]

$=$ RHS

Normal form in Logic

1) CNF = Conjuctive Normal form (Product of elementary sum)
2) DNF = Disjunctive Normal form (Sum of elementary product)
3) PCNF = Principle Conjuctive Normal form.
4) PDNF = Principle Disjunctive Normal form.

1) CNF (Product $(\wedge)$ of elementary sum $(\vee)$)

Rules for Conversion -

i) Remove $\to$, $\leftrightarrow$ (implication & biconditional)
ii) Remove $\sim$ / $\sim$ (negation) using demorgan law
iii) Distributive law is used to get required form.

Q. $P \wedge (P \to Q)$, convert into CNF.

$= P \wedge (\sim P \vee Q)$

$= P \wedge (\sim P \vee Q)$

It is required CNF

Q. $[Q \vee (P \wedge R)] \wedge \sim[(P \vee R) \wedge Q]$ convert into CNF.

$= [Q \vee (P \wedge R)] \wedge [\sim(P \vee R) \vee \sim Q]$ [Demorgans]

$[Q \vee (P \wedge R)] \wedge [(\sim P \wedge \sim R) \vee \sim Q]$

$[(Q \vee P) \wedge (Q \vee R)] \wedge [(\sim P \vee \sim Q) \wedge (\sim R \vee \sim Q)]$

It is required CNF or product of elementary sum.

2) DNF (Sum $(\vee)$ of elementary product $(\wedge)$)

Rules are same as CNF

1) Convert into required DNF

$P \to ((P \to Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((P \to Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((\sim P \vee Q) \wedge \sim(\sim Q \vee \sim P))$

$\sim P \vee ((\sim P \vee Q) \wedge \sim(\sim(Q \wedge P)))$

$\sim P \vee ((\sim P \vee Q) \wedge (Q \wedge P))$

$\sim P \vee ((\sim P \wedge (Q \wedge P)) \vee (Q \wedge (Q \wedge P))$

$\sim P \vee ((F \wedge Q) \vee (Q \wedge P))$

$\sim P \vee F \vee (P \wedge Q)$

$\sim P \vee (P \wedge Q)$

It is required DNF or Sum of elementary product

3) PDNF (Principle Disjunctive Normal form)

Rules

i) Remove $\to$, $\leftrightarrow$ (implication, biconditional)
ii) Put missing element using $Q \vee \sim Q = 1$
eg if α is given.
then $\alpha \wedge (Q \vee \sim Q)$ can be written as $Q \vee \sim Q = 1$
iii) Simplify using Demorgans law
iv) Convert using distributive law to get required form.

Q. Convert to PDNF

$p \rightarrow q$

$(\sim p \vee q)$

$= [\sim p \wedge (q \vee \sim q)] \vee [q \wedge (p \vee \sim p)]$

$= [(\sim p \wedge q) \vee (\sim p \wedge \sim q)] \vee [(q \wedge p) \vee (q \wedge \sim p)]$

$= (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge q)$

It is required PDNF

now using truth table

p	q	$p \rightarrow q$ $(\sim p \vee q)$
F	F	T
F	T	T
T	F	F
T	T	T

For DNF and PDNF we see only True

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q) \vee (p \wedge q)$

It is required PDNF

4) PCNF (Product of elementary sum)

- Remove $\rightarrow$ $\Leftrightarrow$
- Add missing elements
  $\alpha \vee (q \wedge \sim q)$
- De Morgan law
- Distributive law.

Q. $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

$[(\sim p \vee r) \wedge (q \rightarrow p) \wedge (p \rightarrow q)]$

$\Rightarrow (p \vee r) \wedge [(\sim q \vee p) \wedge (\sim p \vee q)]$

$\Rightarrow (p \vee r) \vee (q \wedge \sim q) \wedge (\sim q \vee p) \vee (r \wedge \sim r) \wedge (\sim p \vee q) \vee (r \wedge \sim r)$

$\Rightarrow (p \vee r \vee q) \wedge (p \vee r \vee \sim q) \wedge (\sim q \vee p \vee r) \wedge (\sim q \vee p \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

$(p \vee r \vee q) \wedge (p \vee r \vee \sim q) \wedge (\sim q \vee p \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

required PCNF

now using truth table

p	q	r	$\sim p$	$\sim p \rightarrow r$	$q \leftrightarrow p$	$(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$
F	F	F	T	T	T	F
F	F	T	T	T	T	T
F	T	F	T	T	F	F
F	T	T	T	T	F	F
T	F	F	F	F	F	F
T	F	T	F	F	F	F
T	T	F	F	T	T	T
T	T	T	F	T	T	T

$(p \vee r \vee q) \wedge (p \vee r \vee \sim q) \wedge (\sim q \vee p \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (\sim p \vee q \vee \sim r)$

Q. $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r))$ Check for tautology.

p	q	r	$\sim p$	$\sim r$	$q \rightarrow \sim r$	$q \vee (q \rightarrow \sim r)$	$(\sim p \rightarrow (q \vee (q \rightarrow \sim r))$
F	F	F	T	T	T	T	T
F	F	T	T	F	T	T	T
F	T	F	T	T	F	T	T
F	T	T	T	F	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	T	T	F	F	T	T	T

It is Tautology.

② check the equivalence

$P \leftrightarrow q \equiv (P \wedge q) \vee (\neg P \vee \neg q)$

3) Check for the Tautology

$(P \wedge q) \vee \neg(P \wedge r) \vee (q \vee r)$

4) Solve using Algebra of proposition & Truth Table.

i) $P \leftrightarrow q \equiv (P \vee q) \rightarrow (P \wedge q)$

ii) $\sim(\sim P \vee q) \wedge (P \vee q) \equiv P$

5) Prove Equivalence without using TT

i) $P \vee (P \wedge q) \equiv P$

ii) $(P \vee q) \rightarrow r \equiv (P \rightarrow r) \vee (q \rightarrow r)$

iii) $(P \wedge q) \vee (P \wedge \sim q)$

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2. $P \leftrightarrow q$ LHS RHS

P	q	$P \leftrightarrow q$
F	F	T
T	T	F
F	F	F
T	T	T

P	q	$\sim P$	$\sim q$	$P \wedge q$	$\sim P \vee \sim q$	Ans
F	F	T	T	F	T	T
F	T	T	F	F	T	T
T	F	F	T	F	T	T
T	T	F	F	T	F	T

LHS ≠ RHS It is not equivalent

3) $(P \wedge q) \vee \sim(P \wedge r) \vee (q \vee r)$

P	q	r	$P \wedge q$	$P \wedge r$	$\sim P \wedge r$	$q \vee r$	$(P \wedge q) \vee \sim(P \wedge r)$	Ans
F	F	F	F	F	T	F	T	T
F	F	T	F	F	T	T	T	T
F	T	F	F	F	T	T	T	T
F	T	T	F	F	T	T	T	T
T	F	F	F	F	T	T	T	T
T	F	T	F	T	F	T	T	T
T	T	F	T	F	T	T	T	T
T	T	T	T	F	T	T	T	T

4(i) $P \leftrightarrow q \equiv (P \vee q) \rightarrow (P \wedge q)$

LHS $P \leftrightarrow q$

$= (P \rightarrow q) \wedge (q \rightarrow P)$

$= (\sim P \vee q) \wedge (\sim q \vee P)$

$= (\sim P \wedge (\sim q \vee P)) \vee (q \wedge (\sim q \vee P))$ [Distributive]

$= ((\sim P \wedge \sim q) \vee (\sim P \wedge P)) \vee ((q \wedge \sim q) \vee (q \wedge P))$ [Distributive]

$= ((\sim P \wedge \sim q) \vee F) \vee (F \vee (q \wedge P))$ [complement]

$= (\sim P \wedge \sim q) \vee (P \wedge q)$ [commutative]

$= \sim(P \vee q) \vee (P \wedge q)$ [De morgan]

$= (P \vee q) \longrightarrow (P \wedge q)$ [implication]

RHS

Truth table

P	q	$P \rightarrow q$	$q \rightarrow P$	$(P \rightarrow q) \wedge (q \rightarrow P)$	$P \vee q$	$(P \wedge q)$	$(P \vee q) \rightarrow (P \wedge q)$
T	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	F	F
F	F	T	T	T	F	F	T

ii) $\sim(\sim P \wedge q) \wedge (P \vee q) \equiv P$

P	q	$P \vee q$	$\sim P \wedge q$	$\sim(\sim P \wedge q)$	$\sim(\sim P \wedge q) \wedge (P \vee q)$	P
T	T	T	F	T	T	T
T	F	T	F	T	T	T
F	T	T	T	F	F	F
F	F	F	F	T	F	F

5(i) $P \vee (P \wedge Q) \equiv P$

LHS $P \vee (P \wedge Q)$

$= (P \wedge T) \vee (P \wedge Q)$ [Identity]

$P \wedge (T \vee Q)$ [Distributive]

$P \wedge (T)$ [Identity]

P " RHS

It is equivalence

ii) $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$

LHS $(P \vee Q) \rightarrow R$

$\equiv \sim(P \vee Q) \vee R$

$\equiv (\sim P \wedge \sim Q) \vee R$

$\equiv (\sim P \vee T) \wedge (\sim Q \vee R)$

$\equiv (P \rightarrow R) \wedge (Q \rightarrow R)$

RHS It is equivalence

iii) $(P \wedge Q) \vee (P \wedge \sim Q) \equiv P$

LHS $(P \wedge Q) \vee (P \wedge \sim Q)$ [Distributive]

$= P \wedge (Q \vee \sim Q)$ "

$= P \wedge T$ "

$= P$

It is equivalence

Proposition & Predicate Calculus

i.e. Ram is a bachelor → Proposition

$P(x)$: x is a bachelor → Predicate calculus

Predicate calculus -

1) Universal Quantifier $(\forall x)$ → all, for all, any, every, each

2) Existential Quantifier $(\exists x)$ → there exist, some, for some.

Ex (1) Each person is poorious (Ram every x, if x is a person then x is poorious)

x $\quad$ $P(x)$ $\quad$ $Q(x)$

$\forall x \; P(x) \rightarrow Q(x)$

2) All human beings are mortal

$P(x)$ $\quad$ $Q(x)$

$\forall x \quad P(x) \rightarrow Q(x)$

3) Some students are clever

$P(x)$ $\quad$ $Q(x)$

$\exists x \quad P(x) \wedge Q(x)$

4) Some students are not successful

$P(x)$ $\quad$ $Q(x)$

$\exists x \quad P(x) \wedge \sim Q(x)$

Difference b/w Proposition & Predicate Calculus

Proposition	Predicate Calculus
1) It defines individual statement	It defines a class of statements
2) In this quantifiers does not exist	Two types of quantifiers i.e. universal & existential

* for every x used $\Rightarrow \forall x\, P(x) \rightarrow Q(x)$
* for some used $\Rightarrow \exists x\, P(x) \wedge Q(x)$

$\Rightarrow \sim\forall x\, P(x) = \sim\forall x \sim P(x)$
$= \exists x \sim P(x)$ — (1)

$\Rightarrow \sim\exists x\, P(x) = \sim\exists x \sim P(x)$
$= \forall x \sim P(x)$ — (2)

$\Rightarrow \exists x \sim P(x) = \forall x\, P(x)$ — (3)

$\Rightarrow \forall x \sim P(x) = \exists x\, P(x)$ — (4)

$P_1\ P_2\ P_3\ P_4 - - - - P_n =$ Conclusion
Premises

if T.T $P_1 \wedge P_2 \wedge P_3 \wedge - - - P_n \rightarrow$ Tautology

Rule of Inferance

Rule of Inferance	Tautological form	Name
1 (a) $\dfrac{P}{P \vee Q}$	a. $P \Rightarrow P \vee Q$	Addition
b) $\dfrac{Q}{P \vee Q}$	b. $Q \Rightarrow P \vee Q$	
2 (a) $\dfrac{P \wedge Q}{P}$	a. $(P \wedge Q) \Rightarrow P$	Simplification
b) $\dfrac{P \wedge Q}{Q}$	b. $(P \wedge Q) \Rightarrow Q$	
3. $\dfrac{Q}{P \wedge Q}$	$((P) \wedge (Q)) \Rightarrow (P \wedge Q)$	Conjuction
4) $P \Rightarrow Q$	$((P \Rightarrow Q) \wedge (P)) \Rightarrow$	Modus Ponens.

4) $P \Rightarrow Q$ $\quad [(P \Rightarrow Q) \wedge \sim Q] \Rightarrow \sim P$ — Modus Tollens
$\sim Q$
$\sim P$

6) $P \Rightarrow Q$ $\quad [(P \Rightarrow Q) \wedge (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)$ — Hypothetical Syllogism
$Q \Rightarrow R$
$P \Rightarrow R$

7) $P \vee Q$ $\quad [(P \vee Q) \wedge \sim P] \Rightarrow Q$ — Disjunction syllogism.
$\sim P$
Q

8) $(P \Rightarrow Q) \wedge (R \Rightarrow S)$ $\quad [(P \Rightarrow Q) \wedge (R \Rightarrow S) \wedge (P \vee R)] \Rightarrow (Q \vee S)$ — Constructive Dilemma

9) $P \Rightarrow Q \wedge (R \Rightarrow S)$ $\quad [(P \Rightarrow Q) \wedge (R \Rightarrow S) \wedge (\sim Q \vee \sim S)] \Rightarrow (\sim P \vee \sim R)$ — Destructive dilemma
$\sim Q \vee \sim S$
$\sim P \vee \sim R$

Preposition	Predicate
1) Modus Tollens $P \rightarrow Q$ $\sim Q$ $\sim P$	1) Universal Modus Tollens $\forall x\ P(x) \rightarrow Q(x)$ $\sim Q(a)$ $\sim P(a)$
2) Modus Ponens $P \rightarrow Q$ P Q	2) Universal modus Ponens $\forall x\ P(x) \rightarrow Q(x)$ $P(a)$ $Q(a)$

[illegible]

i) $P \vee Q \to \neg r$ (given)

ii) $r \to S$ (given)

iii) $\neg S$ (given)

using modus tollens in (ii) & (iii)

iv) $\neg r$

using modus tollens in (i) & (iv)

v) $\neg(P \vee Q)$

vi) $\neg P \wedge \neg Q$ [De morgan's]

vii) $\neg P$

2) Show that t is a valid conclusion from the premises $P \to Q$, $Q \to r$, $r \to S$, $\neg S$ and $P \vee t$

i) $P \to Q$ (given)

ii) $Q \to r$ (given)

iii) $r \to S$ (given)

using hypothetical syllogism in (i) & (ii)

iv) $P \to r$

v) $P \to S$ [hypothetical syllogism in (iii) & (iv)]

vi) $\neg S$ (given)

vii) $\neg P$ [modus tollen using 5.6]

viii) $P \vee t$ (given)

ix) t Disjunction syllogism in (vii) & (viii)

3) [illegible] valid conclusion from the premises $P \to Q$, $P \to r$, $\neg(Q \wedge r)$ and $r \vee P$

i) $P \to Q$ (given)

ii) $P \to r$ (given)

iii) $\neg(Q \wedge r)$ (given)

iv) $r \vee P$ (given)

v) $(P \to Q) \wedge (P \to r)$ [using (i) & (ii)]

vi) $\neg r \vee \neg Q$ [De morgan's in (iii)]

vii) $\neg r \vee P$ [Distributive dilemma in (v) & (vi)]

viii) $\neg P$ [Tautology in (vii)]

ix) r [Disjunction syllogism in (iv) & (viii)]

4) If I get the job and work hard, then I will get promotion. If I get promoted then I will be happy. I will not be happy therefore either I will get the job or I will not work hard

$P \wedge Q \to r$

$r \to S$

$\neg S$

$\neg P \vee \neg Q$

P: I get the job

Q: I work hard

r: I will get promoted

S: I will be happy

$[((P \wedge Q) \to r) \wedge (r \to S) \wedge (\neg S)] \to (\neg P \vee \neg Q)$

1. $(P \wedge Q) \to r$ (Given premises)

2. $r \to S$

3. $\neg S$

4. $\neg r$ [using modus tollens in 2, 3]

5. $\neg(P \wedge Q)$ [modus tollen 1, 4]

6. $(\neg P \vee \neg Q)$ [De morgan's]

5) All healthy person eat an apple a day. Ram does not eat apple a day. Therefore Ram is not a healthy person.

$P(x)$: x is a healthy person.

$Q(x)$: x eat apple a day

$\forall x \; P(x) \rightarrow Q(x)$

$$\frac{\sim Q(y)}{\sim P(y)}$$

By using universal Modus Tollen, the given statement is a valid statement.

---

6) find negation of "If I woke early in the morning then I will be healthy"

P: I woke up early in the morning.

Q: I will be healthy.

$P \rightarrow Q \Rightarrow \sim P \vee Q$

to find $\sim(\sim P \vee Q)$

$= P \wedge \sim Q$

I woke up early in the morning and I will not be healthy

---

7) Use rule of inferences to justify these hypothesis -

i) if it does not rain or if it is not foggy then the sailing race will be held and the life saving demonstration will go on.

ii) If the sailing race is held then the trophy will be awarded.

iii) The trophy was not awarded.

iv) Therefore it rained.

P: It rain

Q: It is foggy

R: Sailing race will be held

T: Trophy will be awarded

S: life saving will go on

1. $(\sim P \vee \sim Q) \rightarrow (R \wedge S)$
2. $R \rightarrow T$
3. $\sim T / S$
4. $\sim R$ (modus tollen of 2,3)
5. $\sim R \vee S$ (addition)
6. $\sim(R \wedge S)$ (de morgans)
7. $\sim(\sim P \vee \sim Q)$ (modus tollen 1,6)
8. $(P \wedge Q)$ (de morgans)
9. P (Simplification)

8. Justify the following premises are inconsistent

i) If Nisemala misses many classes through illness then she fails in high school

ii) if Nisemala fails high school then she is uneducated

iii) if Nisemala reads a lot of book then she is not uneducated

iv) Nisemala misses many classes through illness and reads a lot of book

P: Nisemala misses many classes through illness

Q: She fails in high school.

R: She is uneducated.

S: Nisemala reads lot of books.

1. $P \rightarrow Q$
2. $Q \rightarrow R$
3. $S \rightarrow \sim R$
4. $P \wedge S$
5. $P \rightarrow R$ (hypothetical syllogism in 1,2)
6. $R \wedge \sim R$ (by 3,4,5)

Since the statement is not valid and hence the premises are inconsistent

4) Translate the following statement in symbolic form. -

i) The sum of two positive integer is always positive

ii) Everyone is love by someone

iii) Some people are not admired by everyone

iv) if a person is female and is a parent then the person is someone's mother.

i) $\exists x \quad P(x) \quad \wedge \quad P(x)$

ii) $\forall x \quad P(x) \rightarrow Q(x)$

iii) $\exists x \quad P(x) \quad \wedge \quad \sim Q(x)$

iv) P: x is a female

Q: x is a parent

r: x is someone's mother

$(P \wedge Q) \rightarrow r$

i) $\forall x \forall y \; [(x>0) \wedge (y>0)] \rightarrow (x+y) > 0]$

ii) $\forall x \exists y$ loves (y, x)

iii) There is a person who is not admired by some person.

$\exists x \exists y \; P(x) \wedge P(y) \wedge - Adm(x, y)]$

iv) for every person x if x is female and x is a parent then there exist a person y such that x is the mother of person y.

$\forall x \; P(x) \wedge f(x) \rightarrow \exists y \; M(x, y)$

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Unit-5

Recurrence relations:-

$$a_n = a_{n-1} + 2$$

i) Homogeneous Recurrence relation $\rightarrow$ Ex $a_n - a_{n-1} - 2 = 0$

ii) Non-homogeneous Recurrence relation $\rightarrow$ Ex $a_{n-2} + 5a_{n-1} + a_n = f(n)$

Recurrence Relation Solution

(i) Iteration method

ii) Characteristic root method

iii) Generating function.

Iteration method -

$a_n = a_{n-1} + 2 \quad n \geq 2, \; a_1 = 3$

Let $n = n-1$

$a_n = a_{n-2} + 2$ — (2) $\qquad a_n = a_{n-1} + 2$ — (1)

$a_{n-2} = a_{n-3} + 2$ — (3) .... $a_{n-k+1} = a_{n-k} + 2$ — (4)

Now, Put eq^n in above equation

$a_n = a_{n-2} + 2 + 2$

$a_n = a_{n-3} + 2 + 2 + 2$

$\boxed{a_n = a_{n-k} + 2}$

Q1 $a_n = 2a_{n-1} + 1$ — (1) $\quad n > a_1 = 1$

$n = n-1$

$a_{n-1} = 2a_{n-2} + 1$

$n = n-2 \quad a_{n-2} = 2a_{n-3} + 1$

$a_n = 2(2a_{n-2} + 1) + 1$

$= 2^2 \{ a_{n-2} + 2 + 1$

$= 2^2 (2a_{n-3} + 1) + 2 + 1$

$= 2^3 a_{n-3} + 2^2 + 2 + 1$

$a_1 = 1$

$n - k = 1$

$n - 1 = k$

$a_n = 2^k a_{n-k} + \cdots 2^{k-1} + 2^2 + 2^1 + 2^0$

$a_1 = 1$

$n - k = 1$

$k = n-1$

$a_n = 2^{n-1} + 2^{n-2} \cdots + 2^2 + 2^1 + 2^0$

$a_n = 2^{n-1} a_{n-n+1} + 2^{n-1-1} + 2^2 + 2^1 + 2^0$

$a_n = 2^{n-1} \cdot 1 + 2^{n-2} + 2^2 + 2^1 + 2^0$

$\boxed{a_n = \Sigma 2^{n-1}}$

Characteristic Root method-

Step 1. write characteristic equation

Step 2. find Roots and lets roots be $R_1, R_2 --- R_K$

Step 3 (a) If all roots are different than general solution is

$$a_n = C_1 r_1^n + C_2 r_2^n --- C_K r_K^n$$

~~Step 4~~ (b) if two of its roots are same then the general solution is

$$a_n = (C_1 + C_2 n) r_1^n + C_3 r_2^n + --- C_K r_{K-1}^n$$

Q $a_n - 6a_{n-1} + 8a_{n-2} = 0$ $\boxed{a_0 = 4,\ a_1 = 10}$ given

let $a_n = r^n$

$r^n - 6r^{n-1} + 8r^{n-2} = 0$ (characteristic eq)

divide both side by r^{n-2}

$r^2 - 6r + 8 = 0$

$r^2 - 4r - 2r + 8 = 0$

$r(r-4) - 2(r-4) = 0$

$(r-4)(r-2) = 0$

$r = 4, 2$

~~Put n=0~~

$a_n = C_1 2^n + C_2 4^n$ —(1)

Put $n=0$

$a_0 = C_1 2^0 + C_2 4^0$ (as $a_0 = 4$ given)

$4 = C_1 + C_2$ —(2)

Put $n=1$

$a_1 = C_1 2^1 + C_2 4^1$

$10 = 2C_1 + 4C_2$ —(3)

multiply eq(1) by 2 and sub from eq(2)

~~Sub eq(2) from (1)~~

$\boxed{C_2 = 1}$

$4 = C_1 + C_2$

$4 = C_1 + 1$ $\boxed{C_1 = 3}$

Put in the general solution-

$a_n = C_1 2^n + C_2 4^n$

$\boxed{a_n = 3 \cdot 2^n + 1 \cdot 4^n}$