

# ADVANCE STATISTICS PROJECT

## DSBA – 2021 BATCH

SAUMYA RAMANAN

MAY 2021

## TABLE OF CONTENTS

Problem 1 .....	3
PROBLEM 1 QUESTION .....	3
PROBLEM 1.1 SOLUTIONS .....	3
1.1. State the null and the alternate hypothesis for conducting one-way ANOVA for both Education and Occupation individually. ....	3
1.2. Perform one-way ANOVA for Education with respect to the variable 'Salary'. State whether the null hypothesis is accepted or rejected based on the ANOVA results. ....	3
1.3. Perform one-way ANOVA for OCCUPATION with respect to the variable 'Salary'. State whether the null hypothesis is accepted or rejected based on the ANOVA results. ....	4
1.4. What is the interaction between the two treatments? Analyze the effects of one variable on the other (Education and Occupation) with the help of an interaction plot. ....	5
1.5. Perform a two-way ANOVA based on the Education and Occupation (along with their interaction Education*Occupation) with the variable 'Salary'. State the null and alternative hypotheses and state your results. How will you interpret this result? .....	6
1.6. Explain the business implications of performing ANOVA for this particular case study. ....	7
Problem 2 QUESTION .....	7
PROBLEM 2 SOLUTION .....	7
2.1. Perform Exploratory Data Analysis [both univariate and multivariate analysis to be performed]. What insight do you draw from the EDA? .....	7
2.2. Is scaling necessary for PCA in this case? Give justification and perform scaling. ....	17
2.3. Comment on the comparison between the covariance and the correlation matrices from this data.[on scaled data] .....	17
2.4. Check the dataset for outliers before and after scaling. What insight do you derive here? ....	21
2.5. Extract the eigenvalues and eigenvectors.[print both] .....	38
2.6. Perform PCA and export the data of the Principal Component (eigenvectors) into a data frame with the original features .....	40
2.7. Write down the explicit form of the first PC (in terms of the eigenvectors. Use values with two places of decimals only) .....	41
2.8. Consider the cumulative values of the eigenvalues. How does it help you to decide on the optimum number of principal components? What do the eigenvectors indicate? ..	41
2.9. Explain the business implication of using the Principal Component Analysis for this case study. How may PCs help in the further analysis? [Hint: Write Interpretations of the Principal Components Obtained] .....	41
REFERENCES .....	42

## PROBLEM 1

### PROBLEM 1 QUESTION

Salary is hypothesized to depend on educational qualification and occupation. To understand the dependency, the salaries of 40 individuals [SalaryData.csv] are collected and each person's educational qualification and occupation are noted. Educational qualification is at three levels, High school graduate, Bachelor, and Doctorate. Occupation is at four levels, Administrative and clerical, Sales, Professional or specialty, and Executive or managerial. A different number of observations are in each level of education – occupation combination.

[Assume that the data follows a normal distribution. In reality, the normality assumption may not always hold if the sample size is small.]

### PROBLEM 1.1 SOLUTIONS

---

#### **1.1. STATE THE NULL AND THE ALTERNATE HYPOTHESIS FOR CONDUCTING ONE-WAY ANOVA FOR BOTH EDUCATION AND OCCUPATION INDIVIDUALLY.**

##### **For Education:**

Null Hypothesis: #H0 - The mean of Salary is same for all 3 levels of Treatment (Education).

Alternate: H1 - For atleast one level of education, the mean of Salary is different.

##### **For Occupation:**

Null: #H0 - The mean of Salary is same for all 4 levels of Treatment (Occupation).

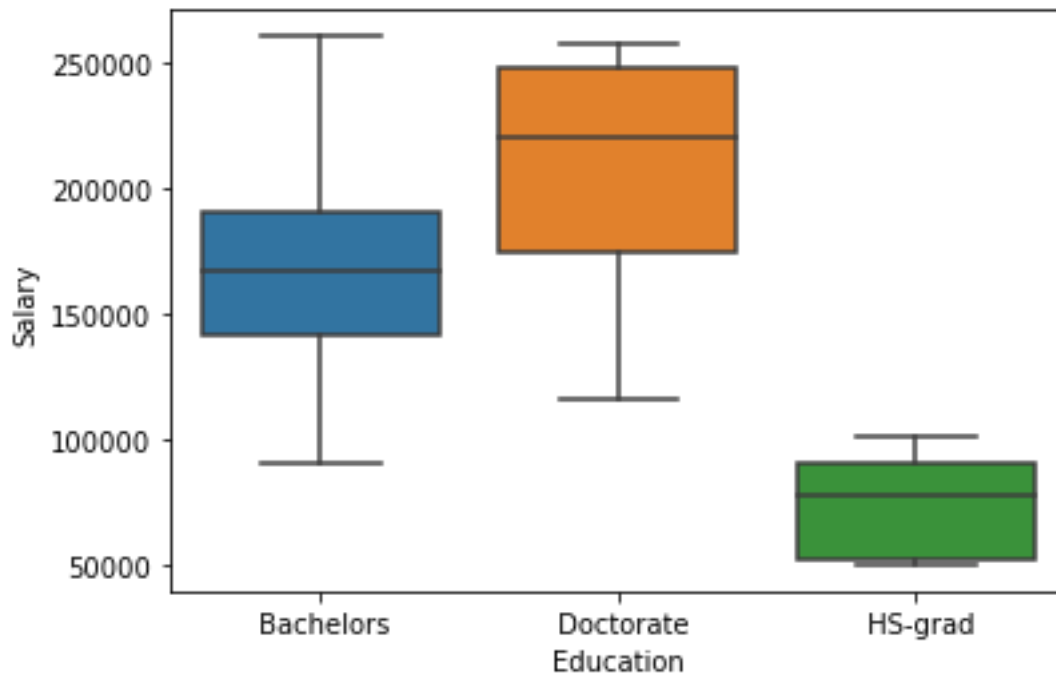
#H1 - For atleast one level of Occupation, the mean of Salary is different.

---

#### **1.2. PERFORM ONE-WAY ANOVA FOR EDUCATION WITH RESPECT TO THE VARIABLE 'SALARY'. STATE WHETHER THE NULL HYPOTHESIS IS ACCEPTED OR REJECTED BASED ON THE ANOVA RESULTS.**

##### **Data Quality:**

It was observed that the data had no nulls or duplicate. The distribution via box plot showed normal distribution with varying means:



Performing 1 way Anova for Test Hypothesis conditions mentioned above, we get the following result:

	df	sum_sq	mean_sq	F	PR(>F)
<b>C(Education)</b>	2.0	1.026955e+11	5.134773e+10	30.95628	1.257709e-08
<b>Residual</b>	37.0	6.137256e+10	1.658718e+09	NaN	NaN

Since the P value is  $< 0.05$  ( we assume  $\alpha = 0.05$ ), the null hypothesis is rejected.

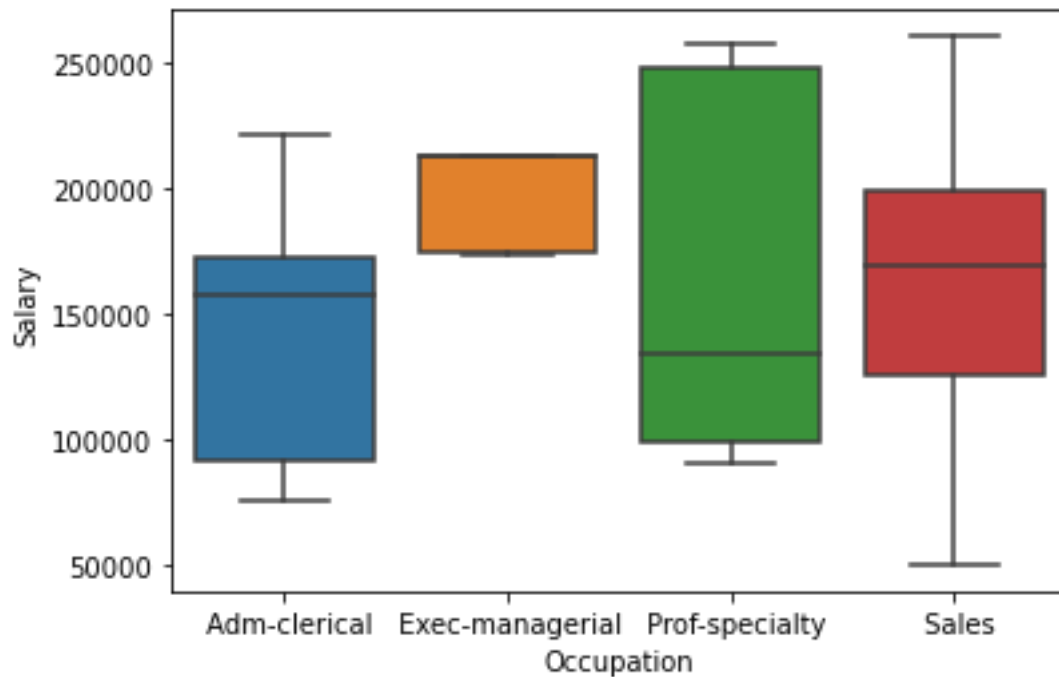
Indicating that : For atleast one level of education, the mean of Salary is different.

---

### 1.3. PERFORM ONE-WAY ANOVA FOR OCCUPATION WITH RESPECT TO THE VARIABLE 'SALARY'. STATE WHETHER THE NULL HYPOTHESIS IS ACCEPTED OR REJECTED BASED ON THE ANOVA RESULTS.

#### Data Quality:

It was observed that the data had no nulls or duplicate. The distribution via box plot showed normal distribution with varying means:



Performing 1 way Anova for Test Hypothesis conditions mentioned above, we get the following result:

	df	sum_sq	mean_sq	F	PR(>F)
<b>C(Occupation)</b>	3.0	1.125878e+10	3.752928e+09	0.884144	0.458508
<b>Residual</b>	36.0	1.528092e+11	4.244701e+09	NaN	NaN

Since the P value is  $< 0.05$  ( we assume  $\alpha = 0.05$ ), we fail to reject the null hypothesis.  
Indicating that : The mean of Salary is same for all 4 levels of Treatment (Occupation).

---

#### 1.4. WHAT IS THE INTERACTION BETWEEN THE TWO TREATMENTS? ANALYZE THE EFFECTS OF ONE VARIABLE ON THE OTHER (EDUCATION AND OCCUPATION) WITH THE HELP OF AN INTERACTION PLOT.

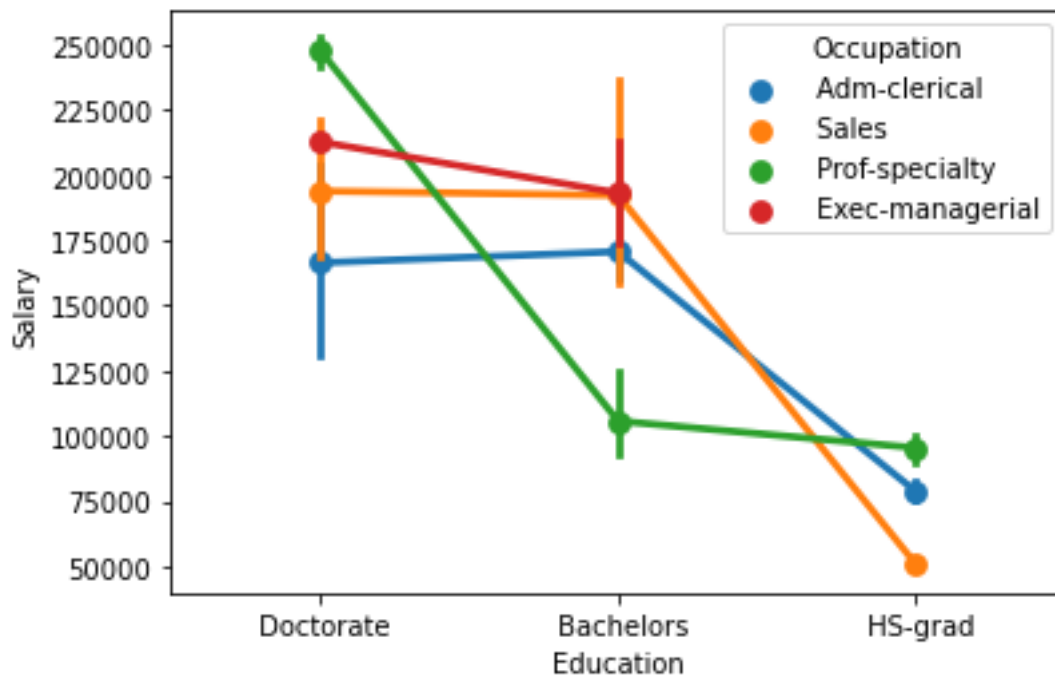
On performing Anova on the interaction of the 2 variables we get the following output:

	df	sum_sq	mean_sq	F	PR(>F)
C(Occupation)	3.0	1.125878e+10	3.752928e+09	2.284576	9.648715e-02
C(Education)	2.0	9.695663e+10	4.847831e+10	29.510933	3.708479e-08

Residual 34.0 5.585261e+10 1.642724e+09 NaN NaN

The p-value in the one of the treatments is less than alpha (0.05)

Let us check whether there is any interaction effect between the treatments.



We observe that there is some interaction between education and occupation (esp in Prof Spl).

---

**1.5. PERFORM A TWO-WAY ANOVA BASED ON THE EDUCATION AND OCCUPATION (ALONG WITH THEIR INTERACTION EDUCATION\*OCCUPATION) WITH THE VARIABLE 'SALARY'. STATE THE NULL AND ALTERNATIVE HYPOTHESES AND STATE YOUR RESULTS. HOW WILL YOU INTERPRET THIS RESULT?**

H0 – Null Hypothesis: There is no interaction of effect of Education and Occupation on each other .

H1: Alternate Hypothesis: There is an interaction of occupation & education

	df	sum_sq	mean_sq	F	PR(>F)
C(Education)	2.0	1.026955e+11	5.134773e+10	72.211958	5.466264e-12
C(Occupation)	3.0	5.519946e+09	1.839982e+09	2.587626	7.211580e-02

C(Education):C(Occupation)	6.0	3.634909e+10	6.058182e+09	8.519815	2.232500e-05
Residual	29.0	2.062102e+10	7.110697e+08	NaN	NaN

The P value  $2.23 \times 10^{-5}$  is less than alpha, hence we can conclude there is no interaction between occupation and education that impacts the mean Salary value.

---

### 1.6. EXPLAIN THE BUSINESS IMPLICATIONS OF PERFORMING ANOVA FOR THIS PARTICULAR CASE STUDY.

There are 2 treatments in this use case – Occupation and Education. We saw interference between the 2 treatments and we needed to find the impact to Salary due to interactions of these treatment. ANOVA is a great technique to achieve this.

ANOVA can also be used to forecast trends by analyzing patterns in data to better understand the Salary trends (in this case). It's also a widely used statistical technique for comparing the relationship between factors that cause a change in Salary, such as educational and occupational impact on salaries and this helps take measures for the future.

### PROBLEM 2 QUESTION

The dataset Education - Post 12th Standard.csv contains information on various colleges. You are expected to do a Principal Component Analysis for this case study according to the instructions given. The data dictionary of the 'Education - Post 12th Standard.csv' can be found in the following file: Data Dictionary.xlsx.

### PROBLEM 2 SOLUTION

---

#### 2.1. PERFORM EXPLORATORY DATA ANALYSIS [BOTH UNIVARIATE AND MULTIVARIATE ANALYSIS TO BE PERFORMED]. WHAT INSIGHT DO YOU DRAW FROM THE EDA?

#### DATA QUALITY CHECKS:

DATA QUALITY CHECKS WERE DONE FOR NULLS, DUPLICATES, AND DATA WAS FOUND TO BE CLEAN. ALSO EXCEP FOR THE NAMES FIELD THE REST OF FIELDS ARE INT (ONLY S.F.RATIO BEING FLOAT).

## **UNIVARIATE ANALYSIS:**

THE MEAN, MODE AND MEDIAN WAS DONE FOR ALL INT & FLOAT FIELDS.

THE DATA WAS FOUND TO BE CLOSE TO NORMAL DISTRIBUTION for INTEGER CONTINUOUS VARIABLES Like Top10perc, Top25perc, Perc.Alumni – these indicate the percentage of students applying from top(10,25 %)and also % of Alumni who denote and graduation Rate .

ON expenditure – the Expend, Personal, Books showed normal distribution.

```
count      777.000000
mean       3001.638353
std        3870.201484
min         81.000000
25%        776.000000
50%       1558.000000
75%       3624.000000
max       48094.000000
Name: Apps, dtype: float64
count      777.000000
mean       2018.804376
std        2451.113971
min         72.000000
25%         604.000000
50%       1110.000000
75%       2424.000000
max       26330.000000
Name: Accept, dtype: float64
count      777.000000
mean       779.972973
std        929.176190
min         35.000000
25%       242.000000
50%       434.000000
75%       902.000000
max       6392.000000
Name: Enroll, dtype: float64
count      777.000000
mean       27.558559
std        17.640364
min          1.000000
25%       15.000000
50%       23.000000
75%       35.000000
max       96.000000
Name: Top10perc, dtype: float64
count      777.000000
mean       55.796654
std        19.804778
min          9.000000
```



```

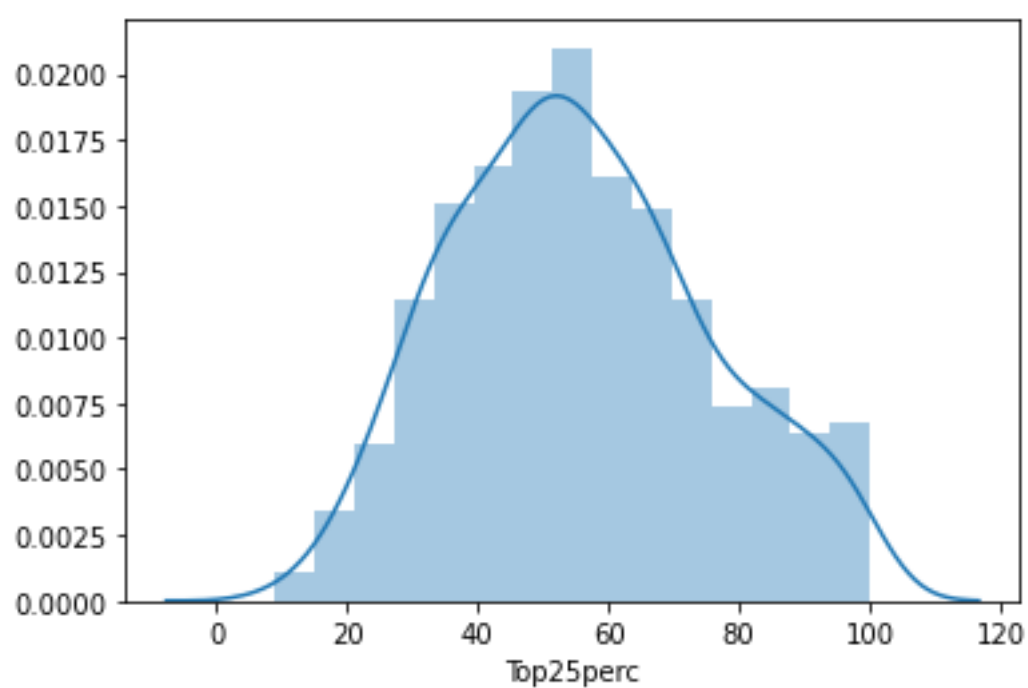
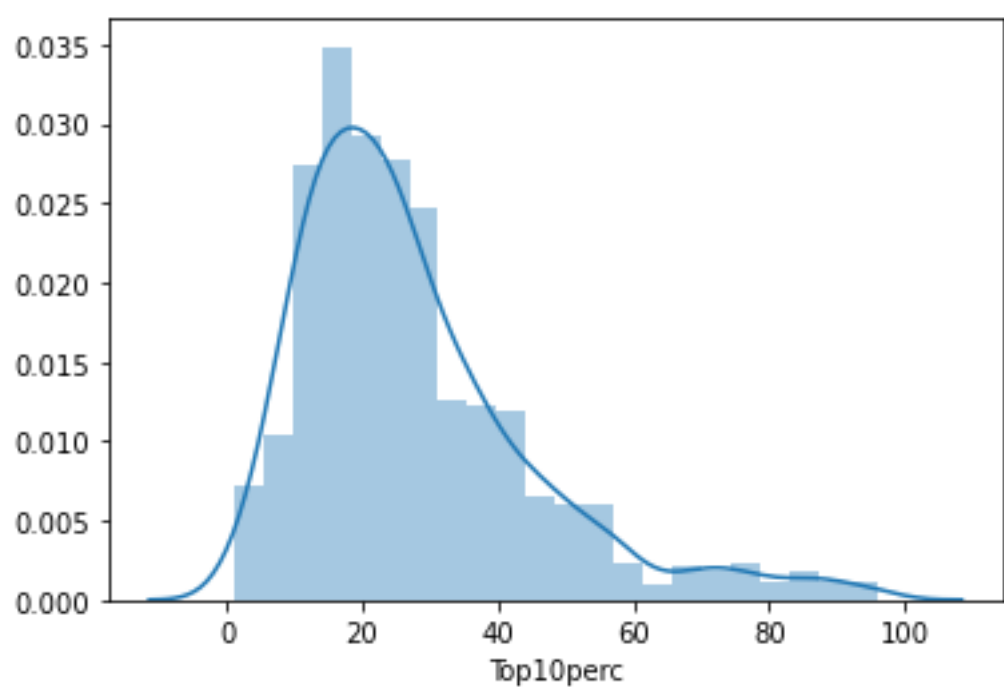
25%      41.000000
50%      54.000000
75%      69.000000
max      100.000000
Name: Top25perc, dtype: float64
count    777.000000
mean     3699.907336
std      4850.420531
min      139.000000
25%      992.000000
50%     1707.000000
75%     4005.000000
max     31643.000000
Name: F.Undergrad, dtype: float64
count    777.000000
mean      855.298584
std     1522.431887
min        1.000000
25%      95.000000
50%     353.000000
75%     967.000000
max    21836.000000
Name: P.Undergrad, dtype: float64
count    777.000000
mean    10440.669241
std     4023.016484
min     2340.000000
25%     7320.000000
50%     9990.000000
75%    12925.000000
max    21700.000000
Name: Outstate, dtype: float64
count    777.000000
mean     4357.526384
std     1096.696416
min     1780.000000
25%     3597.000000
50%     4200.000000
75%     5050.000000
max     8124.000000
Name: Room.Board, dtype: float64
count    777.000000
mean     549.380952
std      165.105360
min       96.000000
25%     470.000000
50%     500.000000
75%     600.000000
max     2340.000000
Name: Books, dtype: float64
count    777.000000
mean     1340.642214
std      677.071454
min      250.000000
25%      850.000000
50%     1200.000000
75%     1700.000000
max     6800.000000

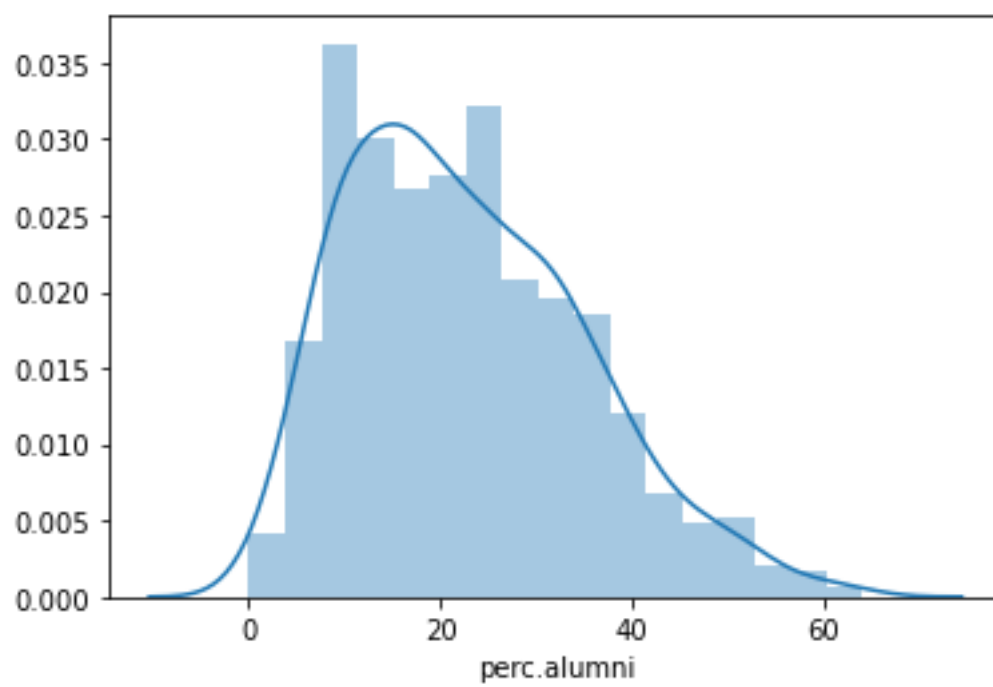
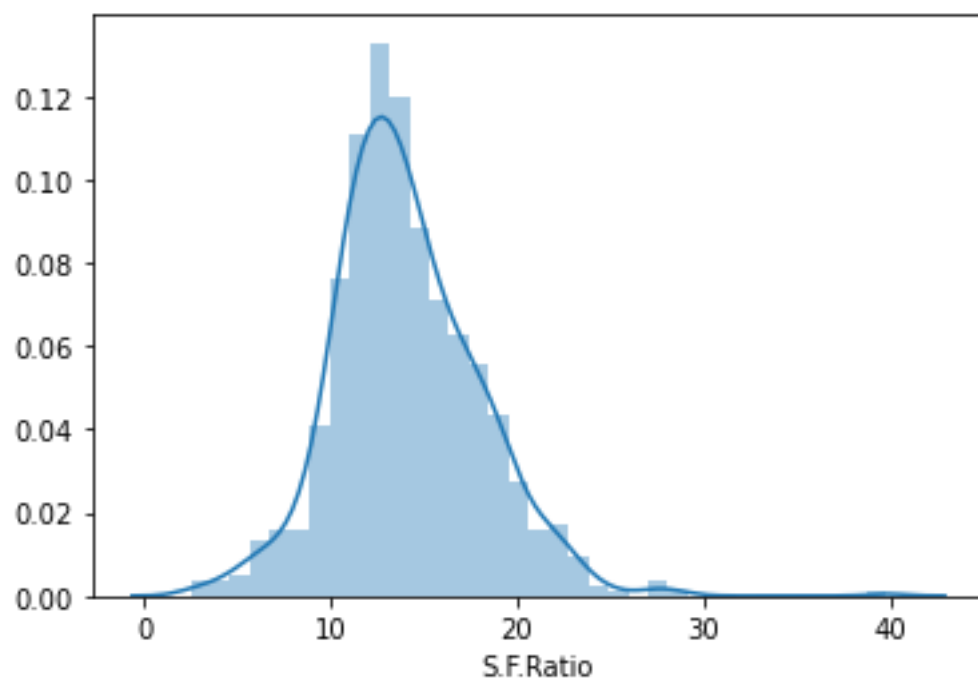
```

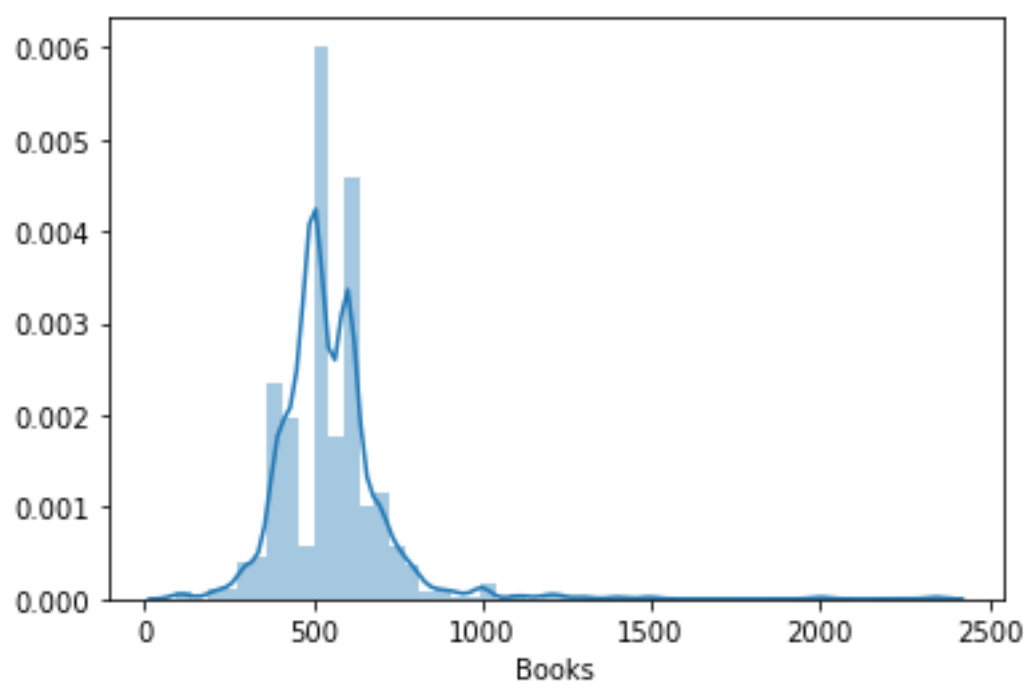
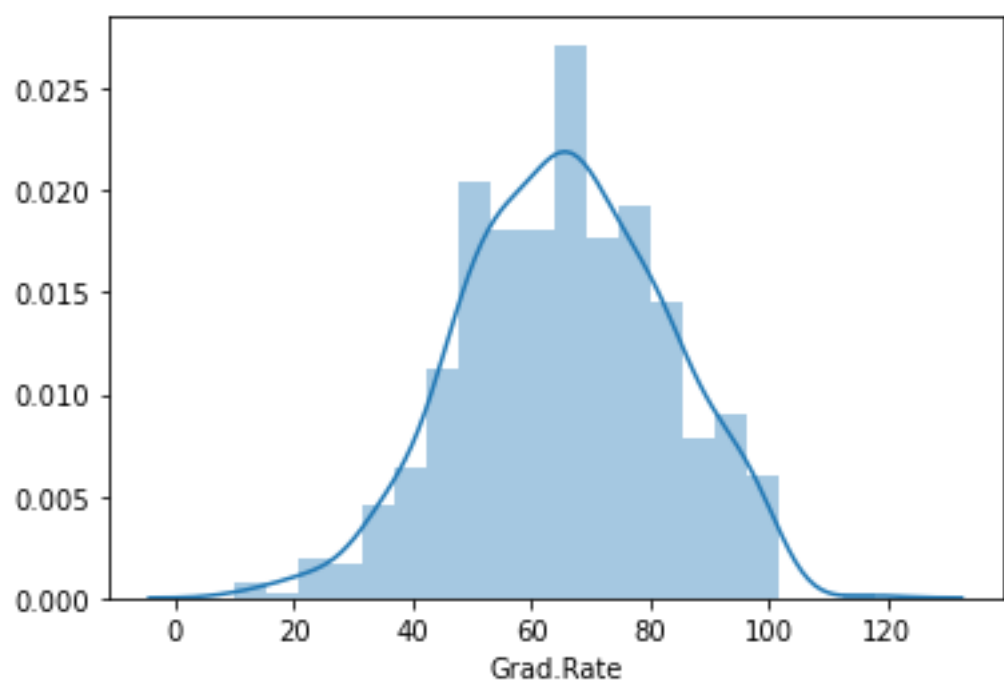
```

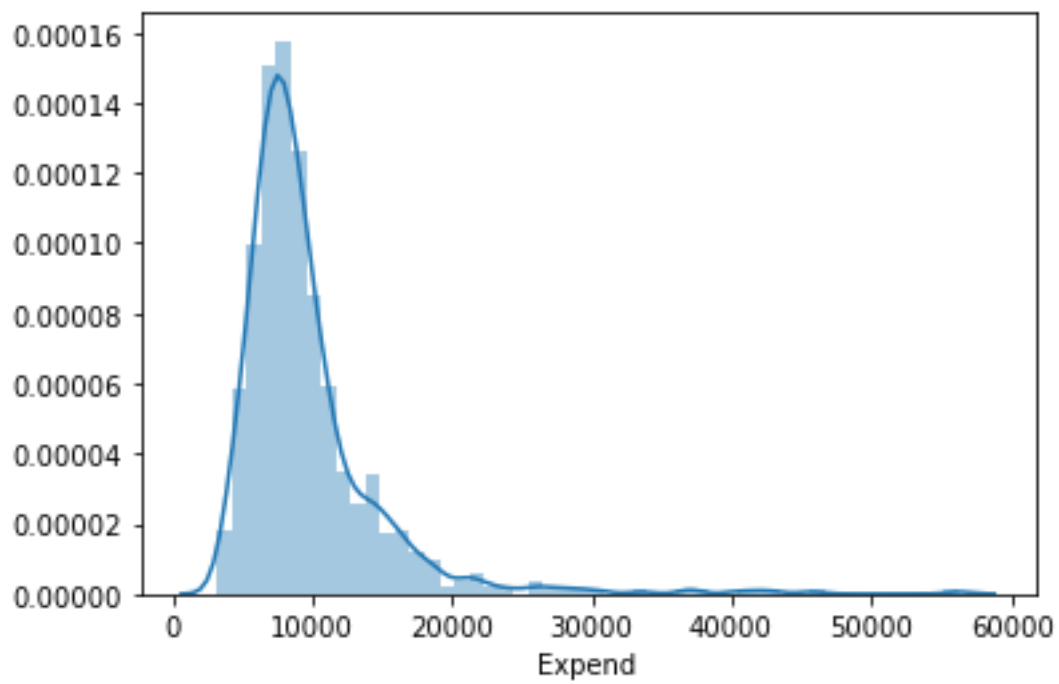
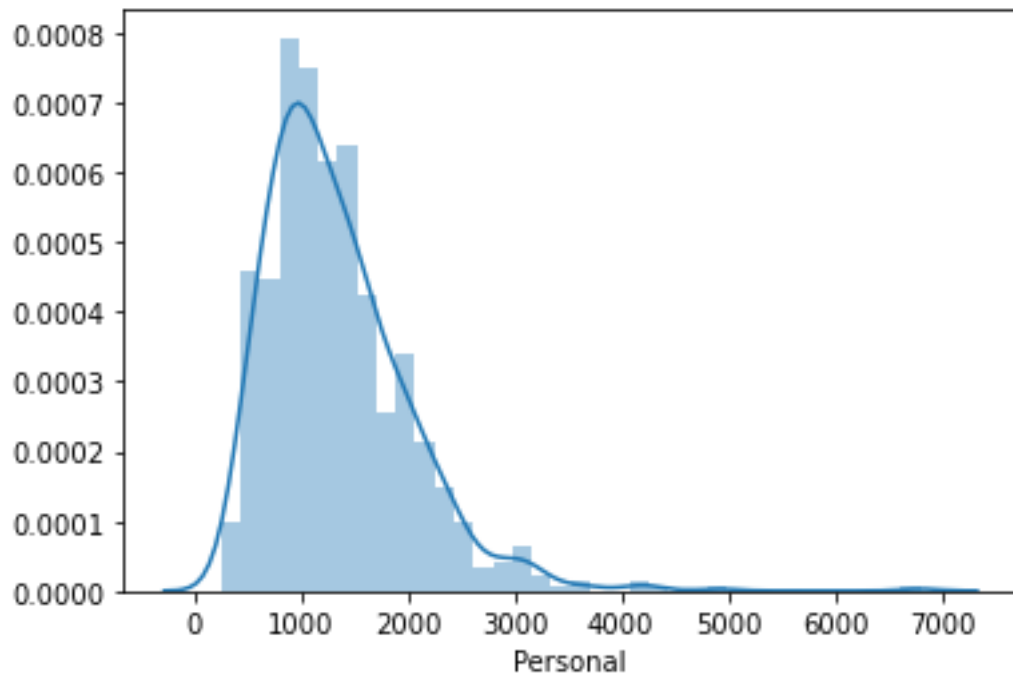
Name: Personal, dtype: float64
count      777.000000
mean       72.660232
std        16.328155
min         8.000000
25%        62.000000
50%        75.000000
75%        85.000000
max       103.000000
Name: PhD, dtype: float64
count      777.000000
mean       79.702703
std        14.722359
min        24.000000
25%        71.000000
50%        82.000000
75%        92.000000
max       100.000000
Name: Terminal, dtype: float64
count      777.000000
mean       14.089704
std         3.958349
min         2.500000
25%        11.500000
50%        13.600000
75%        16.500000
max        39.800000
Name: S.F.Ratio, dtype: float64
count      777.000000
mean       22.743887
std        12.391801
min         0.000000
25%        13.000000
50%        21.000000
75%        31.000000
max        64.000000
Name: perc.alumni, dtype: float64
count      777.000000
mean      9660.171171
std       5221.768440
min       3186.000000
25%      6751.000000
50%      8377.000000
75%     10830.000000
max     56233.000000
Name: Expend, dtype: float64
count      777.000000
mean       65.46332
std        17.17771
min        10.00000
25%        53.00000
50%        65.00000
75%        78.00000
max       118.00000
Name: Grad.Rate, dtype: float64

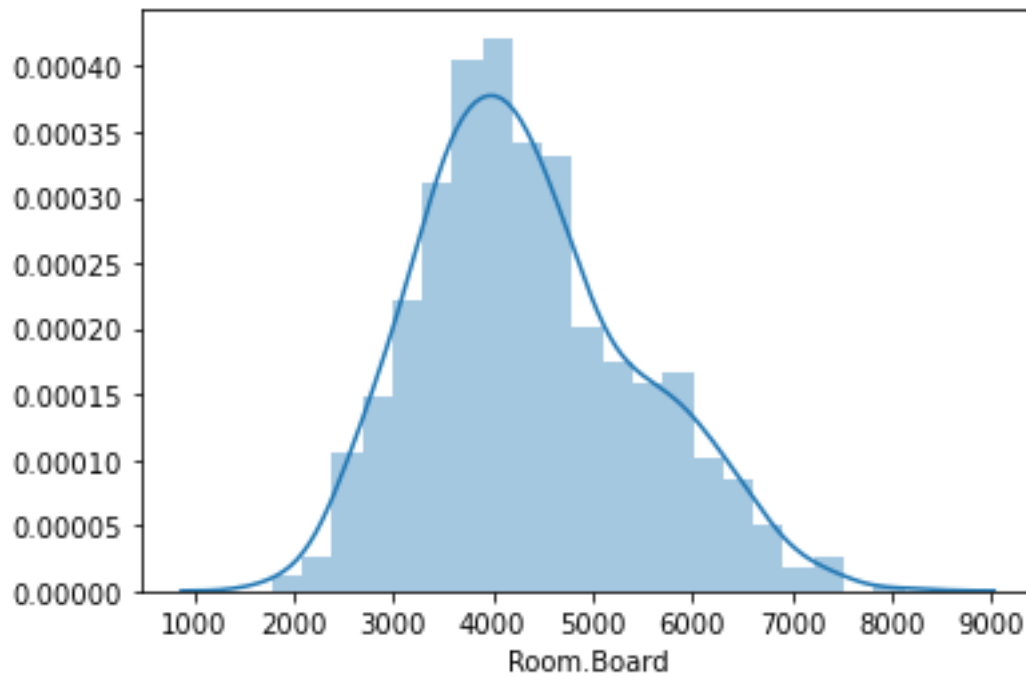
```











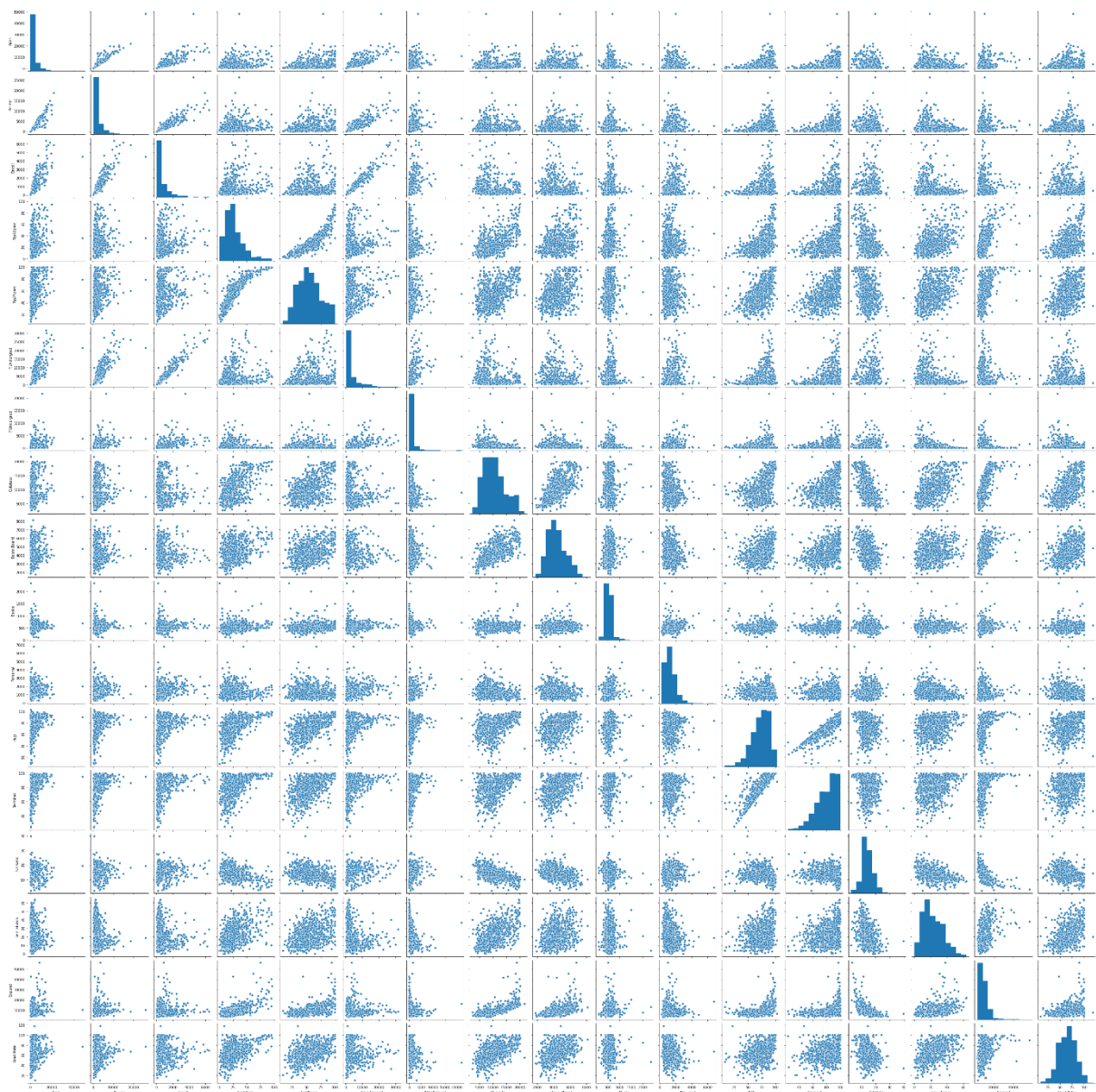
Multivariate Analysis:

A pairplot made for all variable is below:

Following variable pairs – show linear relationship.

Variables	Interpretation
Outstate Vs Expend	An increase in no. of outstation students shows increase in expenditure
Apps Vs Accept	The acceptance number increased with no. of applications
Accept Vs Enroll	The enrollment number increased with no. of acceptance
AppsVs F.undergrad	The application numbers and the numbers of Full time graduates are propotional
Accept Vs F.undergrad	The acceptance rate and the number of Full time graduates are linearly propotional
Enroll Vs F.undergrad	The enrollment rate and the number of Full time graduates are linearly propotional

Top 10% Vs phd	The students from Top 10 pct Higher sec school was propotional to number of PHD teachers (they were probably allocated to such students conciously)
Top 25% Vs phd	The students from Top 25 pct Higher sec school was propotional to number of PHD teachers (they were probably allocated to such students conciously)
Room.board Vs expend	The Room expenses directly correlated to expenses
Terminal Vs Phd	Terminal(last degree) was sharply directly propotional to PhD(indicating PHD was highest degree)





**2.2. IS SCALING NECESSARY FOR PCA IN THIS CASE? GIVE JUSTIFICATION AND PERFORM SCALING.**

**SOLUTION:** YES. SCALING IS NECESSARY AS THERE ARE 16 INT VARIABLES – they are at different scales – some represent number of applications, enrollments etc while some others represent the expenditure and few fields on percent of graduates.

Some of the popular Scaling methods that can be used are : Simple Scalar , Min-Max (adjusts for outliers) or z-score.

In our solution we replace all numeric fields with their **z-score**. This is a widely accepted standards for PCA.

It scales the data in such a way that the mean value of the features tends to 0 and the standard deviation tends to 1.

### 2.3. COMMENT ON THE COMPARISON BETWEEN THE COVARIANCE AND THE CORRELATION MATRICES FROM THIS DATA.[ON SCALED DATA].

Below table shows the covariance and correlation Table for raw-data – with outliers and without scaling.

Apps	Accept Outstate perc.alumni	Enroll Room.Board Expend	Top10perc Books Grad.Rate	Top25perc Personal	F.Undergrad PhD	P.Undergrad Terminal	S.F.Ratio
Apps	1.497846e+07	8.949860e+06	3.045256e+06	23132.773138	26952.663479		
	1.528970e+07	2.346620e+06	7.809704e+05	7.000729e+05	84703.752639		
	4.683468e+05	24689.433666	21053.067602	1465.060576	-4327.122381		
	5.246171e+06	9756.421641					
Accept	8.949860e+06	6.007960e+06	2.076268e+06	8321.124872	12013.404757		
	1.039358e+07	1.646670e+06	-2.539623e+05	2.443471e+05	45942.807867		
	3.335566e+05	14238.201489	12182.093828	1709.838189	-4859.487022		
	1.596272e+06	2834.162918					
Enroll	3.045256e+06	2.076268e+06	8.633684e+05	2971.583415	4172.592435		
	4.347530e+06	7.257907e+05	-5.811885e+05	-4.099706e+04	17291.199742		
	1.767380e+05	5028.961166	4217.086027	872.684773	-2081.693787		
	3.113454e+05	-356.587977					
Top10perc	2.313277e+04	8.321125e+03	2.971583e+03	311.182456	311.630480		
	1.208911e+04	-2.829475e+03	3.990718e+04	7.186706e+03	346.177405	-	
	1.114551e+03	153.184870	127.551581	-26.874525	99.567208	6.087931e+04	
	149.992164						

Top25perc 2.695266e+04 1.201340e+04 4.172592e+03 311.630480 392.229216  
 1.915895e+04 -1.615412e+03 3.899243e+04 7.199904e+03 377.759266 -  
 1.083605e+03 176.518449 153.002612 -23.097199 102.550946 5.454648e+04  
 162.371398

F.Undergrad 1.528970e+07 1.039358e+07 4.347530e+06 12089.113681 19158.952782  
 2.352658e+07 4.212910e+06 -4.209843e+06 -3.664582e+05 92535.764728  
 1.041709e+06 25211.784197 21424.241746 5370.208581 -13791.929691  
 4.724040e+05 -6563.307527

P.Undergrad 2.346620e+06 1.646670e+06 7.257907e+05 -2829.474981 -1615.412144  
 4.212910e+06 2.317799e+06 -1.552704e+06 -1.023919e+05 20410.446674  
 3.297324e+05 3706.756219 3180.596615 1401.302563 -5297.337090 -  
 6.643512e+05 -6721.062488

Outstate 7.809704e+05 -2.539623e+05 -5.811885e+05 39907.179832 38992.427500 -  
 4.209843e+06 -1.552704e+06 1.618466e+07 2.886597e+06 25808.242145 -8.146737e+05  
 25157.515051 24164.147673 -8835.253539 28229.553066 1.413324e+07  
 39479.681796

Room.Board 7.000729e+05 2.443471e+05 -4.099706e+04 7186.705605 7199.903568 -  
 3.664582e+05 -1.023919e+05 2.886597e+06 1.202743e+06 23170.313390 -1.480838e+05  
 5895.034749 6047.299735 -1574.205914 3701.431379 2.873308e+06  
 8005.360183

Books 8.470375e+04 4.594281e+04 1.729120e+04 346.177405 377.759266  
 9.253576e+04 2.041045e+04 2.580824e+04 2.317031e+04 27259.779946  
 2.004303e+04 72.534242 242.963918 -20.867207 -82.263132  
 9.691258e+04 3.008837

Personal 4.683468e+05 3.335566e+05 1.767380e+05 -1114.551186 -1083.605065  
 1.041709e+06 3.297324e+05 -8.146737e+05 -1.480838e+05 20043.025650  
 4.584258e+05 -120.898783 -305.154186 365.415770 -2399.310824 -  
 3.460978e+05 -3132.614944

PhD 2.468943e+04 1.423820e+04 5.028961e+03 153.184870 176.518449  
 2.521178e+04 3.706756e+03 2.515752e+04 5.895035e+03 72.534242 -  
 1.208988e+02 266.608636 204.231332 -8.436492 50.383230 3.689806e+04  
 85.557109

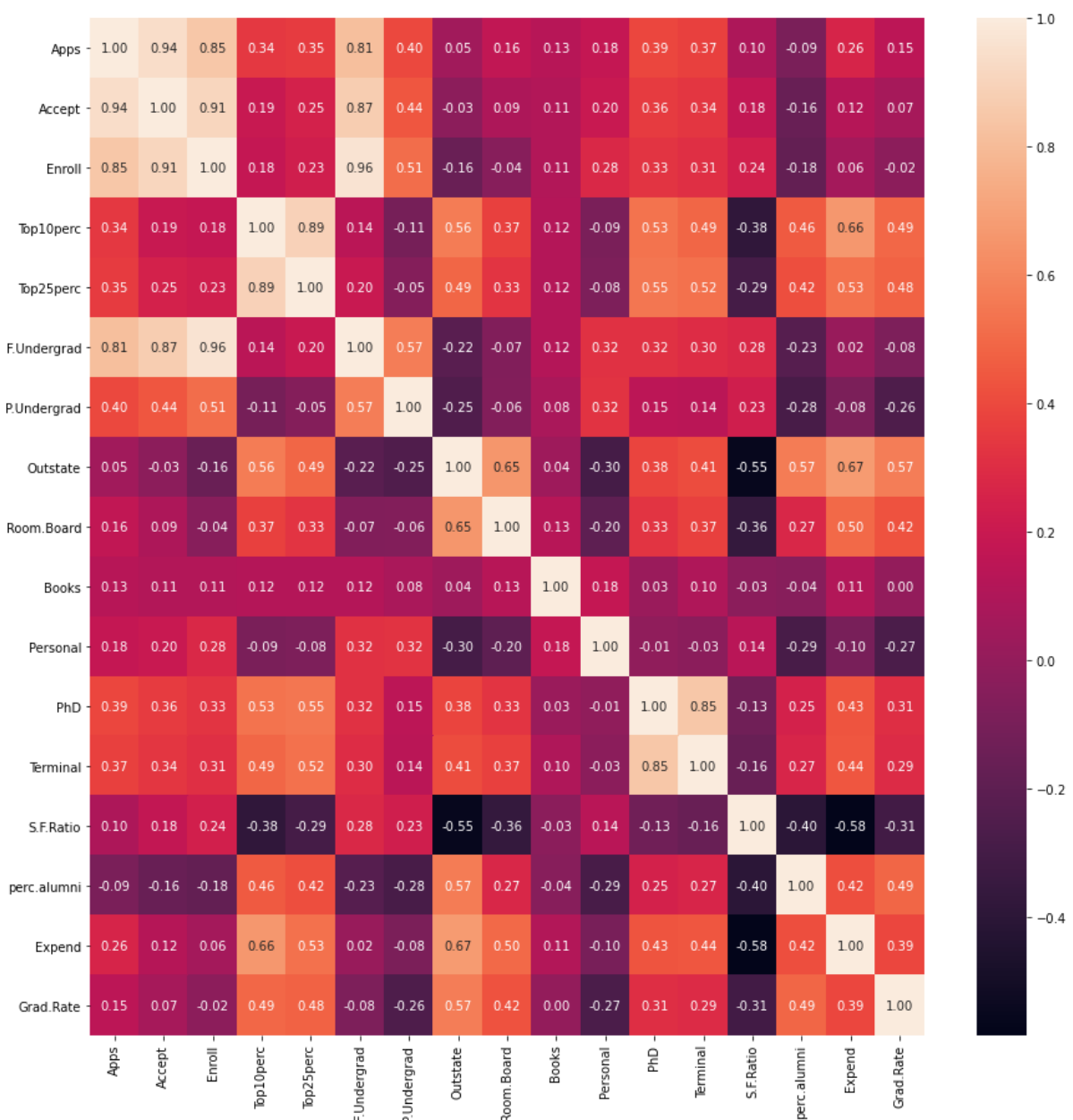
Terminal 2.105307e+04 1.218209e+04 4.217086e+03 127.551581 153.002612  
 2.142424e+04 3.180597e+03 2.416415e+04 6.047300e+03 242.963918 -  
 3.051542e+02 204.231332 216.747841 -9.330256 48.734327 3.373346e+04  
 73.220396

S.F.Ratio 1.465061e+03 1.709838e+03 8.726848e+02 -26.874525 -23.097199  
 5.370209e+03 1.401303e+03 -8.835254e+03 -1.574206e+03 -20.867207  
 3.654158e+02 -8.436492 -9.330256 15.668528 -19.764109 -  
 1.206756e+04 -20.854888

perc.alumni -4.327122e+03 -4.859487e+03 -2.081694e+03 99.567208 102.550946 -  
1.379193e+04 -5.297337e+03 2.822955e+04 3.701431e+03 -82.263132 -2.399311e+03  
50.383230 48.734327 -19.764109 153.556744 2.702892e+04 104.493815

Expend 5.246171e+06 1.596272e+06 3.113454e+05 60879.310196 54546.483305  
4.724040e+05 -6.643512e+05 1.413324e+07 2.873308e+06 96912.580326 -  
3.460978e+05 36898.058233 33733.456882 -12067.564601 27028.921473 2.726687e+07  
35012.968271

Grad.Rate 9.756422e+03 2.834163e+03 -3.565880e+02 149.992164 162.371398 -  
6.563308e+03 -6.721062e+03 3.947968e+04 8.005360e+03 3.008837 -3.132615e+03  
85.557109 73.220396 -20.854888 104.493815 3.501297e+04 295.073717



Some of the specific High correlation fields are as below:

Set 1 of highly correlated variables: 'Apps','Accept','Enroll','F.Undergrad', 'P.Undergrad'

Set 2 of highly correlated variables : 'Room.Board','Outstate','Expend', 'Books'

	Top10perc	Top25perc	Grad.Rate	PhD	Terminal
Top10perc	1.000000	0.891995	0.494989	0.531828	0.491135
Top25perc	0.891995	1.000000	0.477281	0.545862	0.524749
Grad.Rate	0.494989	0.477281	1.000000	0.305038	0.289527
PhD	0.531828	0.545862	0.305038	1.000000	0.849587
Terminal	0.491135	0.524749	0.289527	0.849587	1.000000

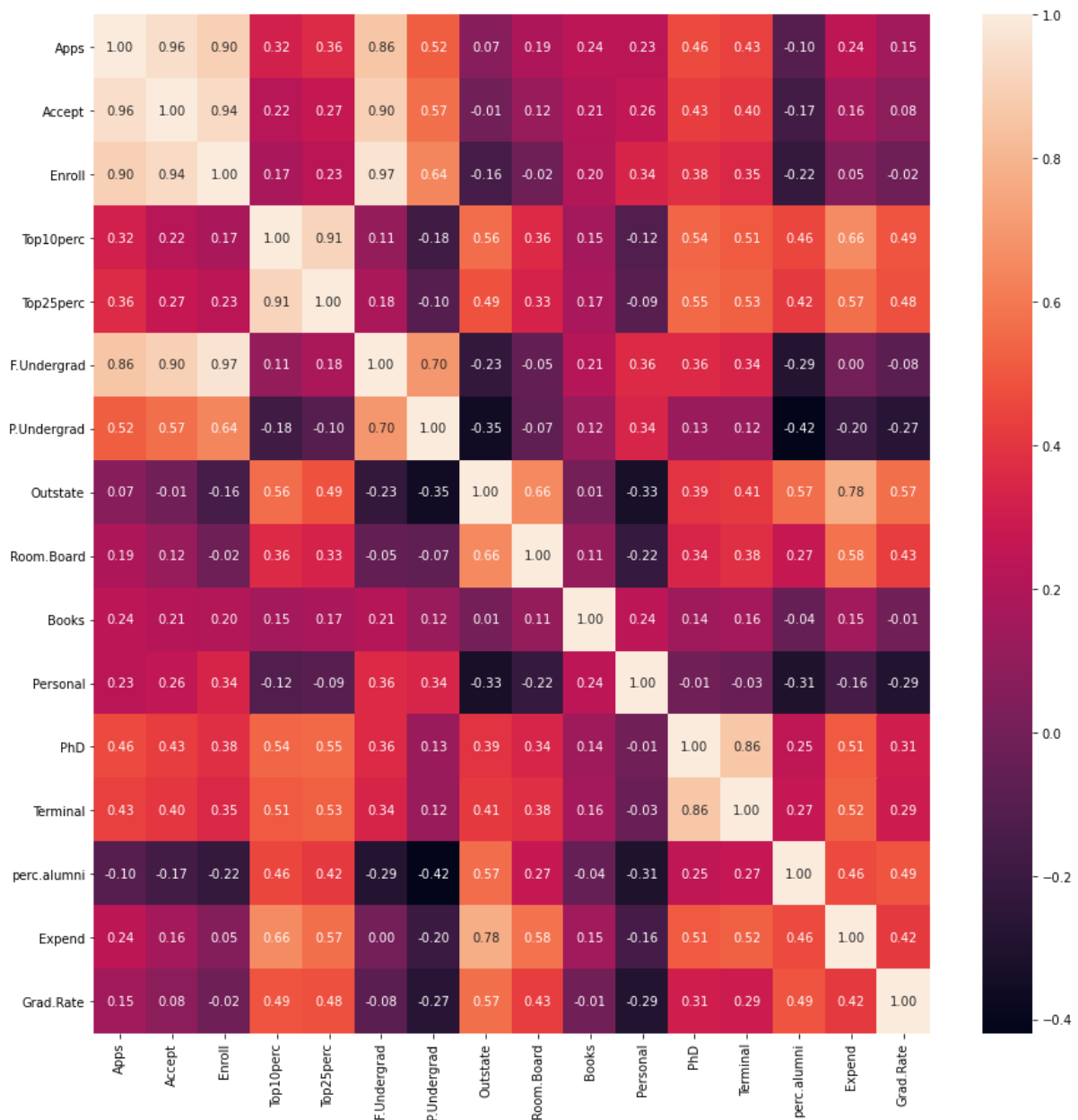
	Room.Board	Outstate	Expend	Books	Personal
Room.Board	1.000000	0.654256	0.501739	0.127963	-0.199428
Outstate	0.654256	1.000000	0.672779	0.038855	-0.299087
Expend	0.501739	0.672779	1.000000	0.112409	-0.097892
Books	0.127963	0.038855	0.112409	1.000000	0.179295
Personal	-0.199428	-0.299087	-0.097892	0.179295	1.000000

### **AFTER SCALING:**

BELOW IS THE COVARIANCE AND CORRELATION AFTER Z-SCORE SCALING AND REMOVING OUTLIERS.

THE **COVARIANCE ON SCALED DATA BECOMES THE CORRELATION.**

THERE IS NO SIGNIFICANT DIFFERENCE IN CORRELATION/COVARIANCE AFTER SCALING.



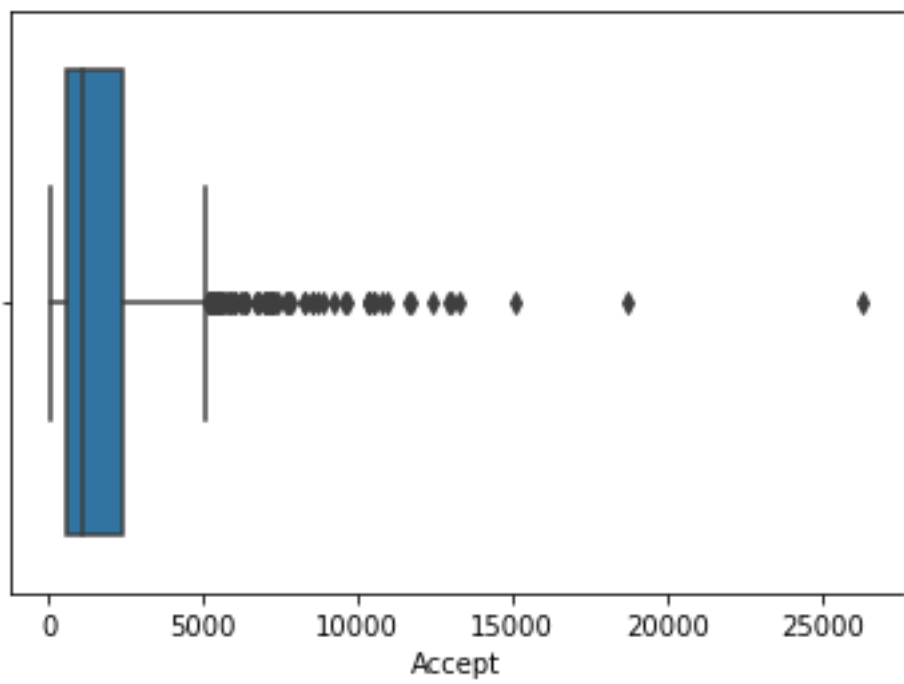
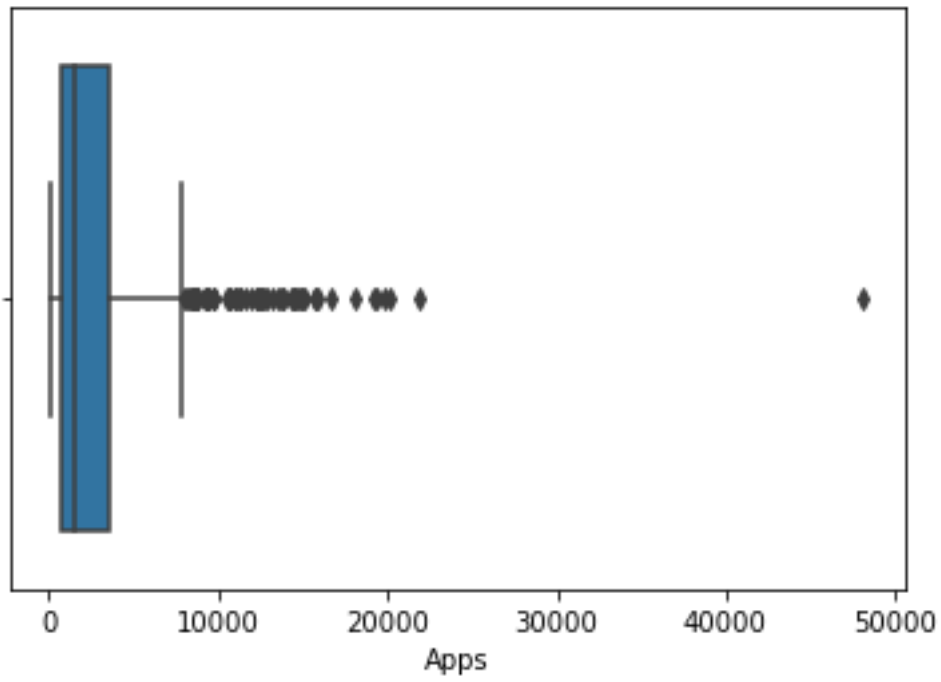
## 2.4. CHECK THE DATASET FOR OUTLIERS BEFORE AND AFTER SCALING. WHAT INSIGHT DO YOU DERIVE HERE?

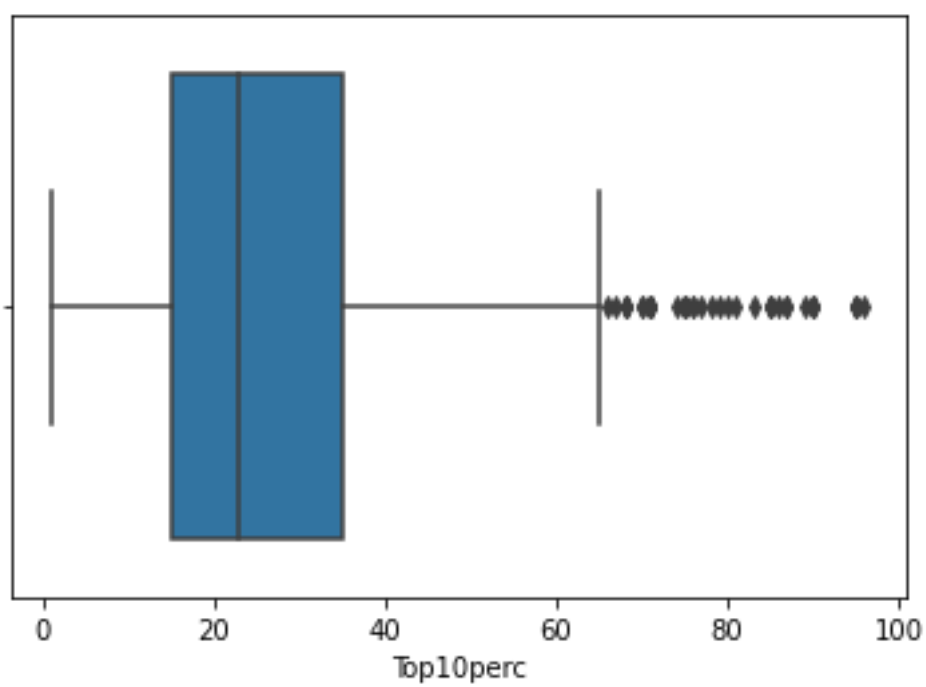
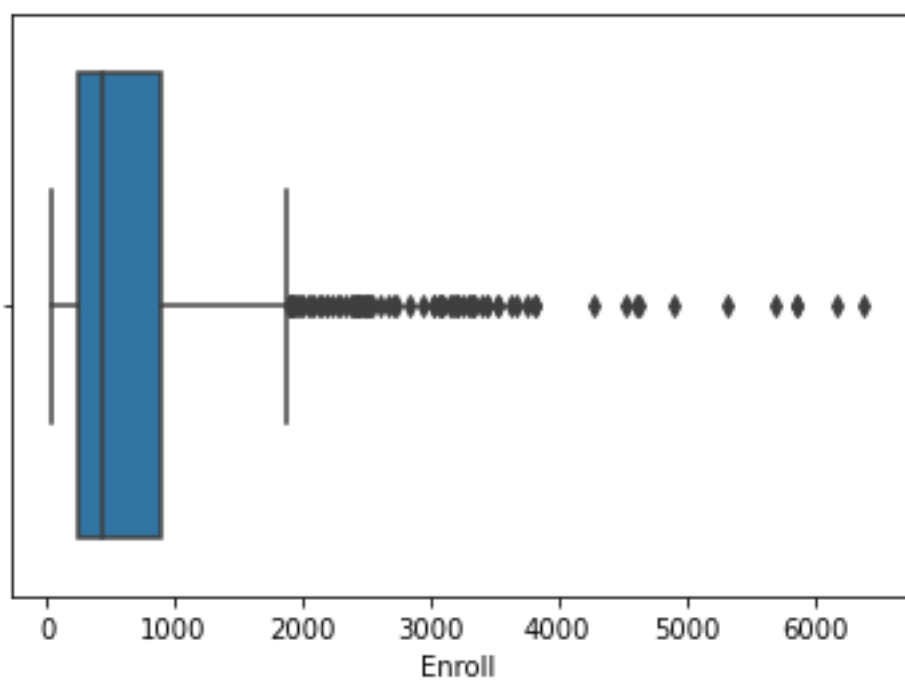
The following is the Boxplot for all integer variables before scaling.

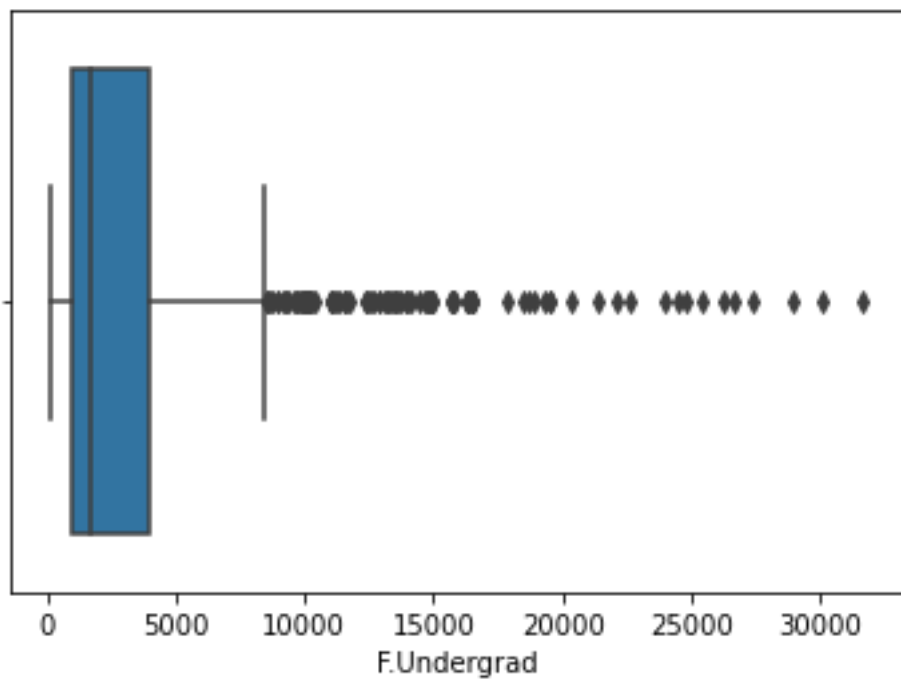
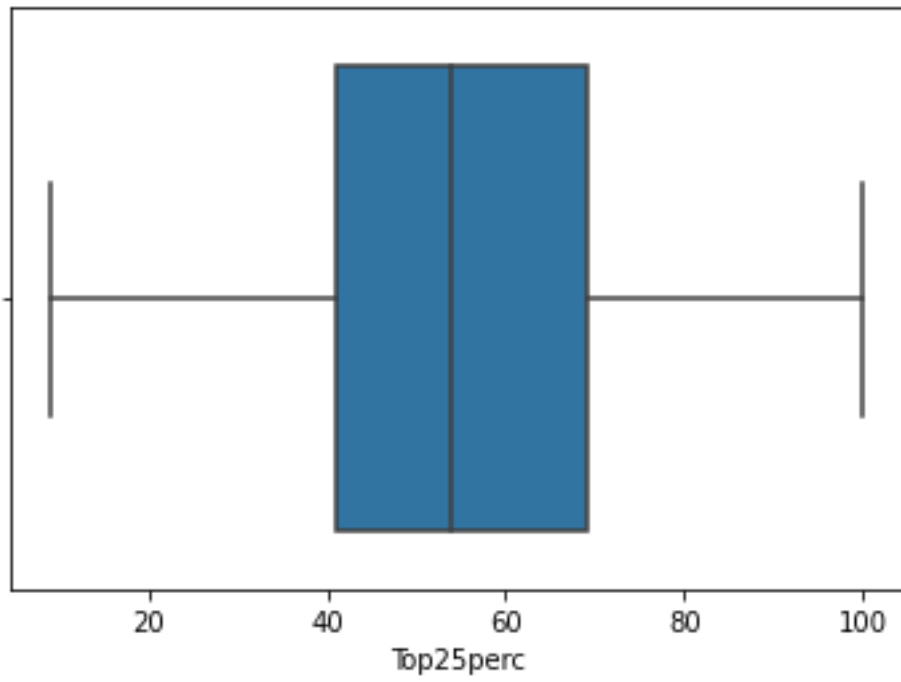
There are several outliers in the data.

Adjusting for outliers by replacing the Max values with upper whisker ( $Q3 + 1.5 \text{ IQR}$ ) and Min value with ( $Q1 - 1.5 \text{ IQR}$ ) lower whisker removes outliers.

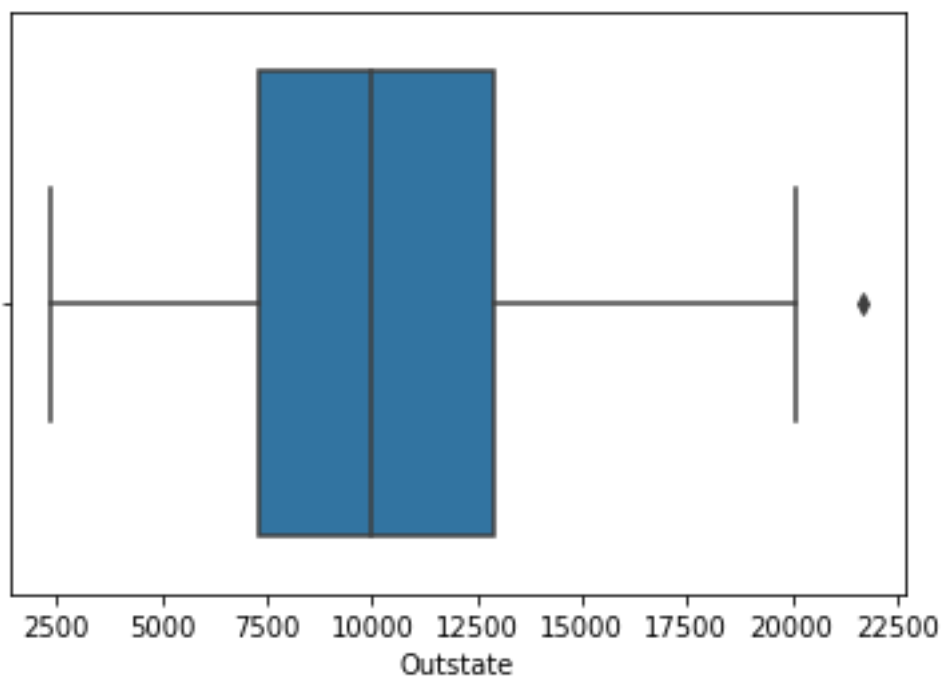
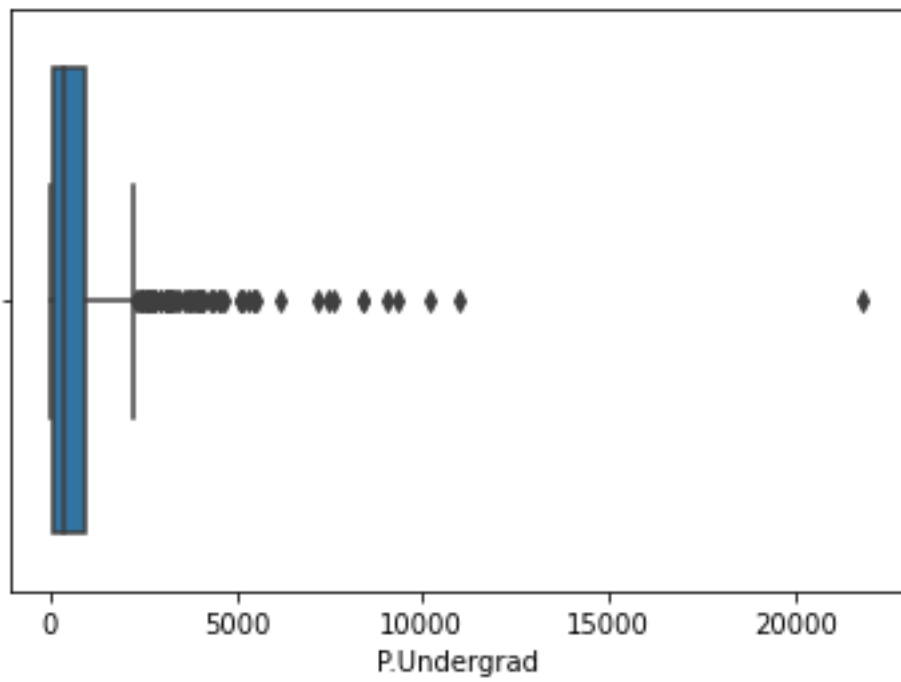
### Box-Plot before scaling:

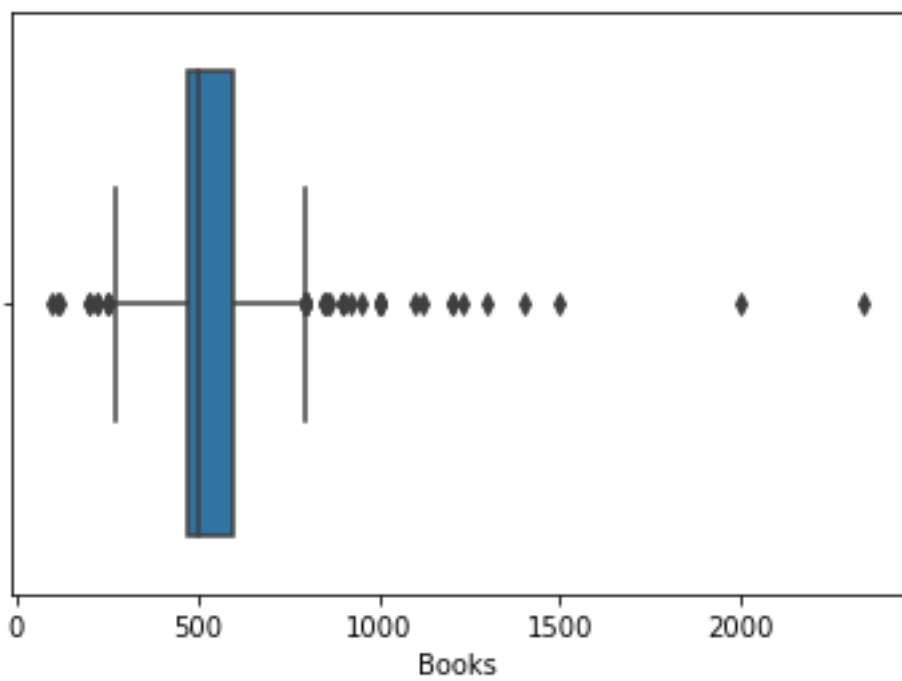
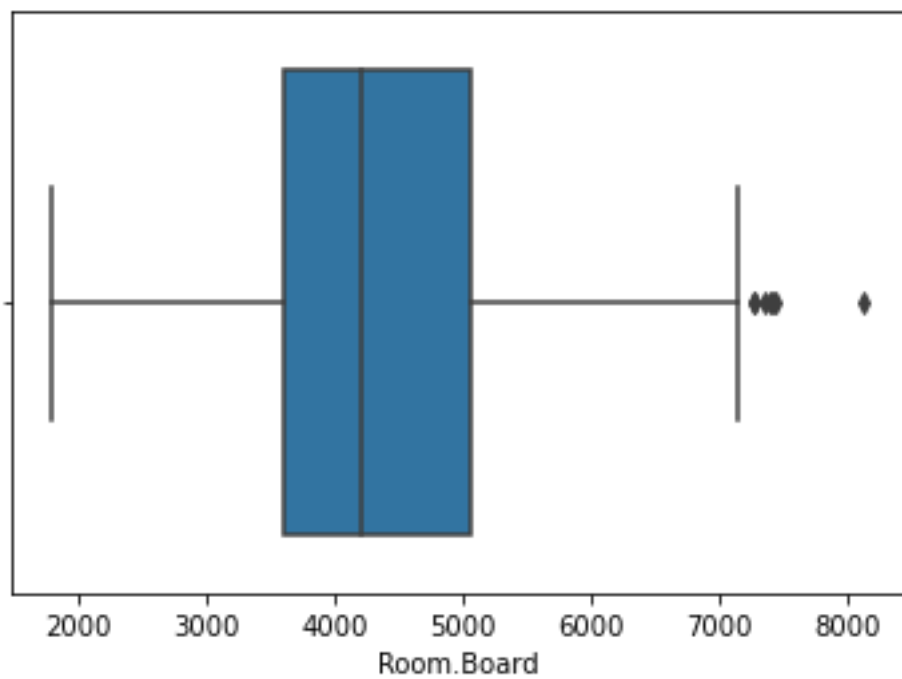


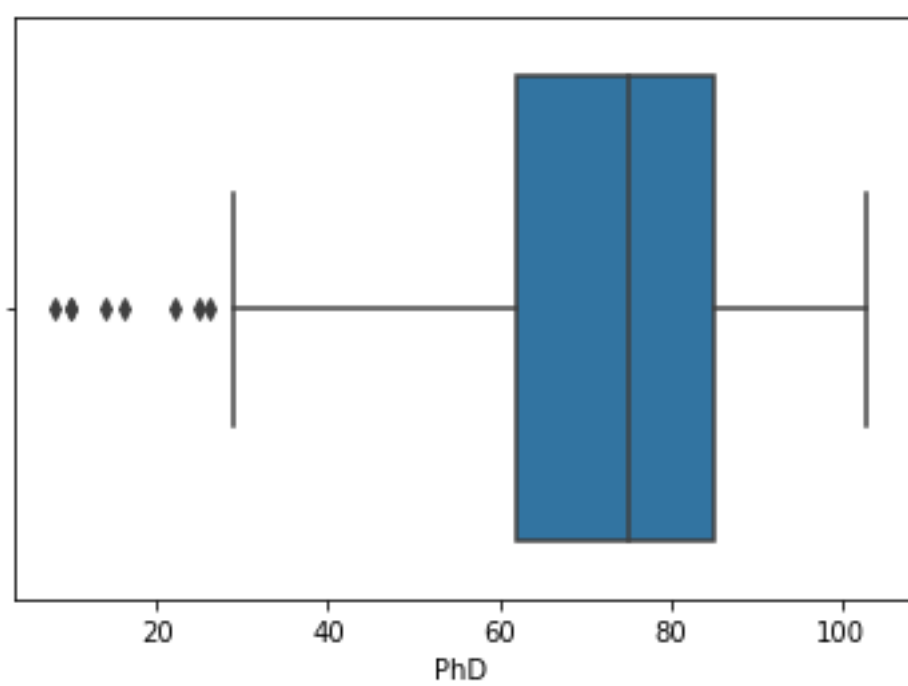
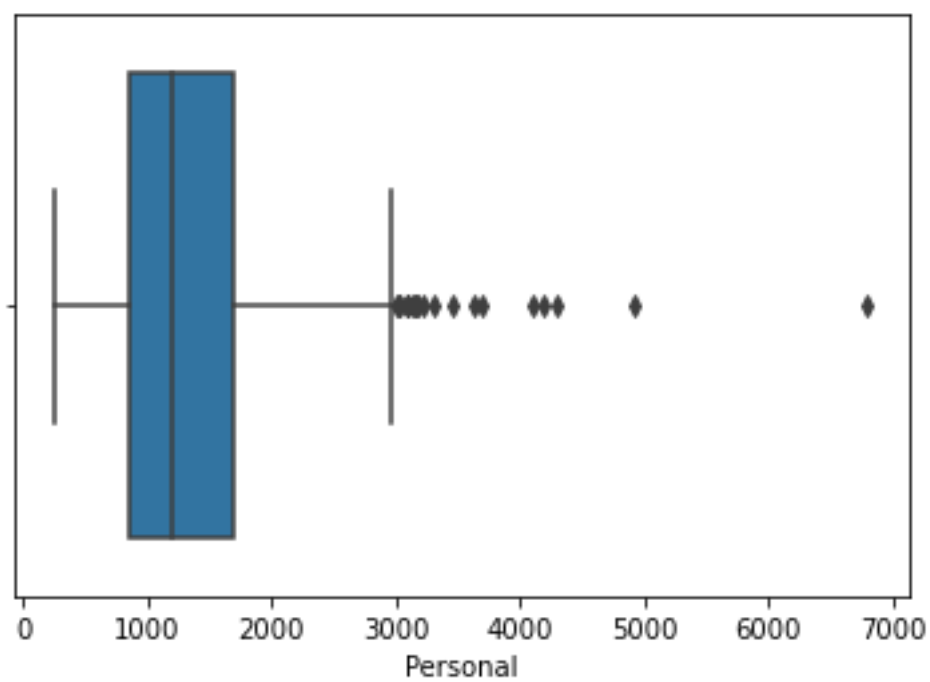


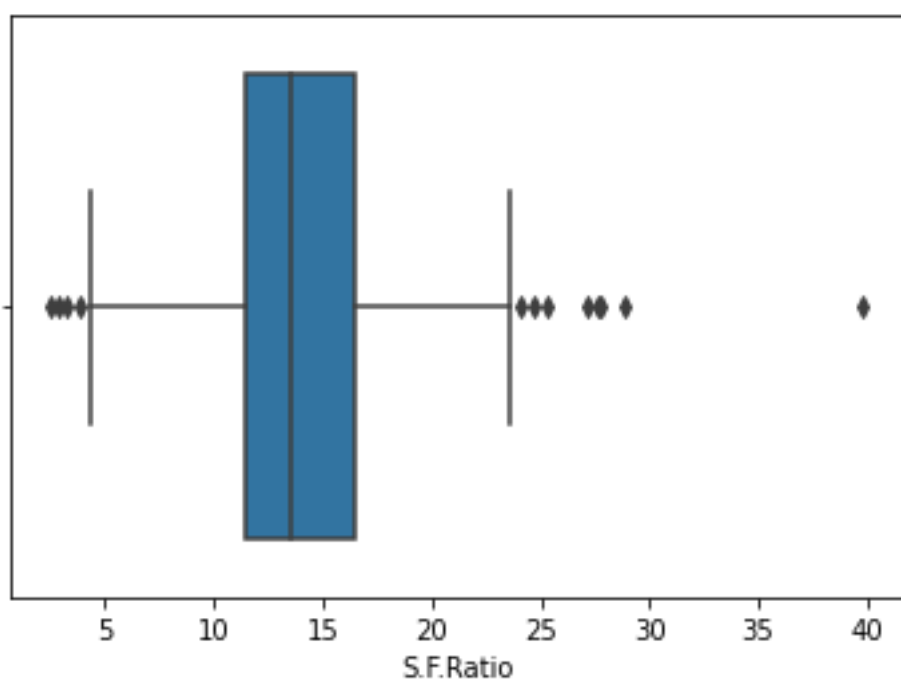
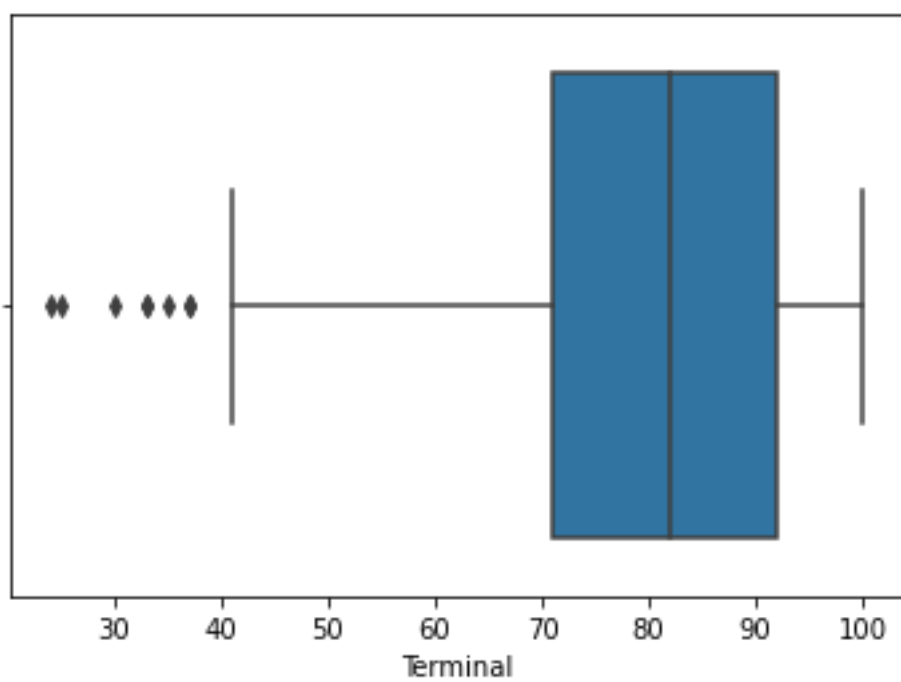


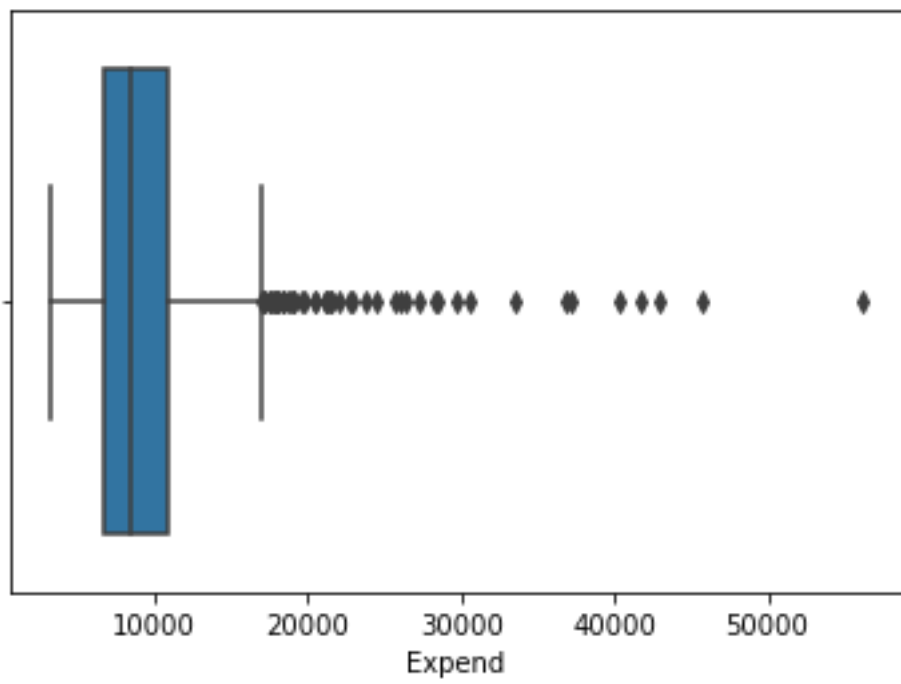
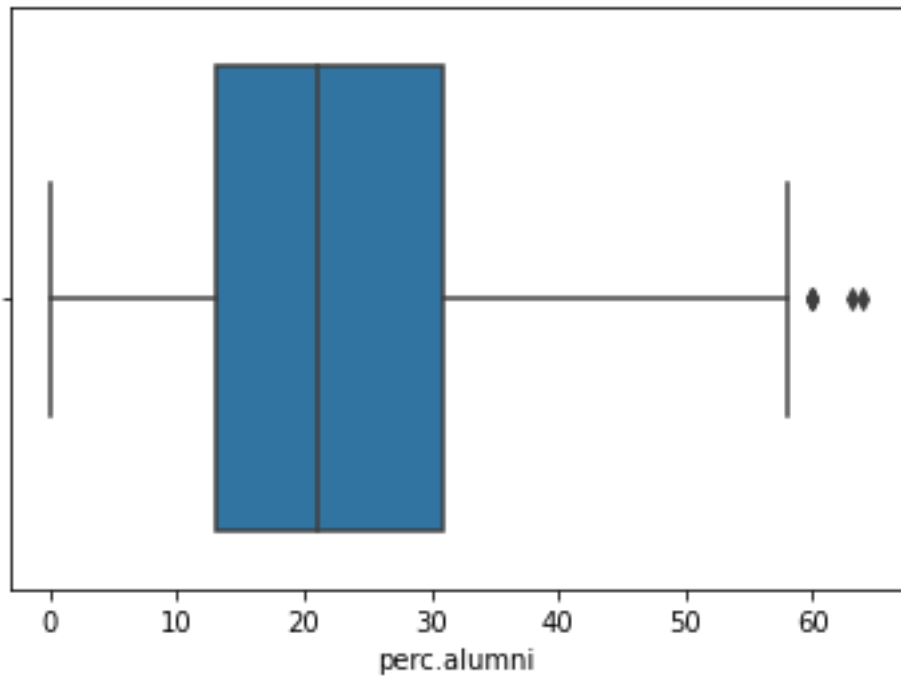


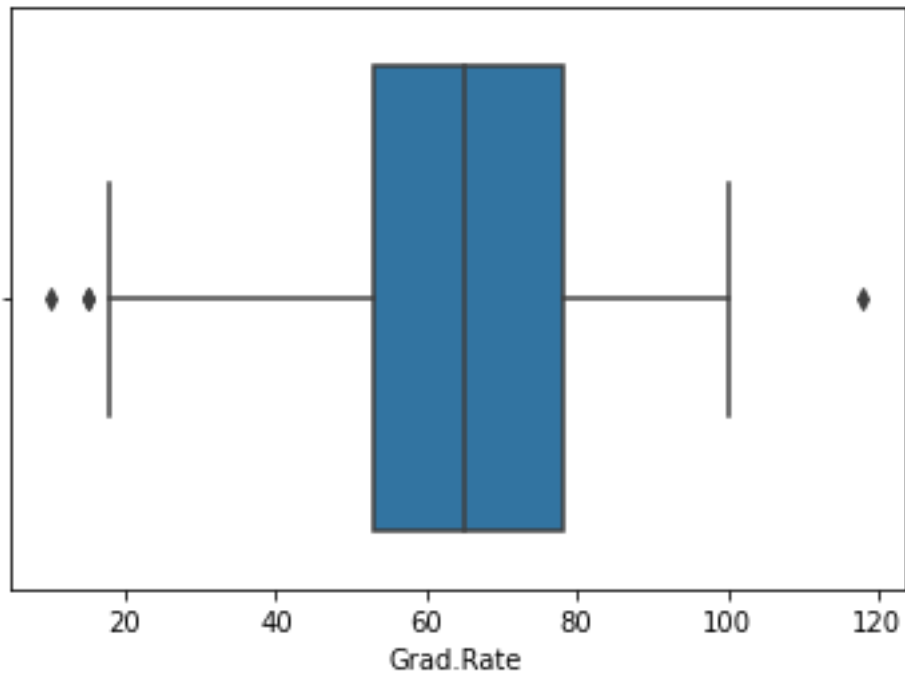




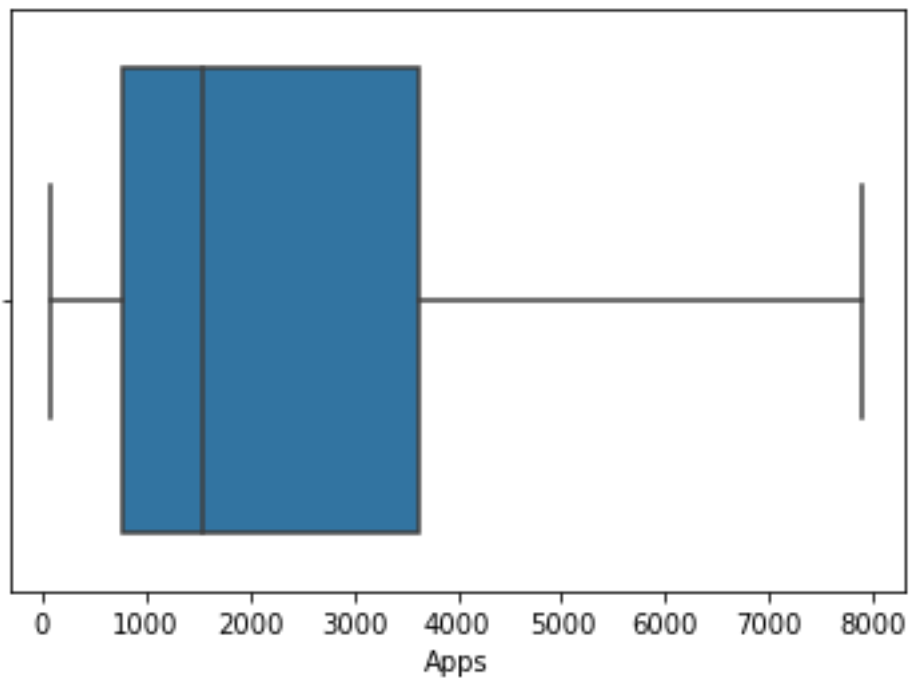


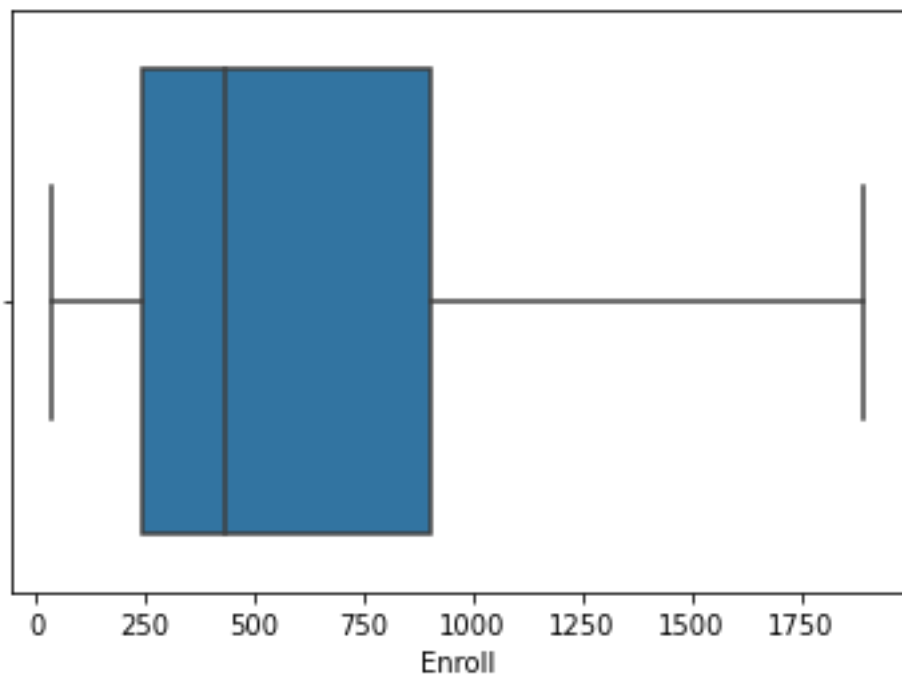
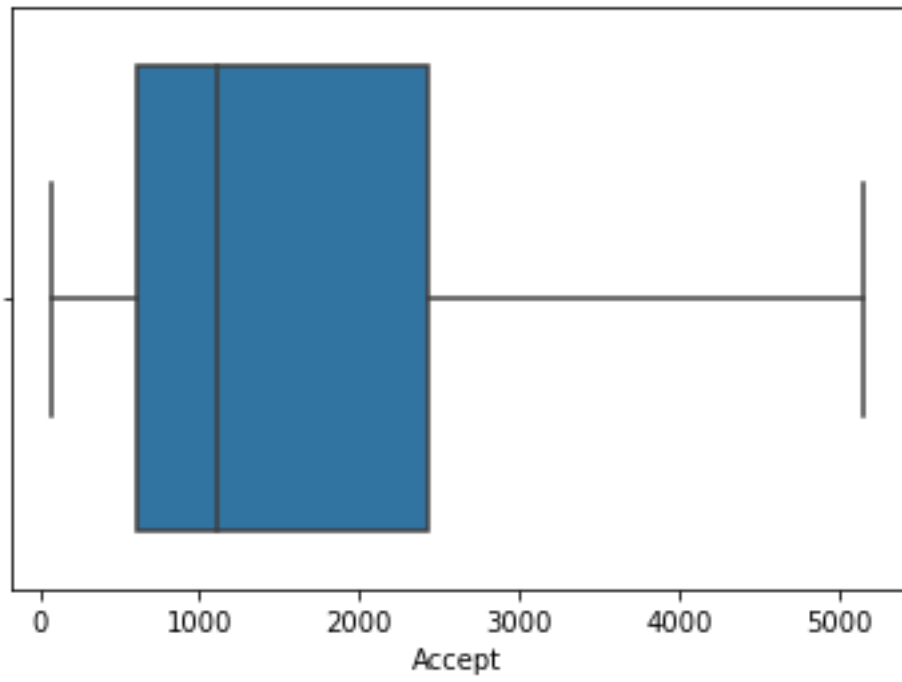


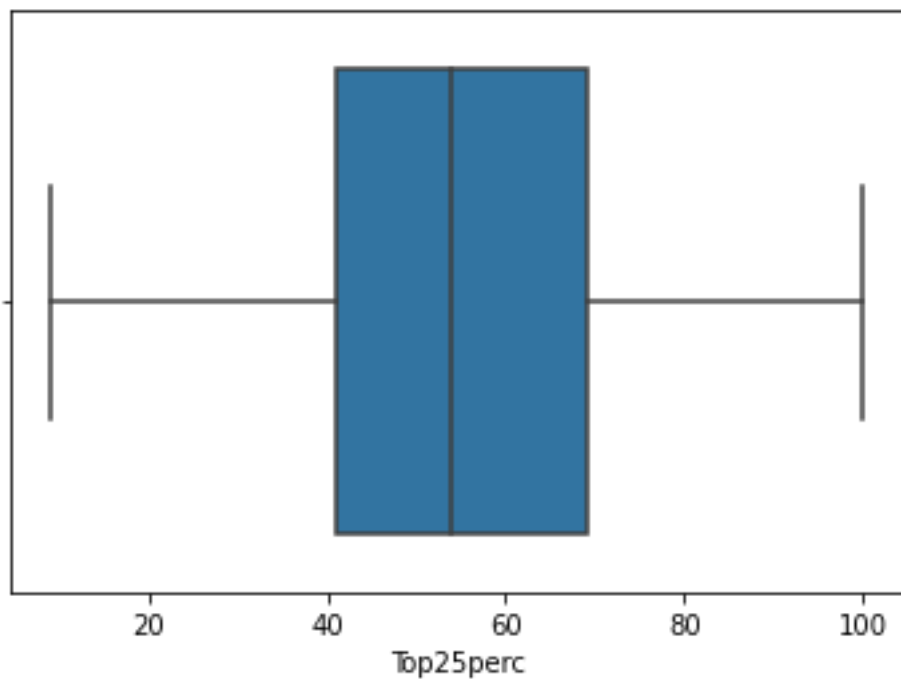
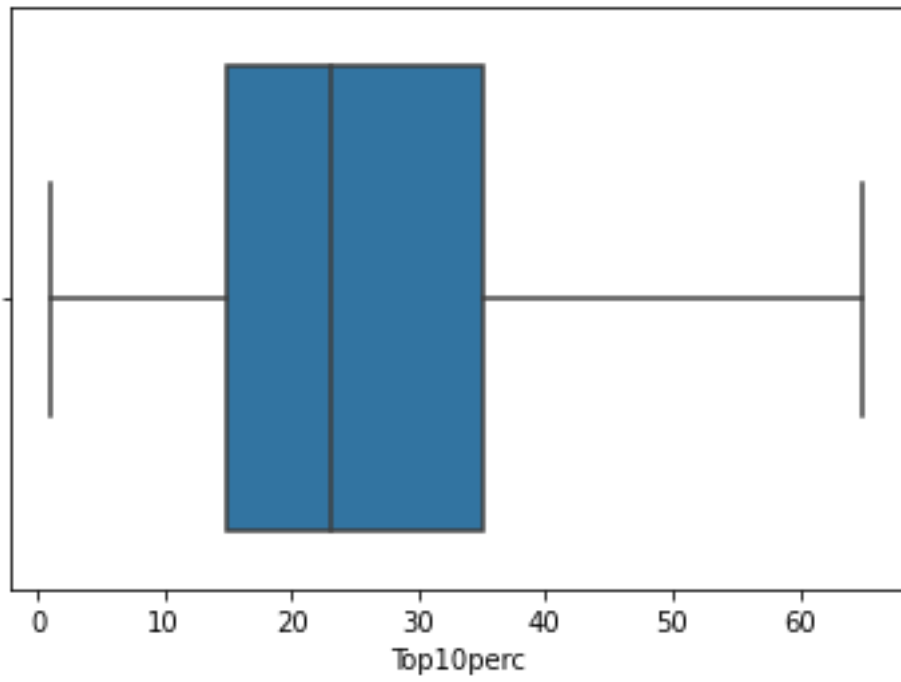




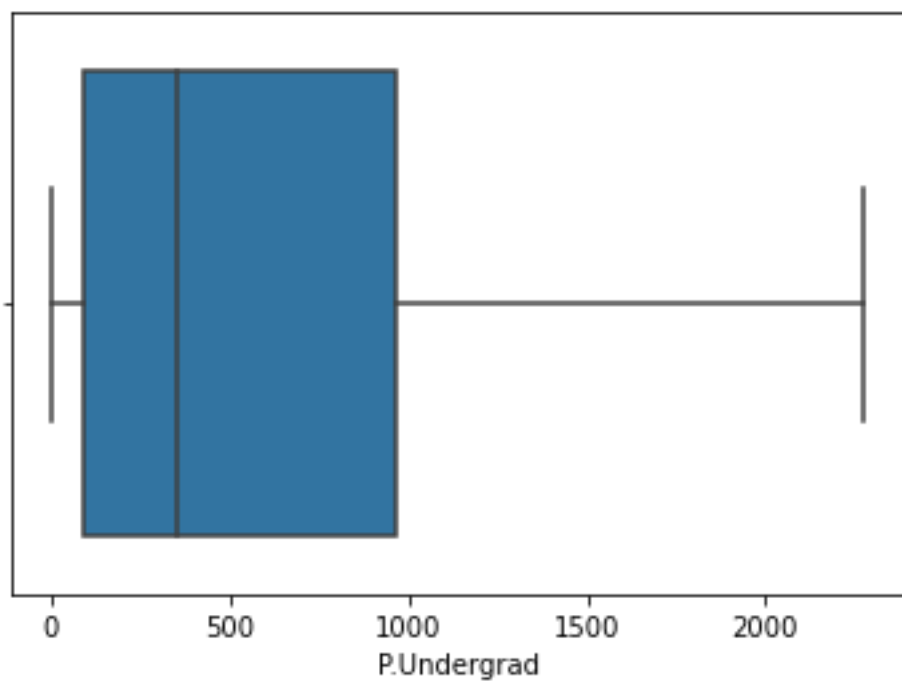
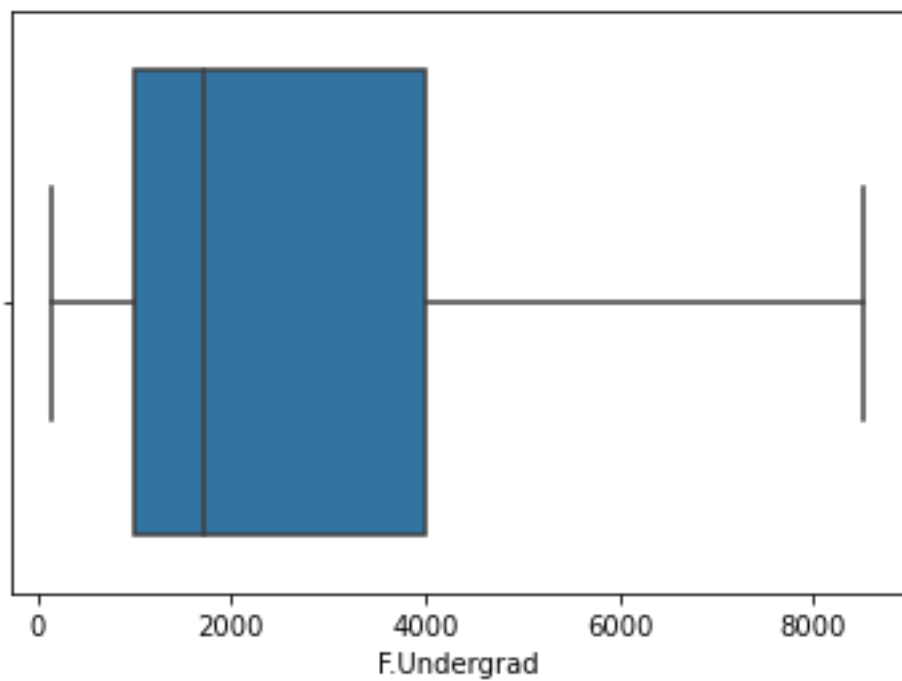
**Box-Plots after adjusting for Outliers:**

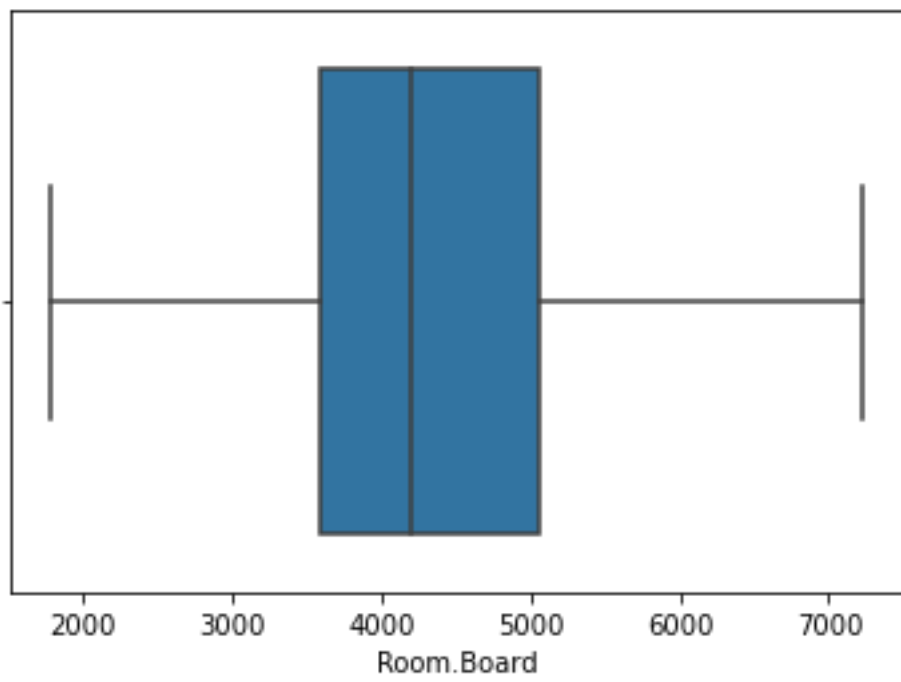
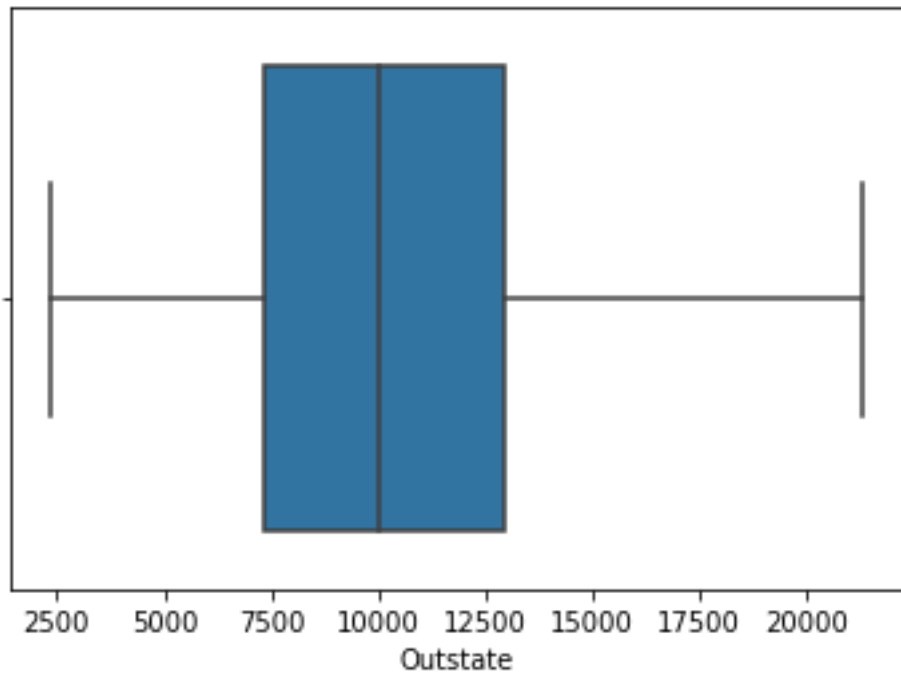


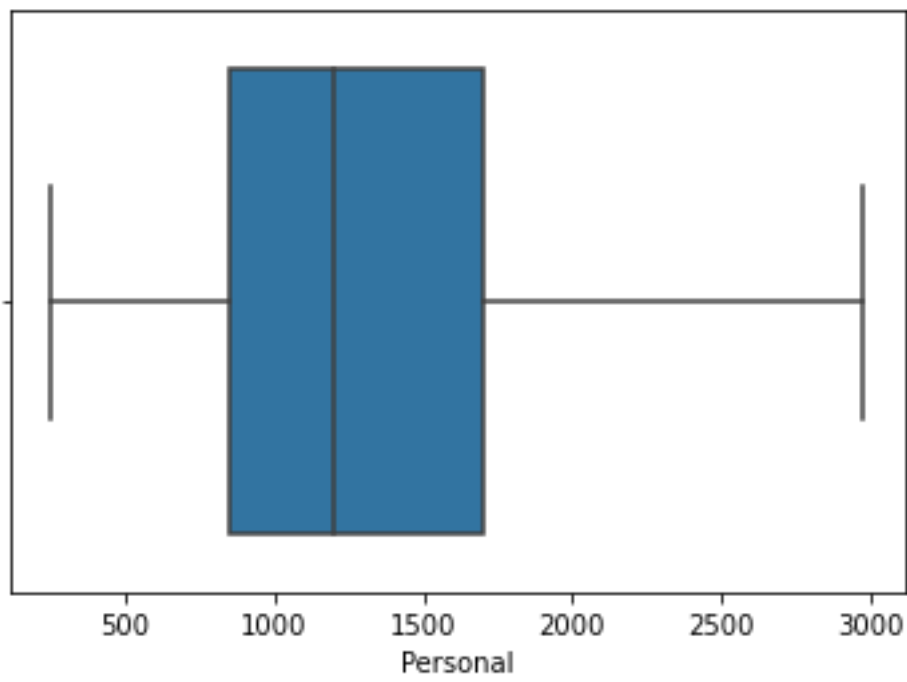
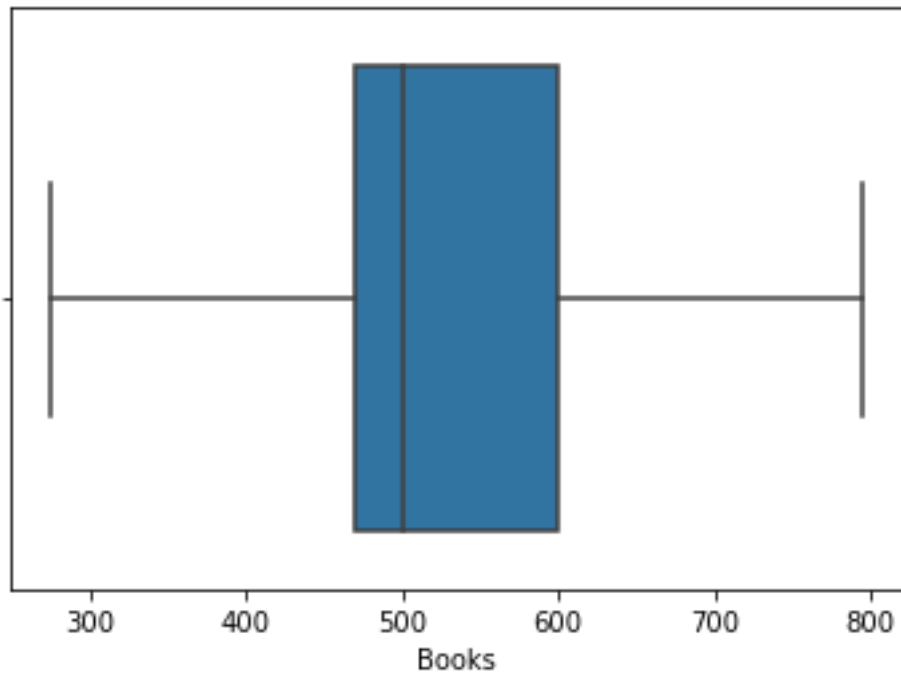


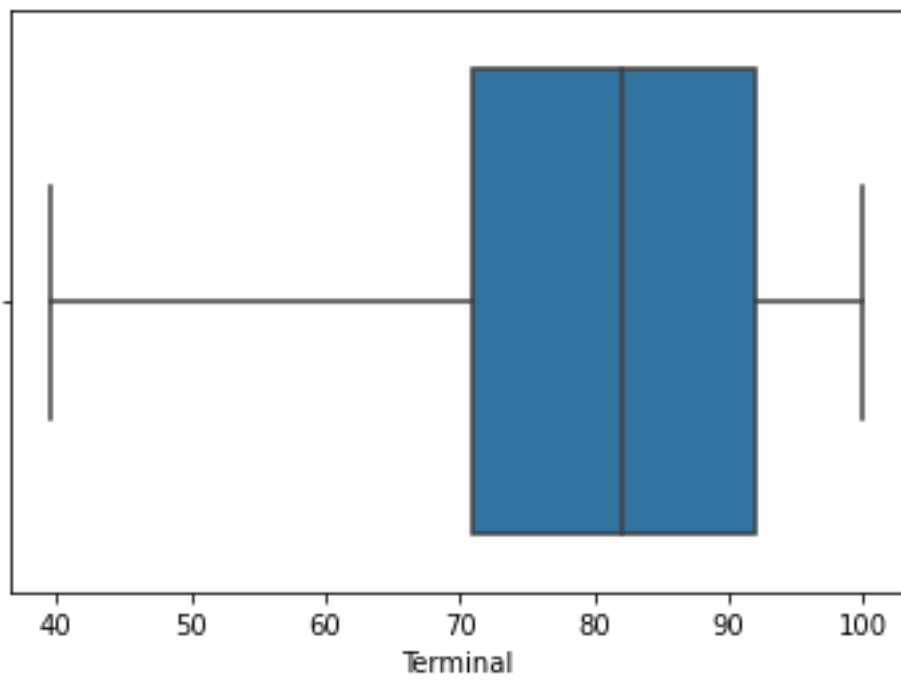
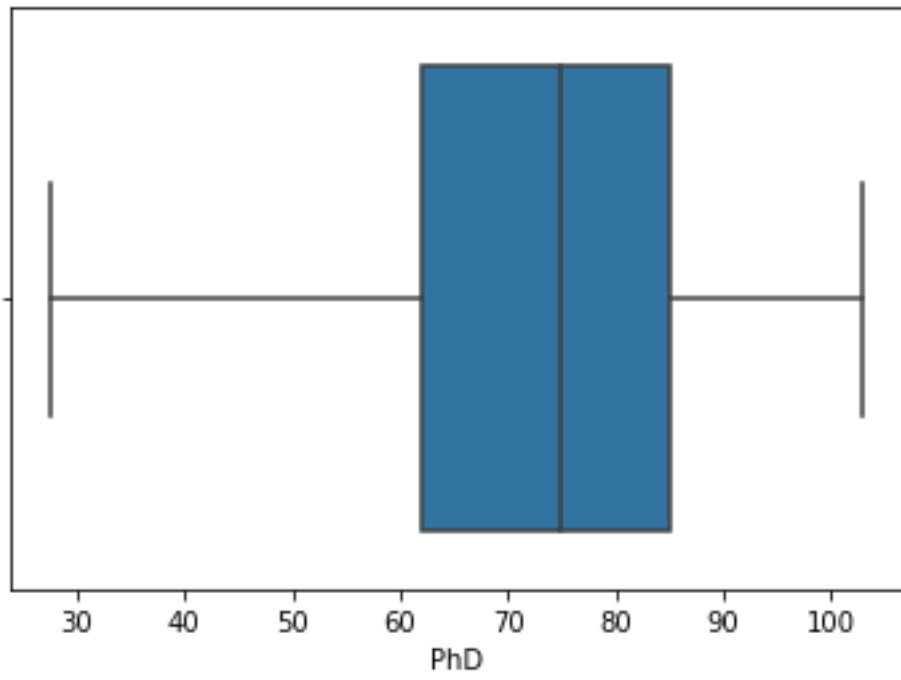


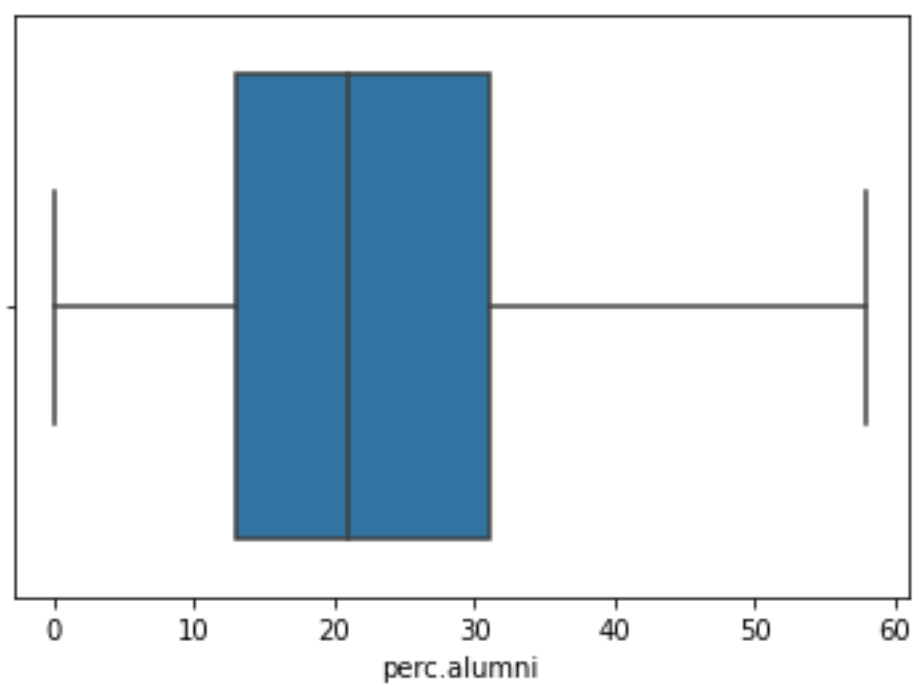
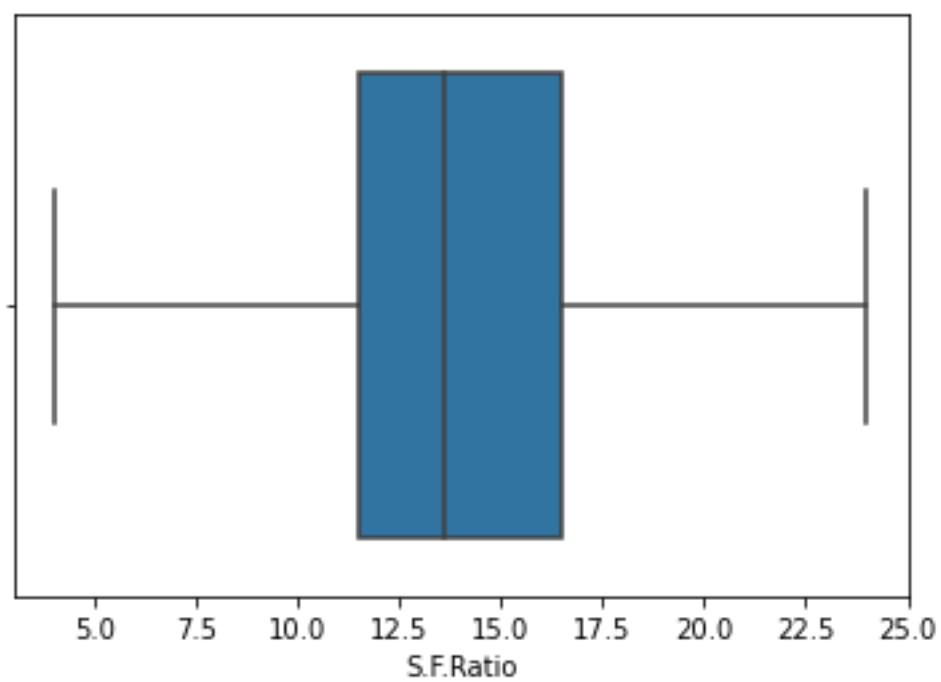


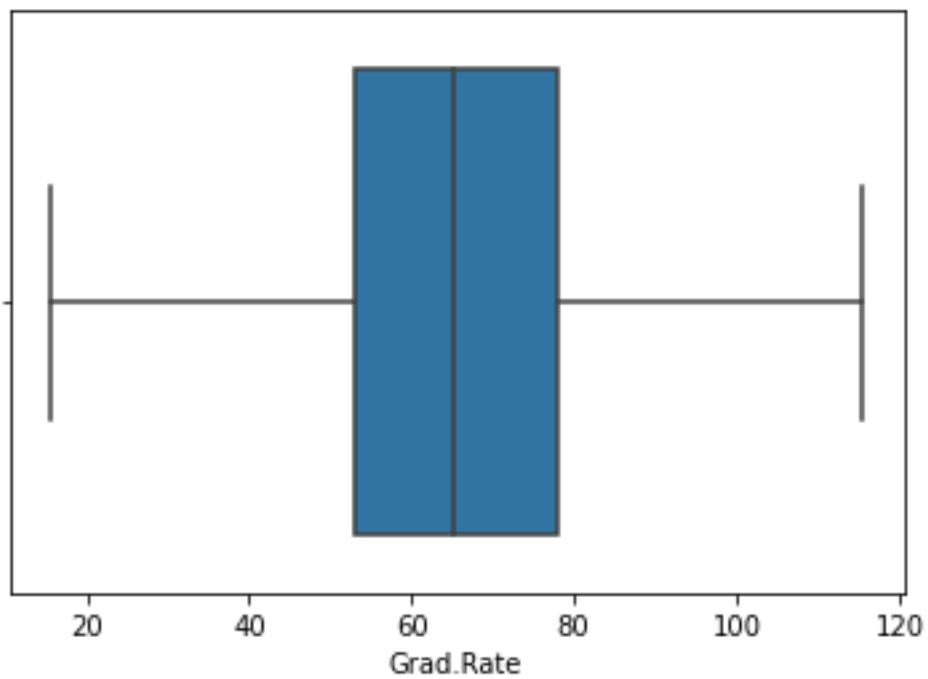
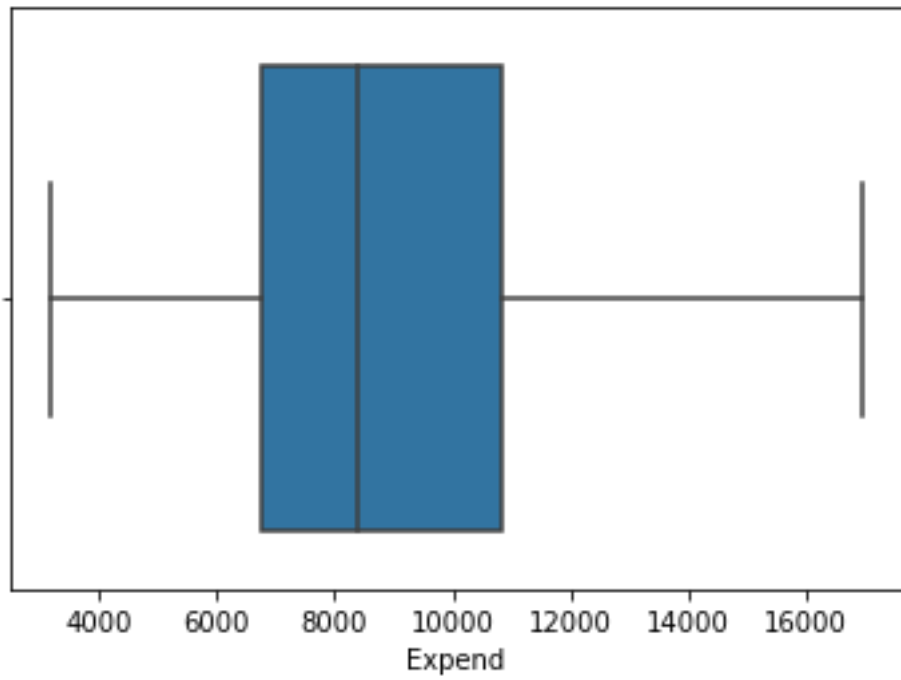












---

**2.5. EXTRACT THE EIGENVALUES AND EIGENVECTORS.[PRINT BOTH]**

## EIGEN VECTORS:

Eigen Vectors

```
%s [[ 3.17814172e-01  2.61253505e-01  1.25956560e-01 -2.02659037e-02
-2.13785276e-01  5.09494393e-03 -2.73447826e-03 -1.11637097e-01
-1.77030063e-01 -1.23575352e-01  1.84110996e-01 -5.99971829e-01
-4.87133853e-02 -7.06078585e-02  5.55352927e-01 -3.95889770e-03]
[ 2.92315664e-01  2.99321411e-01  1.60243348e-01 -2.79594866e-02
-1.78571638e-01  1.07523051e-02 -3.73351890e-02 -1.41061491e-01
-1.88987598e-01 -8.90716147e-02 -3.93425293e-01  6.60457280e-01
-1.23962993e-01 -1.96584922e-02  3.07570126e-01 -1.45901270e-02]
[ 2.58850257e-01  3.46898358e-01  1.13697029e-01  7.87065035e-02
-1.27108233e-01  4.67087824e-03 -4.00284817e-02 -1.36117130e-01
-7.73009395e-02 -4.41861897e-02  7.16937431e-01  2.39144525e-01
 3.36346969e-02  5.42114993e-02 -4.14379846e-01  4.82679874e-02]
[ 3.16599334e-01 -1.78582523e-01 -1.83242981e-01  3.48227421e-01
-1.04606454e-01 -8.00076650e-02  3.74997172e-01  8.98908162e-02
 1.09456040e-01 -3.67406193e-02 -5.68085940e-02  2.09753184e-02
 1.26218784e-02 -7.57220290e-02  2.32823380e-03  7.23940334e-01]
[ 3.20861634e-01 -1.40230929e-01 -1.77114455e-01  3.84014882e-01
-9.79152138e-02 -2.54501770e-02  3.60898100e-01  1.88530825e-01
 2.06643913e-01 -1.48418667e-01  2.01109558e-02  3.36076369e-02
-1.13022087e-01  1.14797372e-01  1.23989286e-03 -6.55974108e-01]
[ 2.36348432e-01  3.68172581e-01  9.99029714e-02  5.79713465e-02
-7.39896854e-02  2.65839910e-02 -1.92282699e-02 -8.98461954e-02
-7.39932829e-03 -1.16424466e-02 -5.40066143e-01 -3.67027764e-01
 1.09610419e-01  6.43306226e-02 -5.80695926e-01 -2.63077132e-02]
[ 8.88648643e-02  3.60184404e-01  1.18358177e-01 -1.89371030e-01
 8.30286874e-02 -1.94799589e-02  4.83987906e-02  1.45932136e-01
 7.69196536e-01  3.98309572e-01  2.95389969e-02  2.65058857e-02
-6.44171208e-02 -3.72659320e-02  1.50230289e-01  3.98989652e-02]
[ 2.31872820e-01 -3.16439306e-01  7.18312295e-02 -2.64880914e-01
-8.03766666e-02 -2.11569337e-01 -1.12364200e-02 -1.66920903e-01
-9.10828853e-02  2.87318733e-01  1.69583120e-03 -7.99534563e-02
-7.46526619e-01 -1.50028963e-02 -1.91827380e-01  5.46475817e-04]
[ 2.15986364e-01 -1.87127280e-01  1.00804509e-01 -6.50716853e-01
-7.81239720e-02 -1.41056084e-01  1.06989319e-01  2.19190118e-01
 1.70557528e-01 -5.73971064e-01  9.88332370e-03  2.72636076e-02
 1.66982702e-01 -2.75940596e-02 -1.02207716e-01  2.79891853e-02]
[ 1.12196803e-01  7.74326943e-02 -7.30623688e-01 -2.83581077e-01
-2.15785264e-01  5.47169431e-01 -9.99200107e-02 -4.58210055e-02
 1.09645487e-02  6.15042143e-02  4.68424624e-03  1.13474344e-02
-5.98171876e-02 -2.95201999e-02 -2.41212595e-02  7.78110527e-03]
[ 5.85034366e-03  2.54481517e-01 -5.07909665e-01  3.35832619e-02
 4.39203463e-02 -7.20502530e-01 -3.37202094e-01  1.83689918e-01
-4.73422462e-02 -5.80198275e-02 -1.15214494e-02  3.15110328e-03
-2.83149719e-02  1.14717763e-02  3.03300820e-02 -1.10547667e-03]
[ 3.34901874e-01 -2.41464824e-02 -1.38829609e-03  5.40643633e-02
 5.54359149e-01  1.28616688e-01 -1.43927243e-01  1.43032345e-01
-1.10215433e-01  2.62118958e-03  1.43925051e-02  1.51281130e-02
 8.85807340e-03 -7.05035211e-01 -5.01800528e-02 -8.39148988e-02]
[ 3.29129870e-01 -3.48931907e-02 -1.01396266e-02 -1.96939301e-02
 5.77514829e-01  1.58196495e-01 -1.70624077e-01  8.90283238e-02
-6.58000855e-02 -3.80456586e-02  7.88216419e-03 -1.70859686e-02
-4.54822637e-02  6.80612870e-01  1.00404528e-01  1.13412683e-01]
[ 1.37165124e-01 -3.06986928e-01  2.27098552e-02  2.36751710e-01
-1.00310808e-01  9.55228462e-03 -5.62948261e-01 -4.87032307e-01
 4.46868149e-01 -2.44147454e-01 -2.44696068e-02 -1.05464227e-03]
```

```

6.22141835e-02 -4.04534304e-02 2.54921083e-02 7.54978787e-03]
[ 2.90799895e-01 -2.22651235e-01 -1.00105123e-01 -2.11867513e-01
2.74998024e-02 -2.41500719e-01 1.78965859e-01 -3.70987912e-01
-1.13806162e-01 4.67225038e-01 3.55410679e-03 4.08808051e-02
5.69944557e-01 3.58966824e-02 5.66286984e-02 -1.41230995e-01]
[ 2.06628476e-01 -2.31197517e-01 1.97097008e-01 7.25989726e-02
-4.00800954e-01 9.24261073e-02 -4.35183246e-01 6.00960018e-01
-1.08169618e-01 3.06876947e-01 -2.97216391e-03 1.38133028e-02
1.77797701e-01 4.50178181e-02 -1.69575329e-02 3.63263587e-03]]

```

### **EIGEN VALUES:**

Eigen Values

```

%s [5.56822194 4.56403917 1.09258336 0.96486806 0.86410796 0.64817632
0.57464166 0.50191334 0.41905686 0.28891907 0.02242577 0.03802336
0.1644268 0.13465697 0.09986333 0.0746946 ]

```

---

## **2.6 PERFORM PCA AND EXPORT THE DATA OF THE PRINCIPAL COMPONENT (EIGENVECTORS) INTO A DATA FRAME WITH THE ORIGINAL FEATURES**

Apps	Accept Outstate perc.alumni	Enroll Room.Board Expend	Top10perc Books Grad.Rate	Top25perc Personal	F.Undergrad PhD	P.Undergrad Terminal
0	0.317814 0.088865 0.329130	0.292316 0.231873 0.137165	0.258850 0.215986 0.290800	0.316599 0.112197 0.206628	0.320862 0.005850	0.236348 0.334902
1	0.261254 0.360184 -0.034893	0.299321 -0.316439 -0.306987	0.346898 -0.187127 -0.222651	-0.178583 0.077433 -0.231198	-0.140231 0.254482	0.368173 -0.024146
2	-0.125957 -0.118358 0.010140	-0.160243 -0.071831 -0.022710	-0.113697 -0.100805 0.100105	0.183243 0.730624 -0.197097	0.177114 0.507910	-0.099903 0.001388
3	0.020266 0.189371 0.019694	0.027959 0.264881 -0.236752	-0.078707 0.650717 0.211868	-0.348227 0.283581 -0.072599	-0.384015 -0.033583	-0.057971 -0.054064
4	0.213785 -0.083029 -0.577515	0.178572 0.080377 0.100311	0.127108 0.078124 -0.027500	0.104606 0.215785 0.400801	0.097915 -0.043920	0.073990 -0.554359



---

**2.7 WRITE DOWN THE EXPLICIT FORM OF THE FIRST PC (IN TERMS OF THE EIGENVECTORS. USE VALUES WITH TWO PLACES OF DECIMALS ONLY).**

First PC:

```
[ [ 0.32  0.26  0.13 -0.02 -0.21  0.01 -0.   -0.11 -0.18 -0.12  0.18 -0.
6  -0.05 -0.07  0.56 -0.   ]
```

---

**2.8 CONSIDER THE CUMULATIVE VALUES OF THE EIGENVALUES. HOW DOES IT HELP YOU TO DECIDE ON THE OPTIMUM NUMBER OF PRINCIPAL COMPONENTS? WHAT DO THE EIGENVECTORS INDICATE?**

The table below shows cumulative values:

Eigen Values

```
%s [5.56822194 4.56403917 1.09258336 0.96486806 0.86410796 0.64817632
0.57464166 0.50191334 0.41905686 0.28891907 0.02242577 0.03802336
0.1644268 0.13465697 0.09986333 0.0746946 ]
```

```
Cumulative Variance Explained [ 34.75659769 63.24513048 70.06498798
76.08765221 81.48137628
85.52726451 89.11415261 92.24707374 94.86280831 96.66622848
97.69257335 98.53309624 99.15643876 99.6226792 99.86001933
100.   ]
```

It shows that the cumulative of first 8 PC will account for understanding PCA to the extent of ~92%. Hence we consider only 8 PCs to be fed for ML.

---

**2.9 EXPLAIN THE BUSINESS IMPLICATION OF USING THE PRINCIPAL COMPONENT ANALYSIS FOR THIS CASE STUDY. HOW MAY PCS HELP IN THE FURTHER ANALYSIS? [HINT: WRITE INTERPRETATIONS OF THE PRINCIPAL COMPONENTS OBTAINED]**

PCA is a widely used multivariate data analysis method. It is particularly useful for data with collinearity and more variables than samples. In this example through multivariate exploration we found correlation between Apps/Accept etc and again for Expend/Food etc..besides other fields.

Based on the original variables, PCA calculates a set of new variables that describes as much as possible of the variance in the data. The new 'variables' are named principal components (PCs). The PCs will be ranked according to how much of the original variance they explain: PC1 will explain the most variance, PC2 the second most and so on.

Calculation of PCs may be done with several methods. Here we use Eigen decomposition on covariance matrix. The number of PCs to include in a given case can be based on a criterion for the explained variance. This is calculated for each PCA. A criterion of >90% is normally the default used in the calculation software. Often only one or a few PCs are needed to sufficiently explain the variance in the data, simplifying significantly the evaluation.

#### REFERENCES:

<https://iwaponline.com/ws/article/19/8/2256/69018/Principal-component-analysis-for-decision-support>

<https://www.researchoptimus.com/article/what-is-anova.php#:~:text=ANOVA%20is%20used%20in%20a,the%20future%20performance%20of%20sales.>

