IMPERIAL

Modelling and Simulation Assessment:

The Dynamical System of Stranger Things

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December 2025

GitHub Code: https://github.com/saura-palace/Modelling-and-Simulation

1. Introduction and Context

The small town of Hawkins Indiana was, by all means an unremarkable, ordinary town. With a population of around three thousand, a dedicated chief of police, a disproportionately large number of poorly lit woods, and an unfortunately located federal research laboratory. It would have stayed unremarkable were it not for its repeated and increasingly inconvenient entanglement with a hostile parallel dimension known as The Upside Down.

The Hawkins Post-Event Reconstruction Authority (HPERA) has been tasked with constructing a dynamic model capable of explaining, reproducing and predicting the population-level interactions between humans, Flayed individuals (people whose minds and bodies are taken over by the Mind Flayer), Demogorgon's, and the Mind Flayer's environmental "field strength".

Before constructing a model, we have some historical observations of various behaviours:

- Healthy humans could become Flayed upon contact with infected individuals or environmental agents [1].
- Once Flayed individuals remained infectious until forcibly removed (via rescue, blunt force or containment).
- No evidence suggests Flayed individuals spontaneously recovered without intervention.
- Removed individuals did not re-enter the active population.
- The incident occurred between 1983 – 1986, a short enough timescale that natural births and non-Upside-Down-related deaths can be neglected.

We will begin by constructing a basic mathematical model with a human to flayed transmission process, which will act as a foundation for more complex interactions later such as: Demogorgon predication, Mind-Flayer influence or environmental forcing. By applying numerical and computational methods, the system will be analysed for key properties such as fixed points, stability, and overall complexity.

2. Assumptions and Model Definition

To create a model capable of reproducing the outbreak dynamics observed in Hawkins, multiple assumptions must be made. While the real events certainly involved increased complexity,

2.1 Model Assumption

1. Hawkins is treated as a single, well-mixed community in which individuals encounter each other randomly.
2. A healthy human becomes Flayed through direct or indirect contact with a Flayed person or an Upside-Down being. Mass-action kinetics is used to represent this, so new infections are proportional to $H(t)F(t)$.
3. Once Flayed, individuals do not return to the healthy state naturally. Transition out of the Flayed state occurs only through removal.

4. People who enter the removed state remain there. They do not contribute to any further spread.
5. Long-term demographic processes like births, ordinary deaths or people moving are omitted.

2.2 Variables and States

Based on the SIR model, the human population of Hawkins is divided into three time-dependent states [2]:

- $H(t)$: Healthy susceptible humans
- $F(t)$: Flayed humans
- $R(t)$: Removed humans

The total population is:

$$N(t) = H(t) + F(t) + R(t)$$

2.3 Parameters and Basic Model

The initial model is governed by two primary parameters:

- $\beta > 0$: **Transmission rate**, controls how quickly healthy humans become Flayed after exposure
- $\mu > 0$: **Removal rate**, the rate at which Flayed humans are neutralised through any means.

The simplest useful dynamical model that shows the observed behaviour is:

$$\frac{dH}{dt} = -\beta HF$$

$$\frac{dF}{dt} = \beta HF - \mu F$$

$$\frac{dR}{dt} = \mu F$$

The model can be shown by the diagram in figure 1.

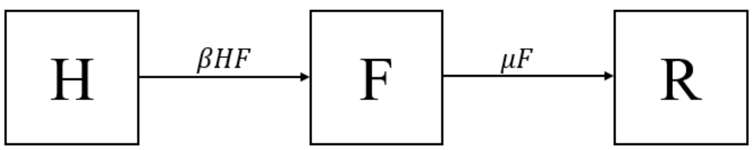

Figure 1 System diagram for the basic HFR model.

2.4 Non-Dimensionalisation

Non-dimensionalising the system will simplify the analysis and reduce the number of parameters. This can be done using a characteristic time scale, let H_0be the initial number of healthy humans:

$$t_0 = \frac{1}{\beta H_0}$$

Which represents the typical timescale of transmission at the start of the outbreak.

We can then introduce dimensionless variables:

$$h = \frac{H}{H_0}, f = \frac{F}{H_0}, r = \frac{R}{H_0}, \tau = \beta H_0 t.$$

Substituting into the original model gives the dimensionless system:

$$\frac{dh}{d\tau} = -hf$$

$$\frac{df}{d\tau} = hf - \delta f$$

$$\frac{dr}{d\tau} = \delta f$$

Where the single dimensionless parameter is:

$$\delta = \frac{\mu}{\beta H_0}$$

This reduces the model to one main control parameter. If $\delta < 1$, infection initially grows, if $\delta > 1$, removal dominates and the outbreak dies out. The variables h, f, r satisfy

$$h + f + r = 1$$

which simplifies numerical simulation and phase-space analysis.

2.5 Fixed Points of the Dimensionless System

With the non-dimensionalised model

$$\frac{dh}{d\tau} = -hf \qquad \frac{df}{d\tau} = hf - \delta f \qquad \frac{dr}{d\tau} = \delta f$$

(where $h + f + r = 1$), we can identify the equilibria by setting

$$h' = f' = r' = 0$$

From $h' = -hf = 0$, an equilibrium requires either

$$h = 0 \quad \text{or} \quad f = 0$$

From $f' = f(h - \delta) = 0$, an equilibrium requires either

$$f = 0 \quad or \quad h = \delta$$

Equilibrium 1: Disease-free state

If $f = 0$, then $h + r = 1$. Any point of the form

$$E_1 = (h^*, f^*, r^*) = (h^*, 0, 1 - h^*)$$

is an equilibrium.

This means that no Flayed individuals remain, and the human + removed population is static!

The most important case is the fully healthy state $h^* = 1,\ f^* = 0,\ r^* = 0$.

Equilibrium 2: Fully removed / infected state

If $h = 0$, then from conservation $f + r = 1$.

This yields a continuum of equilibria

$$E_2 = (0, f^*, 1 - f^*$$

representing outcomes where no healthy humans survive.

No coexistence equilibrium

For a non-zero equilibrium of the form (h^*, f^*, r^*) with both $h^* > 0$ and $f^* > 0$, the system would require

$$h^* = \delta$$

But this would mean

$$h' = -h^* f^* = -\delta f^* \neq 0$$

Unless $f^* = 0$.

So the system cannot sustain coexistence of healthy and Flayed individuals, exactly what we expected given the outcome in Hawkins.

2.6 Stability Analysis

To analyse the stability, we linearise the system around an equilibrium.

Jacobian Matrix

For the two-dimensional subsystem (since r is dependant), define

$$x = \begin{pmatrix} h \\ \overline{f} \end{pmatrix}$$

The vector field is

$$h' = -hf,$$

$$f' = hf - \delta f$$

The Jacobian is:

$$J = \begin{pmatrix} \frac{\partial h'}{\partial h} & \frac{\partial h'}{\partial f} \\ \frac{\partial f'}{\partial h} & \frac{\partial f'}{\partial f} \end{pmatrix} = \begin{pmatrix} -f & -h \\ f & h - \delta \end{pmatrix}$$

Stability of the disease-free equilibrium

At

$$E_1 = (h^*, f^*) = (1,0),$$

the Jacobian becomes

$$J(E_1) = \begin{pmatrix} 0 & -1 \\ 0 & 1 - \delta \end{pmatrix}$$

If $\delta > 1$, then $\lambda_2 < 0$: the Flayed population decays and the disease-free state is stable.

If $\delta < 1$, then $\lambda_2 > 0$: any introduction of a Flayed individual leads to exponential initial growth and the disease-free state is unstable.

This matches the intuitive threshold condition

$$R_0 = \frac{1}{\delta} > 1 \text{ (infection grows)}$$

Stability of the "no human survivors" equilibrium

At

$$E_2 = (0, f^*),$$

The Jacobian becomes

$$J(E_2) = \begin{pmatrix} -f^* & -1 \\ f^* & -\delta \end{pmatrix}$$

Its eigenvalues are:

$$\lambda_2 = -f^* < 0, \qquad \lambda_2 = -\delta < 0,$$

Therefore E_2 is always stable after humans vanish, the system cannot spontaneously regenerate them.

This explains the numerical collapse to zero healthy humans in many simulations.

3 Numerical Demonstration of the Basic Model

To verify the behaviour of the HFR system, the dimensionless model was simulated numerically. The results align closely with the mathematical predictions: rapid infection growth when $\delta < 1$, collapse of the healthy population and eventual stabilisation into a removal-dominated equilibrium.

3.1 Basic Model: Time Evolution

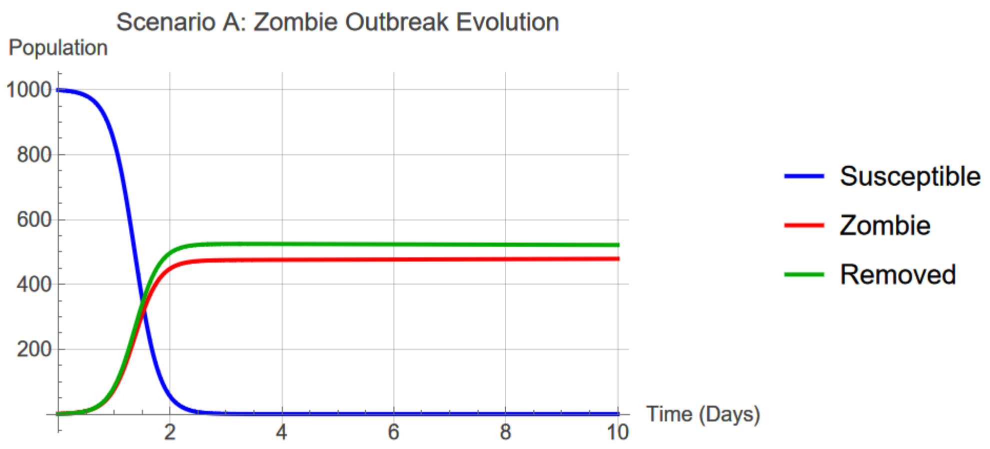

Figure 2 Population time graph of the zombie outbreak evolution.

This simulation demonstrates the characteristic behaviour of the dimensionless system when $\delta < 1$. Initially, the large susceptible population results in a strong infection term hf, causing the Flayed population to grow rapidly. As h approaches zero, transmission slows and the removal term δf becomes dominant, leading to decay in f. The system settles into a state where:

- $\boldsymbol{h \to 0}$ (no healthy individuals remain),
- $\boldsymbol{f}$ stabilises at a small value,
- $\boldsymbol{r}$ increases to approximately the total population.

This matches the analytical result that the only stable equilibrium for $\delta < 1$ is the human-extinction state, consistent with eigenvalue analysis showing the disease-free equilibrium is unstable.

3.2 Basic Model: Phase Space Trajectory

This trajectory visualises the deterministic flow of the model. The monotonic decline in $H$and non-monotonic rise and fall pattern in Freflect the underlying equations:

- $h' = -hf$ ensures healthy individuals decrease whenever Flayed are present.
- $f' = hf - \delta f$initially increases (while $h$is large), then decreases once $h < \delta$.

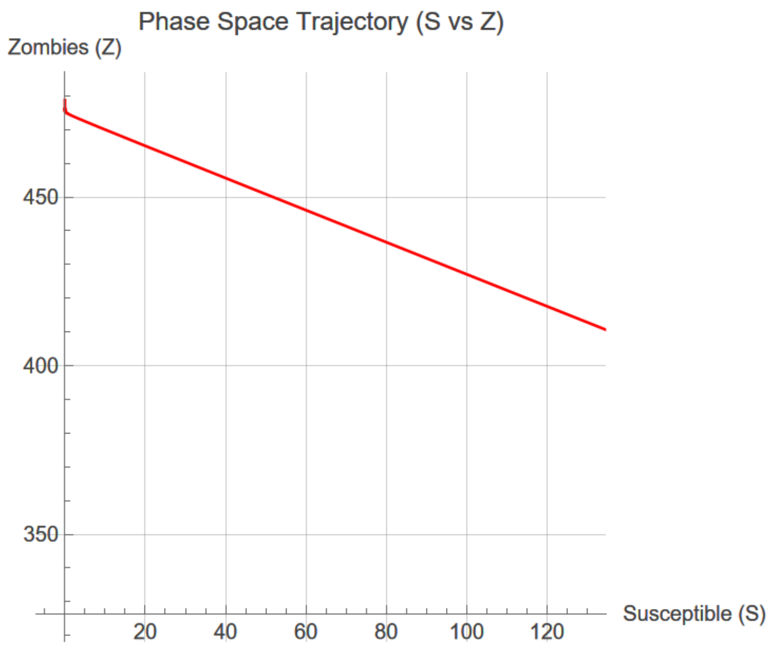

Figure 3 Phase space trajectory for zombie's vs susceptible.

The trajectory's straight, downward-sloping form indicates:

- No oscillations,
- No predator–prey cycles,
- No spiral behaviour or limit cycles.

The system moves directly toward the absorbing boundary $h = 0$, confirming the stability analysis that the Flayed-dominated equilibrium is globally attractive under these parameters.

4. Iterations: Mind Flayer Coming!

Now, the Mind Flayers have arrived. They entered this town through a concealed portal, though the portal cannot be sustained indefinitely and can only transport one Mind Flayer at a time. According to records from the 1985 incident, Mind Flayers transform humans into Flayed Humans. These beings remain under the Mind Flayers' control and cannot revert to their original human form. Some basic settings of the scenario don't change, like no births and natural deaths during the simulation.

4.1 Updated Assumptions and Parameters

- Closed population dynamics: the simulation treats Hawkins as a closed system with a continuous critical event window. We consider migration to be absent (S' = 0 from external sources) and neglect natural births and deaths given that scalping and assimilation happen much more rapidly than demographic processes.
- Irreversibility of the skinned state: while previous events (e.g., the 1984 Will Byers event) demonstrated the possibility of exorcism through extreme heat, the 1985 strain involved severe physiological alterations, including ingestion of corrosive chemicals. We hence modelled the conversion from susceptible to flayed condition as virtually permanent for the population at large without professional support.
- Uniform mixing: We presuppose that the population is thoroughly blended and that every susceptible person has some chance of meeting a Flayed person or a Mind Flayer. While spatial dynamics (diffusion models) provide higher fidelity, the ordinary differential equation approach provides sufficient insight into the global stability and bifurcation properties of the system.
- The Possession ($S \rightarrow F$): Infection is not passive. It requires direct interaction with the Mind Flayer entity.
- The Flayed Human (F): These entities act as "terminators." In this specific model configuration, they do not reproduce; they destroy. Their function is to reduce the human population (S) to eliminate the resistance.
- The Mind Flayer (M): The Mind Flayer is a proxy form. It possesses an intrinsic ability to regenerate (ρ) as long as the portal remains open, but it is vulnerable to high-intensity thermal damage from the human resistance (α_M).

4.2 Model Definition

On the updated model, some parameters would not change, but the value of them will be different. The updated frame of ODEs are shown below,

$$\frac{dS}{dt} = -\beta SF$$

$$\frac{dF}{dt} = \beta SF - \mu F$$

$$\frac{dR}{dt} = \gamma F$$

The S here is the human population. When we introduce the Mind Flayer to our system, a predator-prey dynamics system form too.

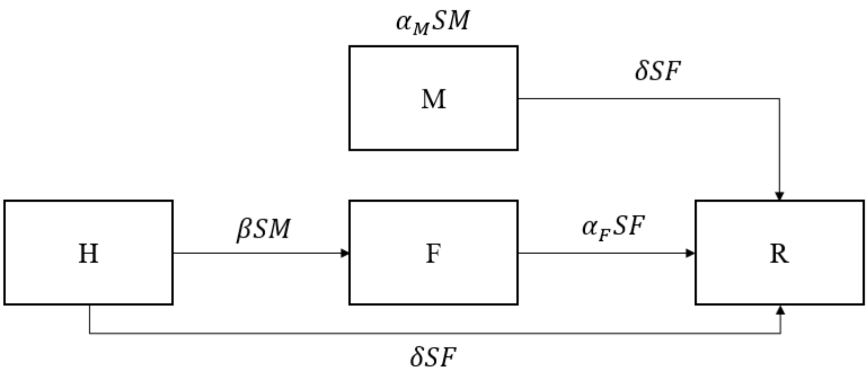

Figure 4 Updated system diagram

4.3 Numerical methods and computational tools

In order to analyse and solve this four-dimensional nonlinear system, we used numerical integration methods. Specifically, the analysis utilises the function within the Wolfram Mathematica environment (NDSolve). The code structure analyses the sensitivity of the system to initial conditions and parameter variations. We calculate the Jacobian matrix to perform linear stability analysis around equilibrium points, determining the eigenvalues to classify the stability of the system's fixed points.

4.4 SFMR Model Derivation

The full equations of four group's population change are shown below.

$$S' = \frac{dS}{dt} = -\beta_F SF - \beta_M SM$$

$$F' = \frac{dF}{dt} = \beta_M SM - \alpha_F SF$$

$$M' = \frac{dM}{dt} = \rho M - \alpha_M SM$$

$$R' = \frac{dR}{dt} = \delta SF + \alpha_F SF + \alpha_M SM$$

Table 1 SFMR parameters

Parameter	Symbol	Physical Meaning
Base Infection Rate	β	The rate at which the Mind Flayer (M) successfully captures and enslaves Humans (S).
Flayed Lethality	δ	The combat effectiveness of the Flayed. The rate at which they kill Humans (S).
Human vs. Flayed	α_F	The human kill rate against Flayed minions. Represents conventional weaponry (guns, axes).

Human vs. Mind Flayer	α_M	The human effectiveness against the main entity. Represents specialised attacks (fireworks, heat).
Mind Flayer Regen	ρ	The intrinsic growth rate of the Mind Flayer population in the absence of resistance.

Table 1. The explaination of parameters

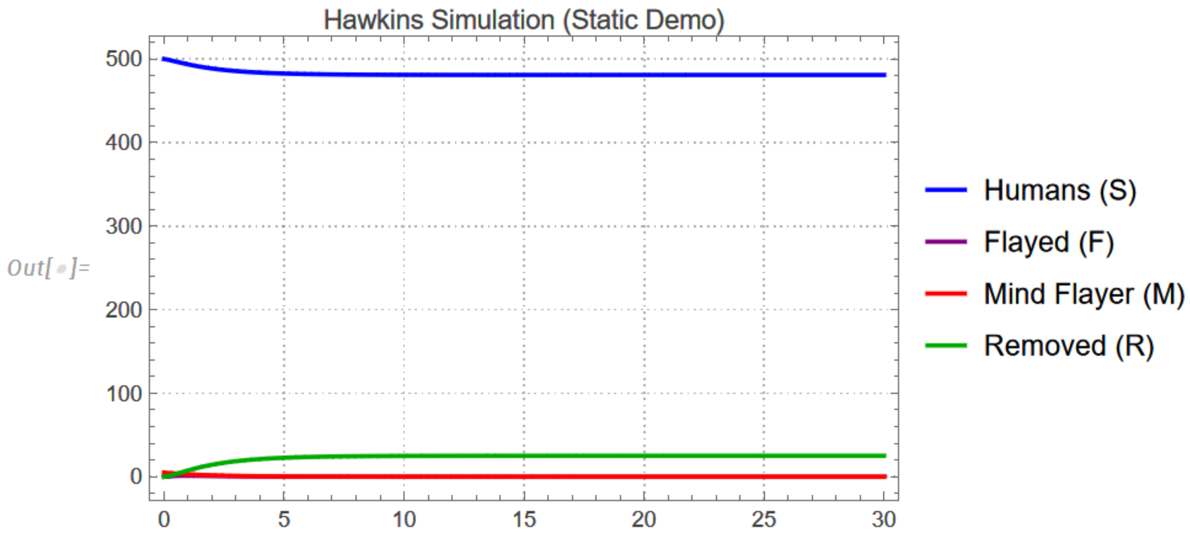

Figure 5 The static demonstration of scenario under β=0.002, δ=0.005, $\alpha_F = 0.005$, $\alpha_M = 0.00125$ and ρ = 0.1.

The initial condition here is 500 human and 5 mind flayers.

4.5 Non-Dimensionalisation Analysis

To identify the fundamental governing relationships, we non-dimensionalise the system.

Scaling Variables:

We scale populations by the initial human population S_0 and time by the Mind Flayer's regeneration rate $\frac{1}{\rho}$.

- Dimensionless Populations: $s = \frac{S}{S_0}, f = \frac{F}{S_0}, m = \frac{M}{S_0}$.
- Dimensionless Time: $\tau = t \cdot \rho$.

Derivation:

Substituting into the S' equation:

$$\frac{d(sS_0)}{d\tau}\rho = -\beta\,(sS_0)(mS_0) - \delta\,(sS_0)(fS_0)$$

$$\frac{ds}{d\tau} = -\left(\frac{\beta S_0}{\rho}\right)sm - \left(\frac{\delta S_0}{\rho}\right)sf$$

This yields the Dimensionless SFMR System:

$$s' = -R_0^{inf}\, sm \; - \; D_L sf$$

$$f' = R_0^{inf} sm \; - \; A_F sf$$

$$m' \; = \; m\,(1 \; - \; A_M s)$$

Dimensionless Groups:

1. $R_0^{inf} \; = \frac{\beta S_0}{\rho}$: The relative speed of infection vs. regeneration.

2. $A_M \; = \frac{\alpha_M S_0}{\rho}$ (The Survival Number): This is the most critical parameter. It is the ratio of Human Attack Power ($\alpha_M\, S_0$) to the Monster's Regeneration (ρ). If $A_M \; > \; 1$, humans can overpower the monster initially.

4.6 Equilibrium and Stability Analysis

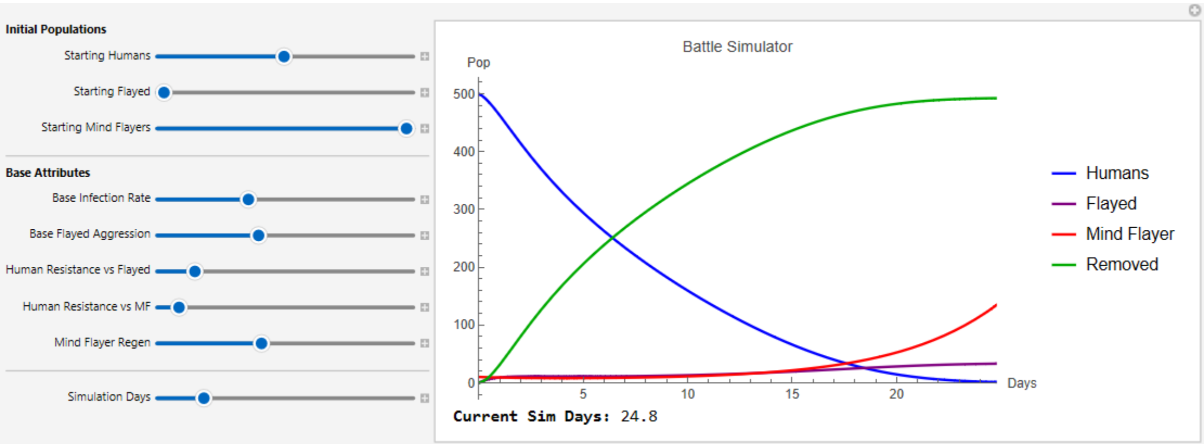

Figure 6 The adjustable combat simulator

Fixed Points

We solve for steady states where $s' \; = \; f' \; = \; m' \; = \; 0$.

From $m' \; = \; m(1 \; - \; A_M s)$:

This equation provides the system's two primary regimes:

1. The Extinction State ($\boldsymbol{m} \neq \boldsymbol{0}$): Requires $s \; = \frac{1}{A_M}$. However, substitution into $s'$and f'often leads to contradictions (negative populations) unless parameters are finely tuned. This suggests co-existence is unlikely.

2. The Survival State ($\boldsymbol{m} \; = \boldsymbol{0}$): If the Mind Flayer is destroyed ($m = 0$), then $f' \; = -A_F sf$. The Flayed population decays to 0.

   - Fixed Point: $E_{survival} \; = \; (s^*, 0, 0)$.
   - This represents a scenario where the entity is banished and a remnant human population s^* survives.

Jacobian Matrix and Stability

To assess the stability of the Survival State, we compute the Jacobian Matrix J:

$$J = \begin{bmatrix} \frac{\partial s'}{\partial s} & \frac{\partial s'}{\partial f} & \frac{\partial s'}{\partial m} \\ \frac{\partial f'}{\partial s} & \frac{\partial f'}{\partial f} & \frac{\partial f'}{\partial m} \\ \frac{\partial m'}{\partial s} & \frac{\partial m'}{\partial f} & \frac{\partial m'}{\partial m} \end{bmatrix} = \begin{bmatrix} -\beta M - \delta F & -\delta S & -\beta S \\ \beta M - \alpha_M F & -\alpha_F S & \beta S \\ -\alpha_M M & 0 & \rho - \alpha_M S \end{bmatrix}$$

Evaluating J at the Survival Equilibrium $E_{survival}$ $(S^*, 0, 0)$:

$$J|E_{survival} = \begin{bmatrix} 0 & -\delta S^* & -\beta S^* \\ 0 & -\alpha_F S^* & \beta S^* \\ 0 & 0 & \rho - \alpha_M S^* \end{bmatrix}$$

Eigenvalue Analysis:

The eigenvalues are the diagonal elements of the block matrix:

1. $\lambda_1 = 0$: Neutral stability along the S-axis.
2. $\lambda_2 = -\alpha_F S^*$: Always negative (Stable). The Flayed die out without the Mind Flayer.
3. $\lambda_3 = \rho - \alpha_M S^*$: The Critical Eigenvalue.

Stability Condition:

For the Survival State to be stable (preventing the Mind Flayer's return), we require $\lambda_3 < 0$.

$$\rho - \alpha_M S^* < 0 \rightarrow S^* > \frac{\rho}{\alpha_M}$$

5. Bifurcation Analysis

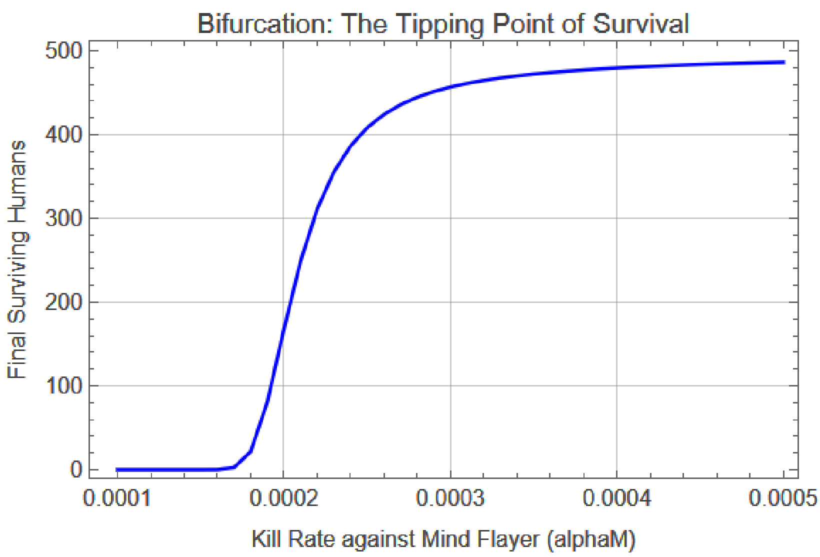

Figure 7 Bifurcation diagram.

The derived stability condition $S > \frac{\rho}{\alpha_M}$ identifies a Transcritical Bifurcation. Using the parameters from the code ($\rho = 0.1, varying\ \alpha_M$):

- The Tipping Point: occurs when $\alpha_M = \frac{\rho}{S_{initial}}$.
  - When $S_0 = 500$, the tipping point is $\alpha_M = 0.1/500 = 0.0002$.
- Interpretation of Diagram:
  - Left of 0.0002 ($S_0 < S_c$): The Final Human Population is 0. The system collapses because humans lack the firepower to overcome regeneration.
  - Right of 0.0002 ($S_0 > S_c$): The Final Human Population jumps to ≈ 500. The humans suppress the monster immediately.
  - This mathematical result perfectly matches the inflection point observed in your provided Bifurcation Diagram.

6. Phase Space Analysis

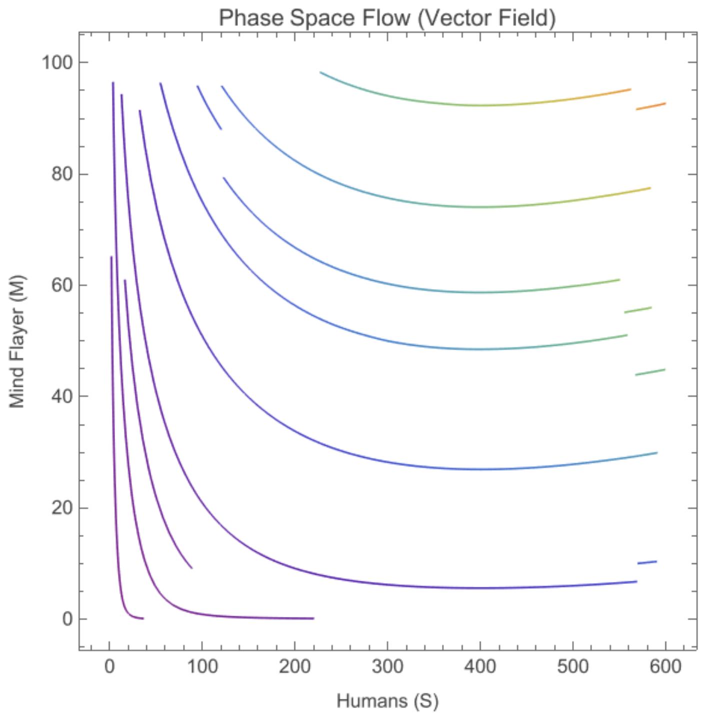

Figure 8 Phase space flow diagram of updated model.

We analyse the (S, M) plane. The flow is dominated by the M' equation:

$$\frac{dM}{dt} = M(\rho - \alpha_M S)$$

Nullcline: The vertical line $S = \frac{\rho}{\alpha_M}$.

Vector Field:

Region $S < \frac{\rho}{\alpha_M}, \frac{dM}{dt} > 0$. Vectors point North (Mind Flayer grows).

Region $S > \frac{\rho}{\alpha_M}, \frac{dM}{dt} < 0$. Vectors point South (Mind Flayer shrinks).

Trajectory: Since S is always decreasing (due to attrition $-\beta SM - \delta SF$), a trajectory starting in the stable region (South) moves left. If it crosses the Nullcline before M hits 0, the vector field flips to North, and the Mind Flayer resurges.

Visual Confirmation: This explains the curves in figure 7, where trajectories dive towards $M = 0$ but can curl back up if S gets too low.

7. Sensitivity Analysis

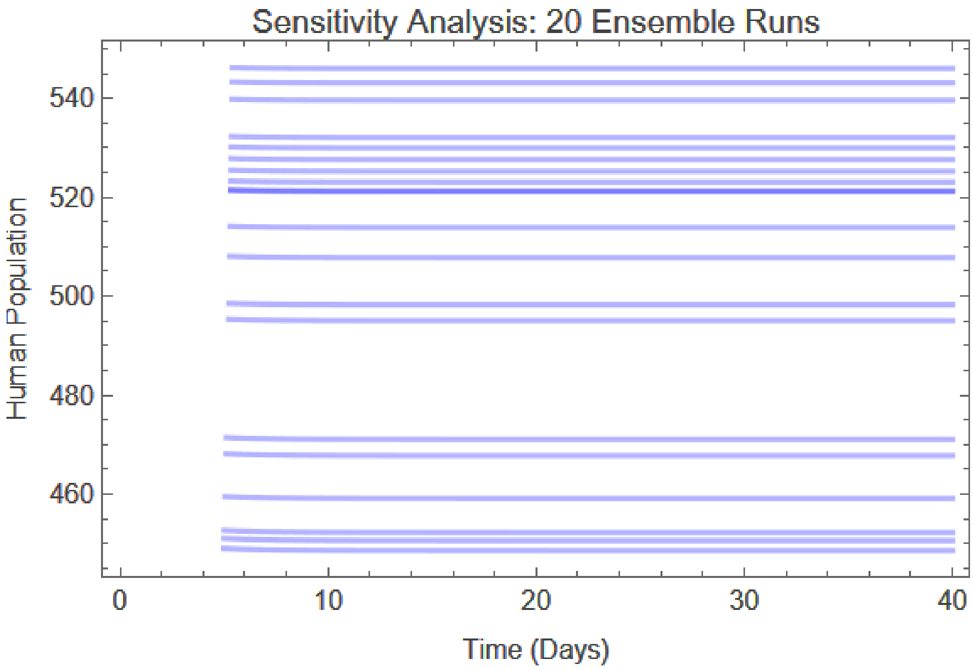

Figure 9 Sensitivity analysis diagram with 20 ensemble runs.

The ensemble simulation adds noise to S_0.

- High Stability: As seen in figure 8, the blue lines remain clustered. This indicates that for the chosen parameters, the system is far from the bifurcation point ($S_0 \gg S_c$). The outcome (survival) is robust against small perturbations in the initial population.
- Chaotic Potential: Sensitivity would only increase if S_0 were extremely close to the threshold $S_c = 80$.

8. Conclusion

The SFMR model essentially describes a dynamic mechanism of "DPS detection" (damage per second detection). This means that the key to winning is not to control the spread of infection, but to establish an absolute combat advantage. The following is a specific model analysis: The Flayed is just an eyesore. From a mathematical point of view, the role of F (The

Flayed) is simply to cut S (human survivors) through the $-\delta SF$ item. They act as "Tanks", consuming human fire output as the Mind Flayer regenerates itself. The survival of the critical threshold Hawkinstown depends entirely on whether the following inequalities are met:

$$S \cdot \alpha_M > \rho$$

(Total human damage output > Mind Flayer regeneration rate)

The most effective intervention is to increase α_M (attack efficiency against Mind Flayer). Even a small increase in thermal damage can significantly shift the bifurcation point to the left. This means that as long as the firepower is strong enough, even if the number of survivors is small, you can win a chance of survival. However, if the population of humans can increase during the simulation for any reason, the whole process will be very different.

References

[1] C. to, “The Mind Flayer,” *Stranger Things Wiki*, 2025. https://strangerthings.fandom.com/wiki/The_Mind_Flayer (accessed Dec. 02, 2025).

[2] T. Kolokolnikov and D. Iron, “Law of mass action and saturation in SIR model with application to Coronavirus modelling,” *Infectious Disease Modelling*, vol. 6, pp. 91–97, 2021, doi: https://doi.org/10.1016/j.idm.2020.11.002. (accessed Dec. 04, 2025).