Lab 8

Q1 A.

Start with an empty stack of integers. You will attempt to do a sequence of pushes and pops so that the sequence of pops will be a specified permutation of 1, 2, 3 ,4 , 5, 6. You will be able to do exactly 6 push operations and 6 pop operations. The first push pushes 1 onto the stack; the next pushes 2; and so forth. The sixth push pushes 6 onto the stack.

For this exercise, we will let S denote a push operation and X a pop operation. Example: The sequence SSSSSSXXXXXX outputs 654321.

a. Describe a sequence of pushes and pops that would produce output 325641 (or explain why it is not possible)
b. Describe a sequence of pushes and pops that would produce output 154623 (or explain why it is not possible)

Q1 B. Suppose we store n keys in a hash table of size m = n^2 using a hash function h randomly chosen from a Universal class H of hash functions. Assume that X is a random variable that counts the number of collisions. Show that the Expected number of Collisions is $< 1/2$.

Q2. For each integer n = 1,2,3,…, 7, determine whether there exists a red-black tree having exactly n nodes, with *all of them black.* Fill out the chart below to tabulate the results:

Num nodes *n*	**Does there exist a red-black tree with *n* nodes, all of which are black?**
1	Yes
2	
3	
4	
5	
6	
7	

Q3. For each integer n = 1,2,3,…, 7, determine whether there exists a red-black tree having exactly n nodes and exactly one red node. Fill out the chart below to tabulate the results:

Num nodes *n*	**Does there exist a red-black tree with *n* nodes that has exactly one red node?**

1	No
2	
3	
4	
5	
6	
7	

Q4.

Show the red-black tree that results after each of the integer keys $21, 32, 64, 75$, and 15 are inserted, in that o
into an initially empty red-black tree. Clearly show the tree that results after **each** insertion (indicating the col
each node), and make clear any rotations that must be performed.