# CS435DE - Lab 8

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Problem 1A: Solution

a.

The required output is 325641.
Here, S = Push, X= Pop
3 = SSSX -> stack: [1,2]
2 = SSSXX -> stack: [1]
5 = SSSXXSSX -> stack : [1,4]
6 = SSSXXSSXSX -> stack : [1,4]
4 = SSSXXSSXSXX -> stack: [1]
1 = SSSXXSSXSXXX -> stack: [ ]

b.

Here, S = Push, X = Pop

1 = SX -> stack: [ ]
5 = SXSSSSX -> stack: [2,3,4]
4 = SXSSSSXX -> stack: [2,3]
6 = SXSSSSXXS -> stack: [2,3,6]
Now to output 2, we need to pop 6, but 6 is not the next expected output.
We cannot reach 2 before popping 6.

Output 154623 is not possible using stack operations.
Because once 6 is pushed, it must be popped before accessing 2 and 3, which violates the required output order.

Problem 1B: Solution

(1) B Ans.

Number of keys = $n$

Table Size = $m = n^2$

Hash Function $h \in H$, a universal class

Let $x$ = number of collisions

For any two distinct keys $x \neq y$

$$Pr[h(x) = h(y)] \leq \frac{1}{m}$$

Total number of pairs: $\frac{n}{2} = \frac{n(n-1)}{2}$

Expected collisions ($E[x]$):

$$E[x] \leq \left(\frac{n}{2}\right) \cdot \frac{1}{m} = \frac{n(n-1)}{2} \cdot \frac{1}{n^2} = \frac{n-1}{2n} < \frac{1}{2}$$

Hence, no. of collisions $< 1/2$.

Q2. Solution

| Num nodes n | Does there exist a red-black tree with n nodes, all of which are black? |
|---|---|
| 1 | Yes |
| 2 | No |
| 3 | Yes |
| 4 | No |
| 5 | No |
| 6 | No |
| 7 | Yes |

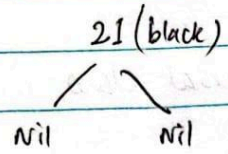
Q3. Solution

| Num nodes n | Exists a red-black tree with exactly 1 red node? |
|---|---|
| 1 | No |
| 2 | Yes |
| 3 | Yes |
| 4 | Yes |
| 5 | Yes |
| 6 | Yes |
| 7 | Yes |

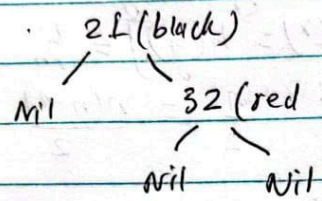
Q4. Solution

(4) Solution: Integer keys: 21, 32, 64, 75 and 15.

Step 1: Insert 21.

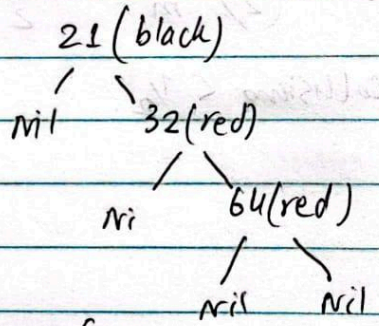



Step 2: Insert 32, 32 > 21, => Right child

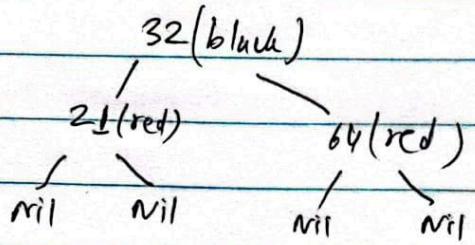



No violations

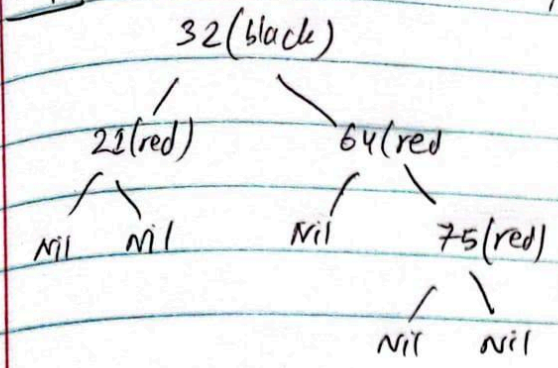
Step 3: Insert 64, 64 > 21, & 64 > 32, => Right child



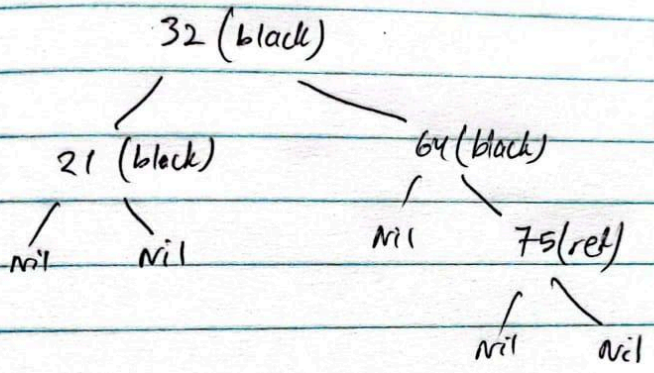

We need to perform a left rotation and recolor



Step 4: Insert 75, 75>32, 75>64 ⇒ Right child




We need to recolor




Step 5: Insert 15

15<32, 15<21 ⇒ left child