Linear Algebra and Matrices

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Abstract—**This document provides a beginner's latex template for math.**

1 POINTS AND VECTORS

1.1. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

$$\mathbf{P} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{1.1.1}$$

Solution: In Fig. 1.1

$$\mathbf{P} - \mathbf{S} = \mathbf{Q} - \mathbf{R} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \tag{1.1.2}$$

$$\mathbf{R} - \mathbf{S} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{1.1.3}$$

Hence *PQRS* is a ||gm ∵ opposite sides are parallel. Also,

$$\|\mathbf{P} - \mathbf{S}\| = \|\mathbf{Q} - \mathbf{R}\| \tag{1.1.4}$$

$$= \|\mathbf{R} - \mathbf{S}\| = \|\mathbf{Q} - \mathbf{P}\| = 2\sqrt{2} \tag{1.1.5}$$

∵ all sides are equal, the ||gm is a rhombus. The angle between *PS* and *RS* is given by

$$\cos\theta = \frac{(\mathbf{S} - \mathbf{P})^\top (\mathbf{S} - \mathbf{R})}{\|\mathbf{S} - \mathbf{P}\|^\top \|\mathbf{S} - \mathbf{R}\|} \tag{1.1.6}$$

∵

$$(\mathbf{S} - \mathbf{P})^\top (\mathbf{S} - \mathbf{R}) = \begin{pmatrix} 2 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 0 \tag{1.1.7}$$

upon substituting from (1.1.2) and (1.1.3),

$$\cos\theta = 0 \implies PS \perp RS \tag{1.1.8}$$
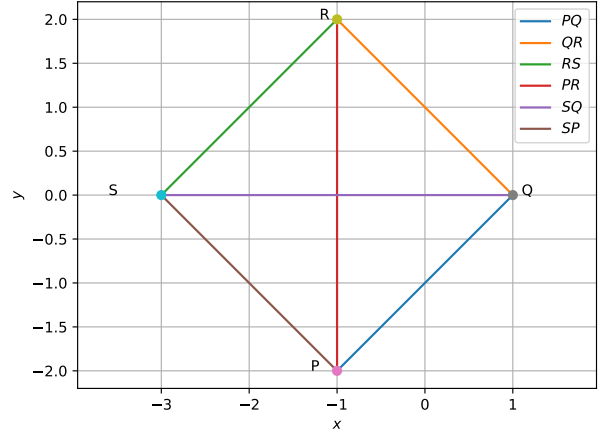
Thus, the rhombus is actually a square.

Fig. 1.1. The given points form a square