

Project Report

Financial risk associated with a diversified stock portfolio.

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ANLY 515-51- A-2024/Spring - Risk Modelling and Assessment

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Executive Summary:

This financial risk modeling project focuses on the construction and analysis of a diversified stock portfolio composed of two Information Technology (IT) stocks, two Financials stocks, and two Consumer Discretionary stocks. The primary objective is to employ quantitative methodologies to assess and manage the inherent financial risks associated with investing in these sectors. The project scope encompasses portfolio construction, risk measurement, and performance evaluation, providing a comprehensive understanding of the potential risks and returns associated with the chosen stocks.

The portfolio's diversification across Information Technology, Financials, and Consumer Discretionary sectors aims to capture a balanced representation of the market while minimizing sector-specific risks. Each sector plays a crucial role in contributing to the overall risk and return profile of the portfolio, allowing for a holistic evaluation of its performance.

Key components of the project include the identification and selection of individual stocks within each sector, consideration of factors such as historical performance, volatility, and correlation among stocks. Risk metrics and models, including various distributions, will be employed to quantify and assess the potential downside risks associated with the portfolio.

In conclusion, this financial risk modeling project aims to provide insights into the risk and return dynamics of a diversified stock portfolio. By leveraging quantitative methodologies and risk management strategies, the analysis seeks to empower readers with the knowledge necessary for informed decision-making in the dynamic and unpredictable world of financial markets.

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1. Introduction

In the realm of financial risk modeling, this project centers on a meticulously curated stock portfolio spanning three distinct sectors: Information Technology (IT), Financials, and Consumer Discretionary.

1.2 Portfolio Under Consideration

1.2.1 Portfolio Sectors and Securities

Each sector is represented by two carefully selected securities, forming the foundation for a comprehensive analysis of risk. The selected sectors and securities are as follows:

1. Information Technology:

1.1 Semiconductors & Semiconductor Equipment: NVIDIA Corporation (NVDA)

1.2 Software & Services: Microsoft Corporation (MSFT)

1.3 Apple Inc. (AAPL)

1.4 Netflix, Inc. (NFLX)

2. Financials:

2.1 Insurance: Berkshire Hathaway Inc. (BRK.B)

2.2 Banks: JPMorgan Chase & Co. (JPM)

2.3 CitiGroup Inc. (C)

2.4 BlackRock, Inc. (BLK)

3. Consumer Discretionary:

3.1 Retailing: McDonald's Corporation (MCD)

3.2 Consumer Durables & Apparel: Skechers U.S.A., Inc. (SKX)

3.3 Starbucks Corporation (SBUX)

3.4 Nike Inc, (NKE)

The time scope of this project encompasses a period through **2015 till 2023**.

1.3 Objectives of the project

The portfolio's objectives are twofold: to provide a balanced representation of the market by diversifying across sectors and to optimize the risk-return trade-off through the strategic selection and management of individual securities.

This project is structured to leverage various tools and techniques acquired from the ANLY 515-51-A-2024/Spring risk modelling and assessment class lectures. The application of quantitative methodologies such as building equal value and value weighted portfolio, Value at Risk (VaR) and Expected Shortfall (ES), Modelling and Portfolio Optimization will be integral in quantifying downside risks associated with the portfolio. Furthermore, risk management strategies, including diversification reconsideration, and optimization, will be employed to enhance the overall risk-adjusted performance.

1.4 Optimization Objectives

The project plan entails a meticulous application of these tools and techniques to the selected portfolio. Each lecture's insights will be strategically integrated to address specific aspects of risk modeling, culminating in a comprehensive understanding of the portfolio's behavior under various market conditions.

2. Project Plan

The project plan is follows:

2.1.1 Data Collection and Cleaning:

- Gather historical stock prices, financial statements, and relevant market data for each security in the portfolio. Clean the data to ensure accuracy and consistency.

2.1.2 Portfolio Construction:

- Utilize Equally Weighted and Value Weighted portfolio strategies to construct an efficient and diversified portfolio.
- Distribute investments across the Information Technology, Financials, and Consumer Discretionary sectors.

2.1.3 Risk Measurement:

- Implement Value at Risk (VaR) and Expected Shortfall (ES) to quantify the potential downside risks associated with individual securities and the overall portfolio.
- Evaluate historical volatility and correlation coefficients between securities to understand their interdependencies.

2.1.4 Performance Evaluation:

- Calculate key performance metrics to assess the risk-adjusted returns of the portfolio.
- Compare the portfolio's performance against relevant benchmarks to gauge its relative strength.

2.1.5 Optimization Techniques:

- Apply optimization models to enhance the risk-return trade-off of the portfolio.

- Explore Mean-Variance Optimization to identify the optimal asset weights that maximize returns for a given level of risk.

2.1.6 Diversification Strategies:

- Evaluate the benefits of diversification by analyzing the impact of adding or removing securities from the portfolio.
- Use diversification metrics to quantify the portfolio's risk reduction through varied asset selection.

2.1.7 Reporting and Visualization:

- Develop clear and concise reports summarizing the findings and insights gained from the analysis.
- Utilize visualizations, including charts and graphs, to communicate complex risk metrics and portfolio performance to stakeholders.

2.1.8 Sensitivity Analysis:

- Conduct sensitivity analysis to understand how changes in external factors, such as interest rates or economic indicators, affect the portfolio's performance.
- Identify key drivers of risk and return in the portfolio.

By systematically applying these tools and techniques throughout the project, the aim is to provide a comprehensive risk modeling framework that enhances the understanding and management of financial risks associated with the selected stock portfolio.

3. Background and Exploratory data analysis

3.1.1 Model description

Statistical Analysis:

- Tools: Regression analysis, time series analysis, correlation analysis.
- Application: Statistical techniques can be used to analyze historical data of individual assets and sectors within the portfolio. For example, regression analysis can help identify relationships between sector performance and macroeconomic factors. Correlation analysis can determine the degree of association between different sectors and the portfolio as a whole.

Portfolio Optimization:

- Tools: Mean-variance optimization, Markowitz portfolio theory, risk-parity.
- Application: Portfolio optimization tools help in determining the optimal allocation of assets within the portfolio. By considering factors such as expected returns, volatilities, and correlations between assets and sectors, these tools can construct portfolios that maximize returns for a given level of risk or minimize risk for a target level of return. Sector-specific constraints can be incorporated to ensure desired sector exposures.

3.1.2 Exploratory Data Analysis

[1]Information Technology:

1.1 Semiconductors & Semiconductor Equipment: NVIDIA Corporation (NVDA)

1.2 Software & Services: Microsoft Corporation (MSFT)

1.3 Software & Services: Apple Inc. (AAPL)

1.4 Software & Services: Netflix Inc. (NFLX)

NVDA. Adjusted	MSFT. Adjusted	AAPL. Adjusted	NFLX. Adjusted
Min. : 4.595	Min. : 34.89	Min. : 20.77	Min. : 45.55
1st Qu.: 26.662	1st Qu.: 59.95	1st Qu.: 33.05	1st Qu.: 144.06
Median : 59.650	Median : 129.13	Median : 51.62	Median : 310.84
Mean : 111.023	Mean : 156.48	Mean : 82.03	Mean : 298.53
3rd Qu.: 159.012	3rd Qu.: 247.19	3rd Qu.: 140.15	3rd Qu.: 409.37
Max. : 504.022	Max. : 381.99	Max. : 197.86	Max. : 691.69

Fig 1: Information Technology sector adjusted prices

[2]Financials:

2.1 Insurance: Berkshire Hathaway Inc. (BRK.B)

2.2 Banks: JPMorgan Chase & Co. (JPM)

2.3 Banks: CitiGroup Inc. (C)

2.4 Insurance: BlackRock Inc. (BLK)

BRK_B. Adjusted	JPM. Adjusted	C. Adjusted	BLK. Adjusted
Min. : 124.1	Min. : 42.25	Min. : 27.62	Min. : 234.5
1st Qu.: 166.3	1st Qu.: 71.61	1st Qu.: 42.29	1st Qu.: 321.8
Median : 205.8	Median : 94.11	Median : 48.20	Median : 435.7
Mean : 220.8	Mean : 97.26	Mean : 49.56	Mean : 485.0
3rd Qu.: 279.8	3rd Qu.: 125.91	3rd Qu.: 57.47	3rd Qu.: 647.6
Max. : 370.5	Max. : 169.26	Max. : 71.63	Max. : 908.3

Fig 2: Financial sector adjusted prices

[3]Consumer Discretionary:

3.1 Retailing: McDonald's Corporation (MCD)

3.2 Consumer Durables & Apparel: Skechers U.S.A., Inc. (SKX)

3.3 Retailing: Starbucks Corp. (SBUX)

3.4 Consumer Durables & Apparel: Nike Inc. (NKE)

MCD. Adjusted	SKX. Adjusted	SBUX. Adjusted	NKE. Adjusted
Min. : 70.11	Min. : 18.42	Min. : 33.27	Min. : 41.18
1st Qu.: 109.85	1st Qu.: 28.31	1st Qu.: 49.64	1st Qu.: 54.11
Median : 169.87	Median : 34.20	Median : 68.68	Median : 80.81
Mean : 172.41	Mean : 35.52	Mean : 70.50	Mean : 86.46
3rd Qu.: 226.21	3rd Qu.: 41.98	3rd Qu.: 91.61	3rd Qu.: 110.72
Max. : 294.83	Max. : 63.81	Max. : 118.91	Max. : 172.45

Fig 3: Consumer Discretionary adjusted prices

[1] Information Technology

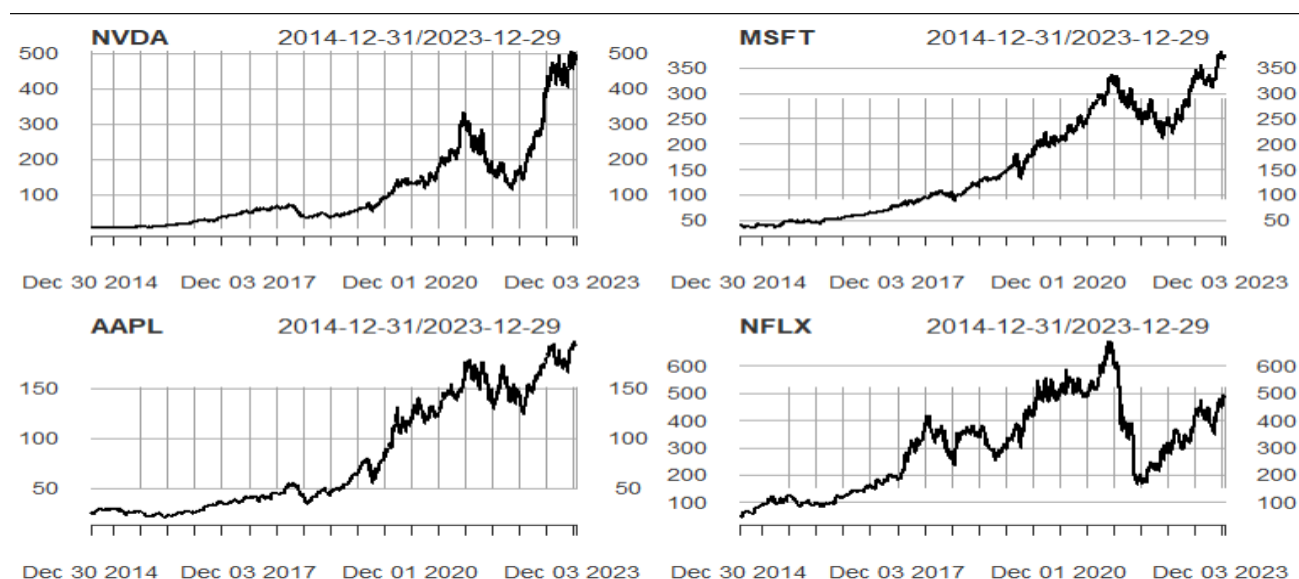


Fig 5: Information Technology adjusted prices plot 2015 - 2023

[2] Financials

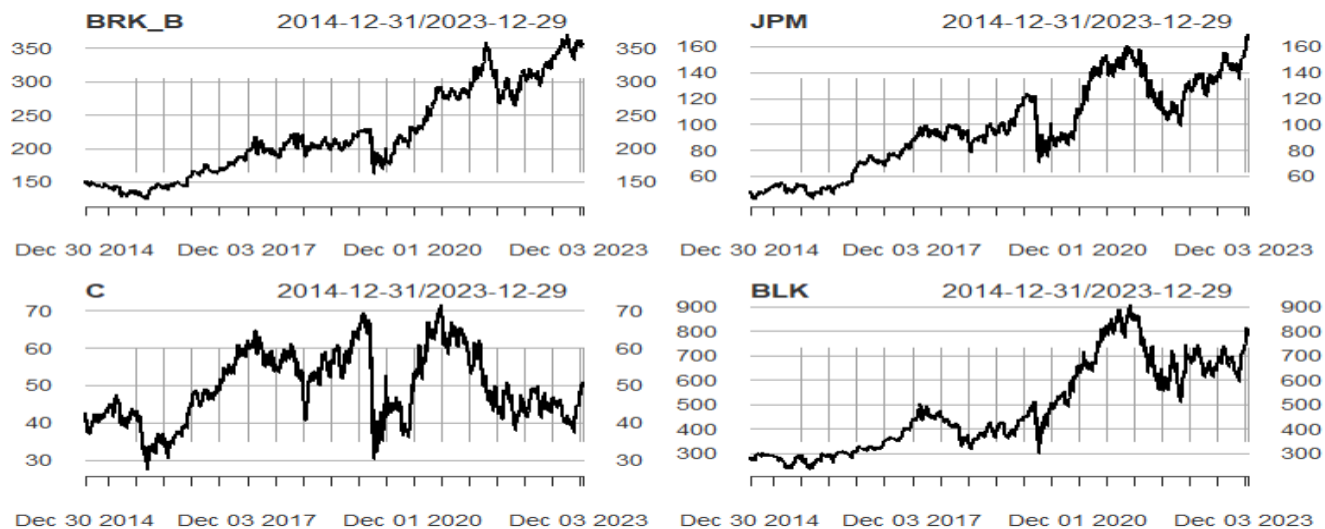


Fig 6: Financial sector adjusted prices plot 2015 - 2023

[3]Consumer Discretionary

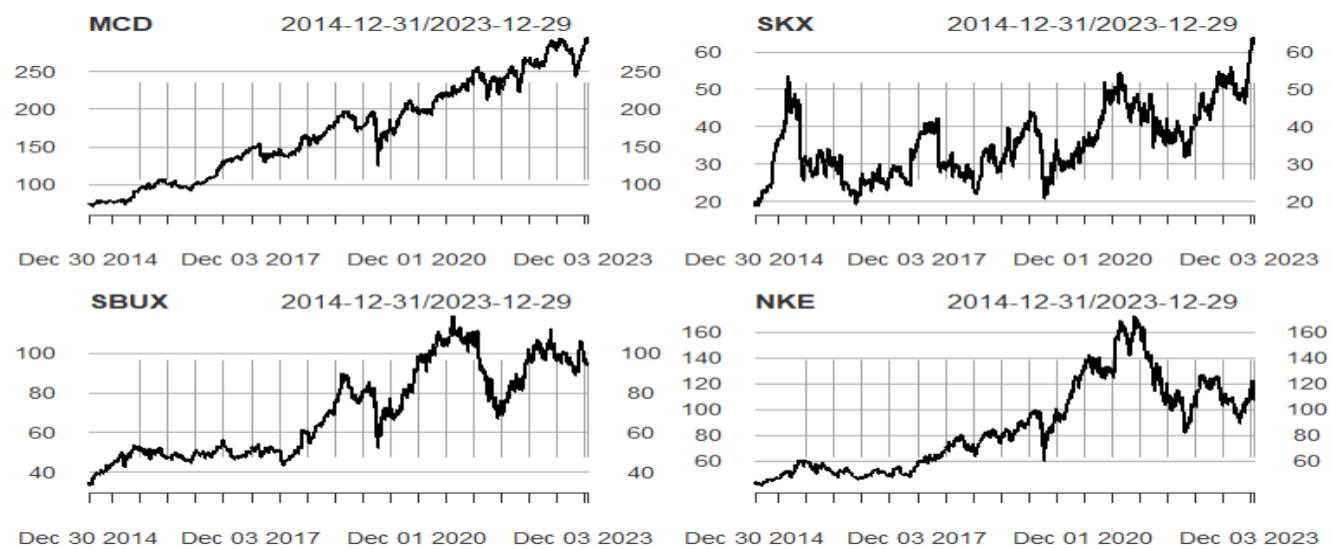


Fig 7: Consumer Discretionary adjusted prices plot 2015 - 2023

3.1.3 VARIANCE - COVARIANCE MATRIX FOR DAILY RETURNS OVER ENTIRE PERIOD

The following figure presents the variance-covariance matrix for daily returns over the entire period, calculated using historical data for the specified portfolio. The portfolio consists of assets from three different sectors: Information Technology, Financials, and Consumer Discretionary. Each sector contains several individual stocks representing prominent companies within their respective industries.

	NVDA	MSFT	AAPL	NFLX	BRK_B	JPM	C	BLK	MCD	SKX	SBUX	NKE
NVDA	2.0824708	0.7439869	0.7128794	0.8167668	0.3483132	0.4290334	0.5222882	0.6090536	0.2786752	0.5725365	0.4637031	0.5299515
MSFT	0.7439869	0.6953464	0.5049643	0.5229391	0.2754086	0.3199794	0.3785007	0.4277112	0.2412554	0.3439514	0.3606239	0.3594673
AAPL	0.7128794	0.5049643	0.7573512	0.4993493	0.2769071	0.3229439	0.3889609	0.4084000	0.2274067	0.3794814	0.3429682	0.3601019
NFLX	0.8167668	0.5229391	0.4993493	1.8120016	0.2255715	0.2673948	0.3553923	0.4069223	0.1736182	0.3645876	0.3496676	0.3844401
BRK_B	0.3483132	0.2754086	0.2769071	0.2255715	0.3518020	0.3826157	0.4295928	0.3539584	0.2030507	0.3400812	0.2691679	0.2662837
JPM	0.4290334	0.3199794	0.3229439	0.2673948	0.3826157	0.6925152	0.7278594	0.4868844	0.2546624	0.5026388	0.3538049	0.3432649
C	0.5222882	0.3785007	0.3889609	0.3553923	0.4295928	0.7278594	1.0033260	0.5699529	0.2984769	0.6215916	0.4277496	0.4205118
BLK	0.6090536	0.4277112	0.4084000	0.4069223	0.3539584	0.4868844	0.5699529	0.7062403	0.2635820	0.4750630	0.3832961	0.4123337
MCD	0.2786752	0.2412554	0.2274067	0.1736182	0.2030507	0.2546624	0.2984769	0.2635820	0.3843484	0.2560332	0.2973829	0.2395553
SKX	0.5725365	0.3439514	0.3794814	0.3645876	0.3400812	0.5026388	0.6215916	0.4750630	0.2560332	1.9916819	0.4438901	0.5641524
SBUX	0.4637031	0.3606239	0.3429682	0.3496676	0.2691679	0.3538049	0.4277496	0.3832961	0.2973829	0.4438901	0.6486750	0.3869846
NKE	0.5299515	0.3594673	0.3601019	0.3844401	0.2662837	0.3432649	0.4205118	0.4123337	0.2395553	0.5641524	0.3869846	0.7786715

Fig 8: Variance - Covariance matrix for daily returns over entire period

The variance-covariance matrix provides insights into the relationships between the daily returns of the individual assets within the portfolio. We leveraged our understanding of these relationships to determine how portfolio optimization works, risk management, and asset allocation decisions.

3.1.4 Correlation matrix

The figure below presents the correlation matrix for daily returns over the entire period, calculated using historical data for the specified portfolio. The portfolio comprises assets from three distinct sectors: Information Technology, Financials, and Consumer Discretionary. Each sector encompasses several individual stocks representing prominent companies within their respective industries.

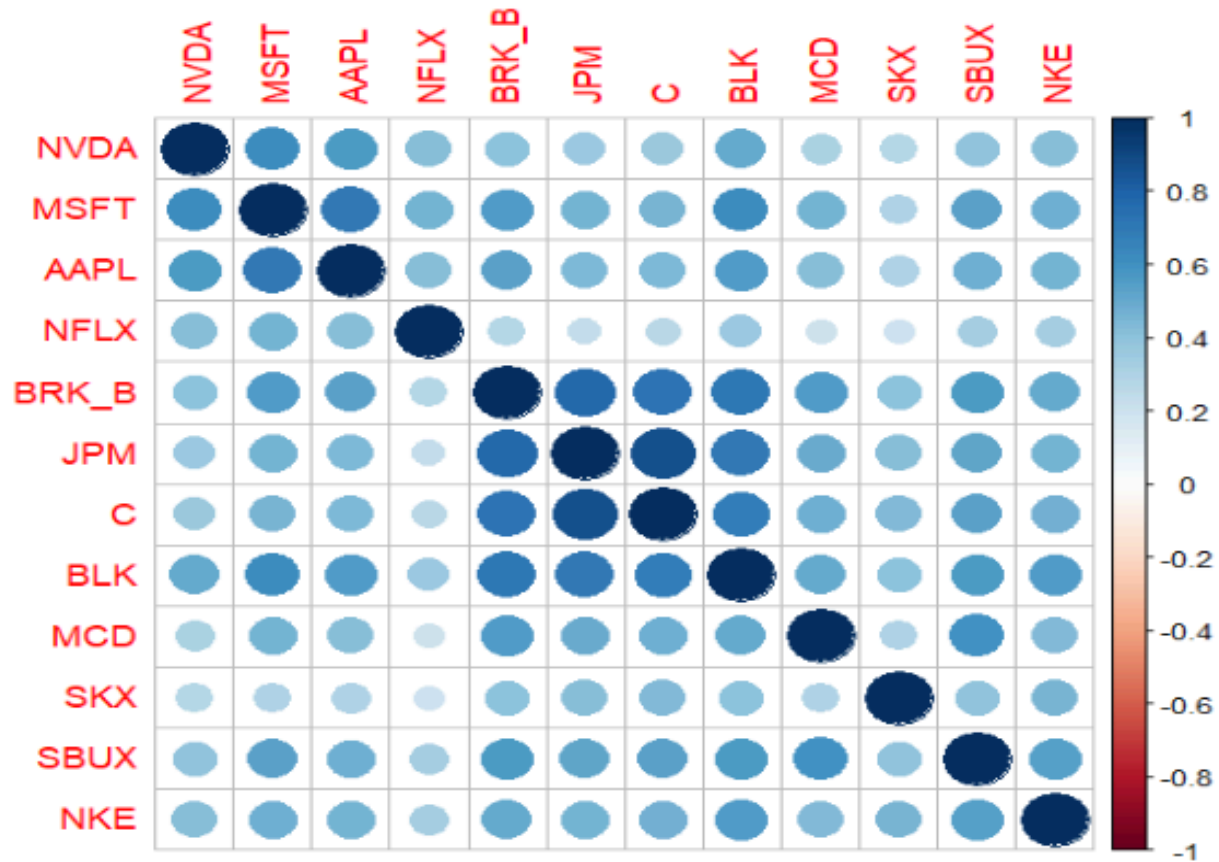


Fig 9: Correlation matrix for daily returns over entire period

The correlation matrix illustrates the pairwise correlations between the daily returns of the individual assets within the portfolio. We utilized these correlations for the purpose of determining diversification benefits, risk management, and identifying potential dependencies among these assets.

3.1.5. Observation:

Based on our analysis, the correlation matrix for daily returns of the various portfolios above, a notable observation emerges regarding the strong positive correlation between two sectors: Information Technology and Financials. Within the Information Technology sector, which comprises companies such as NVIDIA Corporation (NVDA), Microsoft Corporation (MSFT), Apple Inc. (AAPL), and Netflix Inc. (NFLX),

there exists a pattern of closely intertwined returns, indicating a high degree of correlation among these technology-related stocks. Similarly, in the Financials sector, consisting of Berkshire Hathaway Inc. (BRK.B), JPMorgan Chase & Co. (JPM), CitiGroup Inc. (C), and BlackRock Inc. (BLK), a comparable trend of positive correlation is observed, suggesting that the returns of these financial institutions move in tandem. This observation underscores the interconnectedness between certain sectors within the portfolio, highlighting the importance of diversification strategies to mitigate sector-specific risks.

In practical terms, the high positive correlation between Information Technology and Financials sectors implies that movements in one sector are likely to be mirrored by movements in the other. While this correlation may offer opportunities for portfolio managers seeking to capitalize on sector-specific trends, it also poses challenges in terms of risk management. For instance, during periods of market volatility or sector-specific downturns, the portfolio may experience amplified fluctuations due to the synchronized movements of these correlated sectors. Therefore, investors may need to carefully consider the implications of this correlation when constructing or rebalancing their portfolios, incorporating diversification across uncorrelated sectors to better manage risk and enhance overall portfolio resilience.

3.1.6 Returns

Returns analysis is a fundamental aspect of portfolio evaluation, providing insights into the performance and profitability of investments over a specified period. When assessing returns, investors often consider both price returns and total returns, each offering distinct perspectives on investment performance. Additionally, comparing different weighting methodologies, such as value-weighted and equally weighted portfolios, can offer valuable insights into portfolio construction and management strategies. In this context, we delve into two key questions regarding returns analysis for the portfolio: the

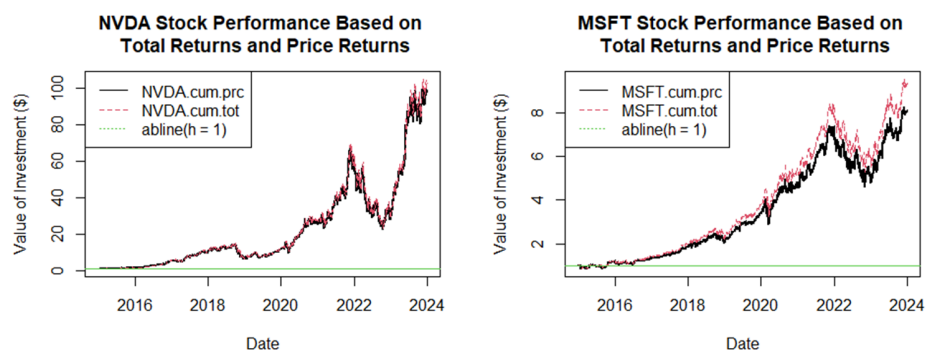
comparison between price returns and total returns over the entire portfolio period, and the contrasting performance of value-weighted and equally weighted portfolios.

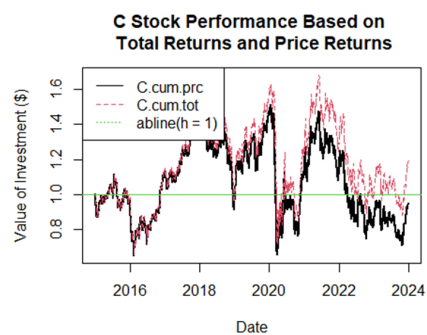
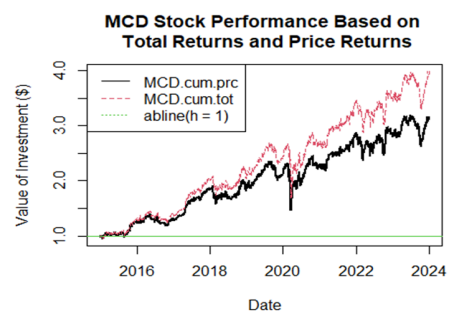
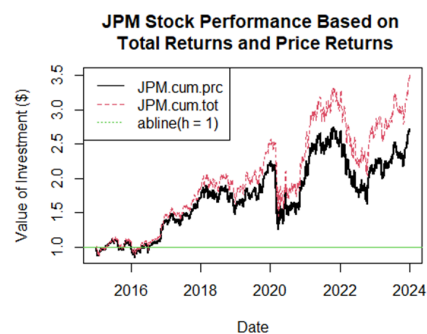
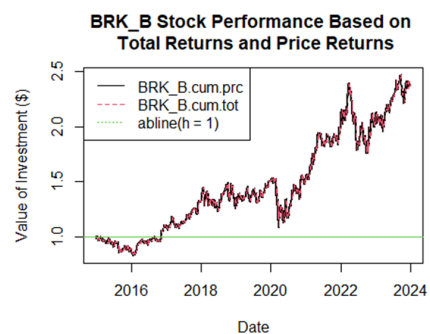
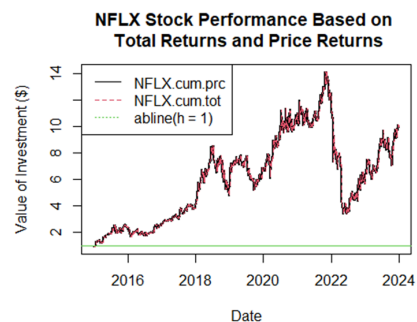
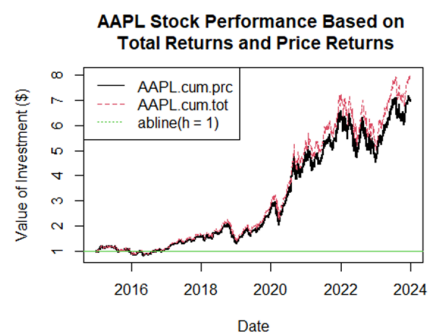
4.1 Returns

Price Returns vs. Total Returns for the Entire Portfolio Period:

Price returns, also known as capital gains or price appreciation, reflect the change in the price of an asset over time. They do not account for factors such as dividends or interest payments. Total returns, on the other hand, incorporate not only price changes but also any income generated by the investment, such as dividends, interest, or distributions. Therefore, total returns provide a more comprehensive measure of investment performance as they capture both price changes and income generated by the investment. Analyzing the price returns versus total returns for the entire portfolio period allows investors to understand the impact of income-generating components on the overall performance of their investments and make more informed decisions regarding portfolio management and strategy.

4.1.1. What are the price returns vs total returns for the entire portfolio period?





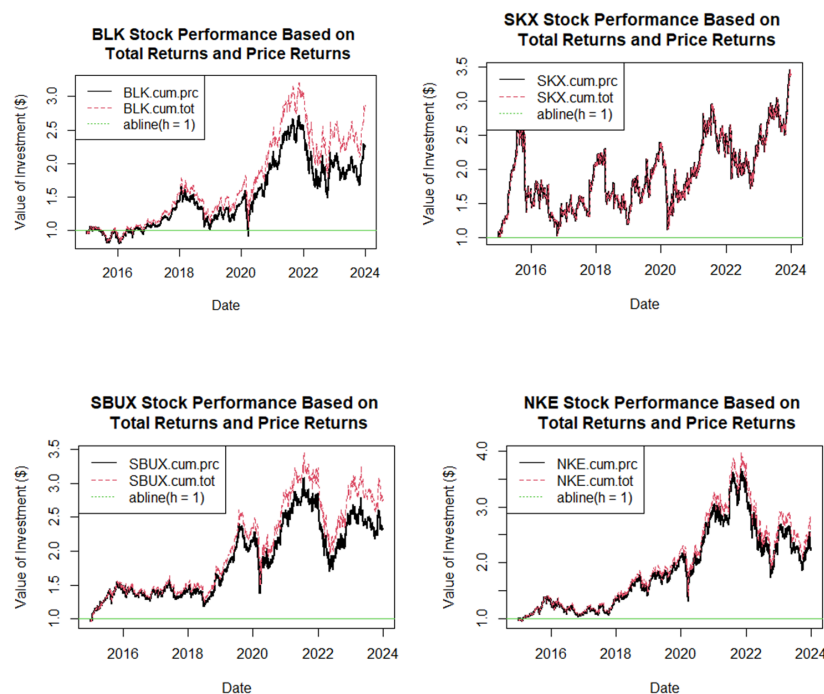


Fig 10: price returns vs total returns over entire period for all securities

4.1.2. How does a Value weighted portfolio and Equally weighted portfolio compare?

Comparison of Value-Weighted and Equally Weighted Portfolios for the worst year 2021:

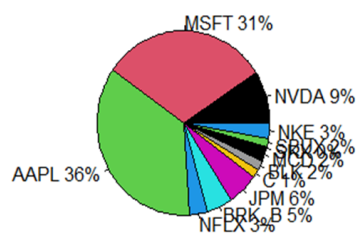
Construction Method:

Value-Weighted Portfolio: In a value-weighted portfolio, stocks are weighted based on their market capitalization. The larger the market capitalization of a stock, the higher its weight in the portfolio. This means that stocks with higher market capitalizations have a greater influence on the performance of the portfolio.

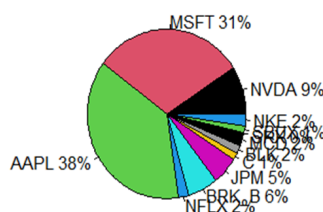
Equally-Weighted Portfolio: In an equally-weighted portfolio, each stock is given the same weight regardless of its market capitalization. This means that each stock contributes equally to the overall performance of the portfolio.

Comparing the performance of these two weighting methodologies provides insights into the impact of portfolio construction techniques on returns and risk. Value-weighted portfolios may offer exposure to larger, more established companies with potentially lower volatility, while equally weighted portfolios may provide greater diversification benefits and potentially higher returns from smaller companies. By analyzing the performance differences between these two approaches, investors can better understand the trade-offs involved in portfolio construction and tailor their investment strategies to align with their objectives and risk preferences.

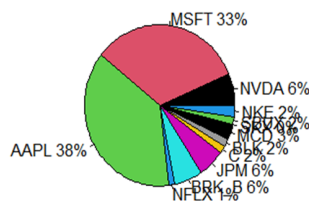
Q1 Value Weigthing



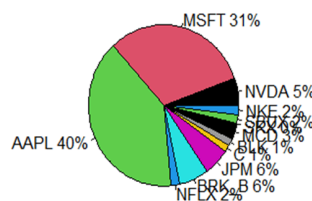
Q2 Value Weigthing



Q3 Value Weigthing



Q4 Value Weigthing



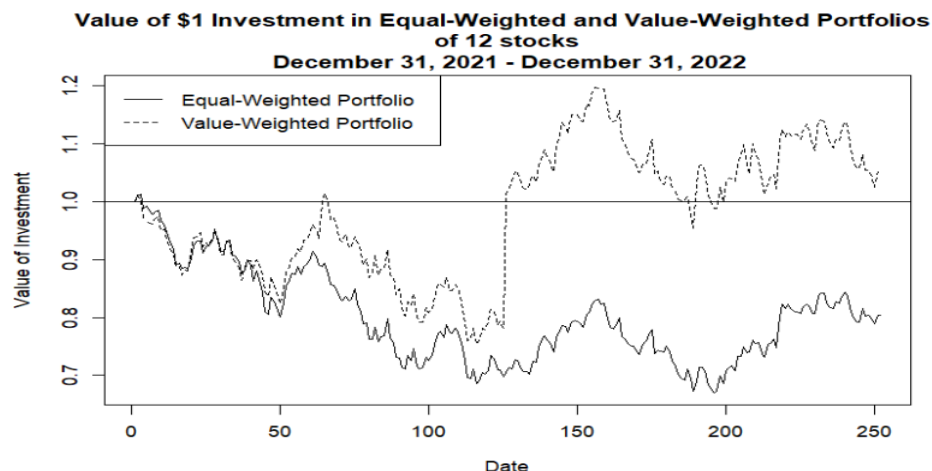


Fig 11: Value weighted portfolio weights and comparison between equal weighted portfolio and value weighted portfolio for the year 2022

4.1.3 Observation

value-weighted portfolios tend to be more influenced by the largest stocks in the market, whereas equally-weighted portfolios offer more diversification and may outperform in certain market conditions. In this case, the weight for large securities changed in Q3 of 2021 for value weighted portfolio. Hence the Value weighted portfolio outperformed the equally weighted portfolio.

5 Suitable Distributions Forecasting

5.1.1) Autoregressive conditional heteroscedasticity (ARCH) - Generalized ARCH Models

In the subject of autoregressive conditional heteroscedasticity (ARCH) and its generalized counterpart, GARCH models, selecting a suitable distribution for forecasting volatility is pivotal for accurate risk assessment and effective portfolio management. Traditionally, ARCH and GARCH models assume the normal distribution for the error terms, leveraging its simplicity and computational efficiency. However, financial data often deviate from normality, exhibiting phenomena like fat tails and excess kurtosis, which are inadequately captured by the normal distribution. To address this limitation, alternative

distributions have gained prominence, such as the student's t-distribution and the generalized error distribution (GED). These distributions offer greater flexibility to model the heavier tails and asymmetry commonly observed in financial returns, making them well-suited for volatility forecasting within the ARCH and GARCH framework. By incorporating parameters like degrees of freedom in the t-distribution or shape parameters in the GED, these models can accommodate various degrees of tail thickness, allowing for more accurate risk assessments and more robust portfolio strategies.

When selecting a distribution for forecasting volatility in ARCH and GARCH models, we carefully weigh the trade-offs between model complexity, computational efficiency, and empirical adequacy. While distributions like the normal and t-distributions offer analytical tractability and interpretability, they may not fully capture the complexities of financial data, particularly during extreme market conditions. On the other hand, the generalized error distribution provides a more flexible and data-driven approach, allowing for a wider range of distributional properties to be captured. However, estimating GARCH models with GED errors can be computationally intensive, especially for large datasets or high-frequency data.

For this analysis an Equally weighted portfolio was considered. Daily returns were calculated for the entire period from 2015 to 2023.

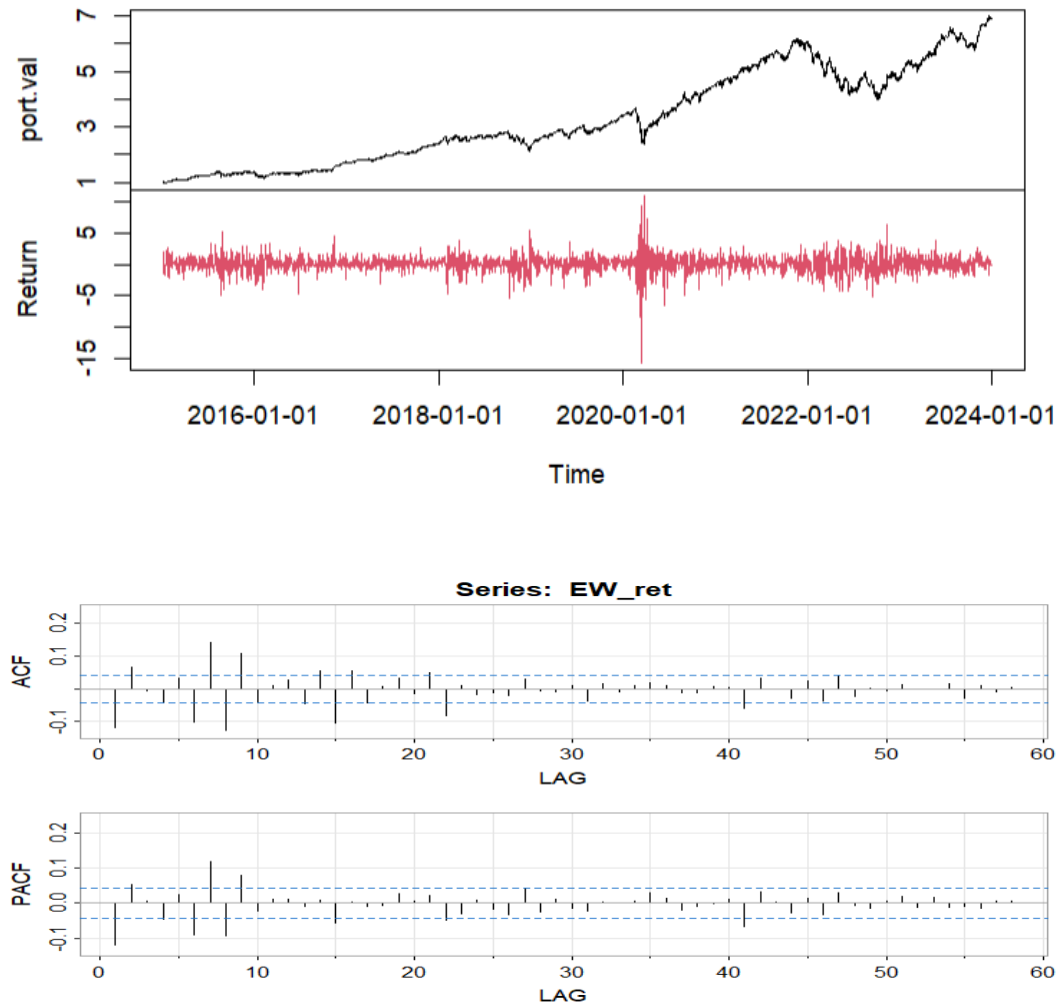
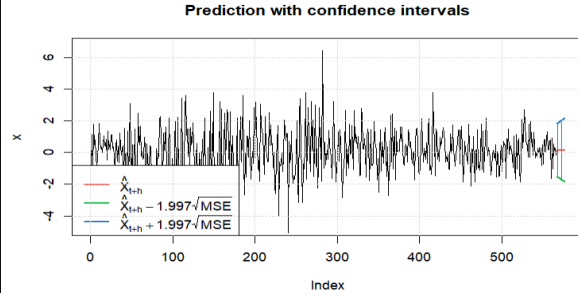


Fig 12: equal weighted portfolio value vs returns and PACF/ACF for the EW portfolio for the entire period 2015-2023

Based on PACF and ACF values the following two models were considered to forecast returns for next 10 periods.

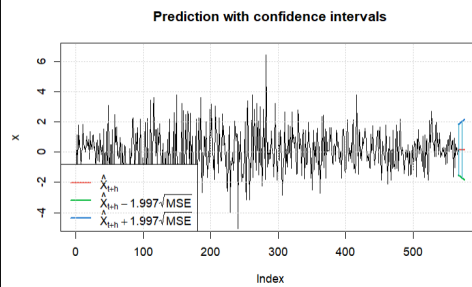
Two GARCH Models were considered for forecasting EW Portfolio returns

meanForecast <dbl>	meanError <dbl>	standardDeviation <dbl>	lowerInterval <dbl>	upperInterval <dbl>
0.1631542	0.8434058	0.8434058	-1.520855	1.847163
0.1484175	0.8641710	0.8634066	-1.577053	1.873888
0.1490524	0.8833336	0.8825486	-1.614679	1.912784
0.1490250	0.9016982	0.9008947	-1.651375	1.949425
0.1490262	0.9193220	0.9185007	-1.686563	1.984615
0.1490262	0.9362544	0.9354162	-1.720371	2.018424
0.1490262	0.9525398	0.9516853	-1.752888	2.050940
0.1490262	0.9682180	0.9673478	-1.784192	2.082245
0.1490262	0.9833249	0.9824397	-1.814356	2.112408
0.1490262	0.9978931	0.9969934	-1.843444	2.141496



```
EW_ret.g1<-garchFit(~arma(1,0)+garch(1,1),data=EW_ret,cond.dist="std")
```

meanForecast <dbl>	meanError <dbl>	standardDeviation <dbl>	lowerInterval <dbl>	upperInterval <dbl>
0.1631605	0.8433411	0.8433411	-1.520697	1.847018
0.1484260	0.8641510	0.8633870	-1.576981	1.873833
0.1490607	0.8833547	0.8825700	-1.614690	1.912811
0.1490334	0.9017569	0.9009536	-1.651460	1.949526
0.1490346	0.9194149	0.9185939	-1.686715	1.984784
0.1490345	0.9363786	0.9355406	-1.720586	2.018655
0.1490345	0.9526927	0.9518384	-1.753159	2.051228
0.1490345	0.9683971	0.9675271	-1.784516	2.082585
0.1490345	0.9835280	0.9826429	-1.814727	2.112796
0.1490345	0.9981182	0.9972186	-1.843858	2.141927



```
EEW_ret.g2<-garchFit(~arma(1,0)+garch(1,2),data=EW_ret,cond.dist="std")
```

Fig 13: GARCH fits for EW portfolio for the entire period 2015-2023

6.1 Risks

Value at Risk (VaR) and Expected Shortfall (ES) are two widely used risk measures that provide insights into the potential losses a portfolio may incur under adverse market conditions. Both historical and simulated approaches can be employed to estimate VaR and ES, each with its strengths and limitations.

VaR quantifies the maximum loss that a portfolio is expected to suffer over a specified time horizon at a given confidence level. Historical VaR relies on historical data to estimate potential losses, making it straightforward and intuitive. However, historical VaR may not capture extreme events or changes in market conditions that have not been observed in the historical data. Simulated VaR, on the other hand, utilizes stochastic models such as Monte Carlo simulations to generate a range of possible outcomes,

allowing for a more comprehensive assessment of risk. Simulated VaR can capture a broader range of scenarios and incorporate factors that may not be fully captured by historical data, providing a more forward-looking perspective on risk. Comparing historical and simulated VaR can reveal discrepancies between past market behavior and potential future outcomes, highlighting areas of uncertainty and guiding risk management strategies accordingly.

6.1 Risks - Suitable Distributions part 1

6.1.1. How does historical VaR and simulated (gaussian) VaR compare?

6.1.2. How does historical ES and simulated (gaussian) ES compare?

Gaussian distributions models are the one the fundamental models in data analysis. They are widely used in statistics and probability theory to describe the behavior of many natural phenomena. They are characterized by a symmetric bell-shaped curve, with the mean (average) at the center and the standard deviation determining the spread of the data around the mean. In Gaussian distributions, approximately 68% of the data falls within one standard deviation of the mean, about 95% within two standard deviations, and nearly 99.7% within three standard deviations. These models are fundamental in fields such as finance, physics, and engineering for modeling random variables and estimating probabilities.

For the analysis of VaR and ES, a gaussian distribution was fitted into a a historical Profit and Loss density.

Simulated Historical Value-at-Risk and Expected Shortfall

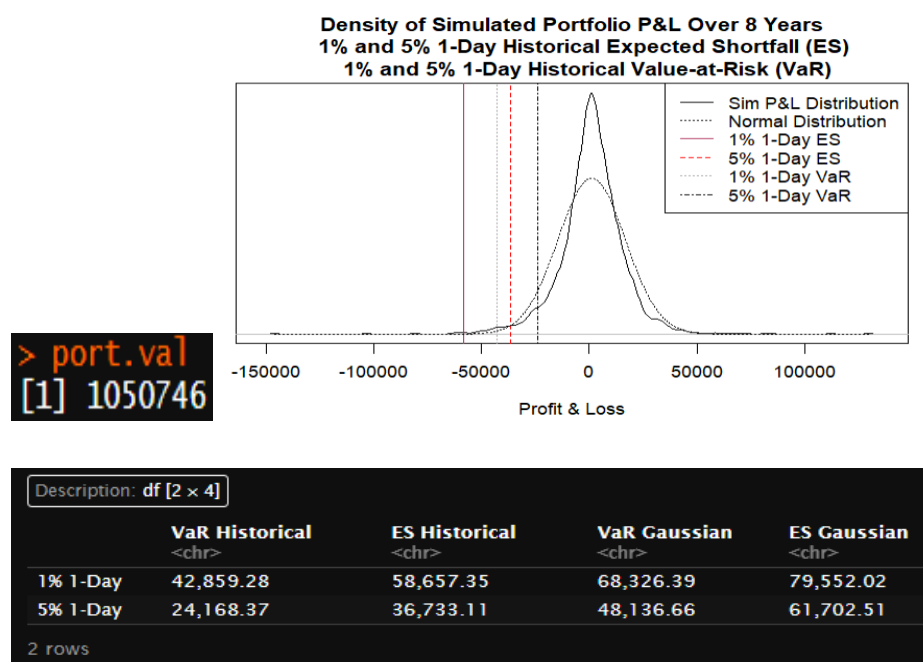


Fig 14: Gaussian Distribution fit for EW portfolio for the entire period 2015-2023. VaR and ER comparison for historical returns vs Simulated Returns

Expected Shortfall (ES), also known as Conditional Value at Risk (CVaR), goes beyond VaR by measuring the average loss of the portfolio given that losses exceed the VaR threshold. Similar to VaR, historical and simulated approaches can be used to estimate ES. Historical ES calculates the average loss based on historical data exceeding the VaR threshold, providing a backward-looking measure of risk. Simulated ES, on the other hand, simulates a wide range of scenarios to estimate the average loss conditional on extreme events, offering a more comprehensive assessment of tail risk. Comparing historical and simulated ES can reveal how different assumptions and modeling techniques impact the estimation of tail risk, helping investors better understand the potential downside and devise more robust risk management strategies to protect their portfolios against extreme market events.

Value-at-Risk - year 2022

date <chr>	VW.cum <dbl>	EW.cum <dbl>	VW.ret <dbl>	EW.ret <dbl>
2021-12-31	1.000000	1.000000	NA	NA
2022-01-03	1.011234	1.0096452	0.0112342861	0.0096451667
2022-01-04	1.001982	1.0133846	-0.0091490065	0.0037037434
2022-12-30	1.050746	0.8040657	-0.0005682855	-0.0006664605

```

314 # {r}
315 port.mean <- mean(port.ret)
316 port.mean
317
318 port.risk <- sd(port.ret)
319 port.risk
320 #
[1] 0.0005645944
[1] 0.0281949

321 Calculate 1 and 5% VaR
322 # {r}
323 VaR01.Gaussian <- -(port.mean+port.risk*qnrm(0.01))*1050746
324 VaR01.Gaussian <- format(VaR01.Gaussian,big.mark=',') #to include a comma
325 VaR01.Gaussian
326 # This means that there is a %1 chance that the portfolio loses more than $68,326.39
327
328 VaR05.Gaussian <- -(port.mean+port.risk*qnrm(0.05))*1050746
329 VaR05.Gaussian <- format(VaR05.Gaussian,big.mark=',') #to include a comma
330 VaR05.Gaussian
331
332 # This means that there is a %5 chance that the portfolio loses more than $48,136.66
333 #
[1] "68,326.39"
[1] "48,136.66"

```

Fig 15: Gaussian Distribution VaR and ER comparison for Simulated Returns

Suitable Distributions

In considering suitable distributions for estimating Value at Risk (VaR) and Expected Shortfall (ES), it's crucial to select models that accurately capture the distributional properties of financial returns. Traditional approaches often assume a normal distribution for simplicity, but financial data frequently exhibit characteristics such as fat tails and skewness that deviate from normality. Therefore, alternative distributions like the student's t-distribution or the generalized error distribution (GED) may be more appropriate. The student's t-distribution offers flexibility in capturing heavier tails, while the GED encompasses a broader range of distributional shapes, including asymmetry and kurtosis. By choosing distributions that better reflect the empirical features of financial returns, VaR and ES estimates can provide more accurate assessments of downside risk and guide risk management decisions effectively.

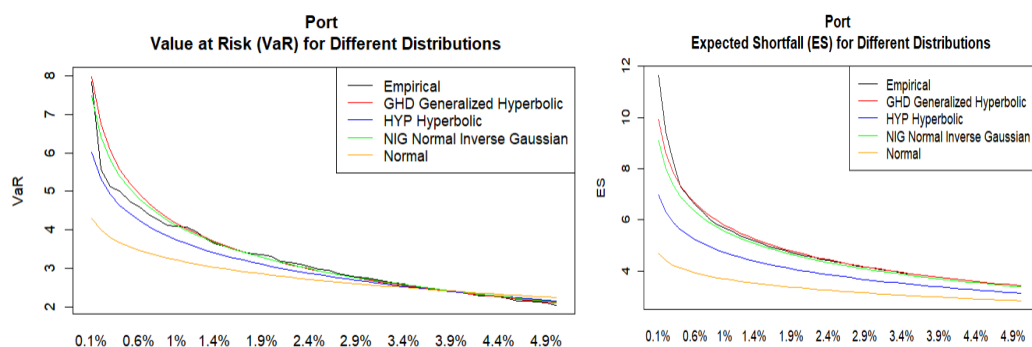
6.2. Risks - Suitable Distributions part 2

6.2.1. How does VaR and ES compare for different distributions?

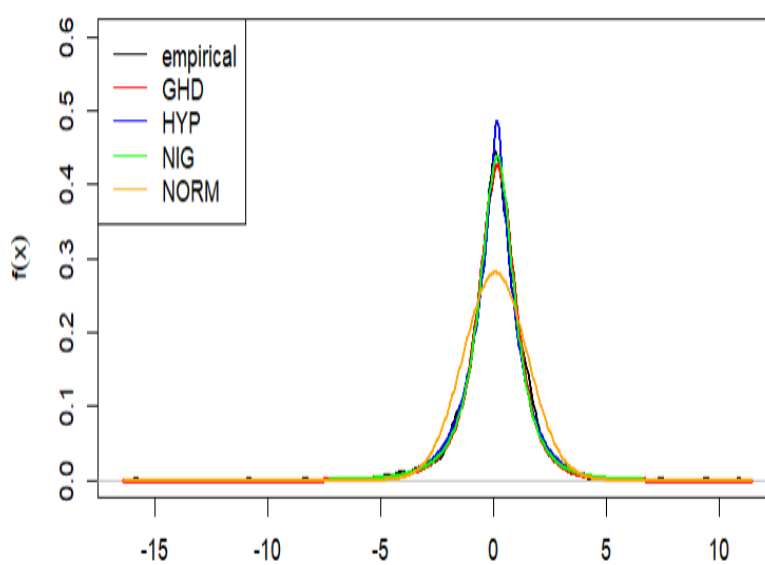
EW Portfolio- Value at Risk and Expected Shortfall

For EW portfolio period 2015 to 2023 various models were fitted and the summary for this analysis is as follows.

- EW Portfolio period 2015-2023
- EW Portfolio rebalanced yearly (12 months).



`density.default(x = PortTimeS_Port, na.rm = TRUE)`



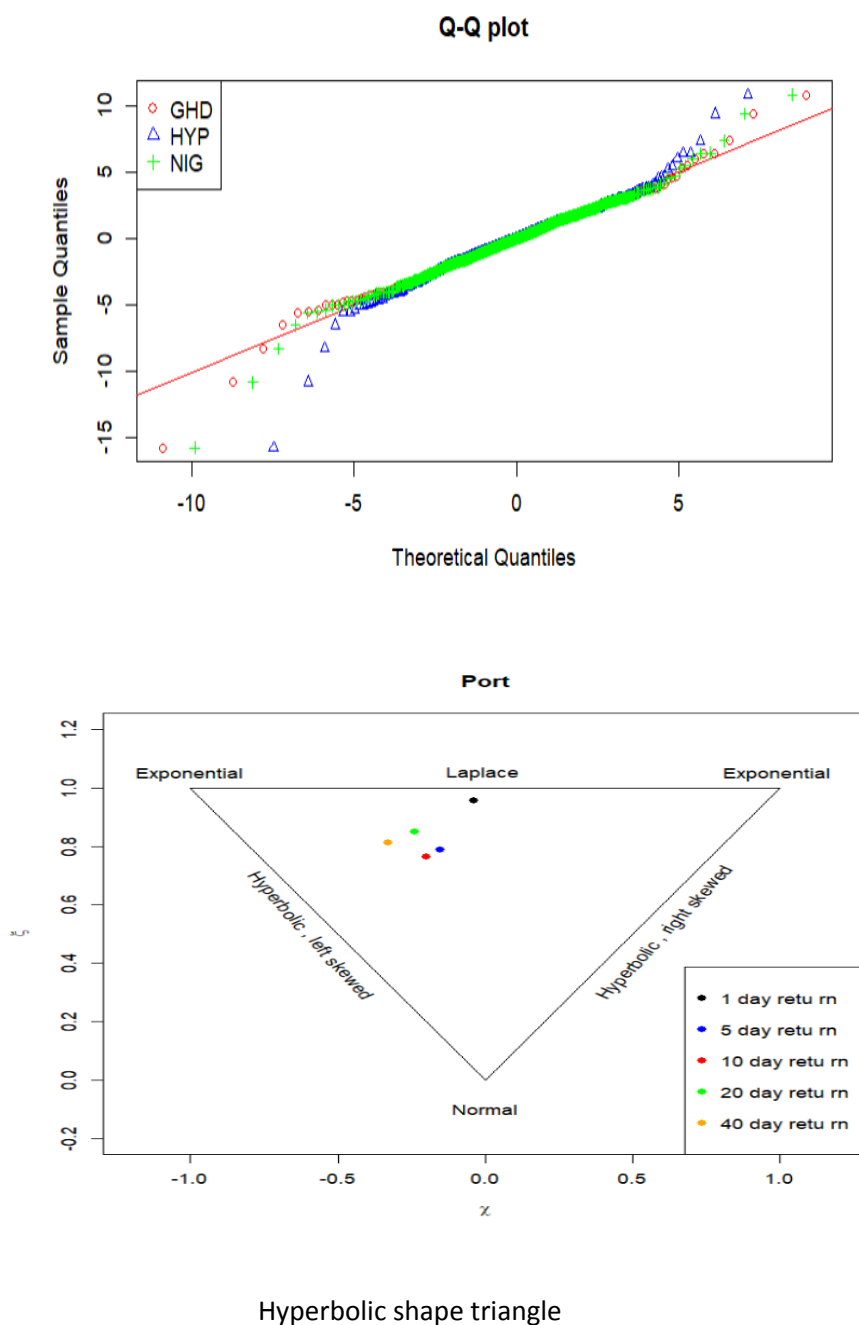


Fig 16: EW portfolio: Various distributions and their fits. Q-Q plot and Hyperbolic Shape Triangle.

When comparing VaR and ES estimates across different distributions, it's essential to assess how well

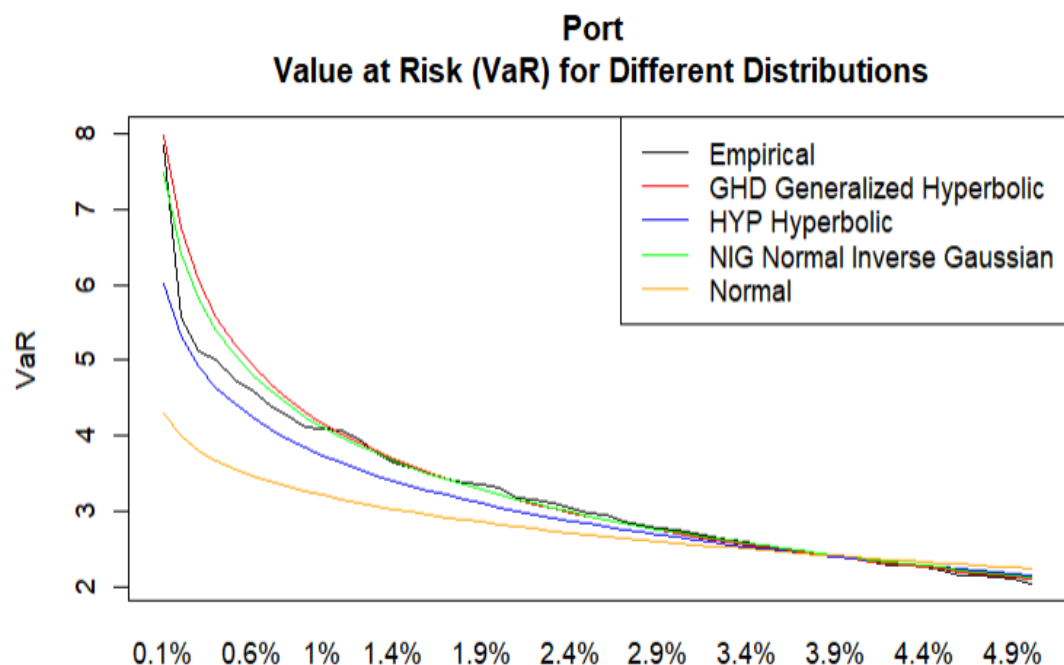
each distribution captures the tail behavior and skewness of financial returns. The choice of distribution can significantly impact the accuracy of risk estimates, particularly in extreme market conditions where tail risk becomes more pronounced. Empirical studies have shown that distributions such as the student's t-distribution or the GED may outperform the normal distribution in capturing extreme events and providing more reliable VaR and ES estimates. However, the best distributional model ultimately depends on the specific characteristics of the data and the objectives of the analysis. Conducting sensitivity analysis and model diagnostics can help identify the most appropriate distribution for estimating VaR and ES and enhance the robustness of risk management practices.

A clear pattern can be detected from this shape triangle: the lower the frequency of the return, the more it approaches the north of the triangle, hence the Laplace distribution could in principle be approximated by a normal distribution for these lower-frequency returns. The weekly returns are more heavily skewed to the left than their daily counterparts.

6.2.2. What are the best models?

EW Portfolio- Value at Risk and Expected Shortfall

- **EW Portfolio period 2015-2023**
- **EW Portfolio rebalanced yearly (12 months).**
- **Best Model: Asymmetric NIG**



```
> AIC_Port$best.model
Asymmetric Normal Inverse Gaussian Distribution:
Parameters:
alpha.bar      mu      sigma      gamma
0.5293661 0.1925684 1.3826918 -0.1080055
log-likelihood:
-3718.242
```

	model <chr>	symmetric <lg>	lambda <dbl>	alpha.bar <dbl>	mu <dbl>	sigma <dbl>	gamma <dbl>	aic <dbl>	llh <dbl>	converged <lg>	n.iter <dbl>
3	NIG	FALSE	-0.5000000	0.5293661	0.1925684	1.382692	-0.1080055	7444.484	-3718.242	TRUE	111
1	ghyp	FALSE	-0.9722317	0.4513983	0.1962518	1.391812	-0.1114128	7445.535	-3717.768	TRUE	658
5	t	FALSE	-1.5645802	0.0000000	0.1924418	1.480469	-0.1149667	7446.868	-3719.434	TRUE	175
8	NIG	TRUE	-0.5000000	0.5163785	0.1305992	1.390173	0.0000000	7448.252	-3721.126	TRUE	80
6	ghyp	TRUE	-0.9425615	0.4400544	0.1314598	1.402661	0.0000000	7449.527	-3720.763	TRUE	305
10	t	TRUE	-1.5425287	0.0000000	0.1316801	1.496534	0.0000000	7450.263	-3722.132	TRUE	122

```
> LRghdnig_Port <- lik.ratio.test(ghdFit_Port, nigFit_Port)
> LRghdnig_Port ##if TRUE there is no relationship between the data sets
$statistic
t
0.6565595
$P.value
[1] 0.3589727
$df
[1] 1
$H0
[1] TRUE

> LRghdhyp_Port <- lik.ratio.test(ghdFit_Port, hypFit_Port)
> LRghdhyp_Port ##if FALSE there is a relationship between the data sets
$statistic
L
0.0002464763
$P.value
[1] 4.575152e-05
$df
[1] 1
$H0
[1] FALSE
```

Fig 18: Asymmetric Normal Inverse Gaussian Distribution - best model for EW portfolio.

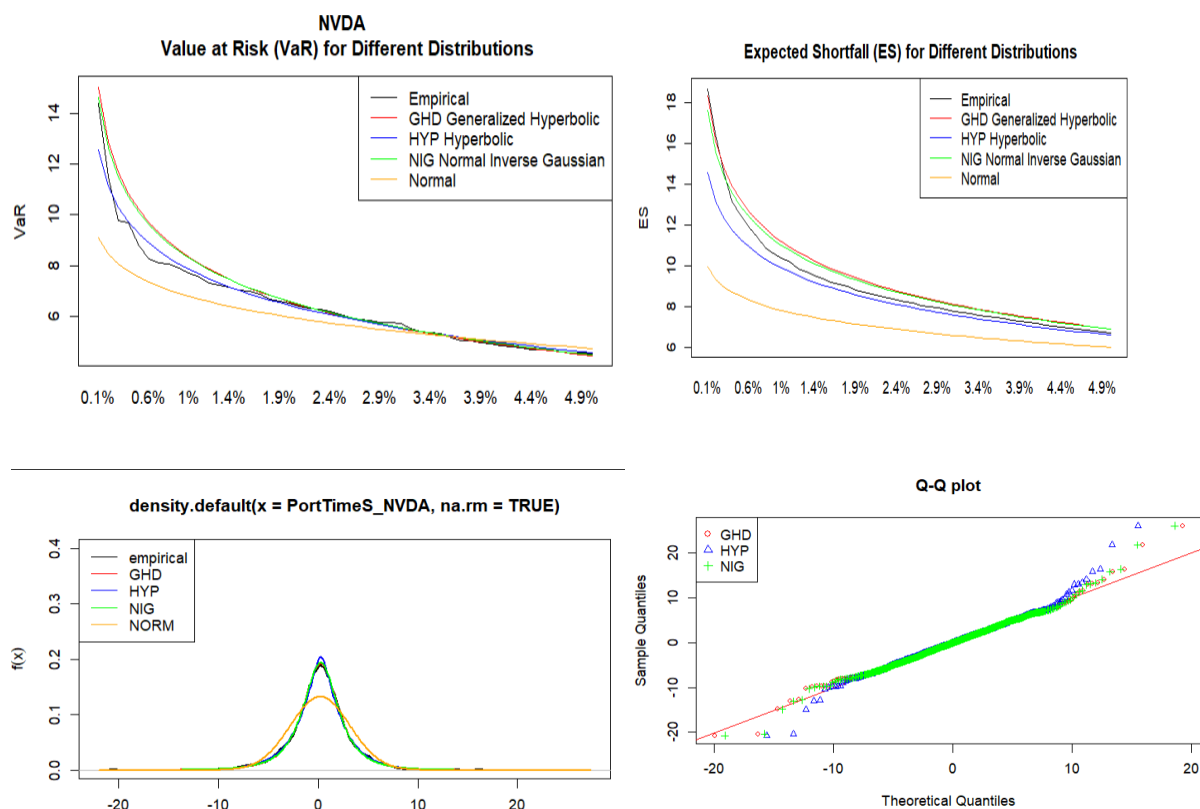
Best model was calculated using AIC table, Like ratio test and best.model function. All 3 methods yielded same results. All 3 methods pointed towards Asymmetric NIG as the best model for this analysis.

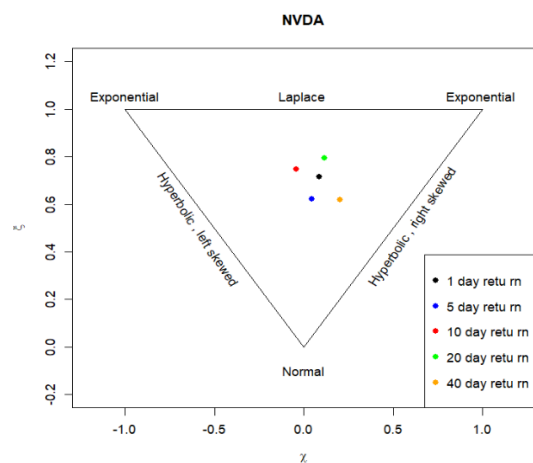
6.2.3 What are the best models per security vs entire portfolio?

Similar to previous section. VaR, ES and Best Models were calculated for each securities and the results are as follows.

NVDA - Value at Risk and Expected Shortfall

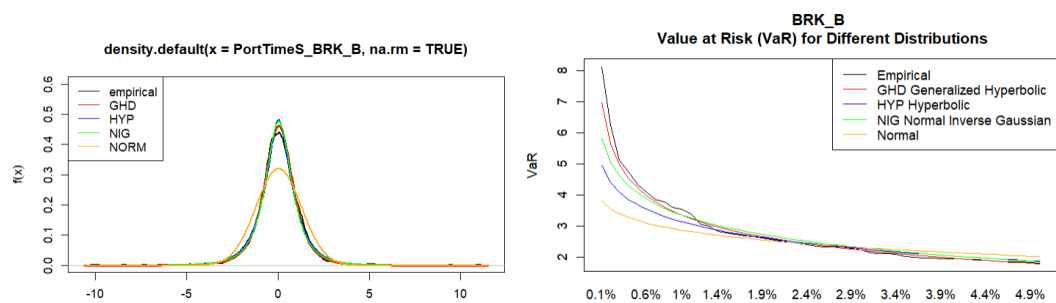
Best Model: Symmetric NIG Distribution

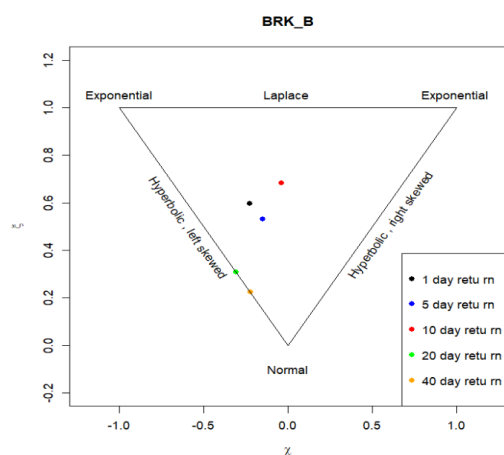
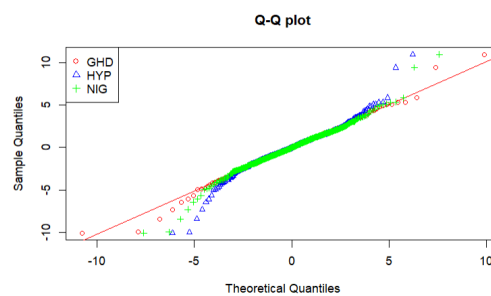
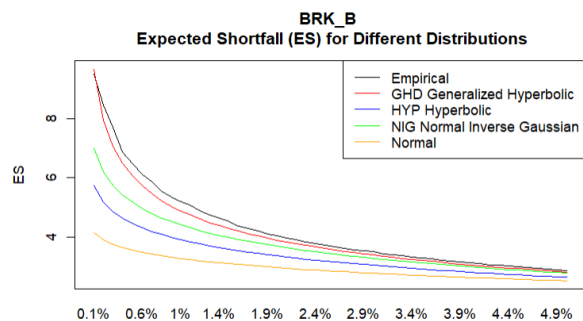




BRK.B - Value at Risk and Expected Shortfall

Best Model: Symmetric Student-t Distribution:





MCD- Value at Risk and Expected Shortfall

Best Model: Symmetric Student-t Distribution:

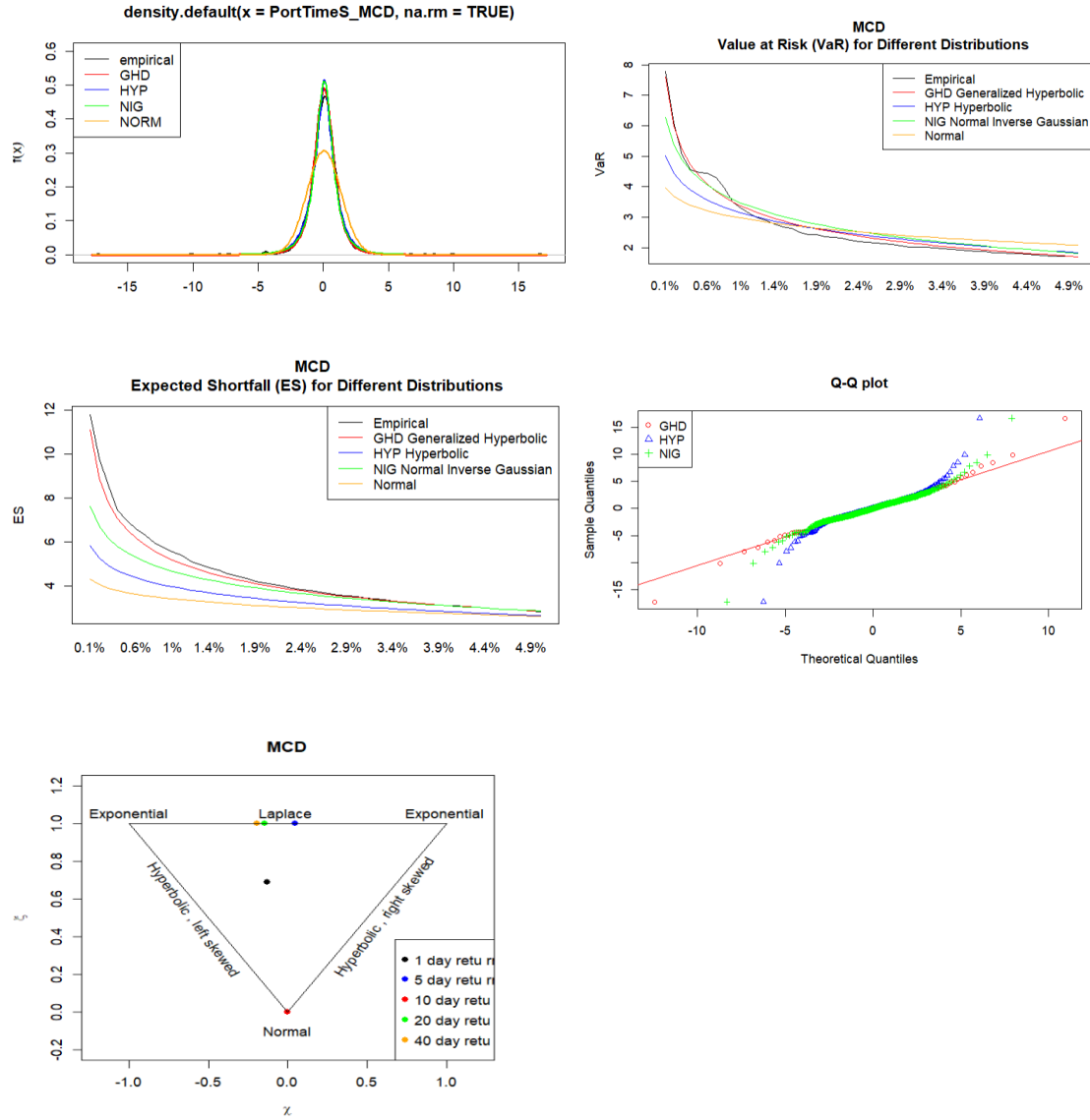


Fig 19: Individual stocks Various distributions and their fits. Q-Q plot and Hyperbolic Shape Triangle.

7. Optimization

Financial portfolio optimization involves the strategic allocation of assets to maximize returns while minimizing risk. By diversifying investments across various asset classes, industries, and geographic regions, investors aim to achieve an optimal balance between risk and return. Through mathematical techniques such as mean-variance optimization

7.1. Markowitz Mean-Variance Optimization

Markowitz Mean-Variance Optimization is a foundational framework in modern portfolio theory for constructing diversified portfolios that balance risk and return. In addressing the question of average monthly returns for each security, Markowitz optimization relies on historical return data to estimate expected returns, which serve as inputs for portfolio optimization. By analyzing historical returns, investors can assess the past performance of individual securities and incorporate this information into the optimization process to construct efficient portfolios.

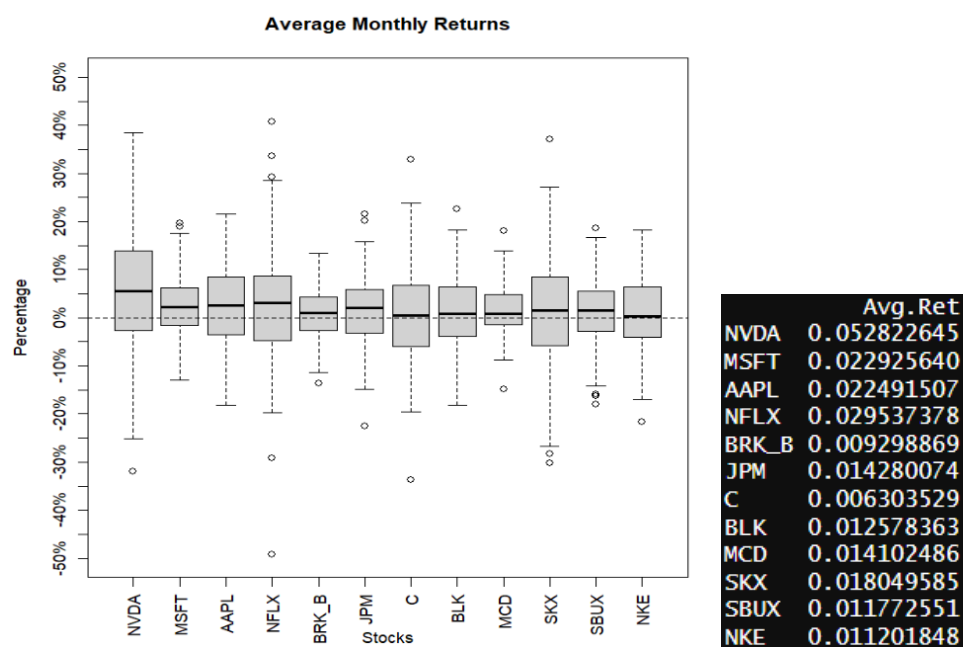
When determining the weights, risks, and returns of the Minimum Variance Portfolio, Markowitz optimization aims to identify the portfolio with the lowest possible volatility given a set of expected returns. This portfolio consists of assets with minimal covariance, effectively diversifying risk and achieving stability. By optimizing for minimum variance, Markowitz optimization ensures that investors can attain a level of risk-adjusted return that aligns with their risk tolerance and investment objectives.

Similarly, in identifying the weights, risks, and returns of the Tangency Portfolio, Markowitz optimization seeks to construct a portfolio that lies on the efficient frontier, offering the highest possible return for a

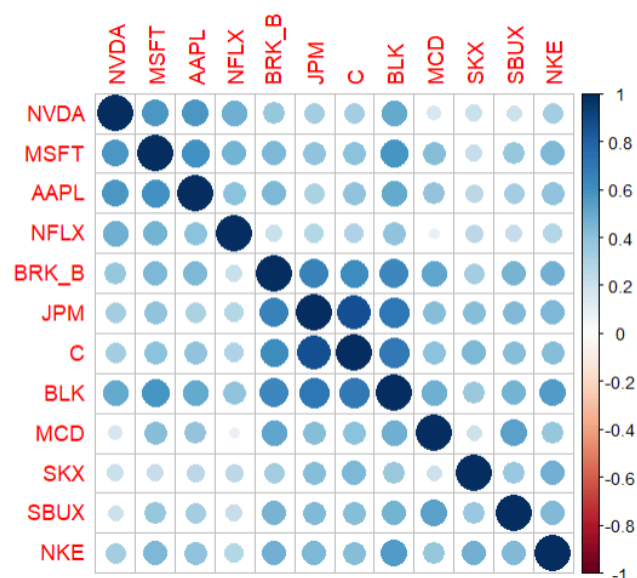
given level of risk. The Tangency Portfolio represents the optimal combination of risky assets and the risk-free asset, maximizing the Sharpe ratio and providing investors with the highest risk-adjusted return. By incorporating the risk-free rate and considering the trade-off between risk and return, Markowitz optimization enables investors to construct portfolios that strike a balance between maximizing returns and minimizing risk.

7.1.1) What are the average monthly returns for each securities?

Monthly Returns - 2015 to 2023



Correlation matrix



Variance-Covariance matrix

	NVDA	MSFT	AAPL	NFLX	BRK_B	JPM	C	BLK	MCD	SKX	SBUX	NKE
NVDA	0.0185	0.0050	0.0064	0.0083	0.0026	0.0032	0.0042	0.0052	0.0011	0.0034	0.0019	0.0034
MSFT	0.0050	0.0041	0.0032	0.0037	0.0014	0.0019	0.0025	0.0029	0.0014	0.0017	0.0016	0.0021
AAPL	0.0064	0.0032	0.0067	0.0042	0.0018	0.0019	0.0031	0.0031	0.0016	0.0025	0.0018	0.0024
NFLX	0.0083	0.0037	0.0042	0.0155	0.0014	0.0026	0.0035	0.0038	0.0005	0.0037	0.0019	0.0026
BRK_B	0.0026	0.0014	0.0018	0.0014	0.0025	0.0024	0.0030	0.0024	0.0013	0.0019	0.0016	0.0018
JPM	0.0032	0.0019	0.0019	0.0026	0.0024	0.0053	0.0060	0.0039	0.0015	0.0036	0.0021	0.0024
C	0.0042	0.0025	0.0031	0.0035	0.0030	0.0060	0.0092	0.0050	0.0020	0.0050	0.0027	0.0030
BLK	0.0052	0.0029	0.0031	0.0038	0.0024	0.0039	0.0050	0.0057	0.0018	0.0032	0.0024	0.0031
MCD	0.0011	0.0014	0.0016	0.0005	0.0013	0.0015	0.0020	0.0018	0.0025	0.0012	0.0018	0.0014
SKX	0.0034	0.0017	0.0025	0.0037	0.0019	0.0036	0.0050	0.0032	0.0012	0.0135	0.0028	0.0041
SBUX	0.0019	0.0016	0.0018	0.0019	0.0016	0.0021	0.0027	0.0024	0.0018	0.0028	0.0046	0.0022
NKE	0.0034	0.0021	0.0024	0.0026	0.0018	0.0024	0.0030	0.0031	0.0014	0.0041	0.0022	0.0054

Fig 20: Monthly returns, Correlation matrix (corplot), Variance-Covariance matrix for all securities for the entire period

7.1.2) What are the weights, risks and returns of Min Variance Portfolio?

Markowitz Mean-Variance Optimization

Min Variance Portfolio

```
> str(minvar.port)
'data.frame': 1 obs. of 14 variables:
 $ tgt.ret : num 0.0138
 $ tgt.sd : num 0.0425
 $ wgt.NVDA : num -2.57e-20
 $ wgt.MSFT : num 0.116
 $ wgt.AAPL : num 5.95e-19
 $ wgt.NFLX : num 0.0351
 $ wgt.BRK_B : num 0.363
 $ wgt.JPM : num -5.75e-18
 $ wgt.C : num 3.8e-17
 $ wgt.BLK : num -4.9e-18
 $ wgt.MCD : num 0.425
 $ wgt.SKX : num 0.00862
 $ wgt.SBUX : num 0.0261
 $ wgt.NKE : num 0.0263
```

Fig 21: Markowitz Mean-Variance Optimization - Min Variance Portfolio - all securities

7.1.3) What are the weights, risks and returns of the Tangency Portfolio?

Tangency Portfolio

```
> str(tangency.port)
'data.frame': 1 obs. of 15 variables:
 $ tgt.ret : num 0.0307
 $ tgt.sd : num 0.0682
 $ wgt.NVDA : num 0.382
 $ wgt.MSFT : num 0.154
 $ wgt.AAPL : num -2.68e-17
 $ wgt.NFLX : num 0.0277
 $ wgt.BRK_B : num 2.97e-17
 $ wgt.JPM : num -3.04e-17
 $ wgt.C : num 3.82e-18
 $ wgt.BLK : num -1.97e-17
 $ wgt.MCD : num 0.418
 $ wgt.SKX : num 0.0189
 $ wgt.SBUX : num 5.5e-18
 $ wgt.NKE : num 5.7e-17
 $ Sharpe : num 0.384
```

Fig 22: Markowitz Mean-Variance Optimization - Tangency Portfolio - all securities

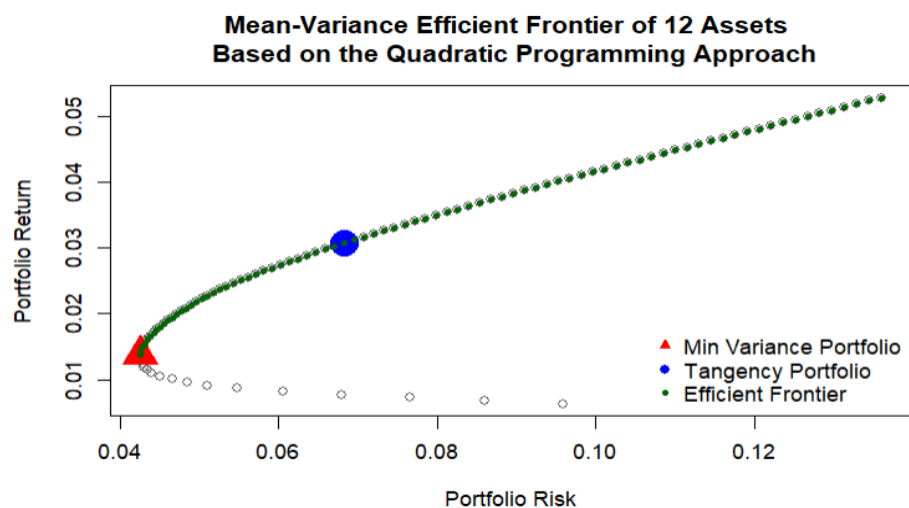
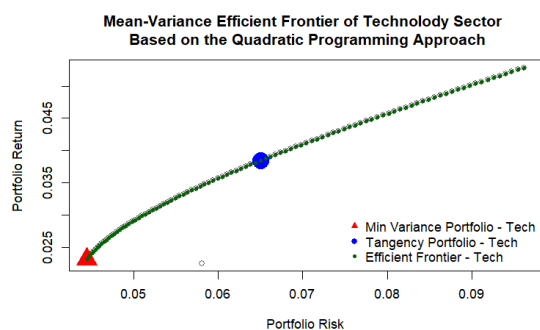


Fig 23: Markowitz Mean-Variance Optimization - Efficient frontier, min variance and Tangency portfolio in a single plot - all securities

7.1.4) Entire portfolio vs sector wise results

Sector: Technology



```
> str(tangency.port.tech)
'data.frame': 1 obs. of 7 variables:
 $ tgt.ret.tech: num 0.0384
 $ tgt.sd.tech : num 0.065
 $ wgt.NVDA   : num 0.515
 $ wgt.MSFT   : num 0.47
 $ wgt.AAPL   : num 0
 $ wgt.NFLX   : num 0.0149
 $ Sharpe     : num 0.522
```

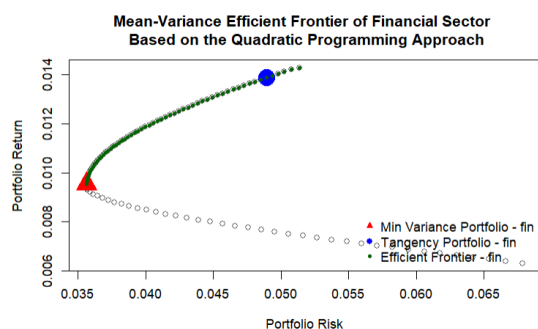
Tangency Portfolio - Technology Sector

```
> str(minvar.port.tech)
'data.frame': 1 obs. of 6 variables:
 $ tgt.ret.tech: num 0.0231
 $ tgt.sd.tech : num 0.0444
 $ wgt.NVDA   : num 0.000318
 $ wgt.MSFT   : num 0.763
 $ wgt.AAPL   : num 0.198
 $ wgt.NFLX   : num 0.0386
```

Minimum Variance Portfolio - Technology Sector

Fig 24: Markowitz Mean-Variance Optimization - Efficient frontier, min variance and Tangency portfolio - Technology sector

Sector: Financial



```
> str(tangency.port.fin)
'data.frame': 1 obs. of 7 variables:
 $ tgt.ret.fin: num 0.0139
 $ tgt.sd.fin : num 0.0489
 $ wgt.BRK_B  : num 0.0342
 $ wgt.JPM    : num 0.829
 $ wgt.C      : num 0
 $ wgt.BLK    : num 0.137
 $ Sharpe     : num 0.192
```

```
> str(minvar.port.fin)
'data.frame': 1 obs. of 6 variables:
 $ tgt.ret.fin: num 0.00953
 $ tgt.sd.fin : num 0.0356
 $ wgt.BRK_B  : num 0.946
 $ wgt.JPM    : num 0.0289
 $ wgt.C      : num 2.81e-17
 $ wgt.BLK    : num 0.0254
```

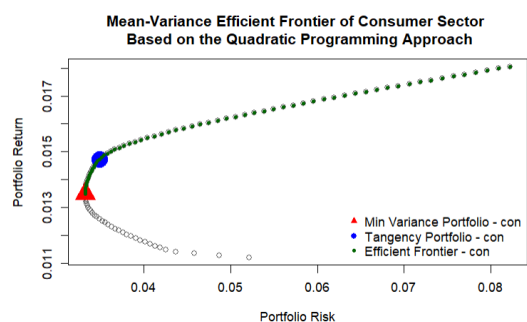
Tangency Portfolio - Financial Sector

Minimum Variance Portfolio - Financial Sector

Fig 26: Markowitz Mean-Variance Optimization - Efficient frontier, min variance and Tangency portfolio -

Financial sector

Sector: Consumer



```
> str(tangency.port.con)
'data.frame': 1 obs. of 7 variables:
 $ tgt.ret.con: num 0.0147
 $ tgt.sd.con : num 0.0349
 $ wgt.MCD   : num 0.841
 $ wgt.SKX   : num 0.159
 $ wgt.SBUX  : num 0
 $ wgt.NKE   : num 0
 $ Sharpe    : num 0.293
```

Tangency Portfolio - Consumer Sector

```
> str(minvar.port.con)
'data.frame': 1 obs. of 6 variables:
 $ tgt.ret.con: num 0.0135
 $ tgt.sd.con : num 0.0332
 $ wgt.MCD   : num 0.698
 $ wgt.SKX   : num 0.0284
 $ wgt.SBUX  : num 0.11
 $ wgt.NKE   : num 0.163
```

Minimum Variance Portfolio - Consumer Sector

Fig 26: Markowitz Mean-Variance Optimization - Efficient frontier, min variance and Tangency portfolio -

Consumersector

7.1.5. Markowitz Mean-Variance Optimization results summary:

Min-Var	EW Portfolio	Technology	Financial	Consumer
Risk	0.0425	0.0444	0.0356	0.0332
Return	0.0138	0.0231	0.00953	0.0135

Fig 27: table of Minimum- Variance EW Portfolios entire portfolio vs individual sectors

Tangency	EW Portfolio	Technology	Financial	Consumer
Risk	0.0425	0.065	0.0489	0.0349
Return	0.0138	0.0384	0.0139	0.0147

Fig 28: table of Tangency EW Portfolios entire portfolio vs individual sectors

Technology sector has the highest risk and highest returns.

7.2 Diversification Reconsideration

The importance of Diversification Reconsideration in portfolio analysis is assessed by examining the marginal risk contributions of individual securities or sectors and evaluating the effectiveness of diversification strategies. When performing an analytical review on the marginal risk contributions for individual securities, we intend to evaluate how each asset contributes to the overall risk of the portfolio. Securities with higher marginal risk contributions may warrant closer scrutiny or potential adjustments to achieve better diversification.

Similarly, when assessing the marginal risk contributions for individual sectors, Diversification Reconsideration is utilized to identify sectors that disproportionately influence portfolio risk. Our understanding of the contributions of different sectors to overall portfolio risk assisted in identifying how investors rebalance their portfolios to achieve better diversification and mitigate concentration risk.

In comparing different portfolios with each other, Diversification Reconsideration involves evaluating the trade-offs between risk and return across various portfolio compositions. Portfolios with lower marginal risk contributions and higher diversification benefits are typically preferred, as they offer a more efficient risk-return trade-off. By considering factors such as portfolio volatility, expected returns, and diversification benefits, investors can make informed decisions to construct portfolios that align with their risk tolerance and investment objectives. Diversification Reconsideration thus serves as a valuable tool for optimizing portfolio performance and enhancing risk management strategies.

- ☐ **GMV** The global-minimum variance portfolio.
- ☐ **MDP** The most diversified Portfolio,
- ☐ **ERC** The equal-risk contribution Portfolio
- ☐ **MTD** Minimum tail-dependent portfolio

GMV (Global-Minimum Variance Portfolio):

The GMV portfolio represents a portfolio allocation strategy aimed at minimizing the portfolio's overall variance or volatility. It seeks to construct a portfolio that achieves the lowest possible level of risk given a set of assets or investments. Mathematically, it is the portfolio with the smallest possible variance among all feasible portfolios.

MDP (Most Diversified Portfolio):

The MDP, also known as the Maximum Diversification Portfolio, is a portfolio construction approach that seeks to maximize diversification. It aims to allocate weights to assets in such a way that the portfolio has the highest possible diversification ratio or the lowest possible concentration risk. The MDP typically achieves this by allocating weights inversely proportional to the assets' pairwise correlations or covariances.

ERC (Equal-Risk Contribution Portfolio):

The ERC portfolio is a portfolio optimization technique that aims to balance the risk contribution of each asset to the overall portfolio risk equally. In other words, every asset in the portfolio contributes an equal amount of risk. This approach ensures that no single asset dominates the portfolio's risk profile, promoting diversification.

MTD (Minimum Tail-Dependent Portfolio):

The MTD portfolio is constructed to minimize tail dependence, which refers to the extent to which extreme events in one asset are correlated with extreme events in another asset. This portfolio aims to reduce the overall tail risk of the portfolio by selecting assets with lower tail dependencies or by optimizing the allocation to minimize tail dependence. It is particularly relevant for investors concerned with extreme market events and tail risk management.

7.2.1) What are the marginal risk contributions for individual securities?

Period = 2015 to 2023

Portfolio rebalance frequency = 1 year (12 months)

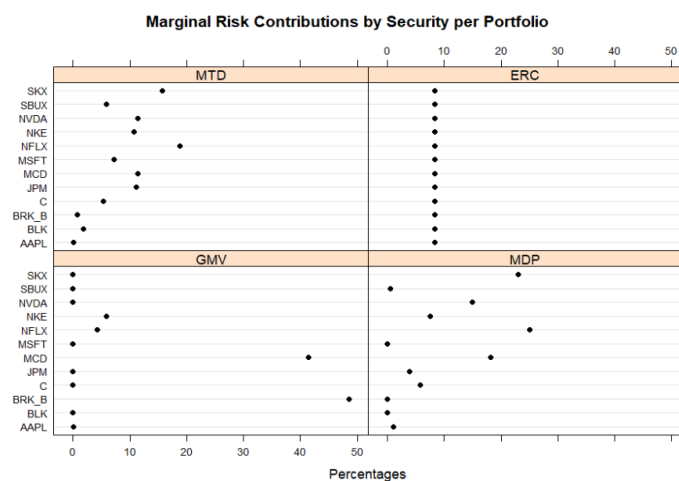


Fig 29: marginal risk contributions for individual securities

7.2.2) What are the marginal risk contributions for individual sectors?

Period = 2015 to 2023

Portfolio rebalance frequency = 1 year (12 months)

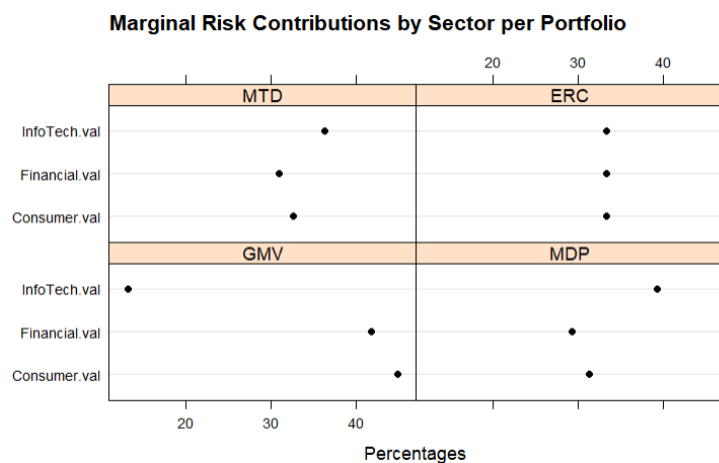
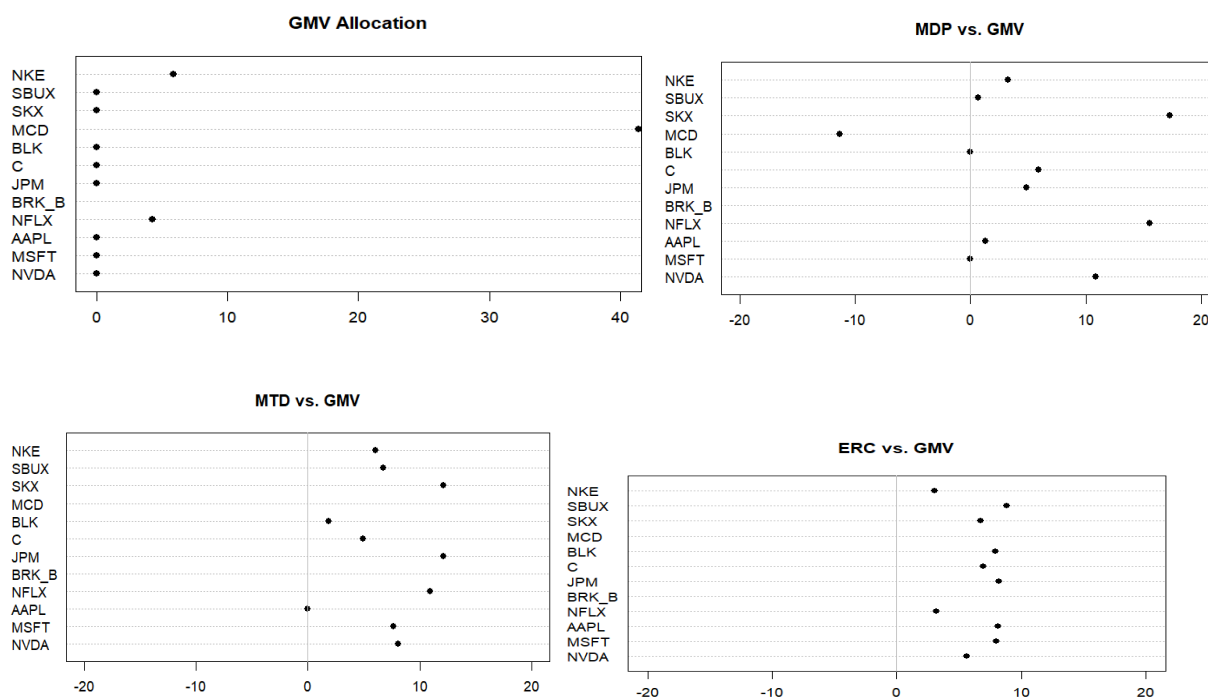


Fig 29: marginal risk contributions for individual sectors

7.2.3) How do different portfolios compare with each other?

Period = 2015 to 2023

Portfolio rebalance frequency = 1 year (12 months)



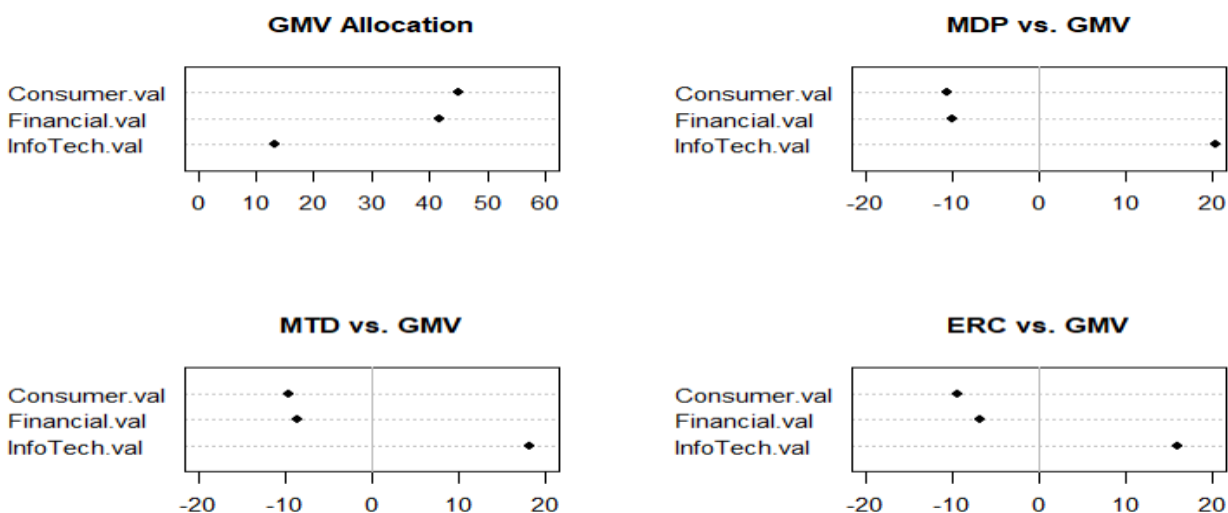


Fig 30: marginal risk contributions comparison for various portfolios

8. Summary and Conclusion:

- Primary sources of risk in a diversified portfolio include market risk, which can impact overall portfolio performance through losses or reduced returns.
- VaR and ES provide quantitative measures of portfolio risk. With VaR focusing on potential losses within a given confidence interval and ES addressing the tail risk beyond VaR, ES and VaR was calculated using various distributions.
- Gaussian Distribution, Generalized Hyperbolic, Hyperbolic and Normal Inverse Gaussian were considered for modeling.
- Key factors for constructing a risk model include asset correlations, volatilities, and distributions, while factor analysis identifies underlying risk drivers and enables diversification benefits. All factors were analyzed in various sections throughout the report.

- Portfolio optimization techniques were utilized to balance risk and return objectives, with mean-variance optimization providing an efficient frontier. Alternative objectives like maximum diversification portfolios were also calculated .
- Asset allocation plays a crucial role in managing risk through diversification across asset classes, and risk parity strategies aim to allocate risk equally among assets for optimal risk-adjusted returns. This was considered in the diversification reconsideration section.

9. Future scope and Recommendations:

9.1 Future Scope:

Following tools could be implemented to refine models and get more control over the weighting criteria.

Factor Modeling:

- Tools: Factor models, principal component analysis (PCA), APT (Arbitrage Pricing Theory).
- Application: Factor models help in understanding the drivers of asset and sector returns. By decomposing returns into systematic factors (e.g., market risk, sector-specific factors) and idiosyncratic factors, analysts can better assess sector exposures and manage risk. PCA can be used to identify underlying factors that explain the co-movements of asset returns within and across sectors.

Monte Carlo Simulation:

- Tools: Monte Carlo simulation, stochastic modeling.
- Application: Monte Carlo simulation can be used to model the uncertainty and variability in portfolio returns. By simulating multiple scenarios based on random sampling of input parameters (e.g., asset returns, volatility, correlations), analysts can assess the potential

range of outcomes and evaluate the impact of different factors on portfolio performance and sector allocations.

Scenario Analysis:

- Tools: Scenario planning, stress testing.
- Application: Scenario analysis involves modeling the portfolio's performance under different economic or market scenarios. By considering various macroeconomic, geopolitical, or industry-specific scenarios, analysts can assess the resilience of the portfolio to different types of risks and identify potential vulnerabilities. Stress testing involves subjecting the portfolio to extreme but plausible scenarios to evaluate its robustness and potential losses.

Machine Learning:

- Tools: Machine learning algorithms (e.g., regression, classification, clustering).
- Application: Machine learning techniques can be applied to analyze large datasets and extract valuable insights for portfolio modeling. For example, classification algorithms can be used to categorize assets into sectors based on their characteristics, while clustering algorithms can identify patterns and group assets with similar risk-return profiles. Regression models can also be used for predictive analytics to forecast sector returns or portfolio performance.

ESG factors :

- Integrating ESG factors into portfolio risk assessments can help mitigate long-term sustainability risks and identify opportunities aligned with societal and environmental goals.

Multi-asset portfolio optimization

- Multi-asset portfolio optimization can be utilized to develop models that optimize risk and return across multiple asset classes, including alternatives like private equity or real assets. We can also consider non-traditional assets or alternative assets like cryptocurrencies or crowdfunded investments.

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- [9] Rstudio library documentation.

7. Appendix:

R-Codes

<https://github.com/Saurabh3494/ANLY-515-Risk-Modeling-Project>