

1 Design of Doubly Reinforced Beam

1.1 Preliminary Design

Sectional characteristic:

Width of the beam(b) = 230mm

Overall depth of the beam(D) = 400mm

Width of the support(w) = 500mm

Clear cover = 30mm

Diameter of the tension bar(d_t) = 25mm

Diameter of the compression bar(d_c) = 20mm

Diameter of the stirrup bar(d_s) = 10mm

Effective depth(d) = $D - \text{clear cover} - \frac{d_t}{2} - d_s$

$$d = 400 - 30 - \frac{25}{2} - 10$$

$$d = 335mm$$

Clear span(l) = 30m

$$\text{Effective span}(l_{eff}) = \min \left\{ \begin{array}{l} l + d = 30 + 335 \\ l + w = 30 + 500 \end{array} \right\} = 6.335m$$

Material characteristic:

Grade of concrete(f_{ck}) = $30 \frac{N}{mm^2}$

Grade of steel(f_y) = $500 \frac{N}{mm^2}$

Grade of stirrup(f_s) = $415 \frac{N}{mm^2}$

Grading characteristic:

$$\text{Dead load}(DL) = 2.3 \frac{kN}{m}$$

$$\text{Imposed load}(LL) = 30 \frac{kN}{m}$$

$$\text{Total Factored load}(w_u) = 1.5 \times (DL + LL) = 48.45 \frac{kN}{m}$$

1.2 Preliminary Design

Shear Force(V_u) :

$$V_u = \frac{w_u l_{eff}}{2}$$

$$V_u = \frac{48.45 \times 6.335}{2}$$

$$V_u = 153.47kN$$

Ultimate bending moment:

$$M_u = \frac{w_u l_{eff}^2}{8}$$

$$M_u = \frac{48.45 \times 6.335^2}{8}$$

$$M_u = 243.05kNm$$

Limiting bending moment:

From similiar triangle

$$\frac{x_{ulim}}{0.0035} = \frac{d - x_{ulim}}{0.002 + \frac{0.87f_y}{E_s}}$$

$$x_{ulim} = 154.86mm$$

$$M_{ulim} = 0.36f_{ck}bx_{ulim}(d - 0.42x_u)$$

$$M_{ulim} = 0.36 \times 30 \times 230 \times 154.86 \times (335 - 0.42 \times 154.86)$$

$$M_{ulim} = 103.84kNm < M_u$$

So, Section should be designed as Doubly reinforced beam.

1.2.1 Tension Reinforcement

Total tensile reinforcement:

$$A_{st} = A_{stlim} + \Delta A_{st}$$

Where A_{stlim} is due to tensile steel corresponding to M_{ulim}

and ΔA_{st} is corresponding to ΔM_u

Limiting Tensile Reinforcement:

$$A_{stlim} = 0.5 \frac{f_{ck}}{f_y} bd \left[1 - \sqrt{1 - \frac{4.6M_{ulim}}{f_{ck}bd^2}} \right]$$

$$A_{stlim} = 0.5 \times \frac{30}{500} \times 230 \times 335 \times \left[1 - \sqrt{1 - \frac{4.6 \times 103.84}{30 \times 230 \times 335^2}} \right]$$

$$A_{stlim} = 884.74mm^2$$

ΔM_u :

$$\Delta M_u = 0.87 f_y \Delta A_{st} (d - d')$$

$$243.05 - 103.84 = 0.87 \times 500 \times \Delta A_{st} \times (335 - 65)$$

$$\Delta A_{st} = 1185.24mm^2$$

Where d' is effective cover

$$A_{st} = 884.74 + 1185.24$$

$$A_{st} = 2069.97mm^2$$

Number of Bars:

$$n = \frac{A_{st}}{\frac{\pi}{4} d_t^2}$$

$$n = \frac{2069.97}{\frac{\pi}{4} \times 25^2}$$

$$n = 4.22$$

So provide 5 bars of 25 mm dia as tensile reinforcement.

Percentage reinforcement:

$$pt = \frac{n \times \frac{\pi}{4} d_t^2}{bd}$$

$$pt = \frac{5 \times \frac{\pi}{4} \times 25^2}{230 \times 335}$$

$$pt = 3.19\%$$

Check:

$$A_{stmax} = 0.04bD$$

$$A_{stmax} = 0.04 \times 230 \times 400$$

$$A_{stmax} = 3680.0 > A_{st}$$

$$A_{stmin} = \frac{0.85bd}{f_y}$$

$$A_{stmin} = \frac{0.85 \times 230 \times 335}{500}$$

$$A_{stmin} = 130.99 < A_{st}$$

OK

OK

1.2.2 Compression Reinforcement

$$\epsilon_{sc} = \left(\frac{x_{ulim} - d'}{x_{ulim}} \right) \times 0.0035$$

$$\epsilon_{sc} = \left(\frac{154.86 - 65}{X_{ulim}} \right) \times 0.0035$$

$$\epsilon_{sc} = 0.00203$$

$$f_{sc} = 375.26 \frac{N}{mm^2}$$

(SP-16 Table A)

$$(f_{sc} - 0.45f_{ck}) A_{sc} = 0.87f_y \Delta A_{st}$$

$$(375.26 - 0.4530) \times A_{sc} = 0.87 \times 500 \times 1185.24$$

$$A_{sc} = 1425.18 mm^2$$

Number of Bars:

$$n = \frac{A_{sc}}{\frac{\pi}{4} d_c^2}$$

$$n = \frac{1425.18}{\frac{\pi}{4} \times 20^2}$$

$$n = 4.54$$

So provide 5 bars of 20 mm dia as compressive reinforcement.

Percentage reinforcement:

$$pc = \frac{n \times \frac{\pi}{4} d_c^2}{bd}$$

$$pc = \frac{5 \times \frac{\pi}{4} \times 20^2}{230 \times 335}$$

$$pc = 2.04\%$$

Check:

$$A_{scmax} = 0.04bD$$

$$A_{scmax} = 0.04 \times 230 \times 400$$

$$A_{scmax} = 3680.0 > A_{sc}$$

OK

1.2.3 Shear Reinforcement

Design shear force(V_d) = $137.23kN$

Nominal shear stress(τ_v) = $\frac{V_d}{bd}$

$$\tau_v = \frac{137.23 \times 1000}{230 \times 335}$$

$$\tau_v = 1.78 \frac{N}{mm^2}$$

Max shear capacity(τ_{max}) = $3.6 \frac{N}{mm^2}$

Permissible shear capacity(τ_c) = $0.6 \frac{N}{mm^2}$

As $\tau_v > \tau_c$

So shear reinforcement is designed for

$$Shear\ force(V_{us}) = (\tau_v - \tau_c) bd$$

$$V_{us} = (1.78 - 0.6) \times 230 \times 335 \times \frac{1}{1000}$$

$$V_{us} = 91.0kN$$

$$A_{sv} = n \times \frac{\pi}{4} d_s^2$$

here n is number of legs of stirrups

$$A_{sv} = 2.0 \times \frac{\pi}{4} 10^2$$

$$A_{sv} = 157.08mm^2$$

Spacing of stirrups

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$S_v = \frac{0.87 \times 415 \times 157.08 \times 335}{91.0}$$

$$S_v = 210.0$$

$$Spacing \leq \min \left\{ \begin{array}{l} S_v = 210.0 \\ 300\ mm \\ 0.75d = 0.75 \times 335 \\ \frac{A_{sv}}{bS_v} \geq \frac{0.4}{0.87f_y} = \frac{157.08}{230 \times S_v} \geq \frac{0.4}{0.87 \times 500} \end{array} \right\}$$

$$Spacing = 200.0mm$$

OK

IS 456 : Table 19

So provide 2.0-legged 10mm stirrups @ 200.0mm c/c