1 Design of Doubly Reinforced Beam

1.1 Preliminary Design

Sectional characteristic:

Width of the beam (b) = 230mm

Overall depth of the beam (D) = 350mm

Width of the support (w) = 230mm

Clear cover = 30 mm

Diameter of the tension bar (d_t) = 20mm

Diameter of the compression bar (d_c) = 12mm

Diameter of the stirrup bar (d_s) = 8mm

Effective depth(d) = D-clear cover-$d_{t\overline{2-d_s}}$

$$d = 350 - 30 - \frac{20}{2} - 8$$

$$d = 292mm$$

Clear span(l) = 8m

Effective span(l_{eff}) = $min \begin{Bmatrix} l+d = 8+292 \\ l+w = 8+230 \end{Bmatrix} = 4.23m$

Material characteristic:

Grade of concrete(f_{ck}) = $30\frac{N}{mm^2}$

$Grade\ of\ steel(f_y)$ = $500\frac{N}{mm^2}$

$Grade\ of\ stirrup(f_s)$ = $415\frac{N}{mm^2}$

Grading characteristic:

$$Dead\ load(DL) = 2.01\frac{kN}{m}$$

$$Imposed\ load(LL) = 0\frac{kN}{m}$$

$$Total\ Factored\ load(w_u) = 1.5 \times (DL + LL) = 3.02\frac{kN}{m}$$

1.2 Preliminary Design

Shear Force(V_u):

$$V_u = \frac{w_u l_{eff}}{2}$$

$$V_u = \frac{3.02 \times 4.23}{2}$$

$$V_u = 21.38kN$$

Ultimate bending moment:

$$M_u = \frac{w_u {l_{eff}}^2}{8}$$

$$M_u = \frac{3.02 \times 4.23^2}{8}$$

$$M_u = 96.75kNm$$

Limiting bending moment:

From similiar triangle

$$\frac{x_{ulim}}{0.0035} = \frac{d - x_{u,lim}}{0.002 + \frac{0.87 f_y}{E_s}}$$

$$x_{ulim} = 134.98mm$$

$$M_{ulim} = 0.36 f_{ck} b x_{ulim} \left(d - 0.42 x_u\right)$$

$$M_{ulim} = 0.36 \times 30 \times 230 \times 134.98 \times (292 - 0.42 \times 134.98)$$

$$M_{ulim} = 78.9kNm < M_u$$

So, Section should be designed as Doubly reinforced beam.

1.2.1 Tension Reinforcement

Total tensile reinforcement:

$$A_{st} = A_{stlim} + \Delta A_{st}$$

Where A$_{stlim}$ *is due to tensile steel corresponding to* M_{ulim}

and ΔA_{st} *is corresponding to* ΔM_u

Limiting Tensile Reinforcement:

$$A_{stlim} = 0.5 \frac{f_{ck}}{f_y} bd \left[1 - \sqrt{1 - \frac{4.6 M_{ulim}}{f_{ck} b d^2}}\right]$$

$$A_{stlim} = 0.5 \times \frac{30}{500} \times 230 \times 292 \times \left[1 - \sqrt{1 - \frac{4.6 \times 78.9}{30 \times 230 \times 292^2}}\right]$$

$$A_{st,lim} = 771.17mm^2$$

ΔM_u :

$$\Delta M_u = 0.87 f_y \Delta A_{st} \left(d - d'\right)$$

$$96.75 - 78.9 = 0.87 \times 500 \times \Delta A_{st} \times (292 - 58)$$

$$\Delta A_{st} = 175.41mm^2$$

Where d' is effective cover

$$A_{st} = 771.17 + 175.41$$

$$A_{st} = 946.58mm^2$$

Number of Bars:

$$n = \frac{A_{st}}{\frac{\pi}{4} d_t^2}$$

$$n = \frac{946.58}{\frac{\pi}{4} \times 20^2}$$

$$n = 3.01$$

So provide 4 bars of 20 mm dia as tensile reinforcement.

Percentage reinforcement:

$$pt = \frac{n \times \frac{\pi}{4} d_t^2}{bd}$$

$$pt = \frac{4 \times \frac{\pi}{4} \times 20^2}{230 \times 292}$$

$$pt = 1.87\%$$

Check:

$$A_{stmax} = 0.04bD$$

$$A_{stmax} = 0.04 \times 230 \times 350$$

$$A_{stmax} = 3220.0 > A_{st}$$

OK

$$A_{stmin} = \frac{0.85bd}{f_y}$$

$$A_{stmin} = \frac{0.85 \times 230 \times 292}{500}$$

$$A_{stmin} = 114.17 < A_{st}$$

OK

1.2.2 Compression Reinforcement

$$\epsilon_{sc} = \left(\frac{x_{ulim} - d'}{x_{ulim}}\right) \times 0.0035$$

$$\epsilon_{sc} = \left(\frac{134.98 - 58}{Xu_l im}\right) \times 0.0035$$

$$\epsilon_{sc} = 0.002$$

$$f_{sc} = 372.83 \frac{N}{mm^2}$$

$$\left(f_{sc} - 0.45 f_{ck}\right) A_{sc} = 0.87 f_y \Delta A_{st}$$

$$(372.83 - 0.4530) \times A_{sc} = 0.87 \times 500 \times 175.41$$

$$A_{sc} = 212.35 mm^2$$

Number of Bars:

$$n = \frac{A_{sc}}{\frac{\pi}{4} d_c^2}$$

$$n = \frac{212.35}{\frac{\pi}{4} \times 12^2}$$

$$n = 1.88$$

So provide 2 bars of 12 mm dia as compressive reinforcement.

Percentage reinforcement:

$$pc = \frac{n \times \frac{\pi}{4} d_c^2}{bd}$$

$$pc = \frac{2 \times \frac{\pi}{4} \times 12^2}{230 \times 292}$$

$$pc = 0.34\%$$

Check:

$$A_{scmax} = 0.04 bD$$

$$A_{scmax} = 0.04 \times 230 \times 350$$

$$A_{scmax} = 3220.0 > A_{sc}$$

OK

1.2.3 Shear Reinforcement

Design shear force(V_d) = $20.5kN$

Nominal shear stress(τ_v) = $\frac{V_d}{bd}$

$$\tau_v = \frac{20.5 \times 1000}{230 \times 292}$$

$$\tau_v = 0.31\frac{N}{mm^2}$$

Max shear capacity(τ_{max}) = $3.6\frac{N}{mm^2}$

Permissible shear capacity(τ_c) = $0.51\frac{N}{mm^2}$

As $\tau_v < \tau_c$

So Nominal shear stirrups are provided