Q.1

$T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2 \quad f(n) = n^2$

a & b are constant & $f(n)$ is the function

master's theorem is applicable

$$c = \log_b a$$
$$= \log_2 3 = 1.58$$

$$n^c = n^{1.58} < n^2$$
$$n^c < f(n)$$

$$T(n) = O(n^2)$$

Q.2

$T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2 \quad f(n) = n^2$

$c = \log_b a = \log_2 4 = 2$

$$n^c = n^2$$
$$n^c = f(n)$$
$$T(n) = \theta(n^2 \log n)$$

Q.3

$T(n) = T(n/2) + 2^n$

$a = 1 \quad b = 2 \quad f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$$n^c = n^0 = 1$$
$$f(n) > n^c$$
$$T(n) = O(2^n)$$

Q4

$T(n) = 2^n T(n/2) + n^n$

$a = 2^n \quad b = 2 \quad f(n) = n^n$

∵ a is not constant, its value depend on n

∴ master's theorem not applicable

Q5

$T(n) = 16\ T(n/4) + n$

$a = 16 \quad b = 4 \quad f(n) = n$

$c = \log_b a = \log_4 16 = 2$

$n^c > f(n)$

$T(n) = \Theta(n^2)$

Q6

$T(n) = 2\ T(n/2) + n \log n$

$a = 2 \quad b = 2 \quad f(n) = n \log n$

$c = \log_b a = \log_2 2 = 1$

$n^c = n$

$f(n) > n^c$

$T(n) = \Theta(n \log n)$

Q7

$T(n) = 2\ T(n/2) + n/\log n$

$a = 2 \quad b = 2 \quad f(n) = \frac{n}{\log(n)}$

$c = \log_2 2 = 1$

$n^c = n$

non Polynomial diff b/w n^c & $f(n)$

∴ masters theorem not applicable

Q8 $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$f(n) > n^c$

$T(n) = \Theta(n^{0.51})$

Q9 $T(n) = 0.5\,T(n/2) + \frac{1}{n}$

$a < 1$ ∴ master's theorem not applicable

Q10 $T(n) = 16\,T(n/4) + n!$

$a = 16 \quad b = n \quad f(n) = n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$f(n) > n^c$

$T(n) = \Theta(n!)$

Q11 $T(n) = 4T(n/2) + \log n$

$a = 4 \quad b = 2 \quad f(n) = \log n$

$c = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

$T(n) = \Theta(n^2)$

Q12 $T(n) = \sqrt{n}\ T(\frac{n}{2}) + \log n$

a - is not constant, therefore master's theorem not applicable

Q13 $T(n) = 3T(n/2) + n$

$a = 3 \quad b = 2 \quad f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58} > f(n)$

$T(n) = O(n^{1.58})$

Q14 $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3 \quad b = 3 \quad f(n) = \sqrt{n}$

$c = \log_b a = 1$

$n^c = n > \sqrt{n}$

$T(n) = O(n)$

Q15 $T(n) = 4(n/2) + c \cdot n$

$a = 4 \quad b = 2 \quad f(n) = c \cdot 2$

$n^c = n^2 > f(n)$

$T(n) = O(n^2)$

Q16 $T(n) = 3T(n/4) + n\log n$

$a = 3 \quad b = 4 \quad f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.78$

$n^c = n^{0.78} < f(n)$

$T(n) = \Theta(n \log n)$

Q17 $T(n) = 3T(n/3) + n/2$

$a = 3 \quad b = 3 \quad f(n) = n/2$

$c = \log_b a = 1$

$n^c = n > f(n)$

$T(n) = \theta(n)$

Q18 $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63} < f(n)$

$T(n) = \theta(n^2 \log n)$

Q19 $T(n) = 4T(n/2) + n\log n$

$c = \log_2 4 = 2$

$n^c = n^2 > f(n)$

$T(n) = O(n^2)$

Q20 $T(n) = 64T(n/8) + n^2 \log n$

$a = 64 \quad b = 8 \quad f(n) = n^2 \log n$

$c = \log_b a = 2$

$n^c = n^2 < f(n)$

$T(n) = \theta(n^2 \lg n)$

Q21 $T(n) = 7\,T\left(\frac{n}{3}\right) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77} < f(n)$

$T(n) = \Theta(n^2)$

Q 22 $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

$\because$ $f(n)$ is not a regular function

$\therefore$ Master's theorem cannot be applied.