

Q 1

for (int n)

{ int j=1, i=0

while (i &lt; n)

{ i = i + j

j++ }

}

$$\frac{k(k+1)}{2} = n$$

$$k^2 = n$$

$$k = \sqrt{n}$$

$$T.C. = O(\sqrt{n})$$

Q 2

$$T(0) = 0$$

$$T(1) = 0$$

$$T(n) = T(n-1) + T(n-2) + 1$$

$$\text{Let } T(n-1) = T(n-2)$$

$$T(n) = 2T(n-1) + 1$$

Using back substn.

$$\begin{aligned} T(n) &= 2 \times 2 (T(n-2) + 1) + 1 \\ &= 4 (T(n-2)) + 3 \end{aligned}$$

$$\begin{aligned} T(n-2) &= 2 T(n-3) + 1 \\ &= 2 (2 (2 (T(n-3) + 1) + 1) + 1) + 1 \\ &= 8 T(n-3) + 7 \end{aligned}$$

$$T(n) = 2^k T(n-k) + 2^k - 1$$

$$T(0) = 0$$

$$n-k = 0$$

$$n = k$$

$$\begin{aligned} T(n) &= 2^n T(n-n) + 2^n - 1 \\ &= 2^n + 2^n \end{aligned}$$

$$\boxed{T.C. = O(2^n)}$$

Q3

$\log(\log n)$

fun (int n)

{ for (int i = n; i >= 2, pow(1, 1/2)

{ some  $O(1)$

}

$n(\log n)$

for (int i = 1; i <= n; i++)

{ for (int j = 1; j <= n; j = j \* 2)

{ for some  $O(1)$

}

$n^3$

for (int i = 1; i < n; i++)

for (int j = 1; j < n; j++)

for (int k = 1; k < n; k++)

some  $O(1)$

Q4

$$T(n) = T(n/4) + T(n/2) + cn^2$$

assume

$$T(n/2) \geq T(n/4)$$

$$T(n) = 2T(n/2) + cn^2$$

$$C = \log_b a$$

$$C = \log_2 2 = 1$$

$$n^C < f(n)$$

$$TC = O(n^2)$$



Q5

i	j
1	n
2	n/2
3	n/3
i	
n	n/n time

$$T.C. = O(n \log n)$$

Q6

$$i = 2, 2^k, (2^k)^k, ((2^k)^k)^k \dots 2^{k \log k (\log n)}$$

$$2^{k \log k (\log n)} = n$$

$$T.C. = O(\log(\log n))$$

Q7

$$T(n) = T\left(\frac{99n}{100}\right) + \frac{n}{100}$$

$$T(1) = 0$$

Putting  $n = \frac{99}{100} n$

$$T\left(\frac{99}{100} n\right) = T\left(\left(\frac{99}{100}\right)^2 n\right) + \frac{99n}{(100)^2}$$

$$T(n) = T\left(\left(\frac{99}{100}\right)^k n\right) + \frac{(99)^{k-1}}{(100)^k} n$$

$$\left(\frac{99}{100}\right)^k n = 1$$

$$n = \left(\frac{100}{99}\right)^k$$

$$k = \log_{\frac{100}{99}} n$$

$$T(n) = n \log n$$

Q8

a)  $100 < \log(\log h) < \log n < \sqrt{n} < n \log n = \log(n!) \\ < n^2 < 2^n < 2^{2n} < 4^n < n!$

b)  $1 < \log(\log(n)) < \sqrt{\log n} < \log(n) < 2n < 4n < \\ 2(2^n) < \log(2n) < 2 \log(n) < n < n \log n = \log(n!) \\ < n!$

c)  $96 < \log_2(n) = \log_8(n) < n \log_6(n) = n \log_2(n) < \\ 5n < 8n^2 < 7n^3 < 8^{2n}$