



Why is
DSA
Important?

⇒ Make you a better
Software Developer.

⇒ Helps you in getting
a Job

⇒ Winning the Sport of
Competitive Coding



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Roadmap to learn DSA

⇒ Learn a programming language

- ⊛ C++
- ⊛ Java
- ⊛ Python
- ⊛ JavaScript

C++
↳ STL

Java
↳ Collections

⇒ Learn DSA basics and implement

⇒ Learn language libraries that have DSA implemented for you

⇒ Do Practice and Learning together

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Analysis of Algorithms (Background)

Example Problem : Sum of n natural numbers

Input : $n = 3$

Output : 6 // $1 + 2 + 3$

Input : $n = 5$

Output : 15 // $1 + 2 + 3 + 4 + 5$

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```
int fun1(int n)
{
    return n * (n+1) / 2;
}
```

```
int fun2(int n)
{
    int sum = 0;
    for (int i = 1; i <= n; i++)
        sum = sum + i;
    return sum;
}
```

1 + 2 + 3

```
int fun3(int n)
{
    int sum = 0;
    for (int i = 1; i <= n; i++)
        for (int j = 1; j <= i; j++)
            sum++;
    return sum;
}
```

sum = 1 + (1+1) + (1+1+1)
= 1 + 2 + 3
= 6

Asymptotic Analysis

- The idea is to measure order of growth.
- Does not depend upon machine, programming language, etc.
- No need to implement, we can analyze algorithms.

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-11:52

1.5x



720p



Idea of
Asymptotic
Analysis

```
int fun1(int n)
{
    return n*(n+1)/2;
}
```

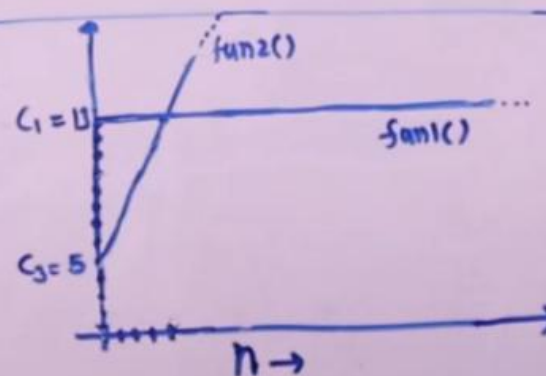
Time Taken : C_1

```
int fun2(int n)
{
    int sum = 0;
    for(int i = 1; i <= n; i++)
        sum = sum + i;
    return sum;
}
```

Time Taken : $C_2n + C_3$

```
int fun3(int n)
{
    int sum = 0;
    for(int i = 1; i <= n; i++)
        for(int j = 1; j <= i; j++)
            sum++;
    return sum;
}
```

Time Taken : $C_4n^2 + C_5n + C_6$

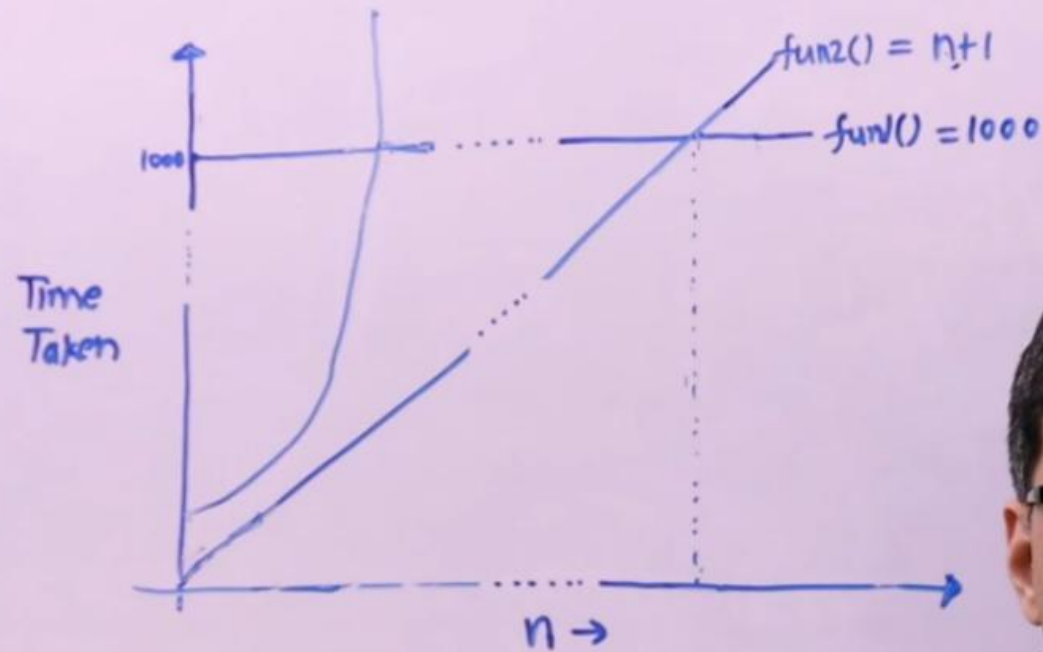


Example Values

$C_1 = 13, C_2 = 2, C_3 = 5$

$fun1() = 13$

$fun2() = 2n + 5$



$$\begin{aligned}n + 1 &\geq 1000 \\ n &\geq 999\end{aligned}$$



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Order of Growth

A function $f(n)$ is said to be growing faster than $g(n)$ if

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = 0$$

OR

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$$

$f(n)$ and $g(n)$ represent
Time Taken.

$$n \geq 0$$

$$f(n), g(n) \geq 0$$

$$\begin{aligned} f(n) &= n+1 \\ g(n) &= 1000 \end{aligned}$$

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Order of Growth

A function $f(n)$ is said to be growing faster than $g(n)$ if

$$\lim_{n \rightarrow \infty} \frac{g(n)}{f(n)} = 0$$

$$f(n) = n^2 + n + 6$$

$$g(n) = 2n + 5$$

$$\lim_{n \rightarrow \infty} \frac{2n + 5}{n^2 + n + 6}$$

$$= \lim_{n \rightarrow \infty} \frac{2/n + 5/n^2}{1 + 1/n + 6/n^2}$$

$$= \lim_{n \rightarrow \infty} \frac{0 + 0}{1 + 0 + 0}$$

$$= 0$$

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Direct Way to Find to Find and Compare Growths

- ① Ignore Lower Order Terms
- ② Ignore Leading Term Constant

Examples: $f(n) = 2n^2 + n + 6$, Order of Growth: n^2 (Quadratic)
 $g(n) = 100n + 3$, Order of Growth: n (Linear)

How do we compare terms?

$$c < \log \log n < \log n < n^{1/3} < n^{1/2} < n \\ < n^2 < n^3 < n^4 < 2^n < n^n$$

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$$\begin{aligned} \textcircled{1} \quad f(n) &= \underset{x}{c_1} \log n + \underset{x}{c_2} \\ g(n) &= \underset{x}{c_3} n + \underset{x}{c_4} \log \log n + \underset{x}{c_5} \\ f(n) &: \log n \\ g(n) &: n \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad f(n) &= \underset{x}{c_1} n^2 + \underset{x}{c_2} n + \underset{x}{c_3} \\ g(n) &= \underset{x}{c_4} n \log n + \underset{x}{c_5} n + \underset{x}{c_6} \\ f(n) &: n^2 \\ g(n) &: n \log n \end{aligned}$$

How do we compare terms?

$$\begin{aligned} c &< \log \log n < \log n < n^{1/3} < n^{1/2} < n \\ &< n^2 < n^3 < n^4 < 2^n < n^n \end{aligned}$$



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Best, Average and Worst Cases

Example 1: Simple
function with same
order of growth for
every input.

```
int getSum(int arr[], int n)
{
    int sum = 0;
    for (int i = 0; i < n; i++)
        sum = sum + arr[i];
    return sum;
}
```

Time Taken : $C_1n + C_2$

Order of Growth : n
(or linear)

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Example 2: Multiple
Orders of growths

Best case: Constant

Average case: Linear
(Under the assumption
that even and odd
cases are equally likely)

Worst case: Linear

```
int getSum(int arr[], int n)
{
    if (n % 2 == 0)
        return 0;
    int sum = 0;
    for (int i = 0; i < n; i++)
        sum = sum + arr[i];
    return sum;
}
```

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Asymptotic

Notations

Big O: Exact on Upper

Theta: Exact

Omega: Exact on lower

```
int search(int arr[], int n,  
           int x)  
{  
    for(int i=0; i<n; i++)  
        if (arr[i] == x)  
            return i;  
    return -1;  
}
```

$arr[] = \{10, 5, 30, 40, 80\}$

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Asymptotic

Notations

Big O: Exact on Upper

Theta: Exact

Omega: Exact on lower

$\theta(1) > \text{constant}$
 $O(1)$

```
int search(int arr[], int n,  
           int x)  
{  
    for(int i=0; i<n; i++)  
        if (arr[i] == x)  
            return i;  
    return -1;  
}
```

arr[] = {10, 5, 30, 40, 80}

$\Omega(1)$

Big O Notation

Direct way * ignore lower order terms
* ignore leading term constant

$$3n^2 + 5n + 6$$

(Blue 'x' marks are under 3n², 5n, and 6)

$$O(n^2)$$

$$3n + 10n \log n + 3$$

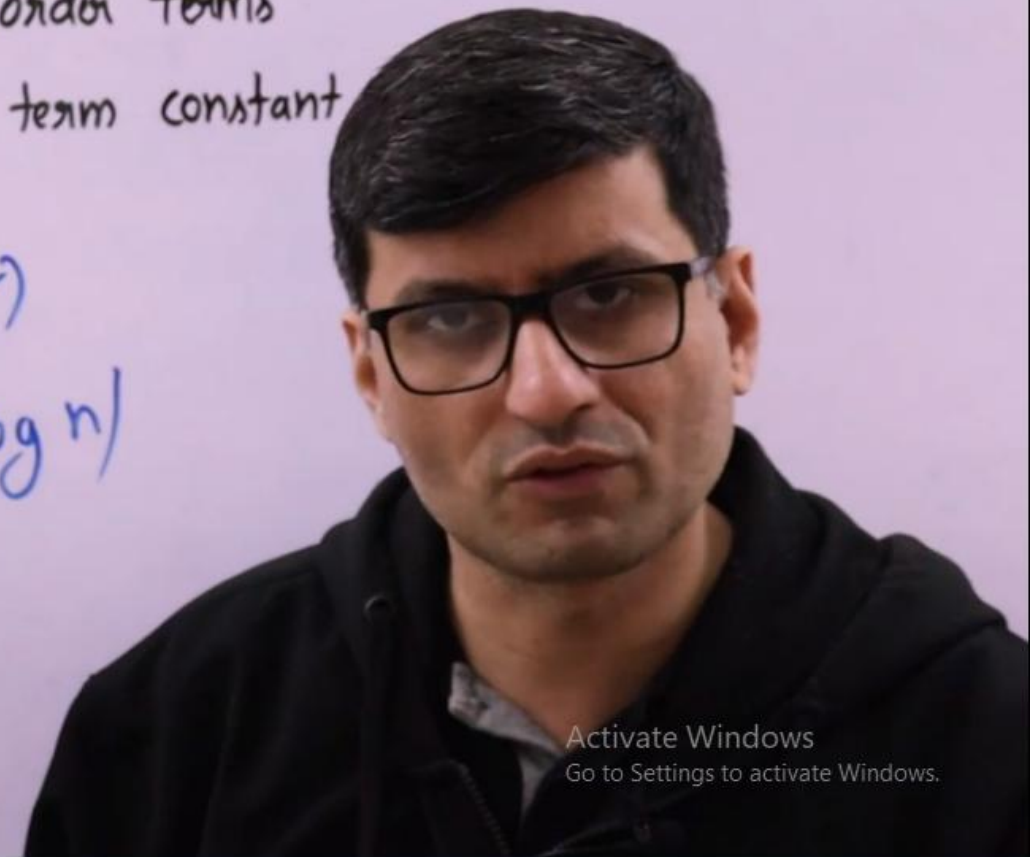
(Blue 'x' marks are under 3n and 3. A blue checkmark is under log n)

$$O(n \log n)$$

$$10n^3 + 40n + 10$$

(Blue 'x' marks are under 10n³, 40n, and 10)

$$O(n^3)$$



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We say $f(n) = O(g(n))$ iff
there exist constants c and n_0
such that $f(n) \leq cg(n)$
for all $n \geq n_0$.

Example

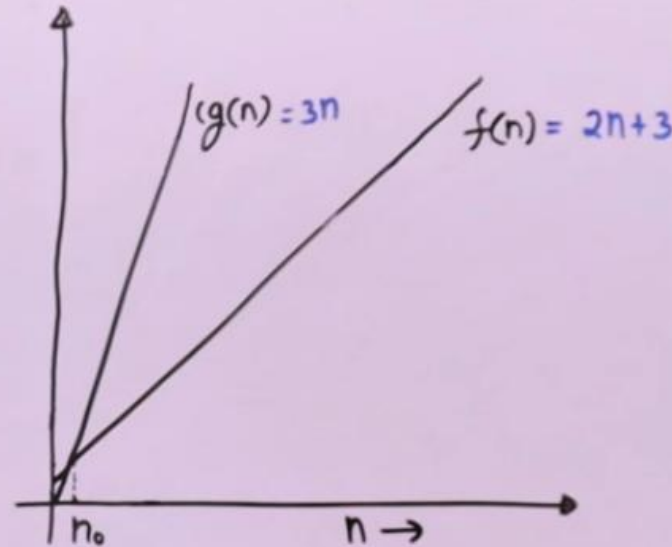
$f(n) = 2n + 3$
can be written as $O(n)$ [$g(n) = n$]

let us take $c = 3$

$$2n + 3 \leq 3n$$

$$3 \leq n$$

We get $n_0 = 3$



$$2n + 3 \leq cn$$

for all $n \geq n_0$

$$\{100, \log 2000, (10)^4, \dots\} \in O(1)$$

$$\cup \left\{ \frac{n}{4}, 2n+3, \frac{n}{100} + \log n, n+10000, \log n + 10, \dots \right\} \in O(n)$$

$$\cup \left\{ n^2+n, 2n^2, n^2+1000n, n^2+n\log n+n, \frac{n^2}{10000}, \dots \right\} \in O(n^2)$$

Big O Notation works for multiple variables also

$$100n^2 + 1000m + n : O(n^2 + m)$$

$$1000m^2 + 200mn + 30m + 20n : O(m^2 + mn)$$

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Applications

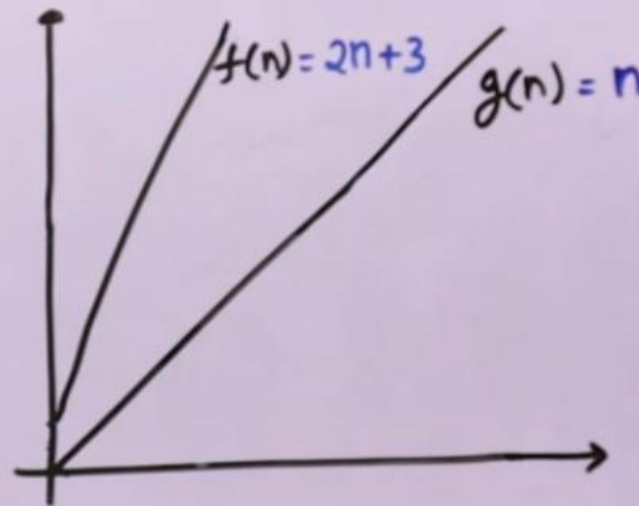
Used when we have an upper bound.

```
bool isPrime(int n)
{
    if (n == 1) return false;
    if (n == 2 || n == 3) return true;
    if (n % 2 == 0 || n % 3 == 0) return false;
    for (int i = 5; i * i <= n; i = i + 6)
        if (n % i == 0 || n % (i + 2) == 0)
            return false;
    return true;
}
```

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Omega Notation

$f(n) = \Omega(g(n))$ iff there exist constants c (where $c > 0$) and n_0 (where $n_0 \geq 0$) such that $cg(n) \leq f(n)$ for all $n \geq n_0$.



$$f(n) = 2n + 3 = \Omega(n)$$

$c = 1$	$n_0 = 0$
$n \leq 2n + 3$	
$-3 \leq n$	

$\Omega(n!)$

$$\textcircled{*} \quad \{n^2, 2n^2 + 5, 1000n^2, 2n^3 + n, \dots\} \in \Omega(n^2)$$

$$\cup \{2n + 3, 100n + \log n, \dots\} \in \Omega(n)$$

$$\cup \{5000, (10)^5, \log 2000, \dots\} \in \Omega(1)$$

$$\textcircled{*} \quad \text{If } f(n) = O(g(n))$$

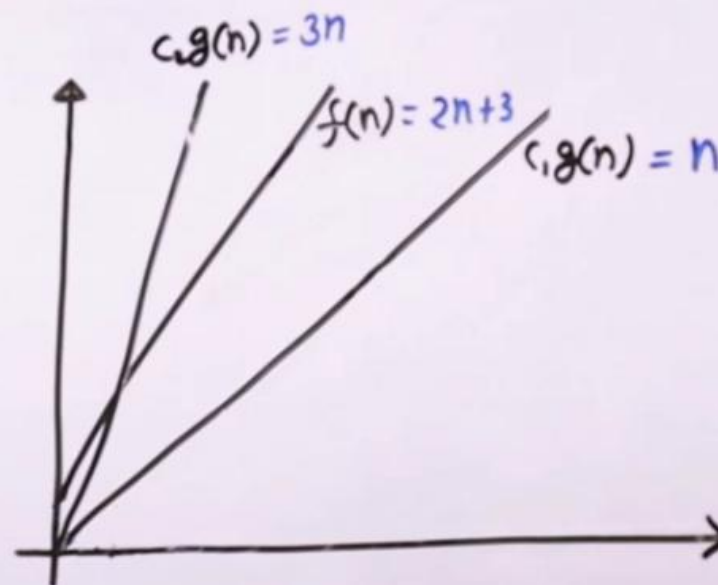
$$g(n) = \Omega(f(n))$$

Theta Notation

$f(n) = \Theta(g(n))$ iff there exist constants c_1, c_2 (where $c_1 > 0$ and $c_2 > 0$) and n_0 (where $n_0 \geq 0$) such that

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

for all $n \geq n_0$.



Example: $f(n) = 2n+3 : \Theta(n)$

$$c_1 = 1, c_2 = 3$$

$$\underbrace{1 \times n}_{n \geq 0} \leq 2n+3 \leq \underbrace{3n}_{n \geq 3}$$

$$n_0 = 3$$

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Direct
Method

$$1000n^2 + 100n \log n + 2n : \Theta(n^2)$$

$$200n^3 + 30n + 5 : \Theta(n^3)$$

$$2000n + 2 \log n : \Theta(n)$$

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$$f(n) = 2n^2 + n : \Theta(n^2)$$

⊛ If $f(n) = \Theta(g(n))$

then $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

and $g(n) = O(f(n))$ and $g(n) = \Omega(f(n))$

⊛ Represents Exact Bound

⊛ $\{100, 10^5, \log 2000, \dots\} \in \Theta(1)$

$\{100n, 2n + \log n, 5n + 3, \dots\} \in \Theta(n)$

$\{2n^2, \frac{n^2}{4} + 5n \log n, \dots\} \in \Theta(n^2)$

Analysis of Common Loops

Example 1:

n : User Input

C : Constant

Loop Runs $\lceil \frac{n}{C} \rceil$ times

Time Complexity: $\Theta(n)$

```
for (int i=0; i<n; i=i+C)
{
    // Some  $\Theta(1)$  Work
}
```

$n=10$	$i=0$
$C=2$	$i=2$
	$i=4$
	$i=6$
	$i=8$

$n=20$	$i=0$
$C=6$	$i=6$
	$i=12$
	$i=18$

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Example 2 :

```
for (int i=n; i>0; i=i-c)
{
    // Some  $\Theta(1)$  Work
}
```

$n=10$	$i=10$
$c=2$	$i=8$
	$i=6$
	$i=4$
	$i=2$

$n=20$	$i=20$
$c=6$	$i=14$
	$i=8$
	$i=2$

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Example 3 :

$c^0, c^1, c^2, \dots, c^{k-1}$

$$c^{k-1} < n$$

$$k < \log_c n + 1$$

Time Complexity : $\Theta(\log_c n)$

```
for (int i=1; i<n; i=i*c)
{
    // Some  $\Theta(1)$  work
}
//  $\lceil \log_c n \rceil$ 
```

$n=33$	$i=1$
$c=2$	$i=2$
	$i=4$
	$i=8$
	$i=16$
	$i=32$

$n=81$	$i=1$
$c=3$	$i=3$
	$i=9$
	$i=27$

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```
for (int i = n; i > 1; i = i/c)
{
    // Some  $\Theta(1)$  Work
}
```

Example 4 :

$n/c, n/c^2, n/c^3 \dots n/c^{k-1}$

$$\frac{n}{c^{k-1}} > 1$$

$$c^{k-1} < n$$

$$k-1 < \log_c n$$

$$k < \log_c n + 1$$

$$\Theta(\log_c n)$$

$n = 33$	$i = 33$
$c = 2$	$i = 16$
	$i = 8$
	$i = 4$
	$i = 2$

$n = 81$	$i = 81$
$c = 3$	$i = 27$
	$i = 9$
	$i = 3$

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Example 5

$2, 2^c, (2^c)^c, \dots, ((2^c)^{c^{k-1}})$
 $2^{c^0}, 2^{c^1}, 2^{c^2}, \dots, 2^{c^{k-1}}$

```

for (int i = 2; i < n; i = pow(i, c))
{
    // Some  $\Theta(1)$  Work
}
    
```

$$2^{c^{k-1}} < n$$

$$c^{k-1} < \log_2 n$$

$$k-1 < \log_c \log_2 n$$

$$k < \log_c \log_2 n + 1$$

$$\Theta(\log_c \log_2 n)$$

$n=33$	$i=2$
$c=2$	$i=4$
	$i=16$

$n=514$	$i=2$
$c=3$	$i=8$
	$i=512$

$$2^3 = 8$$

$$8^3 = 512$$

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Analysis of Multiple

Loops

Example frequent loops

```
void fun(int n)
{
    for(int i = 0; i < n; i++)
    {
        // Some  $\Theta(1)$  work
    }
    for(int i = 1; i < n; i = i * 2)
    {
        // Some  $\Theta(1)$  work
    }
    for(int i = 1; i < 100; i++)
    {
        // Some  $\Theta(1)$  work
    }
}
```

$\left. \begin{array}{l} \text{for(int } i = 0; i < n; i++) \\ \{ \text{// Some } \Theta(1) \text{ work} \} \end{array} \right\} \Theta(n)$
+
 $\left. \begin{array}{l} \text{for(int } i = 1; i < n; i = i * 2) \\ \{ \text{// Some } \Theta(1) \text{ work} \} \end{array} \right\} \Theta(\log n) \times$
+
 $\left. \begin{array}{l} \text{for(int } i = 1; i < 100; i++) \\ \{ \text{// Some } \Theta(1) \text{ work} \} \end{array} \right\} \Theta(1) \times$
 $= \Theta(n)$

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Example 2:

Nested loops

```
void fun(int n)
{
    for(int i=0; i<n; i++) →  $\theta(n)$ 
    {
        for(int j=1; j<n; j=j*2) →  $\theta(\log n)$  *
        {
            // Some  $\theta(1)$  work
        }
    }
}
```

$\theta(n \log n)$

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Example 3:

Mixed Loops

```
void fun(int n)
{
    for(int i=0; i<n; i++)
    {
        for(int j=1; j<n; j=j*2)
        {
            // Some  $\Theta(1)$  work
        }
    }
    for(int i=0; i<n; i++)
    {
        for(int j=1; j<n; j++)
        {
            // Some  $\Theta(1)$  work
        }
    }
}
```

$\Theta(n \log n)$

$\Theta(n^2)$

$\Theta(n^2)$

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Example 4 :

Different Input

```
void fun(int n, int m)
{
    for (int i = 0; i < n; i++)
    {
        for (int j = 1; j < n; j = j * 2)
        {
            // Some  $\Theta(1)$  work
        }
    }
    for (int i = 0; i < m; i++)
    {
        for (int j = 1; j < m; j++)
        {
            // Some  $\Theta(1)$  work
        }
    }
}
```

$\Theta(n \log n)$

$\Theta(m^2)$

$\Theta(n \log n + m^2)$

Analysis of Recursion (Introduction)

Example 1

```
void fun(int n)    n > 0
{
    if (n <= 0)
        return;
    print("GfG");
    fun(n/2);
    fun(n/2);
}
```

$$\begin{aligned} T(n) &= T(n/2) + T(n/2) + \theta(1) \\ &= 2T(n/2) + \theta(1) \\ n &\leq 0 \\ T(n) &= \theta(1) \end{aligned}$$

Analysis of Recursion (Introduction)

Example 1

```
void fun(int n)    n > 0
{
    if (n <= 0)
        return;
    print("GfG");
    fun(n/2);
    fun(n/2);
}
```

$$T(n) = T(n/2) + T(n/2) + \theta(1)$$

$$T(n) = 2T(n/2) + \theta(1)$$

$$T(0) = \theta(1)$$

Analysis of Recursion (Introduction)

Example 2

```
void fun(int n) n > 0
{
    if (n <= 0)
        return;
    for (int i = 0; i < n; i++)
        print("a+h");
    fun(n/2);
    fun(n/3);
}
```

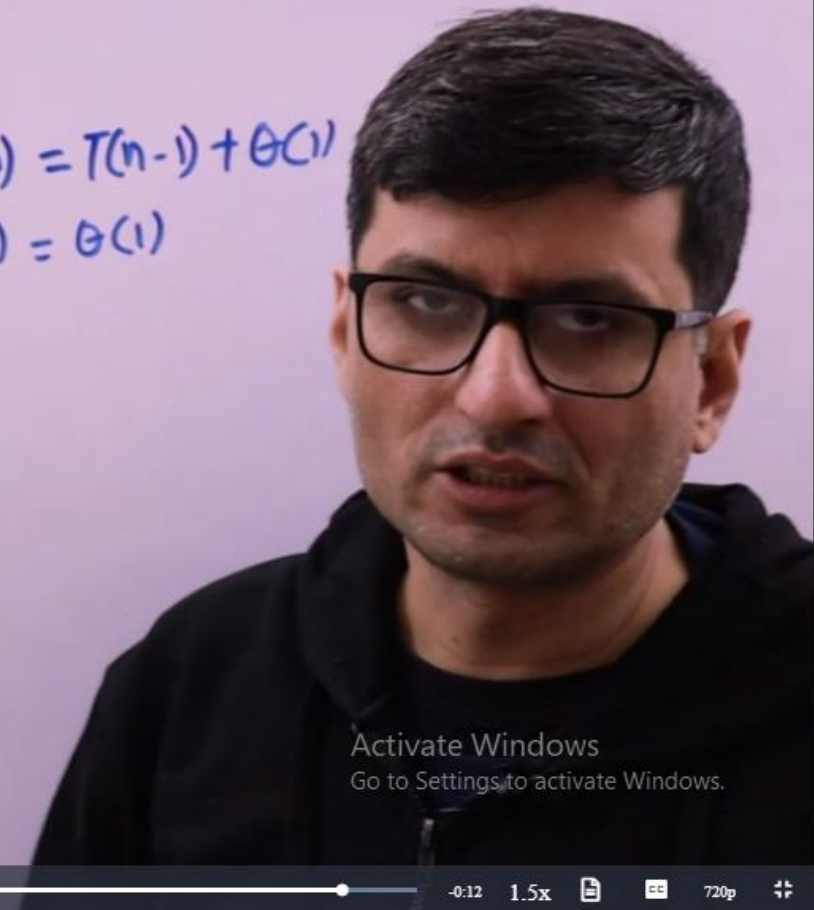
$$T(n) = T(n/2) + T(n/3) + \theta(n)$$
$$T(0) = \theta(1)$$

Analysis of Recursion (Introduction)

Example 3

```
void fun(int n)
{
    if (n <= 1)
        return ;
    print("GfG")
    fun(n-1);
}
```

$$T(n) = T(n-1) + \theta(1)$$
$$T(1) = \theta(1)$$

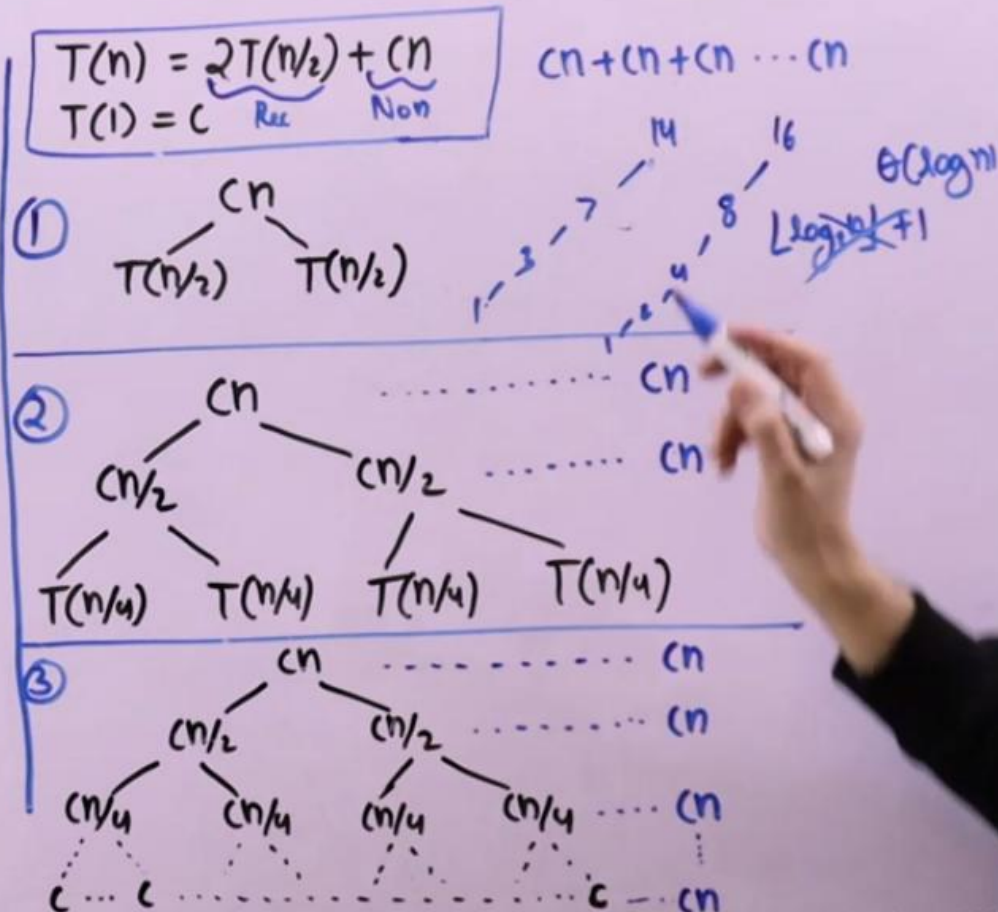


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Recursion Tree

Method for Solving Recurrences.

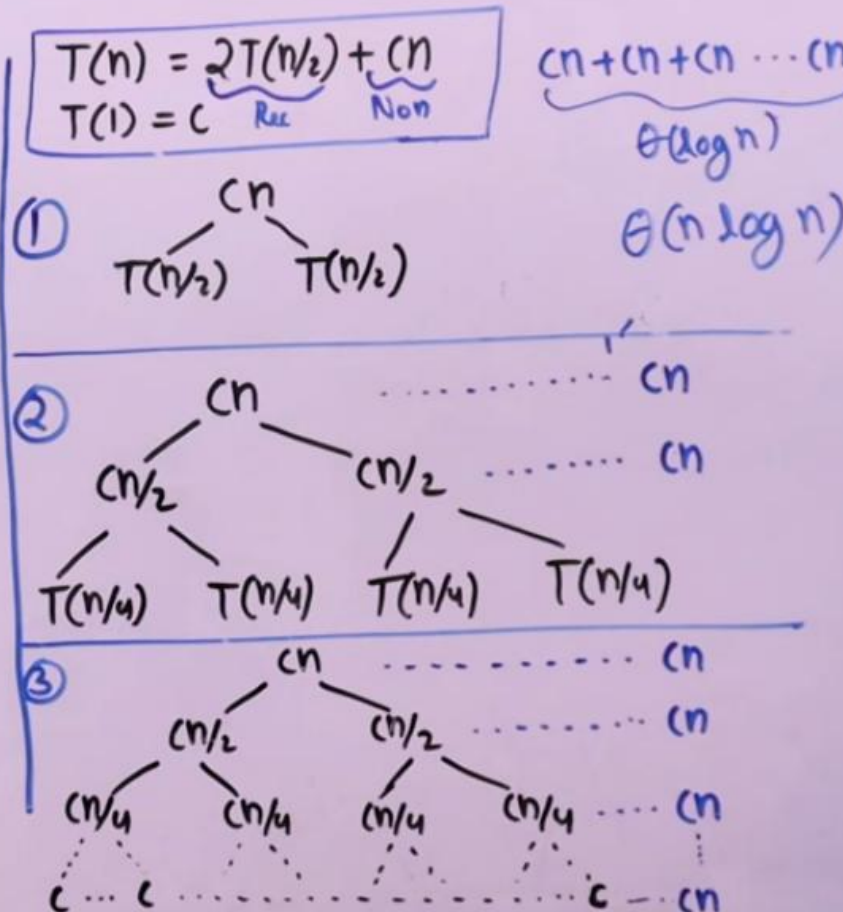
- We consider the recursion tree and compute total work done.
- We write non-recursive part as root of the tree and write the recursive part as children.
- We keep expanding until we see a pattern.



Recursion Tree

Method for Solving Recurrences.

- We consider the recursion tree and compute total work done.
- We write non-recursive part as root of the tree and write the recursive part as children.
- We keep expanding until we see a pattern.



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Recursion Tree

Method for Solving Recurrences.

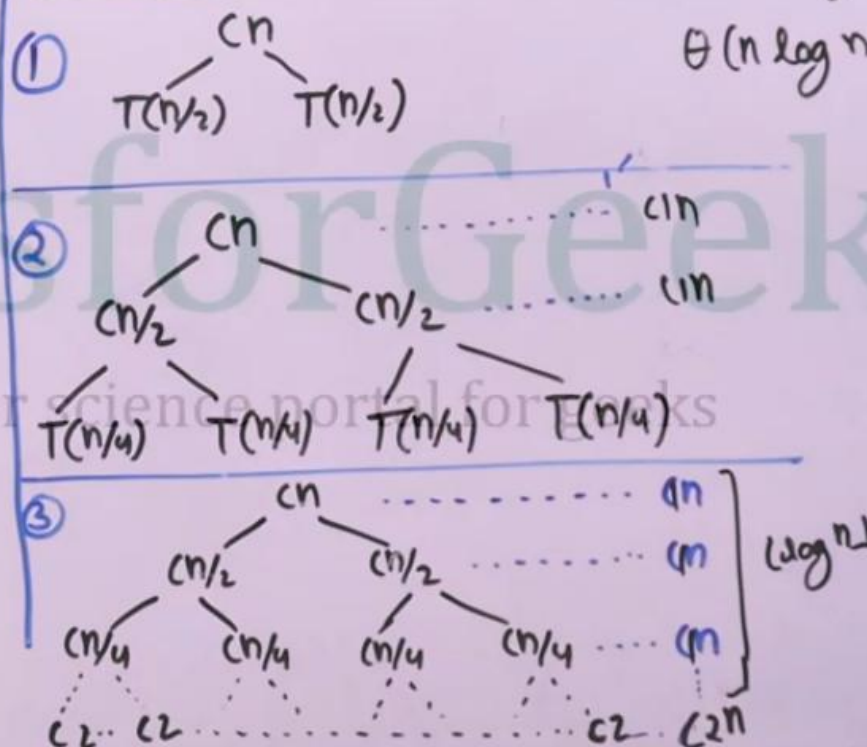
- We consider the recursion tree and compute total work done.
- We write non-recursive part as root of the tree and write the recursive part as children.
- We keep expanding until we see a pattern.

$$T(n) = 2T(n/2) + cn$$

$$T(1) = c$$

$$cn \times \theta(\log n) + \cancel{cn}$$

$$\theta(n \log n)$$



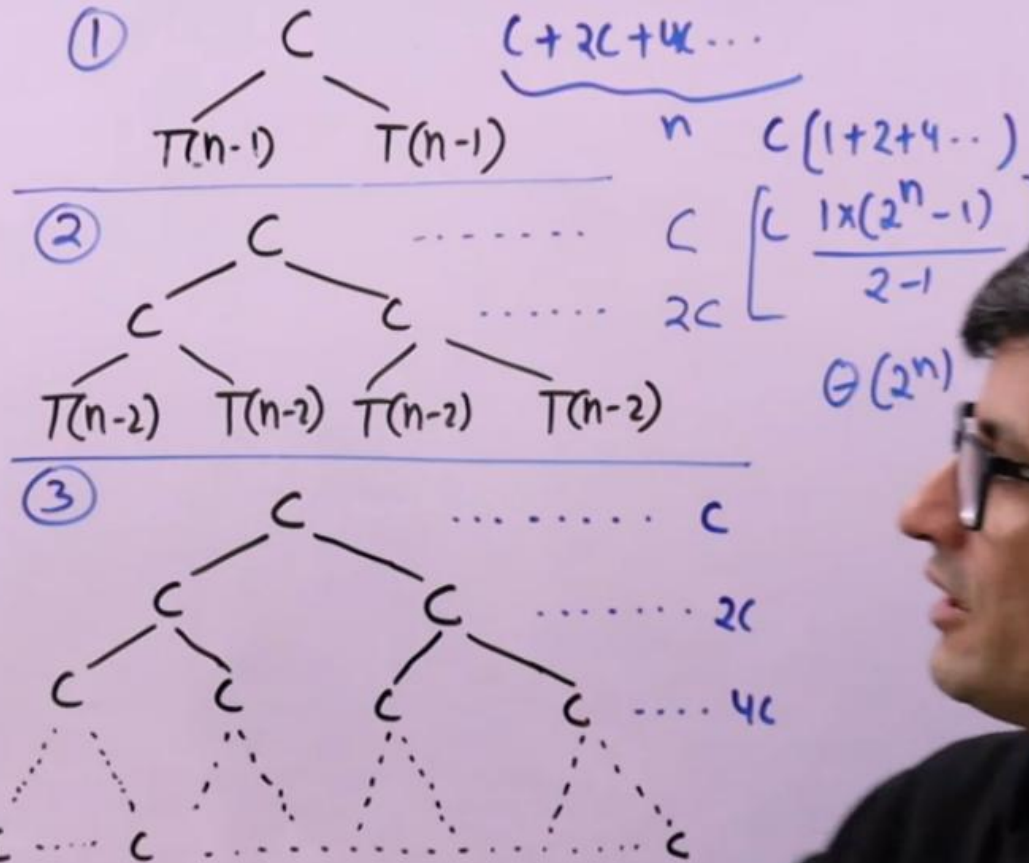
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More Example

Recurrences

$$T(n) = 2T(n-1) + C$$

$$T(1) = C$$



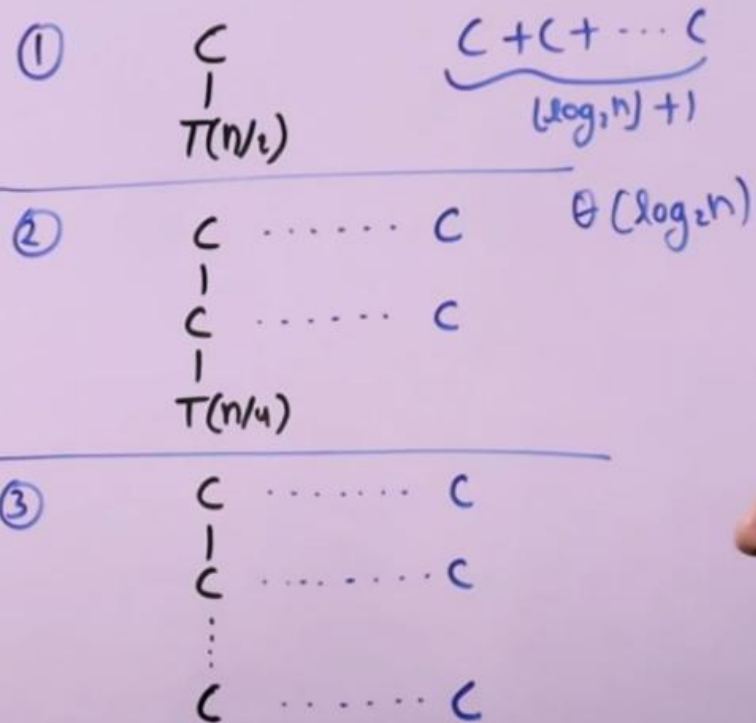
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More Example

Recurrences

$$T(n) = T(n/2) + c$$

$$T(1) = c$$



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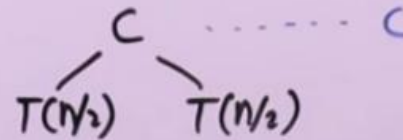
More Example

Recurrences

$$T(n) = 2T(n/2) + c$$

$$T(1) = c$$

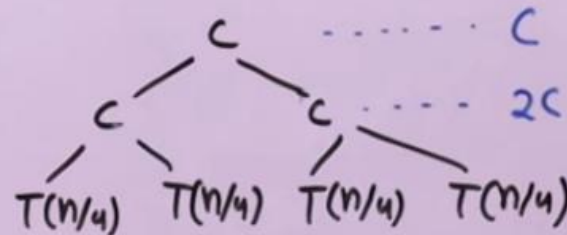
①



$$c + 2c + 4c + \dots$$

$$\Theta(\log_2 n)$$

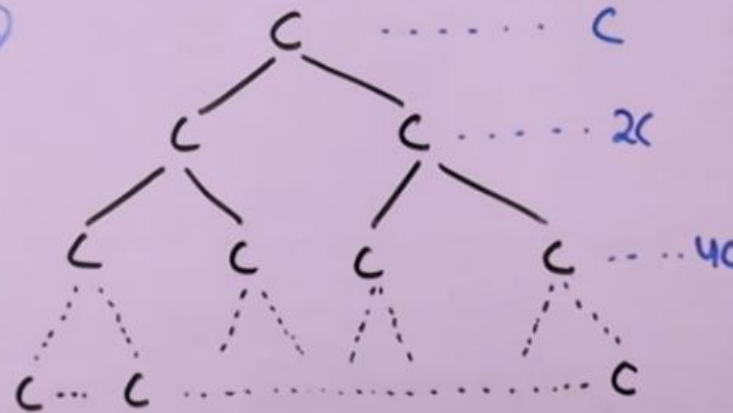
②



$$\Theta\left(\frac{2^{\log_2 n} - 1}{2 - 1}\right)$$

$$\Theta(n)$$

③



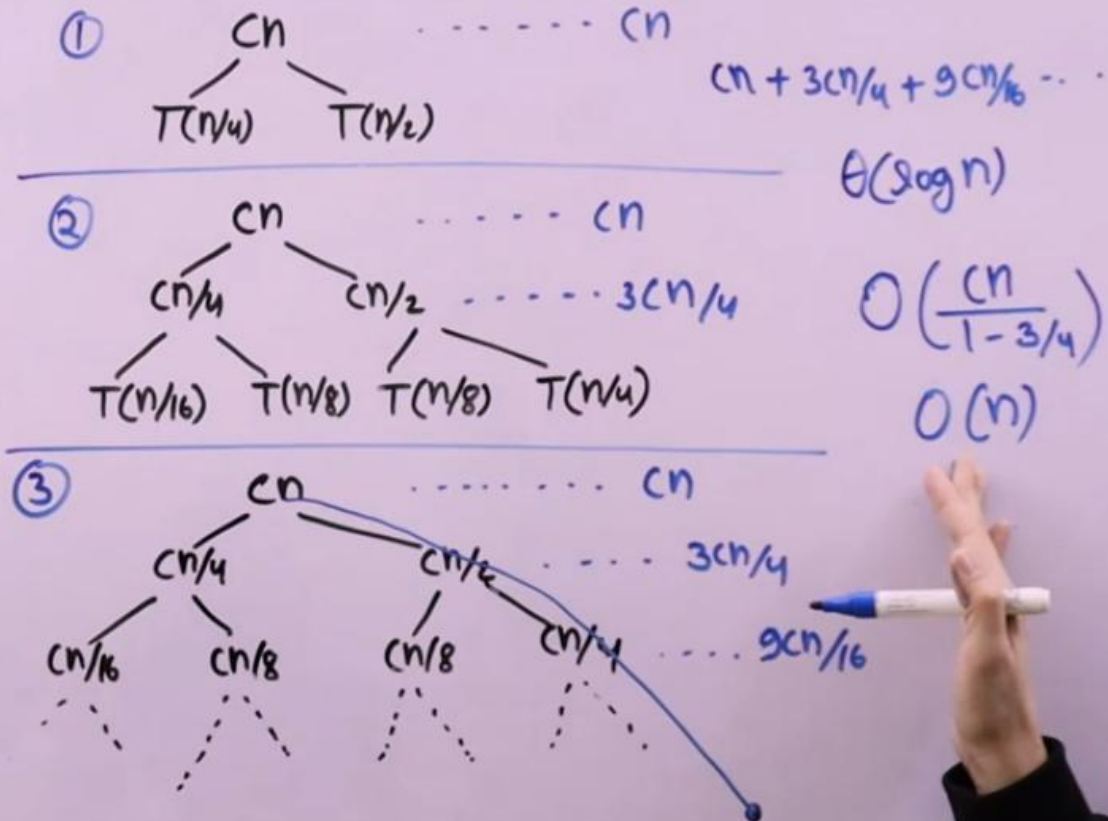
$$4c$$

$$(\log_2 n) + 1$$

Upper Bounds Using Recursion Tree Method

$$T(n) = T(n/4) + T(n/2) + cn$$

$$T(1) = c$$



$$\Theta(\log n)$$

$$O\left(\frac{cn}{1-3/4}\right)$$

$$O(n)$$

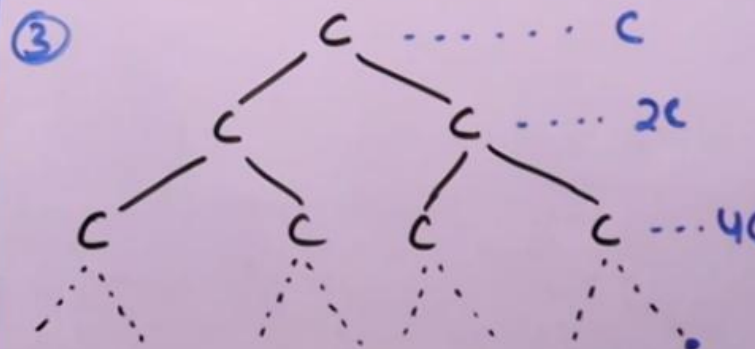
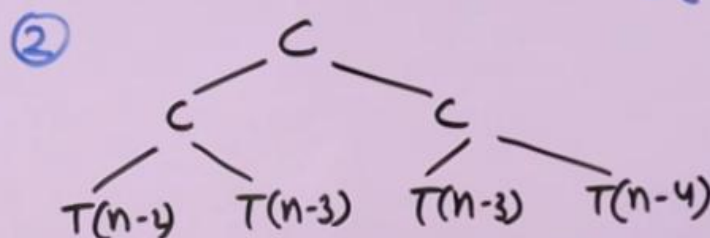
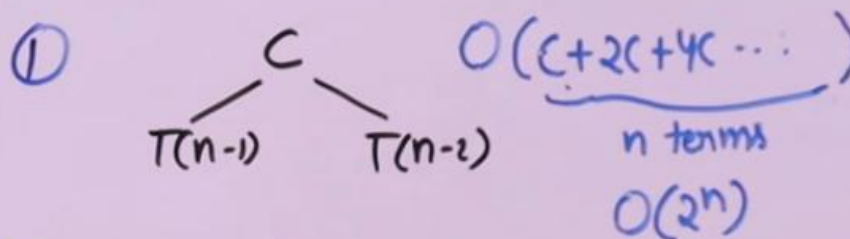
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Upper Bounds Using Recursion Tree Method

$$T(n) = T(n-1) + T(n-2) + C$$

$$T(1) = C$$

$$T(0) = C$$



Space Complexity

Order of growth of memory (or RAM) space in terms of input size.

```
int getSum1(int n)
{
    return n*(n+1)/2;
}
```

$\Theta(1)$ or $O(1)$

```
int getSum2(int n)
{
    int sum = 0;
    for(int i=1; i<=n; i++)
        sum = sum + i;
    return sum;
}
```

$\Theta(1)$ or $O(1)$

Space Complexity

```
int arrSum(int arr[], int n)
{
    int sum = 0;
    for (int i = 0; i < n; i++)
        sum = sum + arr[i];
    return sum;
}
```

$\Theta(n)$

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Space Complexity

Auxiliary Space: Order of growth of extra space or temporary space in terms of input size.

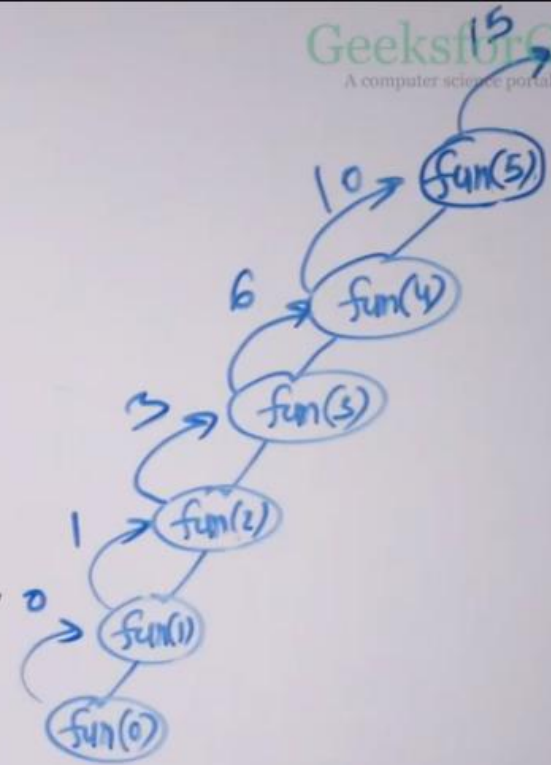
Auxiliary
Space:
 $\Theta(1)$
Space complexity
 $\Theta(n)$

```
int arrSum(int arr[], int n)
{
    int sum = 0;
    for(int i=0; i<n; i++)
        sum = sum + arr[i];
    return sum;
}
```

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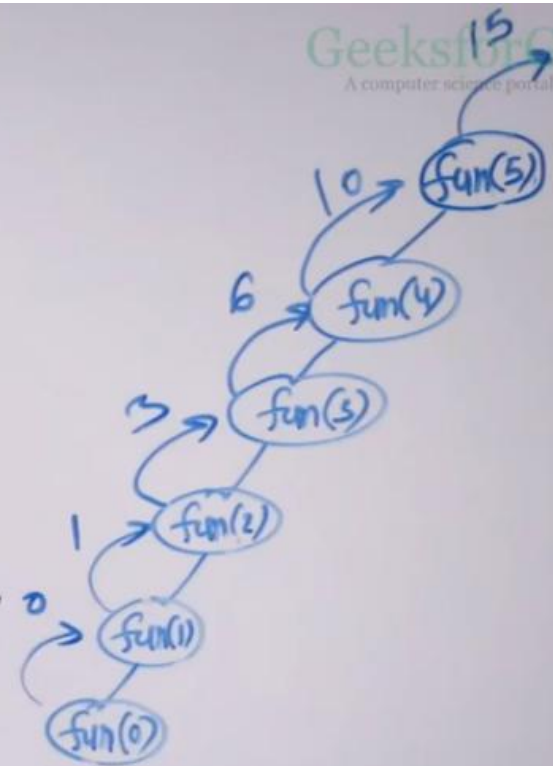
Space Complexity

```
int fun(int n)
{
    if (n <= 0)
        return 0;
    return n + fun(n-1);
}
```



Space Complexity

```
int fun(int n)
{
    if (n <= 0)
        return 0;
    return n + fun(n-1);
}
```



fun(4)
fun(5)

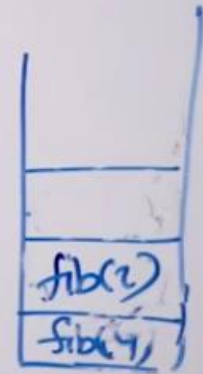
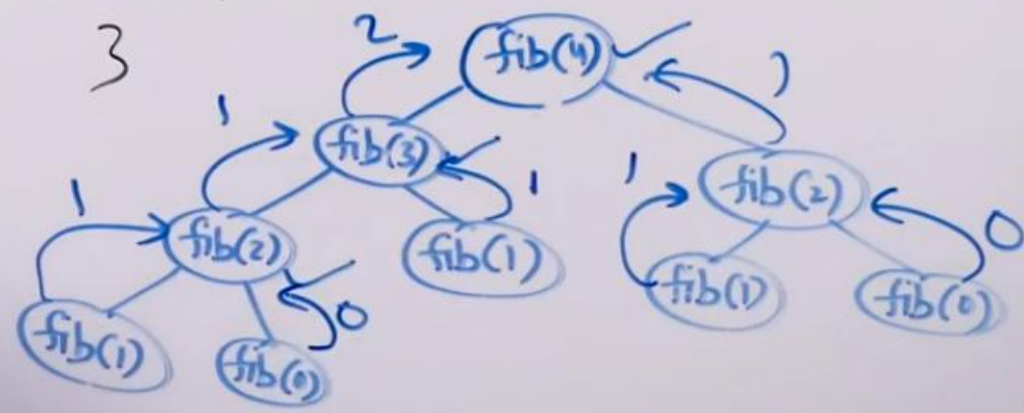
$\Theta(n)$

Space Complexity

```
int fib(int n)
{
    if (n == 0 || n == 1)
        return n;
    return fib(n-1) + fib(n-2);
}
```

0, 1, 1, 2, 3, 5, ...

$n=0$ $n=1$ $n=2$



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Space Complexity

$n = 4$

```
int fib(int n)
{
```

```
    int f[n+1];
```

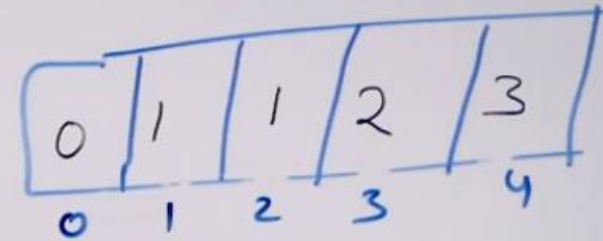
```
    f[0] = 0;
```

```
    f[1] = 1;
```

```
    for(int i=2; i<=n; i++)
```

```
        f[i] = f[i-1] + f[i-2];
```

```
    return f[n];
```



Space Complexity

$n = 4$
 $a = 0, b = 1$

```
int fib(int n)
{
```

```
    if (n == 0 || n == 1)
        return n;
```

```
    int a = 0, b = 1;
```

```
    for (int i = 2; i <= n; i++)
```

```
    {
        c = a + b;
        a = b;
        b = c;
    }
```

```
    return c;
```

$i = 2: c = 1, a = 1, b = 1$

$i = 3: c = 2, a = 1, b = 2$

$i = 4: c = 3, a = 2, b = 3$

$\Theta(1)$

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