

Priority Queue

Queue → FIFO highest priority → first insert

Stack → LIFO highest priority → last insert

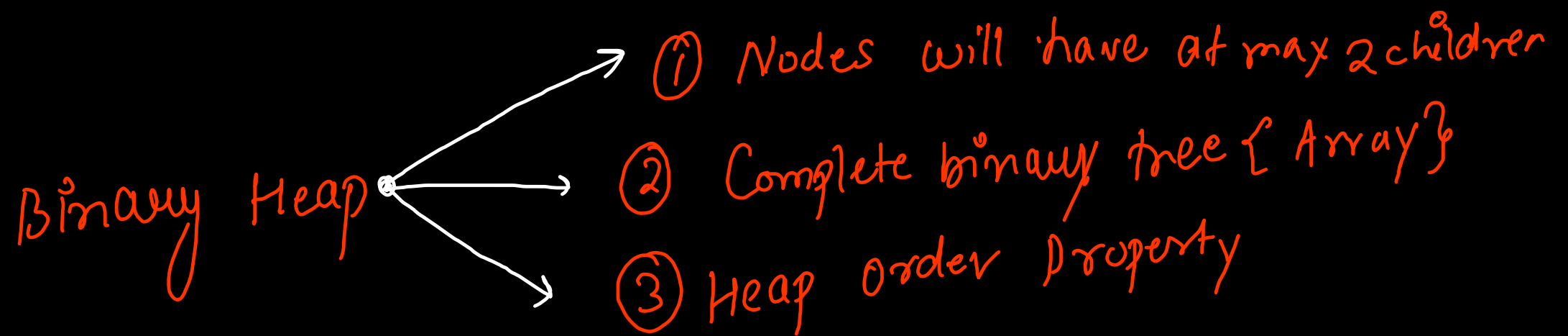
highest priority → custom sorting
(max/min)

Priority Queue
(abstract datatype)

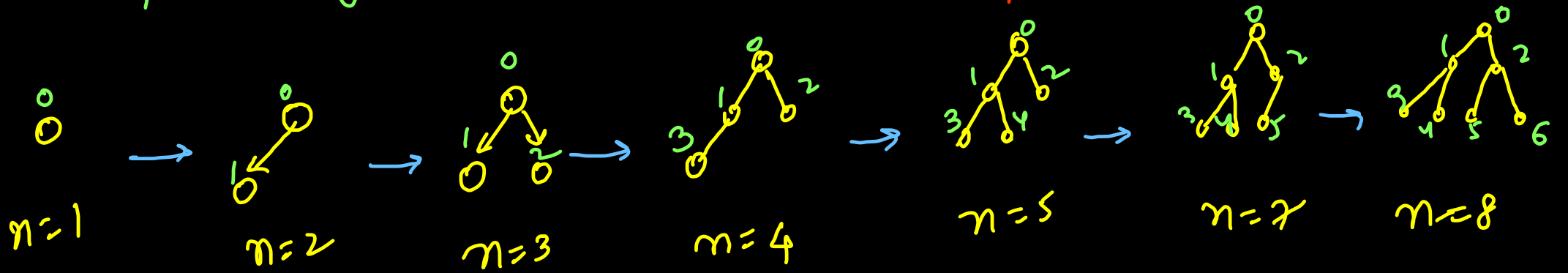
Binary Heap
(concrete data
& structure)

min heap
(by default)

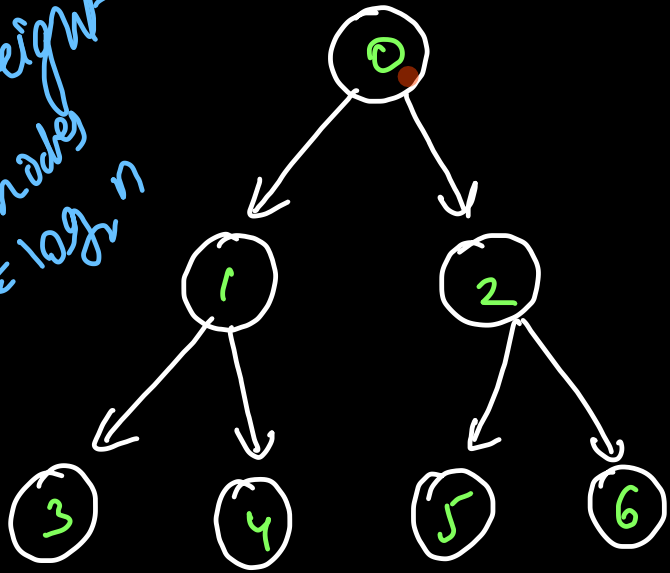
<p>peek()</p> <p>get highest priority element</p> <p>$O(1)$ avg/worst</p>	<p>add()</p> <p>insert elements acc to priority</p> <p>$O(\log n)$ avg/worst</p>	<p>remove()</p> <p>delete highest priority element</p> <p>$O(\log n)$ avg/worst</p>
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Complete binary tree { All levels (maybe except last) are full and in last level, all nodes are as left as possible }



Min height
of n nodes
 $= \log_2 n$



0-based indexing
= indices on CBT

root \rightarrow idx $\begin{cases} \text{left} = 2 \cdot \text{idx} + 1 \\ \text{right} = 2 \cdot \text{idx} + 2 \end{cases}$

root \rightarrow idx \rightsquigarrow parent $= (\text{idx} - 1) / 2$

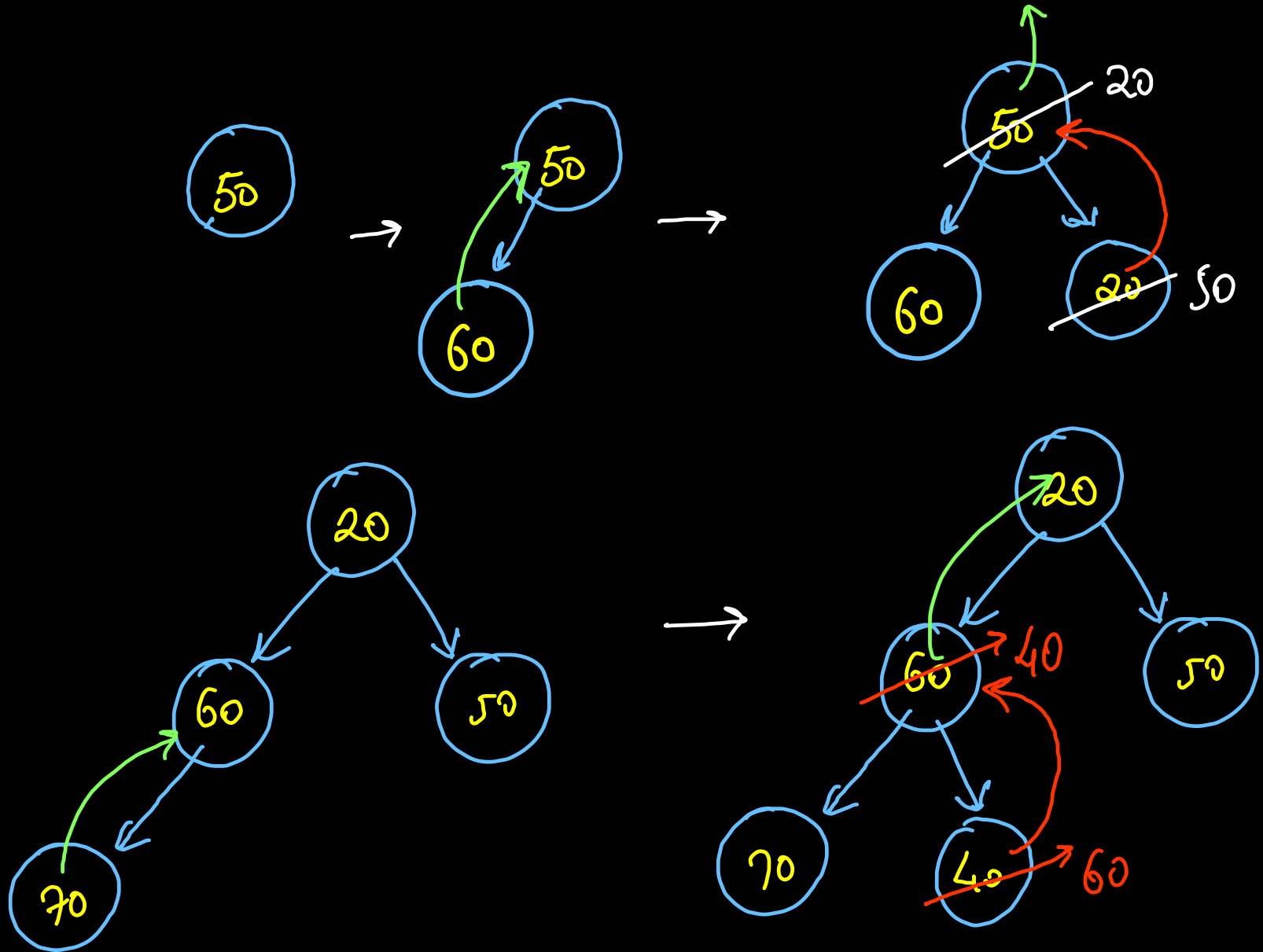
Insertⁿ

0 ✓ 50, 1 ✓ 60, 2 ✓ 20, 3 ✓ 70, 4 ✓ 40, 5 10, 6 30, 7 90, 8 80

Heap order
property

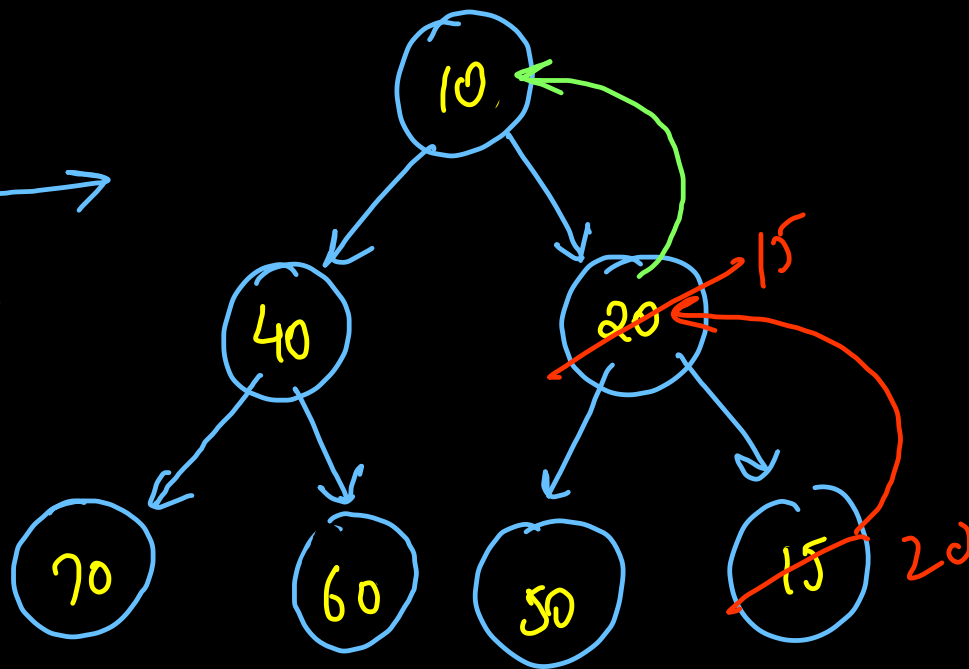
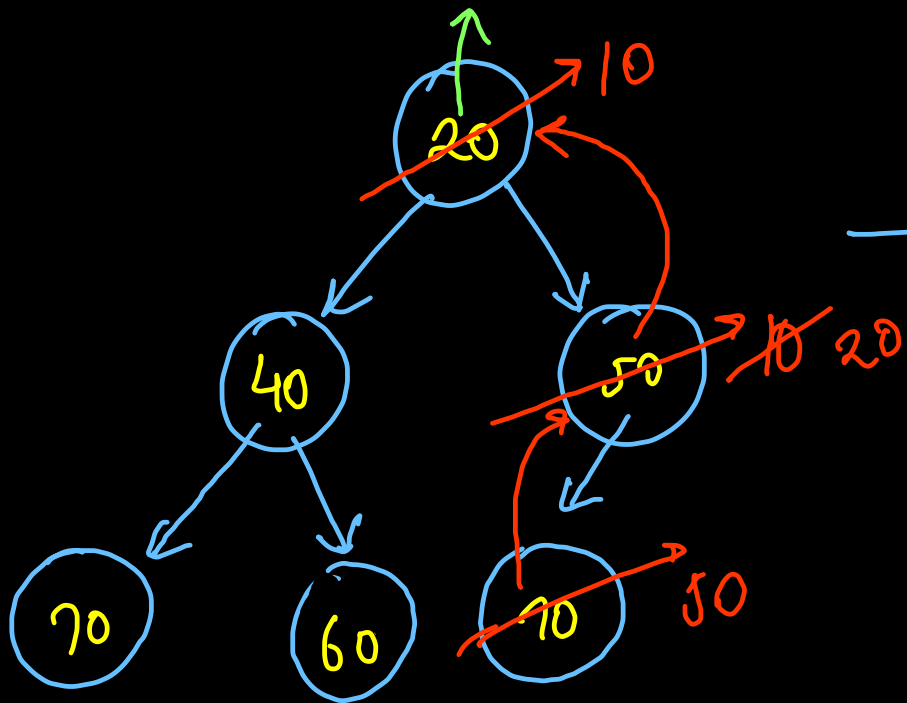
↓
out of all
nodes in
a subtree

↓
highest priority
(min^m value)
should be at
root



Insert

✓ 0 ✓ 1 ✓ 2 ✓ 3 ✓ 4 ✓ 5 ✓ 6
50, 60, 20, 70, 40, 10, 15



upheapify()

```

public static class PriorityQueue {
    ArrayList<Integer> data = new ArrayList<>();

    public void add(int val) {
        data.add(val);
        upheapify(data.size() - 1);
    }

    void upheapify(int idx){
        if(idx == 0) return;

        int par = (idx - 1) / 2;
        if(data.get(par) < data.get(idx)) return;

        Collections.swap(data, idx, par);
        upheapify(par);
    }
}

```

always
balanced for CBT ★

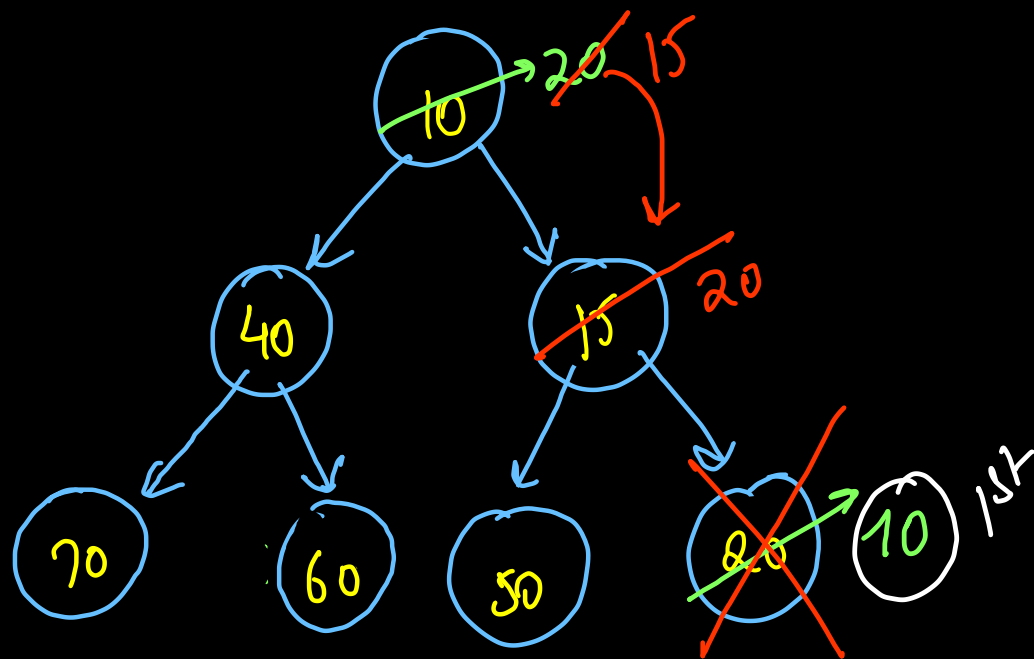
$O(\text{height})$
= $O(\log_2 n)$ → avg
→ worst

```

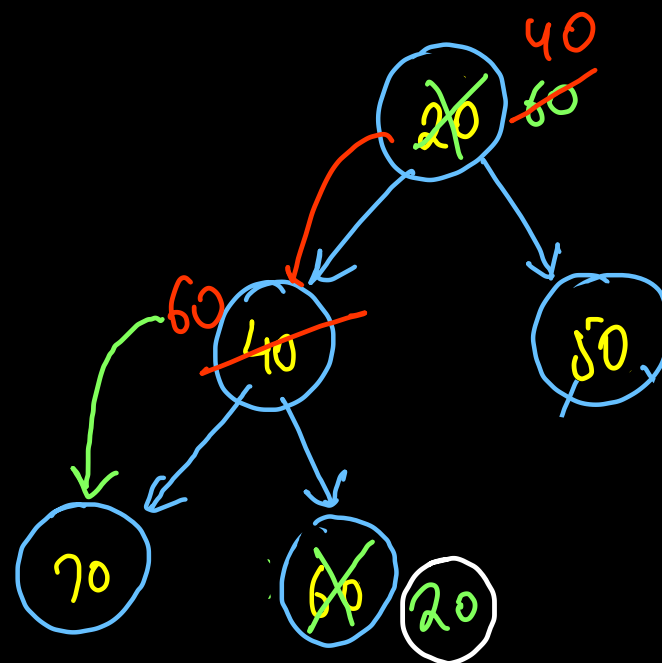
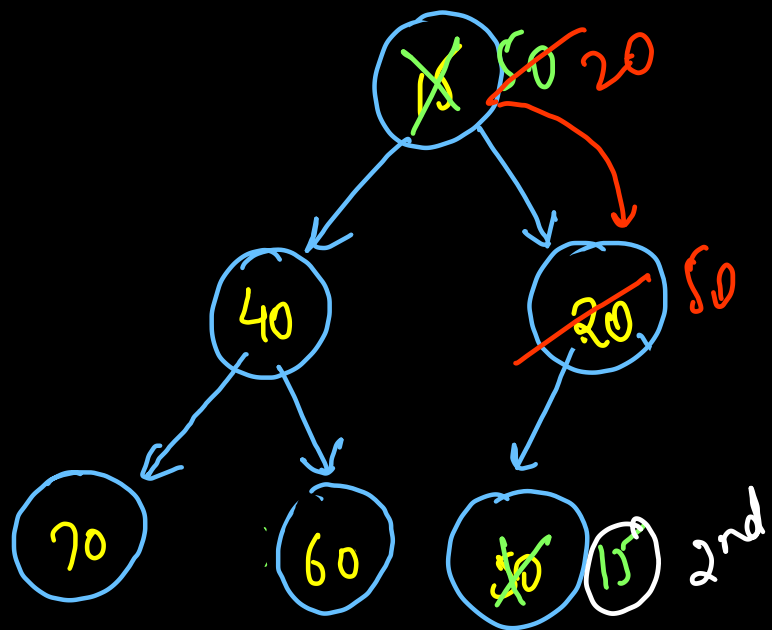
public int peek() {
    if(data.size() == 0) {
        System.out.println("Underflow");
        return;
    }
    return data.get(0);
}

```

$O(1)$
avg/worst



- ① $\text{swap}(\text{data}[0], \text{data}[\text{n}-1]);$
- ② delete the last node
- ③ $\text{down heapify}(0)$



`public static class PriorityQueue {`
`ArrayList<Integer> data = new ArrayList<>();`

`public void add(int val) {`
`data.add(val); $\rightarrow O(1)$`
`upheapify(data.size() - 1); $\rightarrow O(\log n)$`
`}`

$O(\log n)$
`void upheapify(int idx){`
`if(idx == 0) return;`

`int par = (idx - 1) / 2;`
`if(data.get(par) < data.get(idx)) return;`

`Collections.swap(data, idx, par);`
`upheapify(par);`
`}`

$O(1)$
`public int peek() {`
`if(data.size() == 0) {`
`System.out.println("Underflow");`
`return -1;`
`}`
`return data.get(0);`
`}`

`public int size() {`
`return data.size(); $\rightarrow O(1)$`
`}`

\rightarrow min value \rightarrow highest priority
 $O(\log n)$

`public void downheapify(int idx){`
`int l = 2 * idx + 1;`
`int r = 2 * idx + 2;`
`int min = idx;`

`if(l < data.size() && data.get(l) < data.get(min))`
`min = l;`
`if(r < data.size() && data.get(r) < data.get(min))`
`min = r;`

`if(min == idx) return;`
`Collections.swap(data, idx, min);`
`downheapify(min);`
`}`

$\rightarrow \log N$
`public int remove() {`
`if(data.size() == 0) {`
`System.out.println("Underflow");`
`return -1;`
`}`

$O(1)$
 $O(1)$
 $O(\log n)$
`int val = data.get(0);`
`Collections.swap(data, 0, data.size() - 1);`
`data.remove(data.size() - 1);`
`downheapify(0);`
`return val;`
`}`

