Random Rotation Codes are Good Quantum Error Correcting Codes for Loss

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Outline of Defense

- Introduction
- Error Correction
  - Classical Error Correction (EC)
  - Quantum Error Correction (QEC)
- Harmonic Oscillator
- Bosonic Rotation Codes
- My Work
  - Random Code Generation
  - Code Comparison
  - Results
- Conclusion

Quantum Computers and the Need for EC

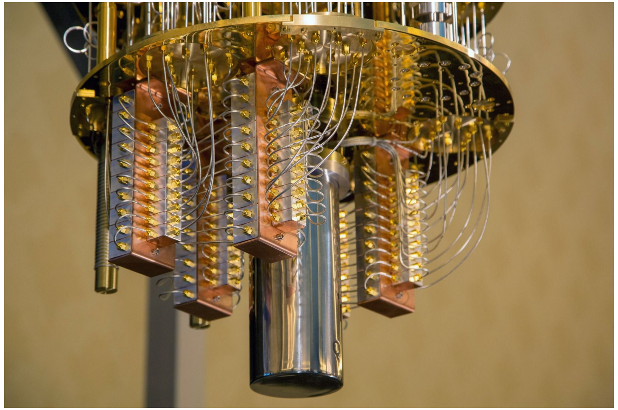


Image from: https://www.engadget.com/2018-01-09-this-is-what-a-50-qubit-quantum-computer-looks-like.html

Image from: https://aws.amazon.com/products/quantum/

Image from: https://ionq.com/

Image from: https://www.quantinuum.com/

Exciting Experimental Realizations of Bosonic QEC

Published: 20 July 2016

Extending the lifetime of a quantum bit with error correction in superconducting circuits

Nissim Ofek, Andrei Petrenko, Reinier Heeres, Philip Reinhold, Zaki Leghtas, Brian Vlastakis, Yehan Liu, Luigi Frunzio, S. M. Girvin, L. Jiang, Mazyar Mirrahimi, M. H. Devoret & R. J. Schoelkopf

Nature **536**, 441–445 (2016) | Cite this article

21k Accesses | **502** Citations | **188** Altmetric | Metrics

Article | Open Access | Published: 22 March 2023

Beating the break-even point with a discrete-variable-encoded logical qubit

Zhongchu Ni, Sai Li, Xiaowei Deng, Yanyan Cai, Libo Zhang, Weiting Wang, Zhen-Biao Yang, Haifeng Yu, Fei Yan, Song Liu, Chang-Ling Zou, Luyan Sun, Shi-Biao Zheng, Yuan Xu & Dapeng Yu

Nature **616**, 56–60 (2023) | Cite this article

11k Accesses | **1** Citations | **75** Altmetric | Metrics

Article | Published: 22 March 2023

Real-time quantum error correction beyond break-even

V. V. Sivak, A. Eickbusch, B. Royer, S. Singh, I. Tsioutsios, S. Ganjam, A. Miano, B. L. Brock, A. Z. Ding, L. Frunzio, S. M. Girvin, R. J. Schoelkopf & M. H. Devoret

Nature **616**, 50–55 (2023) | Cite this article

8349 Accesses | **1** Citations | **161** Altmetric | Metrics

Classical Error Correction - The Repetition Code

Example of Classical Repetition

Original Message	Repeated Message	Noisy Channel	Received Message	Corrected Message
1101	111 111 000 111	→	101 111 001 011	1101
	ENCODING	NOISE		DECODING

NOISE

Quantum States & Channels

$$C : H_A \rightarrow H_B$$

A channel, C, is a function that takes any state to another state. States live in Hilbert Spaces, denoted in this formula as H_A and H_B.

NOISE

Quantum Channels - Kraus Representation

$$C(\rho) = \sum_i K_i \rho K_i^\dagger$$

K_i are the Kraus Operators for channel C .

$$B_q(\rho) = q\hat{X}\rho\hat{X} + (1-q)\hat{I}\rho\hat{I}$$

$$\left\{\sqrt{q}\hat{X}, \sqrt{1-q}\hat{I}\right\}$$

State Fidelity - Similarity of Two States

$$F(\rho, \sigma) = \left(\mathrm{Tr}\left(\sqrt{\rho}\sigma\sqrt{\rho}\right)\right)^2$$

F is the fidelity (similarity) of two states, ρ and σ, between 0 and 1.

NOISE

Gate Fidelity - Similarity of Two Channels

$$F(O, M, \rho) = \left(\mathrm{Tr} \left(\sqrt{\sqrt{O(\rho)} M(\rho) \sqrt{O(\rho)}} \right) \right)^2$$

O and M are operators and ρ is a state once more.

$$\overline{F}(O, M) = \frac{d + \sum_i \left| \mathrm{Tr} \left(K_i M^\dagger \right) \right|^2}{d^2 + d}$$

d is the dimension of the states that are being considered, and the K_i are the Kraus Operators for O.

NOISE

Noise Channel Fidelity

$$\overline{F}(O) = \frac{d + \sum_i |\mathrm{Tr}(K_i)|^2}{d^2 + d}$$

Optimal Recovery

$$\mathcal{R}^\star(\mathcal{E}) = \arg\max_{\mathcal{R}} \overline{F}(\mathcal{R} \circ \mathcal{E})$$

$\mathcal{R}^\star$ is the optimal recovery channel to apply after a noise or error channel $\mathcal{E}$.

Putting It All Together: Code Fidelity

$$\overline{F}\left(\mathcal{R}\circ\mathcal{E}\circ E\right)$$

E encodes information into a code, $\mathcal{E}$ is the noise channel, and $\mathcal{R}$ is the recovery.

Specific Code Fidelities Used

Optimal Fidelity

$$\overline{F}\left(\mathcal{R}^{\star}\left(\mathcal{E}\circ E\right)\circ\mathcal{E}\circ E\right)$$

No Recovery Fidelity

$$\overline{F}\left(E^{\dagger}\circ\mathcal{E}\circ E\right)$$

Classical Harmonic Oscillator - Overview

$$\mathbf{F} = -k\mathbf{x} \implies U = \frac{1}{2}k\left|\mathbf{x}\right|^2$$

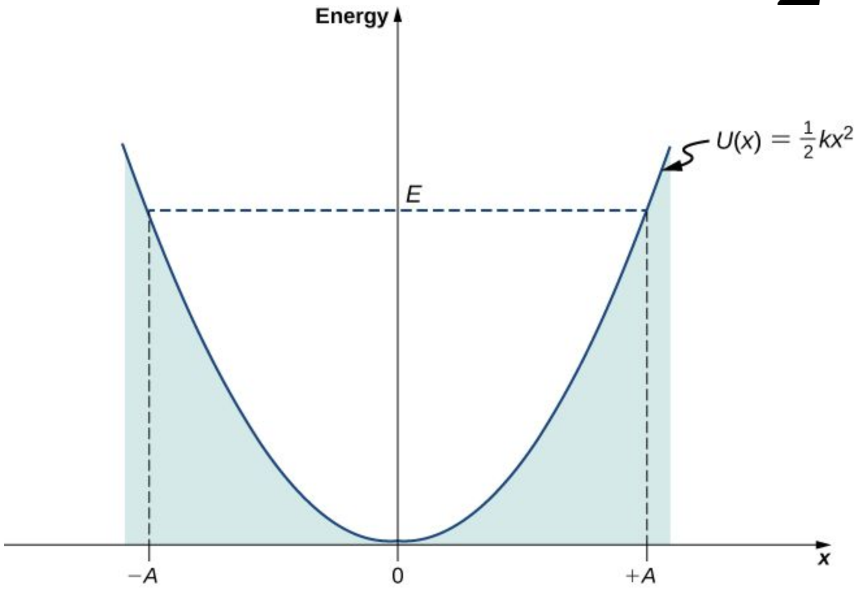


Image from: https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/07%3A_Quantum_Mechanics/7.06%3A_The_Quantum_Harmonic_Oscillator

Quantum Harmonic Oscillator - Overview

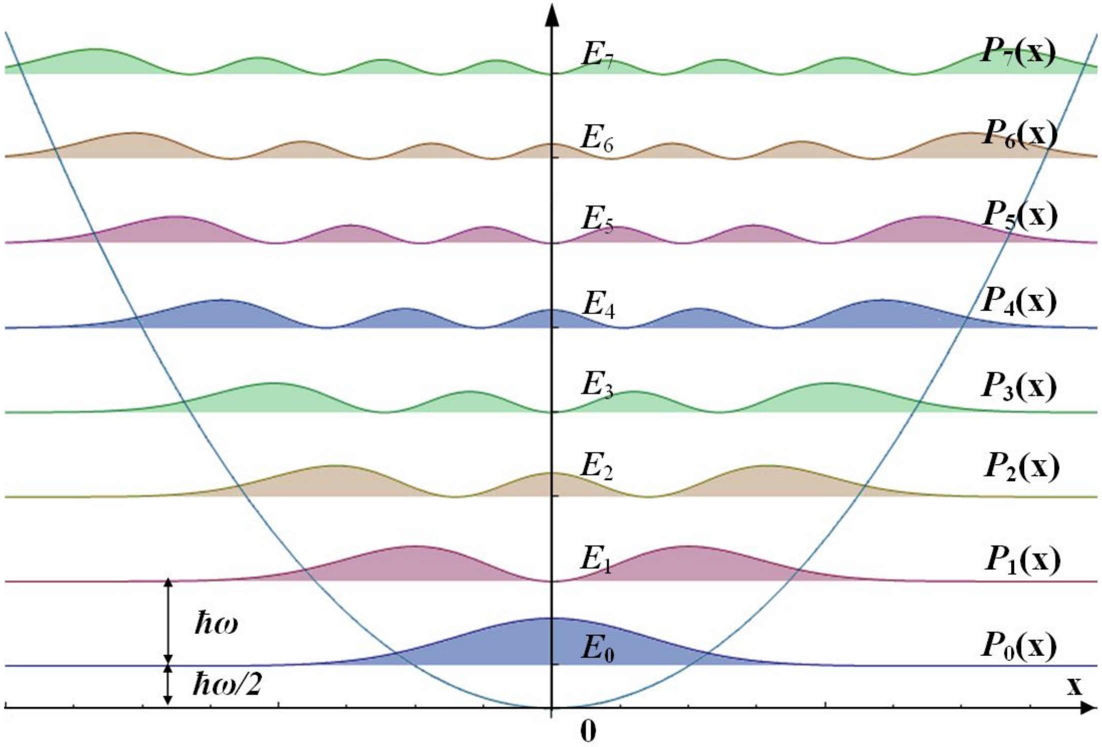
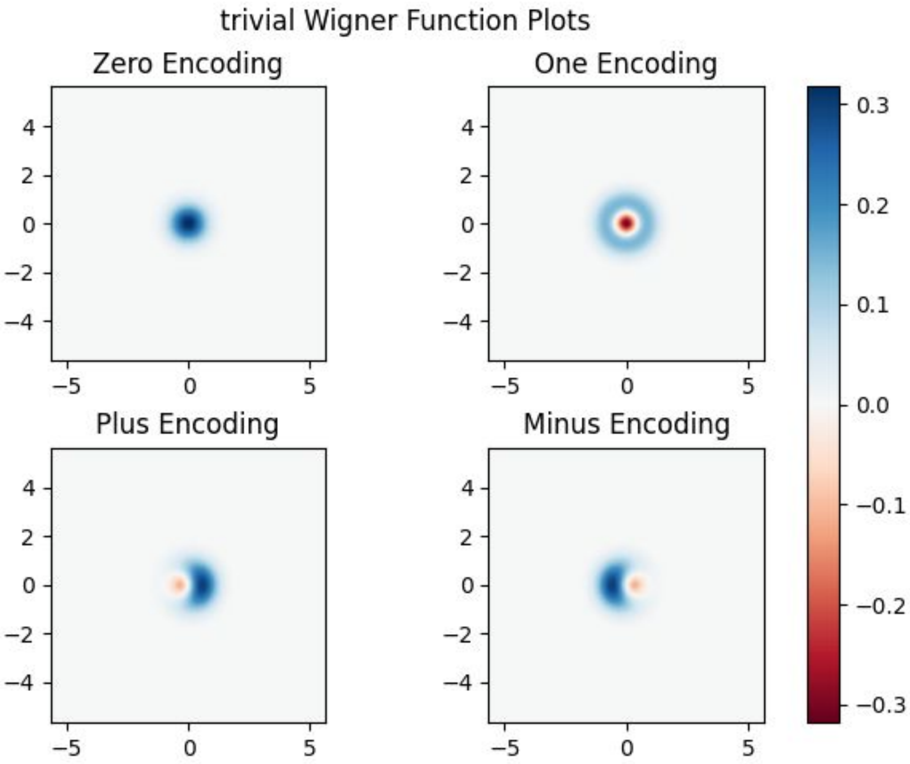


Image from: https://en.wikipedia.org/wiki/Quantum_harmonic_oscillator

Wigner Plots

ENCODING



ENCODING

Bosonic Rotation Codes - Overview

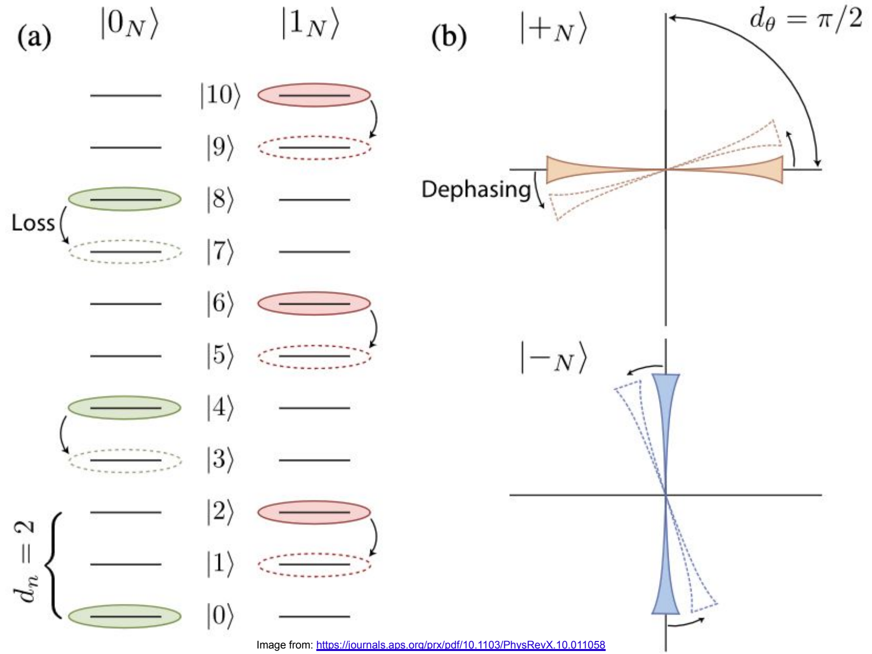
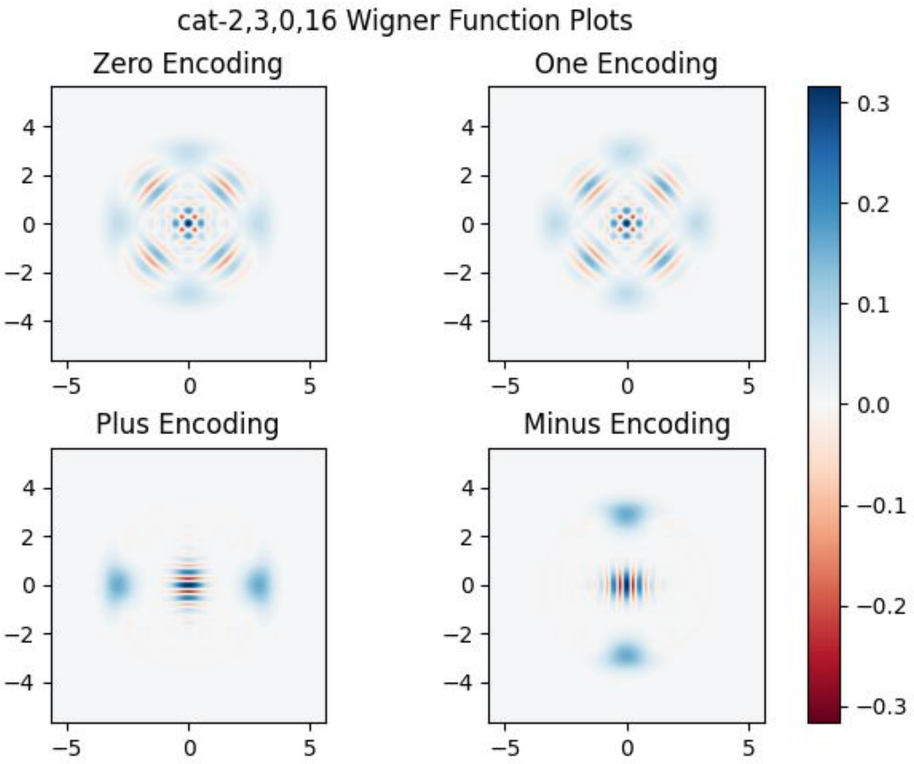


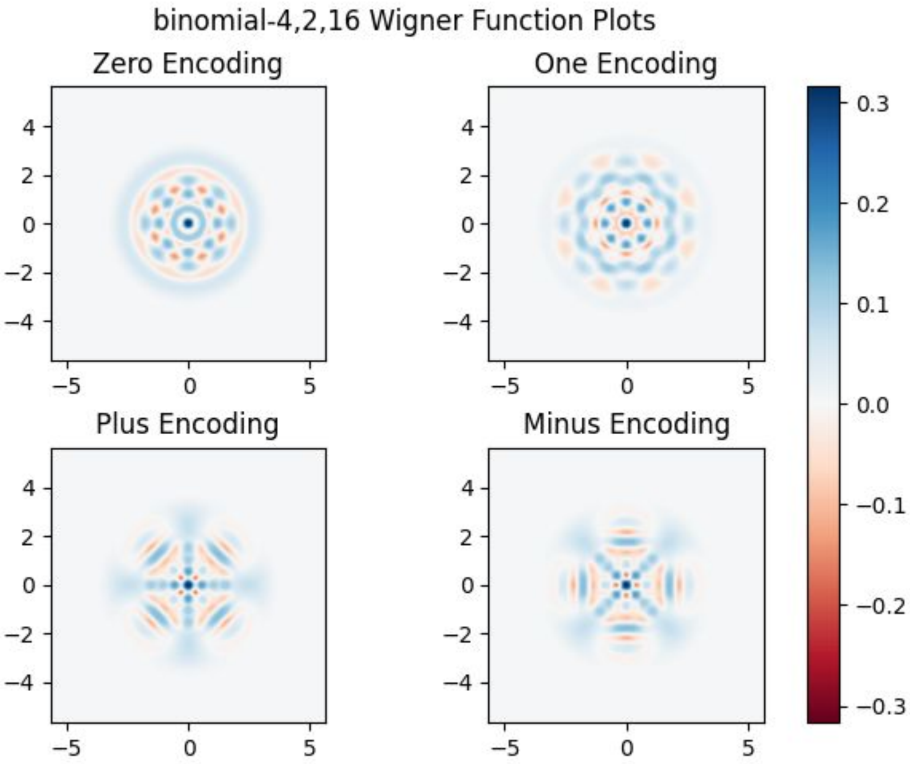
Image from: https://journals.aps.org/prx/pdf/10.1103/PhysRevX.10.011058

N is from hereon used to denote rotation symmetry degree. In this example, $N = 2$.

Bosonic Rotation Codes - Cat Code



Bosonic Rotation Codes - Binomial Code



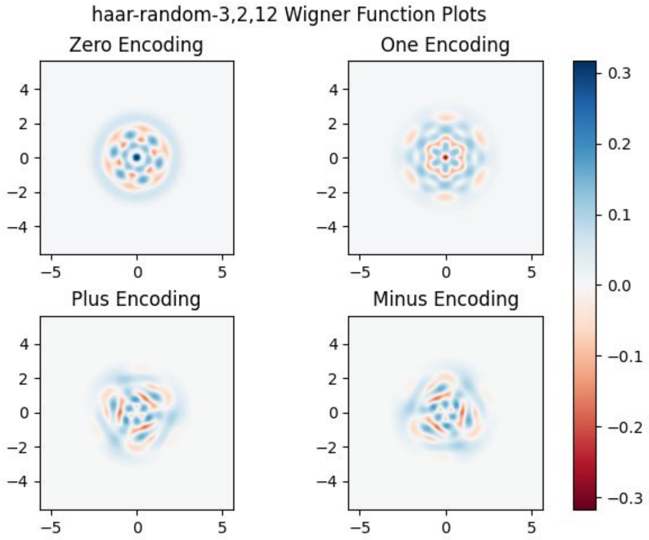
Methodology - General

- Generate random codes
- Compute their optimal fidelity and their no recovery fidelity
- Compare best random codes against binomial codes and cat codes
    - Codes are only compared if they have the same rotation symmetry degree and underwent the same loss channel

Methodology - Generation of Random Codes

$$|0_L\rangle = \frac{1}{\mathcal{N}_0}\left(\sum_{i=0}^{\infty} |2iN\rangle\langle 2iN|\right)|\psi\rangle$$

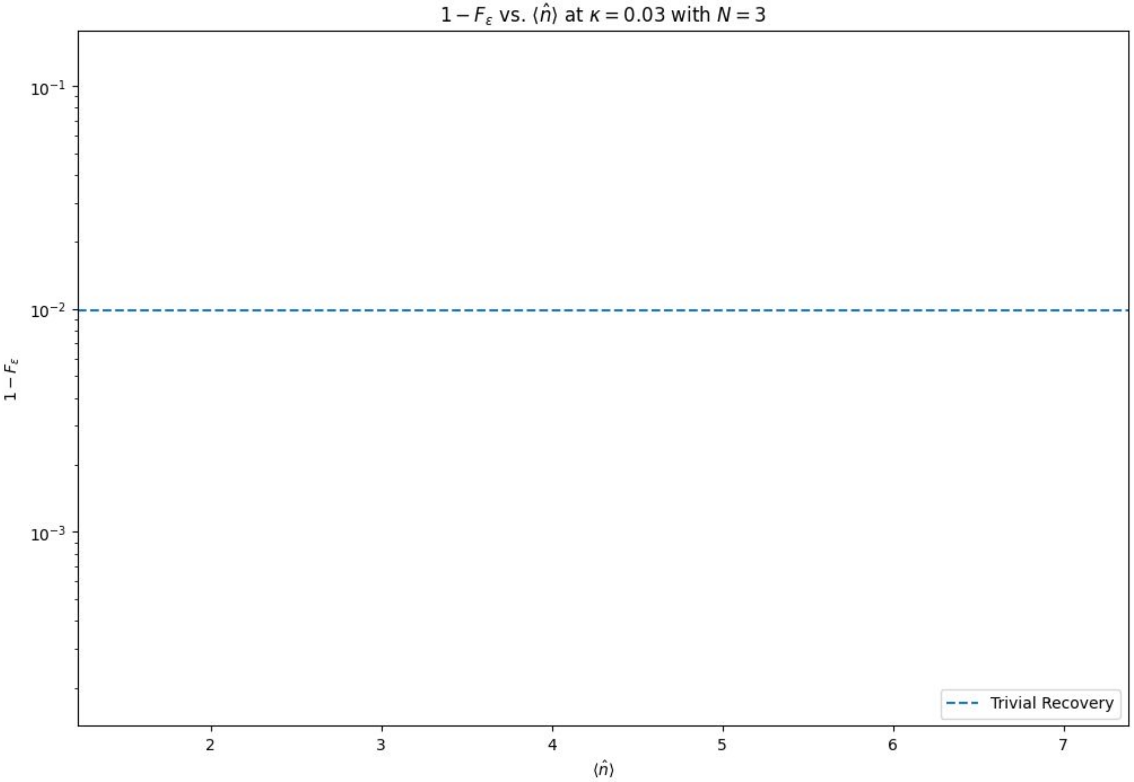
$$|1_L\rangle = \frac{1}{\mathcal{N}_1}\left(\sum_{i=0}^{\infty} |(2i+1)N\rangle\langle(2i+1)N|\right)|\psi\rangle$$



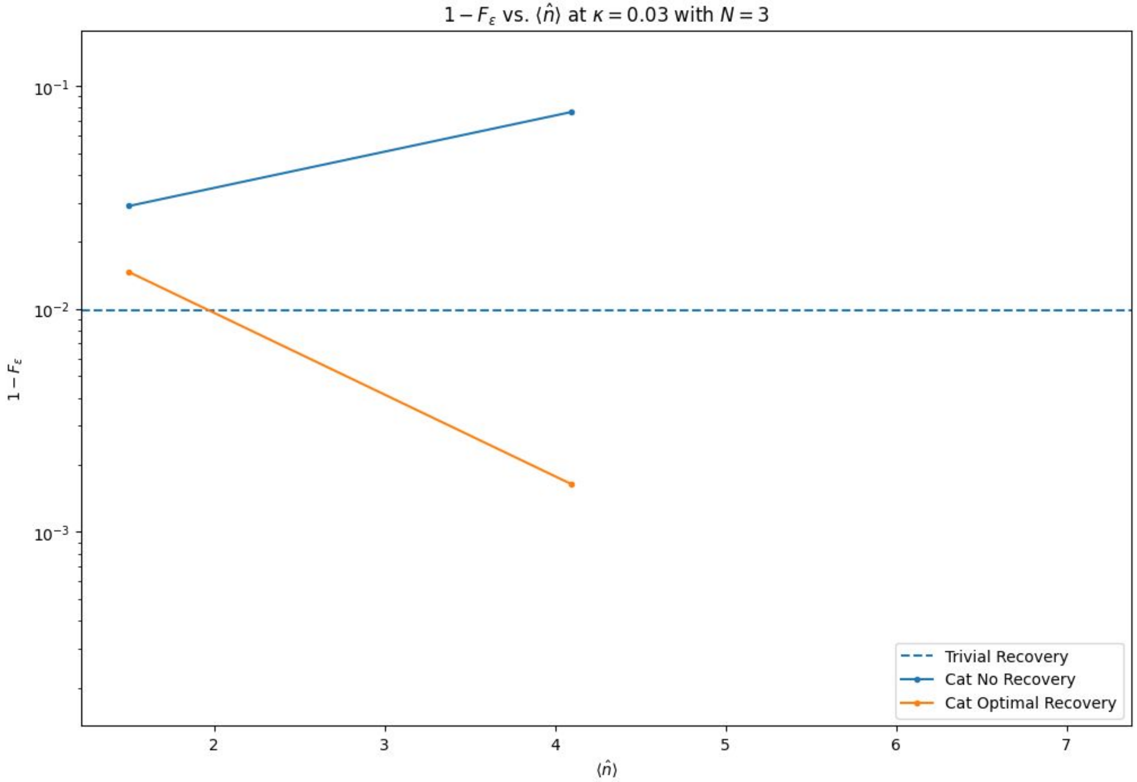
Methodology - Comparison of Codes

$$|\langle 0_b \mid 0_r \rangle|^2 \qquad |\langle 1_b \mid 1_r \rangle|^2 \qquad |\langle +_b \mid +_r \rangle|^2$$

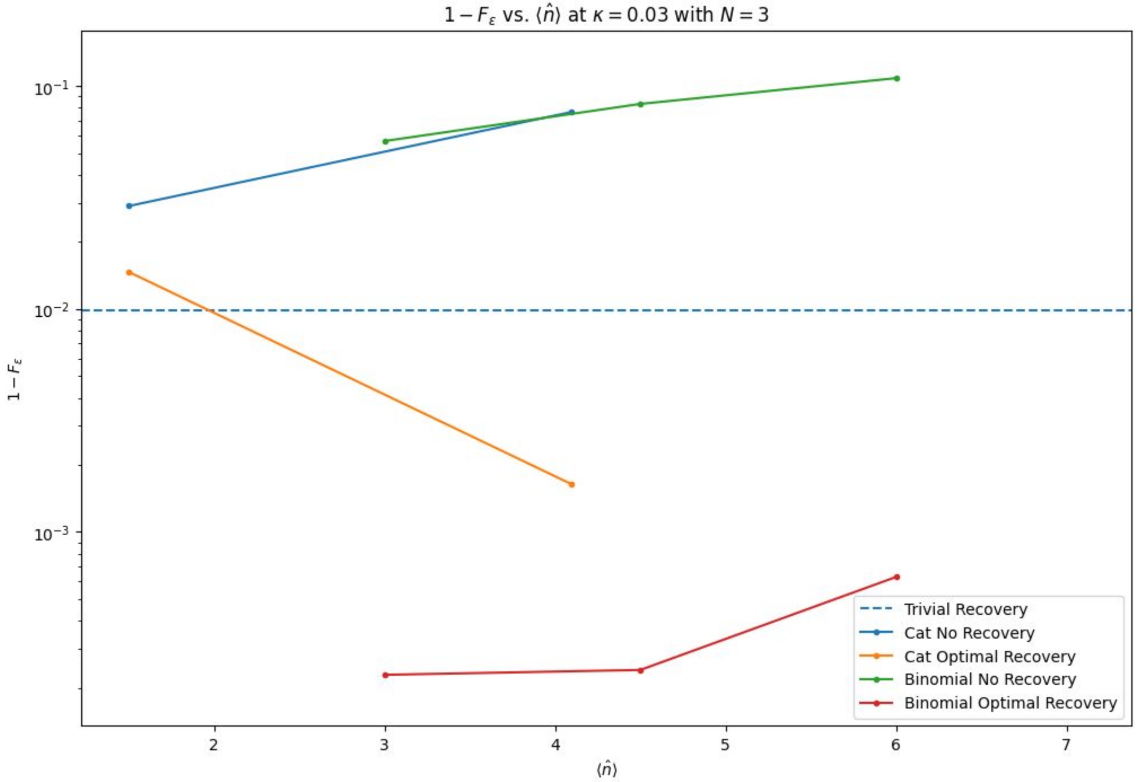
Results - Simulation Data for One Regime



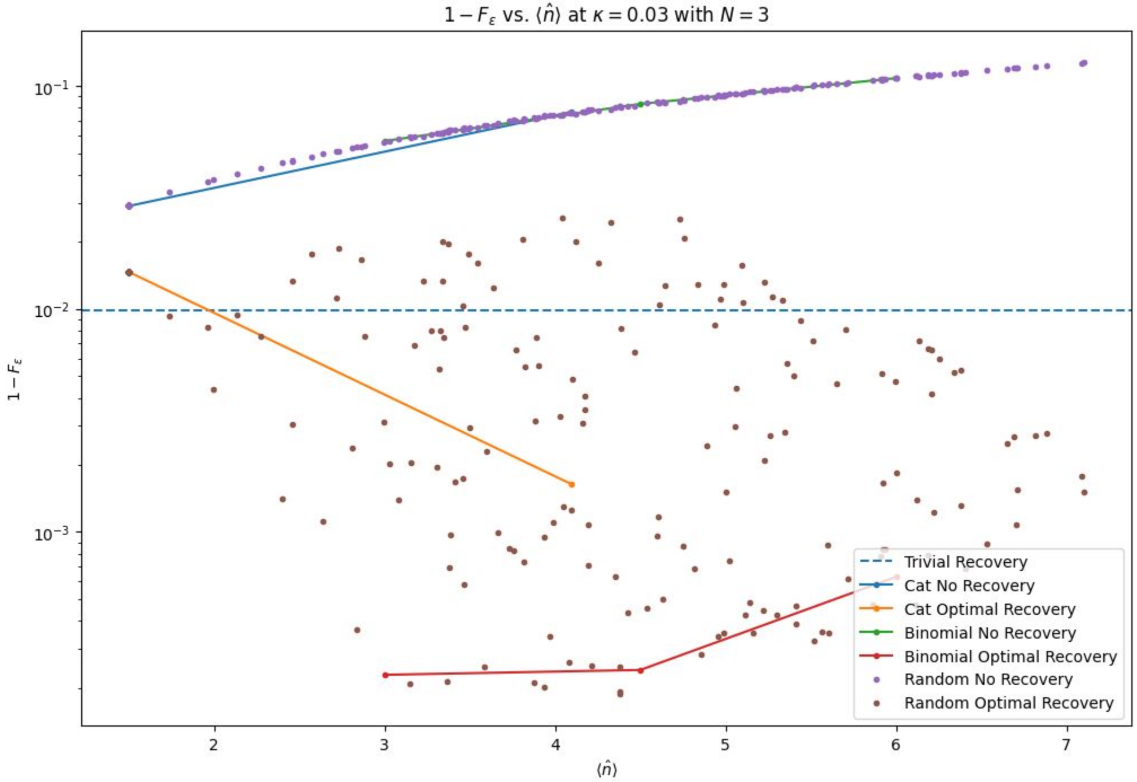
Results - Simulation Data for One Regime



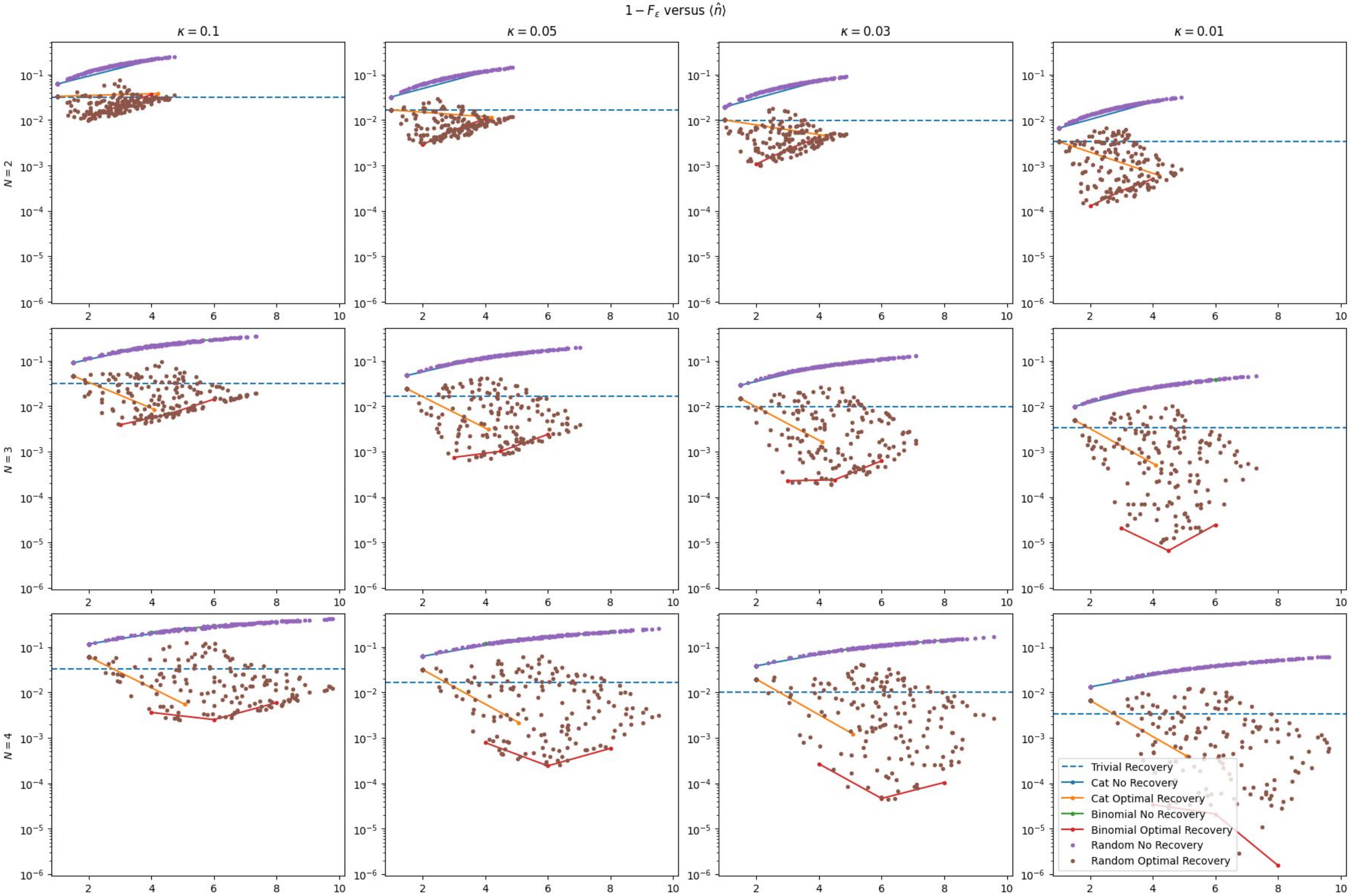
Results - Simulation Data for One Regime



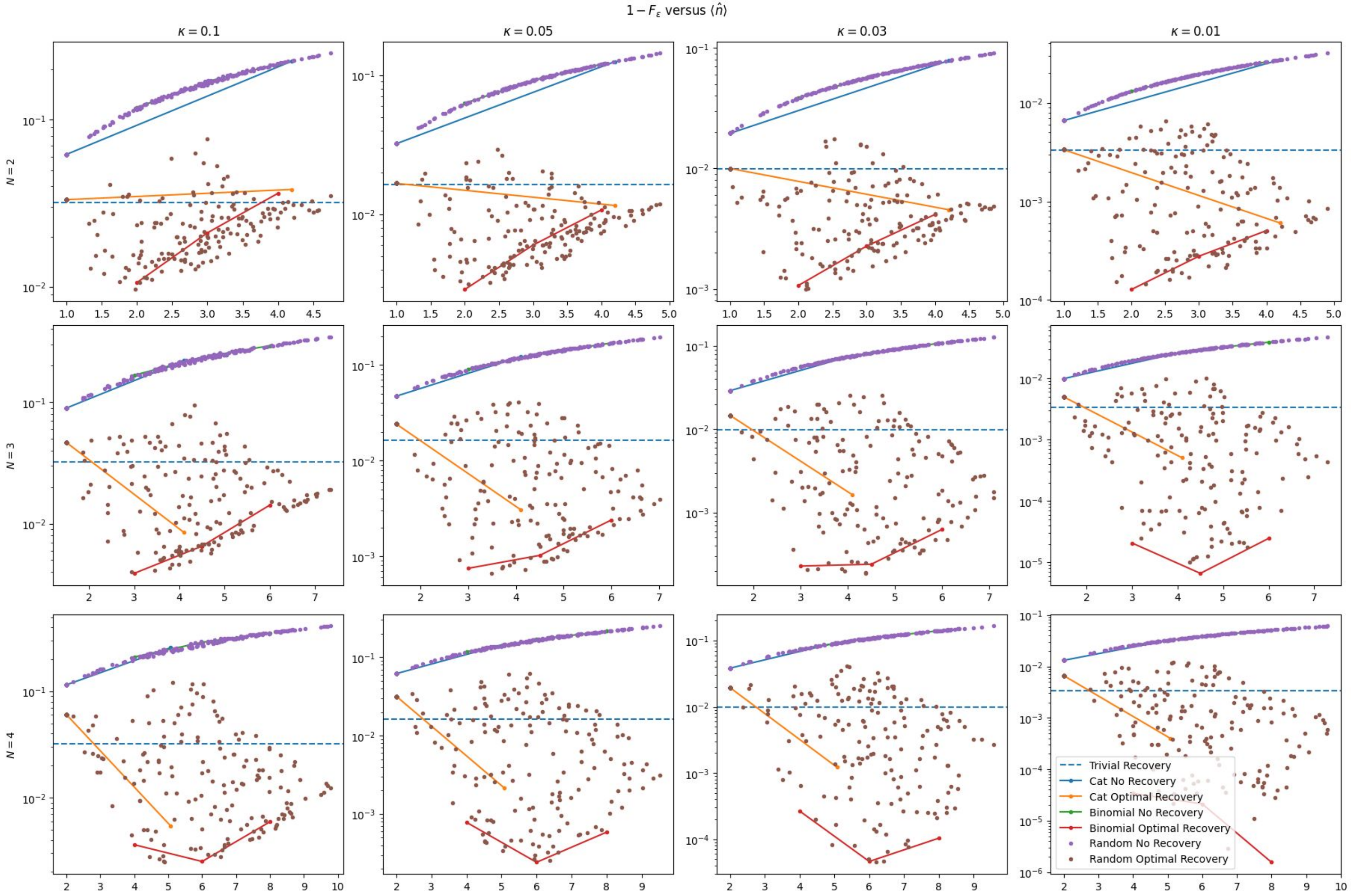
Results - Simulation Data for One Regime



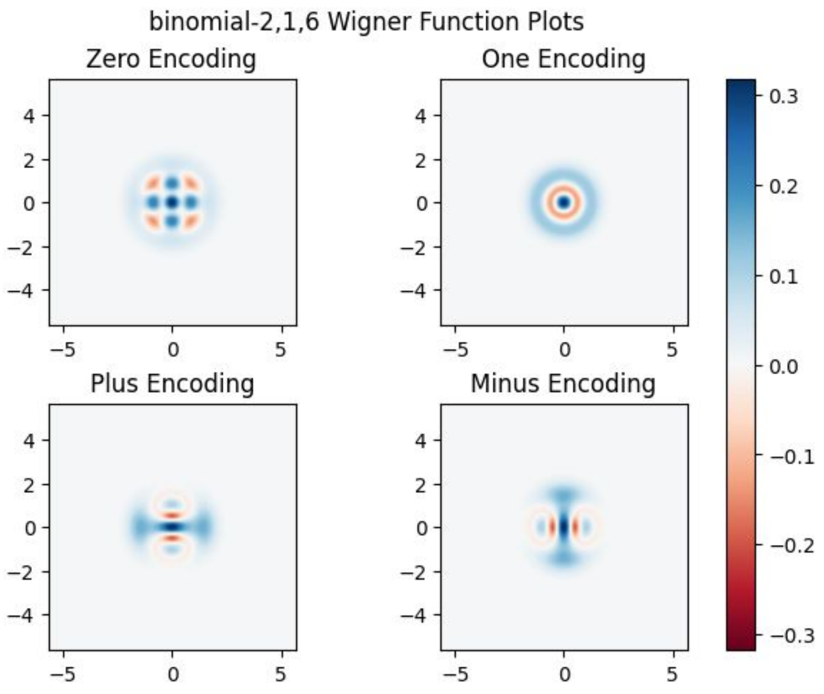
Results - Simulation Data

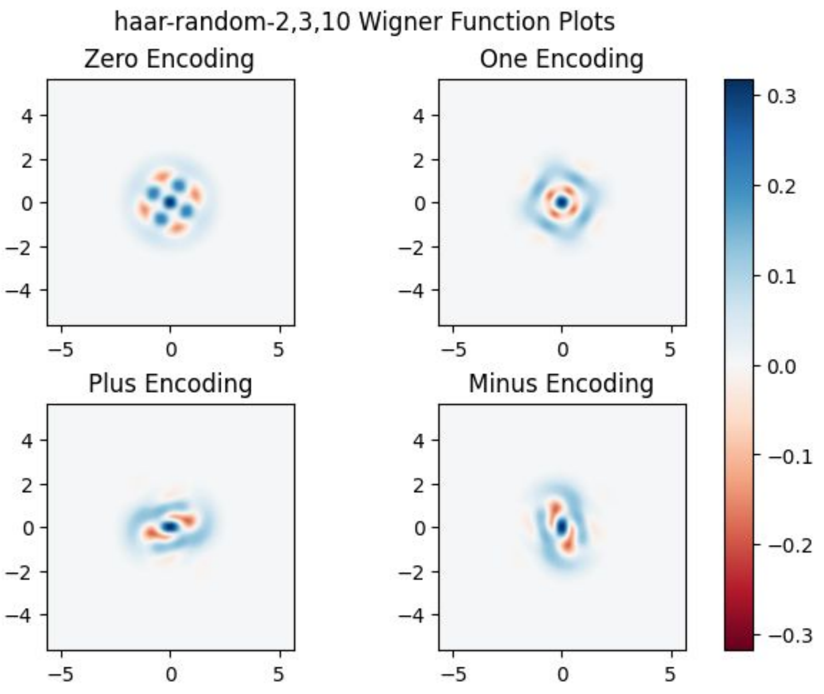


Results - Simulation Data



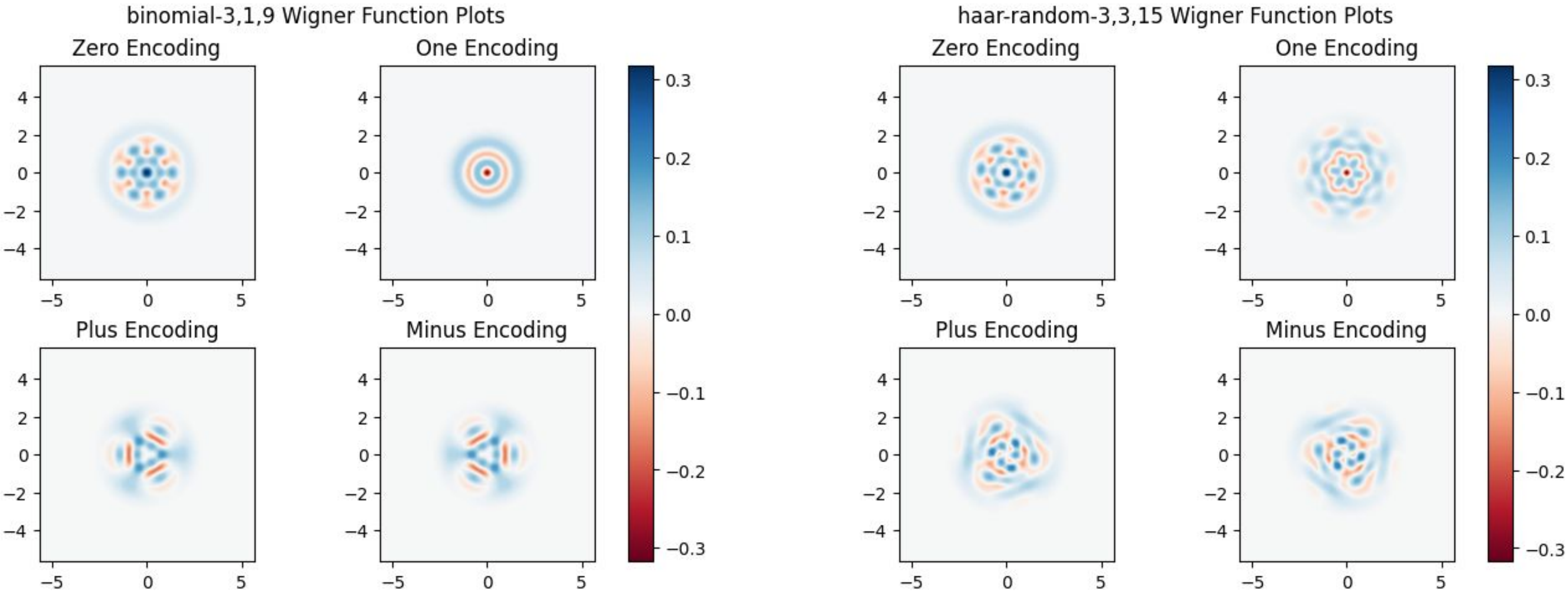
Comparison - Random Code that is Very Similar





Fidelity difference: 7.89×10^{-5}; Dot products: 0.301, 0.948, 0.450; Under $\kappa = 0.03$

Comparison - Random Code that is Very Different



Fidelity difference: 4.19×10^{-5}; Dot products: 0.802, 0.782, 0.020; Under $\kappa = 0.03$

Conclusion - Results Interpretation

- Some random codes beat binomial codes
  - Random codes can be better for any rotation symmetry degree, but binomial codes tend to perform better in lower loss regimes
- When random codes won, they had fidelity differences that were on the order of 10^{-5} to 10^{-4} better
- Winning random codes were generally unlike binomial codes

Further Work

- Check random codes in a setting with dephasing noise in addition to loss
- Use random codes as a benchmark to see how good a bosonic rotation code is
- Check how effective non-optimal recoveries are with random codes
- Use a genetic algorithm to select for the best QEC
- Investigate feasibility of physically implementing specific random codes

Thank You!