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Modified Warshall (All Pair Shortest Path)

Theory :

The Floyd-Warshall algorithm, named after its creators Robert Floyd and Stephen Warshall, is a fundamental algorithm in computer science and graph theory. It is used to find the shortest paths between all pairs of nodes in a weighted graph. This algorithm is highly efficient and can handle graphs with both positive and negative edge weights, making it a versatile tool for solving a wide range of network and connectivity problems.

Algorithm :

step 1 :

Initialize the solution matrix same as the input graph matrix as a first step.

Step 2 :

Then update the solution matrix by considering all vertices as an intermediate vertex.

Step 3 :

The idea is to pick all vertices one by one and updates all shortest paths which include the picked vertex as an intermediate vertex in the shortest path.

Step 4 :

When we pick vertex number k as an intermediate vertex, we already have considered vertices {0, 1, 2, .. k-1} as intermediate vertices.

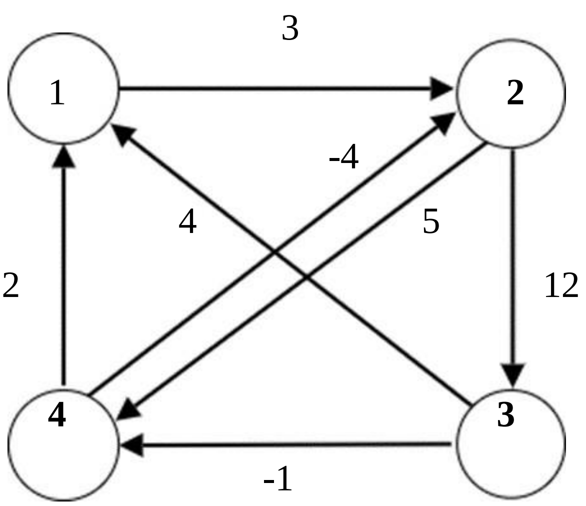
Step 5 :

For every pair (i, j) of the source and destination vertices respectively, there are two possible cases.

1 .k is not an intermediate vertex in shortest path from i to j. We keep the value of dist[i][j] as it is.

2. k is an intermediate vertex in shortest path from i to j. We update the value of dist[i][j] as dist[i][k] + dist[k][j], if dist[i][j] > dist[i][k] + dist[k][j]

Problem : Find the all pair shortest path for the following graph (source vertex is 1)



Solution :

A^0 :

	1	2	3	4
1	0	3	∞	∞
2	∞	0	12	5
3	4	∞	0	-1
4	2	-4	∞	0

Find the matrix for vertex 1

A^1 considering 1 as an intermiadiatary vertex

A^1 :

	1	2	3	4
1	0	3	∞	∞
2	∞	0	**12**	**5**
3	4	**7**	0	**-1**
4	2	**-4**	∞	0

1) $A^0[2,3] \qquad A^0[2,1] + A^0[1,3]$
   $12 \quad < \quad \infty \quad + \quad \infty$

2) $A^0[2,4] \qquad A^0[2,1] + A^0[1,4]$
   $5 \quad < \quad \infty \quad + \quad \infty$

3) $A^0[3,2] \qquad A^0[3,1] + A^0[1,2]$
   $\infty \qquad 4 \quad + \quad 3$
   $\infty \quad > \quad 7$

4) $A^0[3,4] \qquad A^0[3,1] + A^0[1,4]$
   $-1 \quad < \quad 4 \quad + \quad \infty$

5) $A^0[4,2] \qquad A^0[4,1] + A^0[1,2]$
   $-4 \qquad 2 \quad + \quad 3$
   $-4 \quad < \quad 5$

6) $A^0[4,3] \qquad A^0[4,1] + A^0[1,3]$
   $\infty \quad = \quad 2 \quad + \quad \infty$

Find the matrix for vertex 2

A^2 considering 2 as an intermiadiatary vertex

A^2 :

	1	2	3	4
1	0	3	∞	∞
2	∞	0	**12**	**5**
3	4	**7**	0	**-1**
4	2	**-4**	∞	0

1) $A^1[1,3]$ $A^1[1,2] + A^1[2,3]$

∞ $3 + 12$

$\infty > 15$

2) $A^1[1,4]$ $A^1[1,2] + A^1[2,4]$

∞ $3 + 5$

$\infty > 8$

3) $A^1[3,1]$ $A^1[3,2] + A^1[2,1]$

$4 < 7 + \infty$

4) $A^1[3,4]$ $A^1[3,2] + A^1[2,4]$

$-1 < 7 + 5$

5) $A^1[4,1]$ $A^1[4,2] + A^1[2,1]$

$2 < -4 + \infty$

6) $A^1[4,3]$ $A^1[4,2] + A^1[2,3]$

∞ $-4 + 12$

$\infty > 8$

Find the matrix for matrix of vertex 3

A^3 considering 3 as an intermiadiatary vertex

A^3 :

	1	2	3	4
1	0	3	5	8
2	16	0	12	5
3	4	7	0	-1
4	2	-4	8	0

1) A^2 [1,2] $\quad$ A^2 [1,3] + A^2 [3,2]

   $3 \quad < \quad 15 + 7$

2) A^2 [1,4] $\quad$ A^2 [1,3] + A^2[3,4]

   $8 \quad < \quad 15 + (-1)$

3) A^2 [2,1] $\quad$ A^2 [2,3] + A^2 [3,1]

   $\infty \quad 12 + 4$

   $\infty \quad > \quad 16$

4) A^2 [2,4] $\quad$ A^2 [2,3] + A^2 [3,4]

   $5 \quad 12 + (-1)$

   $5 \quad > \quad 11$

5) A^2 [4,1] $\quad$ A^2 [4,3] + A^2 [3,1]

   $2 \quad < \quad 8 + 4$

6) A^2 [4,3] $\quad$ A^2 [4,2] + A^2 [2,3]

   $-4 \quad < \quad 8 + 7$

Find the matrix for matrix of vertex 4

A^4 considering 4 as an intermiadiatary vertex

A^4 :

	1	2	3	4
1	0	3	5	8
2	14	0	12	5
3	1	3	0	-1
4	2	-4	8	0

1) A^4 [1,2] A^4 [1,4] + A^4 [4,2]

3 8 + 4

3 < 12

2) A^4 [1,3] A^4 [1,4] + A^4 [4,3]

5 8 + 8

5 < 16

3) A^4 [2,1] A^4 [2,4] + A^4 [4,1]

16 12 + 2

16 > 14

4) A^4 [2,3] A^4 [2,4] + A^4 [4,3]

12 5 + 8

12 < 13

5) A^4 [3,1] A^4 [3,4] + A^4 [4,1]

4 -1 + 2

4 > 1

6) A^4 [3,2] A^4 [3,4] + A^4 [4,2]

7 -1 + 4

7 > 3