Chapter 2

Quadrotor Modeling

In this chapter, coordinated frames are presented to describe the motion of a quadrotor in three dimensional space. Particularly, inertial and body frames are described. In order to rotate from the body to inertial frames, intermediate frames are required, which are described using Euler angles. Next, a six degrees of freedom model of quadrotor is derived using Newton's law with these coordinated frames. The model involves various parameters that needs to be computed to completely describe the mathematical model. In this direction, thrust vs pulse width modulation (pwm) and torque vs pwm curves are obtained experimentally using force-torque sensors.

2.1 Coordinate Frames

The position of a quadrotor can be best described in an inertial frames. The aerodynamic and thrust force can be best described in the body frame. Therefore, we require different coordinate frames to describe the motion of a quadrotor. This practice is common to aerospace community. The various frames involved in describing the motion of a quadrotor are given next.

- Global inertial frame (i^i, j^i, k^i): This is the fixed earth centered frame with X, Y, and Z axes aligned with North, East, and Downward direction. Newton's equations of motion are derived relative to this frame.

- Vehicle or Local inertial frame (i^v, j^v, k^v): This translating frame is located at the center of mass of the vehicle with the axes x, y and z parallel to global inertial frame. This is an important intermediate frame between global and body frames. Local inertial frame is obtained by translating the global frame.

- Vehicle frame 1: This is located at center of mass of the aircraft, this frame is obtained by rotating the local inertial frame about z axis by an angle ψ.

- Vehicle frame 2: This is also located at center of mass of the aircraft, this frame is obtained by rotating Vehicle Frame 1 about y axis by an angle θ.

- Body frame (i^b, j^b, k^b): This frame is located at the center of mass of the vehicle with the axes x, y and z aligned with the vehicles front, right side and downward directions.The body frame is obtained by rotating Vehicle Frame 2 about x axis by an angle ϕ.

in [32].

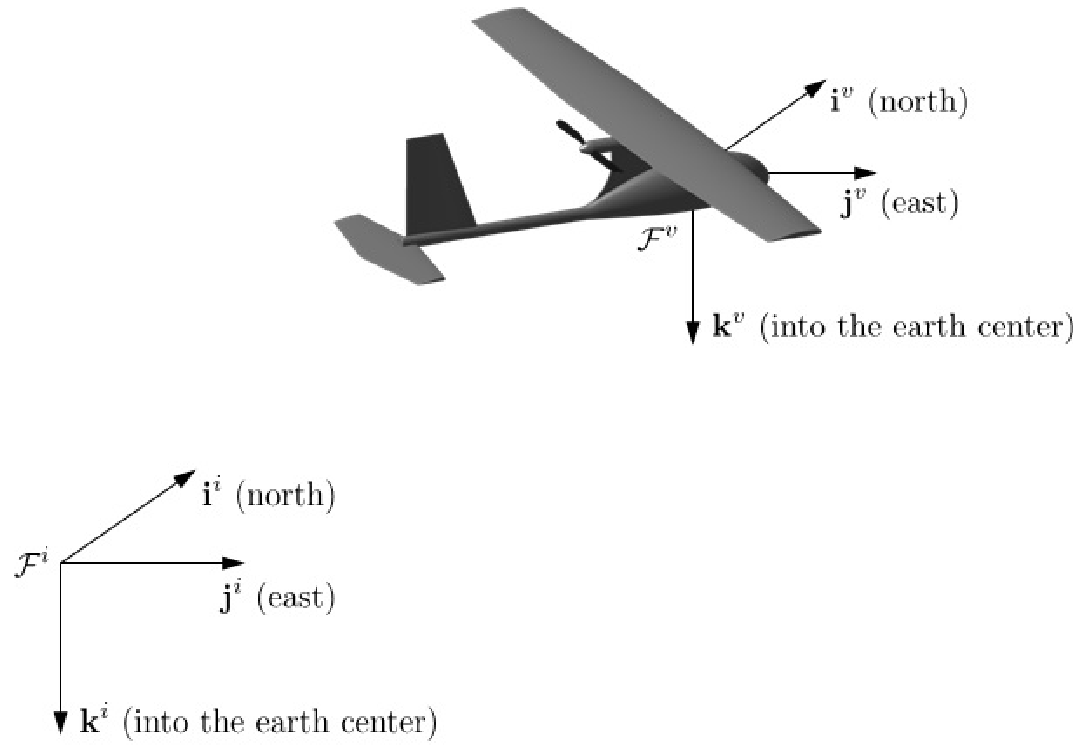

Figure 2.1: Reference frames, as described in [32]

A transformation matrix relating body frame and local inertial frame is obtained by sequentially rotating the local inertial frame to Vehicle frame 1 by ψ, Vehicle frame 2 by θ and then to body frame by ϕ. This sequence of rotation is important and it is not commutative. Note that in this rotation singularity can occur, which depends on the sequence of rotation. It is to be noted that one can use different sequence to reach the body frame like 123 rotation, 121 rotation, 313 rotation etc [29]. The order of rotation cannot be altered once we choose a particular sequence, however one can switch from one sequence to another sequence by using intra Euler angle relation between the sequence. The preliminary material regarding modeling can be refereed in [32].

2.1.1 Rotation of Reference Frame

Rotation about k axis:

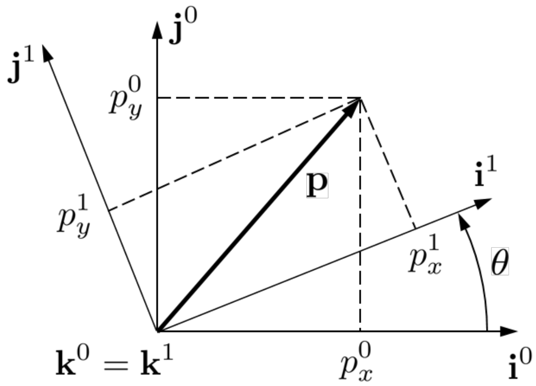

Figure 2.2: Rotation of a reference frame, as described in [32]

$$
\begin{aligned}
p &= p_x^0 i^0 + p_y^0 j^0 + p_z^0 k^0 \\
p &= p_x^1 i^1 + p_y^1 j^1 + p_z^1 k^1
\end{aligned}
\tag{2.1}
$$

Equating the position vector of the two frames, we get

$$
p_x^0 i^0 + p_y^0 j^0 + p_z^0 k^0 = p_x^1 i^1 + p_y^1 j^1 + p_z^1 k^1 \tag{2.2}
$$

$$
\begin{aligned}
\begin{bmatrix} p_x^1 \\ p_y^1 \\ p_z^1 \end{bmatrix} &= \begin{bmatrix} i^1.i^0 & i^1.j^0 & i^1.k^0 \\ j^1.i^0 & j^1.j^0 & j^1.k^0 \\ k^1.i^0 & k^1.j^0 & k^1.k^0 \end{bmatrix} \begin{bmatrix} p_x^0 \\ p_y^0 \\ p_z^0 \end{bmatrix} \\
\implies p^1 &= R_0^1 p^0
\end{aligned}
\tag{2.3}
$$

where,

$$
R_0^1 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.4}
$$

Similarly,

Rotation about j axis:

$$R_0^1 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \tag{2.5}$$

Rotation about i axis:

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{2.6}$$

Having defined transformation (rotation) matrices, we next present direction cosine matrices (DCMs) to transform from the inertial frame to the body frame or vice versa. As discussed, singularity may occur in these transformation matrices. To avoid this, an intra-Euler angle switching strategy based on different sequence of transformations can be used.

2.1.2 Euler Sequence 123 or xyz

Now the transformation matrix corresponding to rotation of a vector from local inertial frame to body frame can be obtained by sequentially rotating the local inertial frame by an angle ψ, θ, ϕ about z, y, x axes, respectively.

$$R_v^b(\phi, \theta, \psi) = R_{v2}^b(\phi) R_{v1}^{v2}(\theta) R_v^{v1}(\psi) \tag{2.7}$$

$$R_v^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_v^b = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} \tag{2.8}$$

The transformation matrix is a square matrix with real entries whose columns and rows are orthogonal unit vectors. Hence, the transformation matrix is an orthogonal matrix. Thus, $[R_v^b]\ [R_v^b]^T = [R_v^b]^T\ [R_v^b] = I \implies [R_v^b]^T = [R_v^b]^{-1}$. Transformation matrix from body frame to local inertial frame is obtained by inverting Eq.(2.8).

$$R_b^v = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \tag{2.9}$$

Since the onboard micro electro-mechanical systems (MEMS) sensor provides the data in body frame we have to convert this to Eulerian frame. Hence the relation between body angular rates and eulerian rates can be obtained as,

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot\phi \\ 0 \\ 0 \end{bmatrix} + R_{v2}^b(\phi) \begin{bmatrix} 0 \\ \dot\theta \\ 0 \end{bmatrix} + R_{v2}^b(\phi) R_{v1}^{v2}(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot\psi \end{bmatrix} \tag{2.10}$$

$$\implies \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & \sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} \tag{2.11}$$

Note: Transformation matrix Eq.(2.8) obtained using this Euler sequence-123 has singularity at $\theta = \pm 90^o$.

2.1.3 Euler Sequence 121 or xyx

Similar to Euler sequence 123, the transformation matrix corresponding to the Euler sequence 121 from local inertial frame to body frame can be obtained by sequentially rotating the local inertial frame by an angle ψ_1, θ_1, ϕ_1 about x, y, x axes, respectively.

$$R_v^b(\phi_1, \theta_1, \psi_1) = R_{v2}^b(\phi_1) R_{v1}^{v2}(\theta_1) R_v^{v1}(\psi_1) \tag{2.12}$$

$$R_v^b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_1 & \sin\phi_1 \\ 0 & -\sin\phi_1 & \cos\phi_1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ 0 & 1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_1 & \sin\psi_1 \\ 0 & -\sin\psi_1 & \cos\psi_1 \end{bmatrix}$$

$$R_v^b = \begin{bmatrix} \cos\theta_1 & \sin\psi_1 \sin\theta_1 & -\cos\psi_1 \sin\theta_1 \\ \sin\phi_1 \sin\theta_1 & \cos\phi_1 \cos\psi_1 - \sin\phi_1 \cos\theta_1 \sin\psi_1 & \cos\phi_1 \sin\psi_1 + \sin\phi_1 \cos\theta_1 \cos\psi_1 \\ \cos\phi_1 \sin\theta_1 & -\sin\phi_1 \cos\psi_1 - \cos\theta_1 \cos\phi_1 \sin\psi_1 & -\sin\phi_1 \sin\psi_1 + \cos\theta_1 \cos\phi_1 \cos\psi_1 \end{bmatrix} \tag{2.13}$$

Also,

$$R_b^v = \begin{bmatrix} \cos\theta_1 & \sin\phi_1 \sin\theta_1 & \cos\phi_1 \sin\theta_1 \\ \sin\psi_1 \sin\theta_1 & \cos\phi_1 \cos\psi_1 - \sin\phi_1 \cos\theta_1 \sin\psi_1 & -\sin\phi_1 \cos\psi_1 - \cos\theta_1 \cos\phi_1 \sin\psi_1 \\ -\cos\psi_1 \sin\theta_1 & \cos\phi_1 \sin\psi_1 + \sin\phi_1 \cos\theta_1 \cos\psi_1 & -\sin\phi_1 \sin\psi_1 + \cos\theta_1 \cos\phi_1 \cos\psi_1 \end{bmatrix} \tag{2.14}$$

Also, the relation between body rates and Eulerian rates corresponding to Euler sequence 121 is,

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cos\theta_1 \\ 0 & \cos\phi_1 & \sin\phi_1 \cos\theta_1 \\ 0 & -\sin\phi_1 & \cos\phi_1 \cos\theta_1 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\theta}_1 \\ \dot{\psi}_1 \end{bmatrix} \tag{2.15}$$

Note: Transformation matrix Eq.(2.13) obtained using this Euler sequence-121 has singularity at $\theta_1 = 0^o$ and 180^o.

2.2 Equations of Motion

In this section, the expression for the kinematics and the dynamics of the quadrotor is derived [32]. The state variables used to describe the state space of the system is provided in Table. 2.1.

Notation	**Description**
x	Inertial North position of vehicle along i^i
y	Inertial East position of vehicle along j^i
z	Inertial Down position of vehicle along k^i
u	Body frame velocity of vehicle along i^b
v	Body frame velocity of vehicle along j^b
w	Body frame velocity of vehicle along k^b
ϕ	Roll angle of vehicle with respect to vehicle frame 2
θ	Pitch angle of vehicle with respect to vehicle frame 1
ψ	Yaw angle of vehicle with respect to vehicle / local inertial frame
p	Roll rate of vehicle measured along i^b
q	Pitch rate of vehicle measured along j^b
r	Yaw rate of vehicle measured along k^b

Table 2.1: State variables

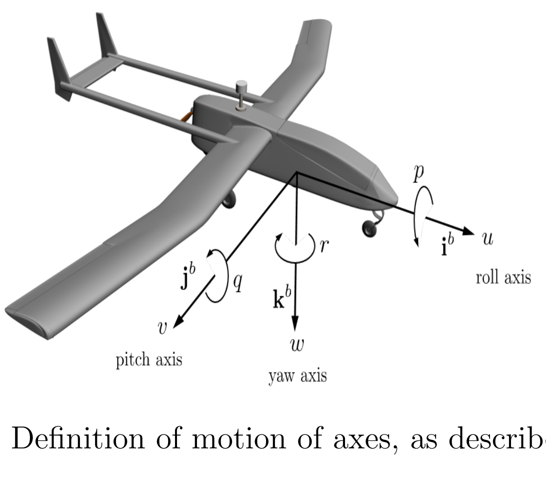

Figure 2.3: Definition of motion of axes, as described in [32]

2.2.1 Kinematics

The translating velocity of quadrotor is generally measured using accelerometer sensor onboard in a body frame. On the other hand, the position of quadrotor is provided in a global inertial reference frame by onboard global positioning system (GPS). Hence, in order to relate the position and velocity information, the velocity information requires differentiation and rotational transformation from body to the inertial frame which can be obtained as,

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_b^v \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{2.16}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{2.17}$$

This is the kinematic relation that relates position and velocity of a quadrotor. These constitute the first three state of equations of motion. Similarly, the rotational transformation relation between body rates and Eulerian rates is obtained as follows,

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2.18}$$

These rotational kinematics equations constitute the next three state of the quadrotor. Note that in this inversion Eq.(2.18), we have $\sec\theta$, which becomes singular at pitch angle 90^o.

2.2.2 Dynamics

The dynamic equations of motion for a quadrotor is obtained by applying Newtons second law to the translational degrees of freedom and then to rotational dynamics of freedom. Newtons laws hold in an inertial frame, hence the motion of the quadrotor is expressed in global inertial frame (Earth).

2.2.2.1 *Translational Motion*

Newtons second law: *The vector sum of the external forces F on an object is equal to the mass m of that object multiplied by the acceleration vector a of the object.*

$$\begin{aligned} F &= m\ a \\ &= m\frac{dV_i}{dt} \end{aligned} \tag{2.19}$$

here, V_i are the translating velocity of a quadrotor. The time derivative of velocity in the inertial frame can be decomposed as follows,

$$\frac{dV_i}{dt} = \frac{dV_b}{dt} + \omega^b_{b/i} \times V_b \tag{2.20}$$

here, V_b is the translating velocity in the body frames (u,v,w are the components of the velocity in the body frame). Also, $\omega^b_{b/i} = p\vec{\mathbf{i}} + q\vec{\mathbf{i}} + r\vec{\mathbf{i}}$ is the angular rate expressed in body frame.

Thus, force in the body frame can be represented as,

$$F_b = m\left(\frac{dV_b}{dt} + \omega^b_{b/i} \times V_b\right) \tag{2.21}$$

here, $F_b = F_x\vec{\mathbf{i}} + F_y\vec{\mathbf{i}} + F_z\vec{\mathbf{i}}$ is the force acting on quadrotor.

$$\frac{dV_b}{dt} = -\omega^b_{b/i} \times V_b + \frac{1}{m}F_b \tag{2.22}$$

$$
\implies \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} rv - qw \\ pw - ru \\ qu - pv \end{bmatrix} + \frac{1}{m} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \tag{2.23}
$$

Basically the force acting on quadrotor can be assumed as the sum of gravitational force, aerodynamic force and propulsion force $F_b = F_g + F_a + F_p$. Since in a quadrotor, propulsion force (Thrust - T) and the gravitational forces are the dominant forces, the aerodynamic force is neglected assuming aerodynamic force is very small. Gravitational force acts in the inertial frame toward the center of earth, this is transformed to body frame using transformation matrix. Also the thrust force (T) always acts normal to the xy plane (i.e. outwards) of quadrotor. Taking into account these assumptions, Eq.(2.23) is written as follows,

$$
\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -T/m \end{bmatrix} + R_i^b \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} rv - qw \\ pw - ur \\ qu - pv \end{bmatrix} \tag{2.24}
$$

Thus,

$$
\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -T/m \end{bmatrix} + \begin{bmatrix} -g \sin\theta \\ g \cos\theta \sin\phi \\ g \cos\theta \cos\phi \end{bmatrix} + \begin{bmatrix} rv - qw \\ pw - ur \\ qu - pv \end{bmatrix} \tag{2.25}
$$

2.2.2.2 Rotational Motion

For rotational motion, Newton's second law states that

$$
\frac{dh_i}{dt} = \mathbf{m} \tag{2.26}
$$

where h_i is the angular momentum in the inertial frame and $\mathbf{m}$ is the sum of all externally applied moments.

Now the angular momentum (h_i) in the inertial frame can be represented as the combination of angular momentum (h_b) and angular velocity ($\omega^b_{b/i}$) in a body frame.

$$\frac{dh_i}{dt} = \frac{dh_b}{dt} + \omega^b_{b/i} \times h_b = \mathbf{m}_b \tag{2.27}$$

Assuming that a quadrotor to be a perfect rigid body, the angular momentum is defined as the product of inertia matrix $\mathbf{I}$ and angular velocity vector $h_b = \mathbf{I}\omega^b_{b/i}$. Also the moment of inertia $\mathbf{I}$ is a constant in body frame, hence $\frac{d\mathbf{I}}{dt} = 0$. Thus the angular momentum equation can be represented as,

$$\mathbf{I}\frac{d\omega^b_{b/i}}{dt} + \omega^b_{b/i} \times \left(\mathbf{I}\omega^b_{b/i}\right) = \mathbf{m}_b \tag{2.28}$$

By re-arranging Eq.(2.28), we get

$$\dot{\omega}^b_{b/i} = \mathbf{I}^{-1}\left[-\omega^b_{b/i} \times \left(\mathbf{I}\omega^b_{b/i}\right) + \mathbf{m}_b\right] \tag{2.29}$$

Since the quadrotor considered in this research is symmetric about x and y axis, the cross product terms of the inertia matrix are zero.

$$\Longrightarrow \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}^{-1} \left[\begin{bmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{bmatrix} \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \begin{bmatrix} l \\ m \\ n \end{bmatrix}\right] \tag{2.30}$$

here, $\mathbf{m}_b = l\vec{\mathbf{i}} + m\vec{\mathbf{j}} + n\vec{\mathbf{k}}$ where l, m, n are the components of the externally applied moments known as rolling pitching and yawing moments respectively.

$$\Longrightarrow \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}}qr \\ \frac{I_{zz}-I_{xx}}{I_{yy}}pr \\ \frac{I_{xx}-I_{yy}}{I_{zz}}pq \end{bmatrix} + \begin{bmatrix} \frac{1}{I_{xx}}l \\ \frac{1}{I_{yy}}m \\ \frac{1}{I_{zz}}n \end{bmatrix} \tag{2.31}$$

2.3 Nonlinear Six Degrees of Freedom Model

Using kinematic and dynamic relation, a complete six degrees of freedom model of quadrotor is given as follows.

2.3.1 Kinematics

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{2.32}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2.33}$$

2.3.2 Dynamics

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -T/m \end{bmatrix} + \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \end{bmatrix} + \begin{bmatrix} rv - qw \\ pw - ur \\ qu - pv \end{bmatrix} \tag{2.34}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}}qr \\ \frac{I_{zz}-I_{xx}}{I_{yy}}pr \\ \frac{I_{xx}-I_{yy}}{I_{zz}}pq \end{bmatrix} + \begin{bmatrix} \frac{1}{I_{xx}}l \\ \frac{1}{I_{yy}}m \\ \frac{1}{I_{zz}}n \end{bmatrix} \tag{2.35}$$

here, T, l, m and n are the control variables (T - thrust, l - Rolling moment, m -pitching moment, n - yawing moment). The demanded thrust and moments by the controller are achieved by changing the angular speed (ω) of the individual brushless

DC motor. The relation between the thrust and moments for the plus“+” configuration of a quadrotor is written as follows,

$$
\begin{aligned}
T &= c(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\
l &= c.d(\omega_1^2 - \omega_3^2) \\
m &= c.d(\omega_2^2 - \omega_4^2) \\
n &= a.d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)
\end{aligned}
\tag{2.36}
$$

here, c, a and d are thrust constant, drag constant and arm length of the quadrotor. ω_i is the angular speed of individual BLDC motor (i = 1,2,3,4). In experiments, the demanded angular speed of the motors is achieved by changing the pwm signals. Thus, the relation between pwm signals and thrust and with moments are as follows.

$$
\begin{bmatrix} pwm_1 \\ pwm_2 \\ pwm_3 \\ pwm_4 \end{bmatrix} = \begin{bmatrix} k & k & k & k \\ k.d & 0 & -k.d & 0 \\ 0 & k.d & 0 & -k.d \\ b & -b & b & -b \end{bmatrix}^{-1} \begin{bmatrix} T \\ l \\ m \\ n \end{bmatrix}
\tag{2.37}
$$

here, k, b and d are thrust coefficient, torque coefficient and arm length of the quadrotor.

2.4 Plant Parameters

For this research work, we consider a commercially available quadrotor shown in Fig. 2.4, FPV250 V2 from Hobbyking. An individual brushless dc motor (1704kv) with 12A electronic speed controller (ESC) and 1000mAh battery provides a maximum thrust of 240 grams. Hence weight plays a critical role in selection of individual components onboard. A powerful ARM based STM 32 microcontroller clocked at

168Mhz weighing only 10 grams from Navstik was used as the onboard controller in experimental purpose. In order to perform simulation and experiment, we need plant parameters. The inertia parameters of the quadrotor shown in Table. 2.2 is obtained by modeling the plant in Solidworks. The schematics of a quadrotor is shown in Fig. 2.5.

Figure 2.4: FPV250 V2 Quadrotor

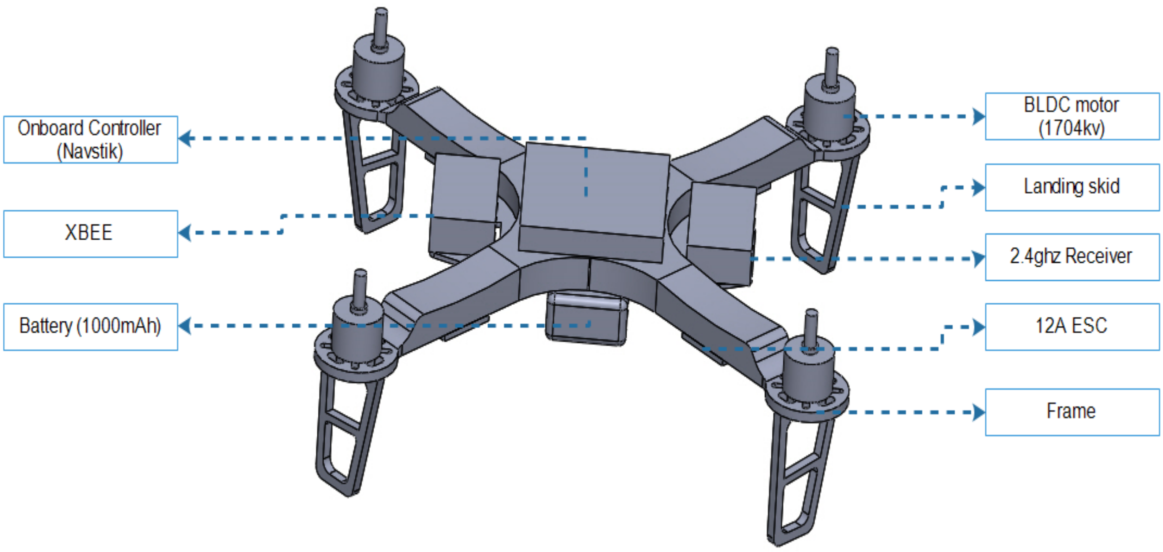

Figure 2.5: Solid works model

Name	Value
mass	340 (grams)
I_{xx}	321862.60 $10^{-9}(Nm^2)$
I_{yy}	305825.42 $10^{-9}(Nm^2)$
I_{zz}	576255.87 $10^{-9}(Nm^2)$
Arm length	0.088388 (m)

Table 2.2: Quadrotor parameters

Next, we determine thrust vs pwm and torque vs pwm curves needed for control allocation block. First, Thrust vs rpm is theoretically determined using dynamic thrust formulae [39] [40].

$$F = 4.392399\ 10^{-8}\ RPM \frac{d^{3.5}}{\sqrt{pitch}}(4.23333\ 10^{-4}\ RPM\ pitch - V_0) \tag{2.38}$$

here, F = Thrust (N), d = propeller diameter (inches) , RPM = propeller rotations/minute , pitch = propeller pitch (inches), V_0 = propeller forward airspeed (m/s).

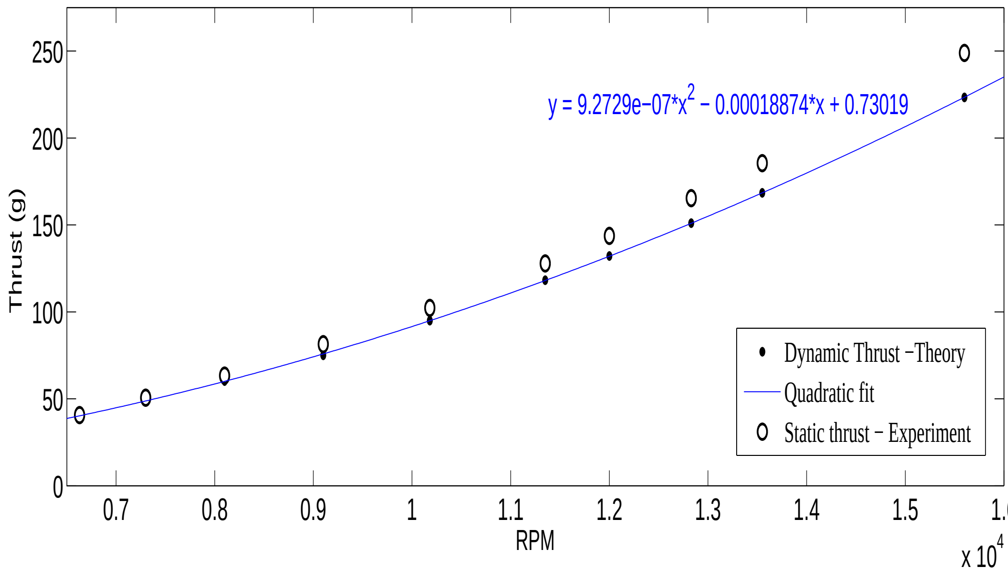

Figure 2.6: Thrust comparison - theory vs experiment

Also, the static thrust vs rpm is obtained from experiments using six degrees of freedom force torque measurement sensor and the results is provided in Fig.2.6. Next, RPM has to mapped against the pwm signals. The RPM vs pwm curve is obtained from experiments and the result is presented in Fig. 2.7.

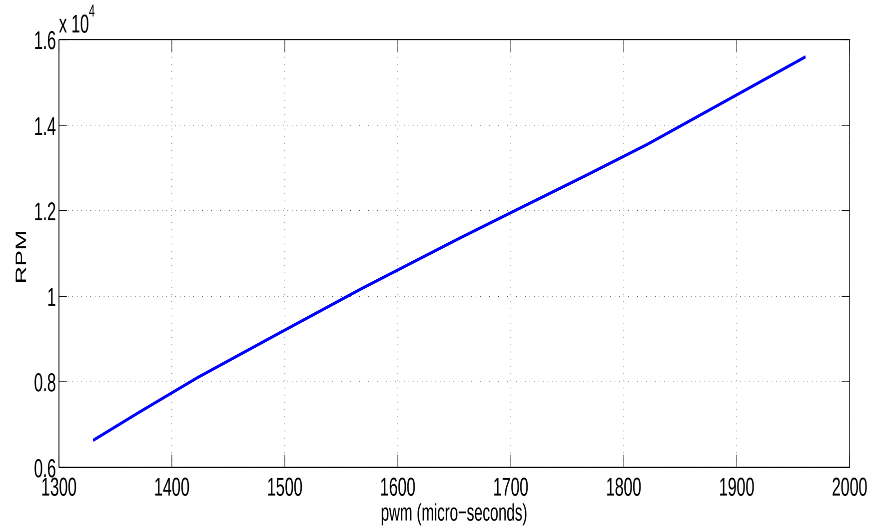

Figure 2.7: RPM Vs pwm

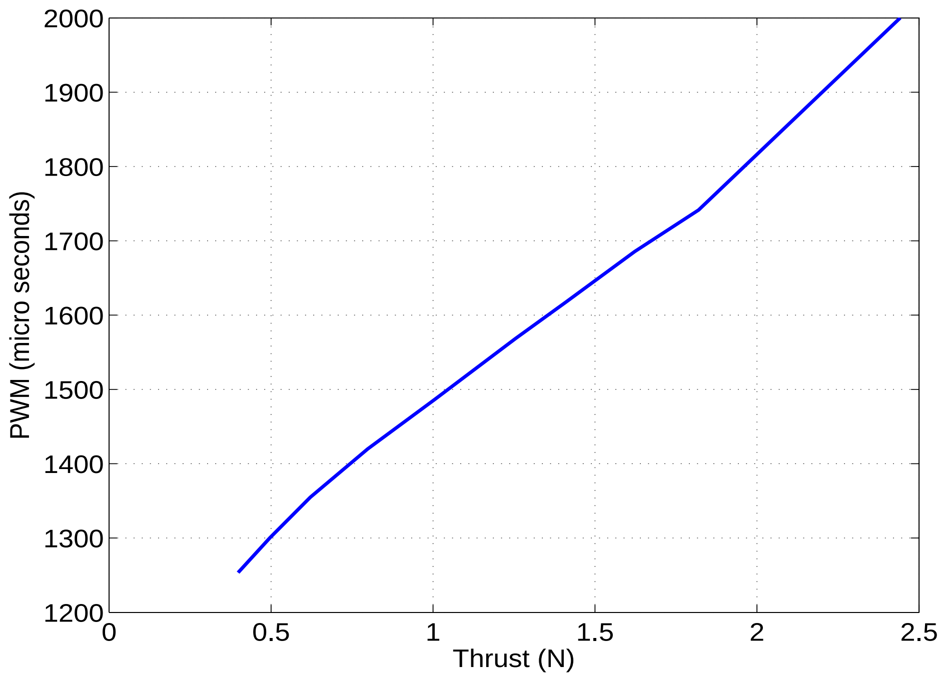

Figure 2.8: Thrust vs pwm

Using Fig. 2.6 and Fig. 2.7, the direct relation between thrust and torque against pwm is obtained which are shown in Fig. 2.8 and Fig. 2.9 respectively.

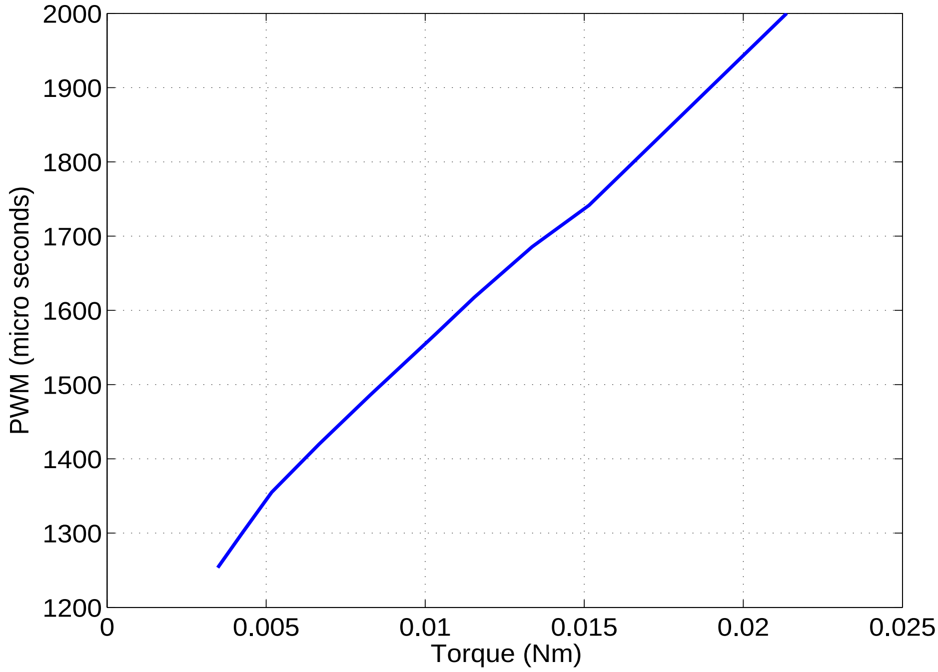

Figure 2.9: Torque vs pwm