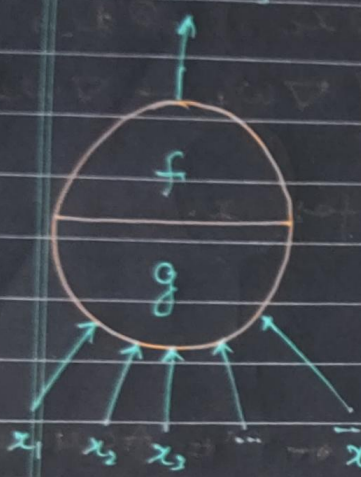
McCulloch Pitts Neuron, Thresholding Logic

neuroscients Logician · proposed highly simplified computational of neuron. (1943)



- g aggregates the inputs and function f takes a decision based on this aggregation.
- if $y=0$; neuron doesn't fire
  if $y=1$; neuron fire

- The inputs can be Excitatory or Inhibitory.
- $y=0$ if any x_i is inhibitory, else

$$g(x_1, x_2, \dots, x_n) = g(x) = \sum_{i=1}^{n} x_i$$

- if Inhibitory input is on, then irrespective of any input, whether it is on or not, output going to be zero, means neurons will not fire.

$$y = f(g(x)) = 1 \quad \text{if } g(x) \geq \theta \leftarrow \text{Thresholding}$$
$$y = f(g(x)) = 0 \quad \text{if } g(x) < \theta \leftarrow \text{Parameter}$$

- And the overall is called Thresholding Logic.

- Implement some Boolean Function using this some McCulloh Pitts (MP) neuron:-

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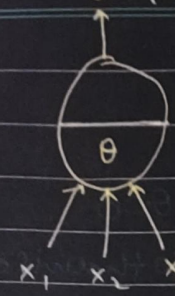
$y = \{0, 1\}$

(1)



What is θ ?

Ans:- $y = 1$ if $\theta \geq 3$ $\theta = 3$

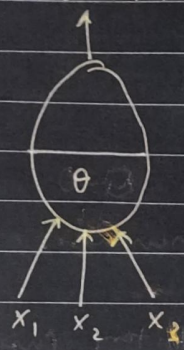
$\Rightarrow$ $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

$y = 0$ if $\theta < 3$ Otherwise

AND Function

(2)

$y = \{0, 1\}$



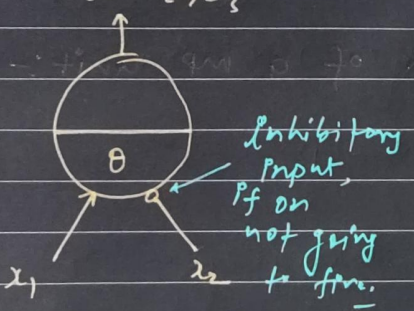
Ans:- $y = 1$ if $\theta \geq 1$ $\theta = 1$

$y = 0$ if $\theta < 1$ Otherwise.

OR Function

(3)

$y \in \{0, 1\}$



x_1	x_2	$!x_2$	x_1 AND $!x_2$
0	0	1	0
0	1	0	0
1	0	1	1 ←
1	1	0	0

x_1 AND $!x_2$

$y = 1$ if $\theta \geq 1$ $\theta = 1$

$y = 0$ If $\theta < 1$ Otherwise.

- In row 2nd and 4th $x_2 = 1$, so remove this row, and checks on and then checks, which has $y = 1$ and by Applying the Original expression, then sum up the Input and give the Output (θ).

④

NOR Function

x_1	x_2	x_1 NOR y_2
0	0	1
x → 0	1	0
x → 1	0	0
x → 1	1	0

$y = 1$ if $\theta \not> 0$ $\theta = 0$

$y = 0$ if $\theta \not< 0$ otherwise

⑤

NOT function

x_1	$\bar{x_1}$
0	1
x → 1	0

$y = 1$ if $\theta = 0$

$y = 0$ otherwise.

NOTE:- Circle at the end indicates inhibitory input. If any inhibitory input is 1 the output will be 0.

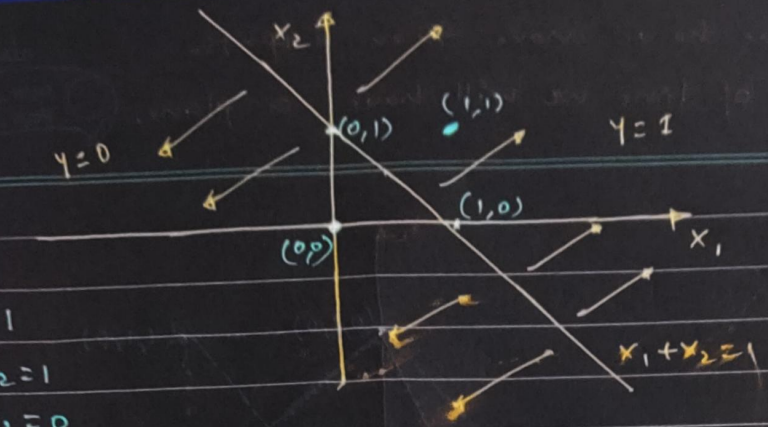
- Geometric Intespretation of a mp unit:-

①

neuron if $x_1 + x_2 = \sum_{i=1}^{2} x_i \geq 1$

fire

Let, just take $x_1 + x_2 = 1$



$x_1 + x_2 = 1$

$x_1 = 0, \; x_2 = 1$

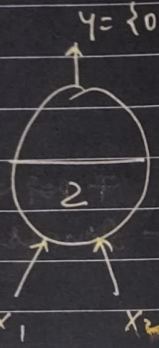
$x_1 = 1 \quad x_2 = 0$

- Linear Separable.

- Boolean x_1, x_2 so only Four i/ps possible

(0,0), (0,1) (1,0), (1,1)

(2)



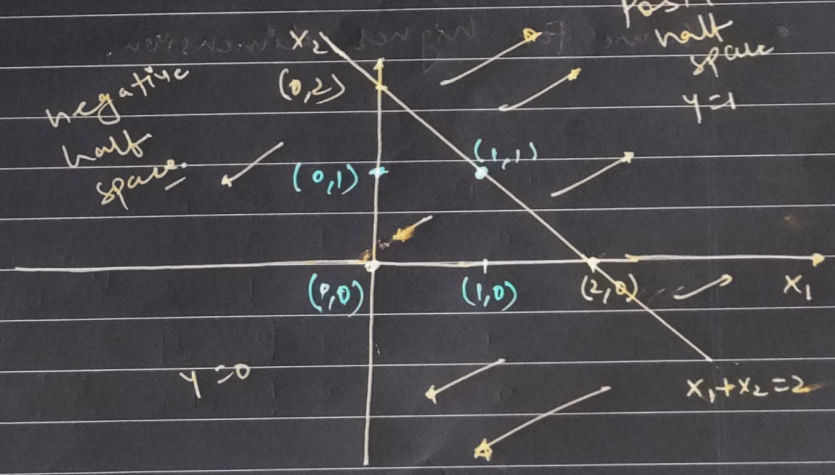
$x_1 + x_2 = 2$

$x_1 = 0, \; x_2 = 2$

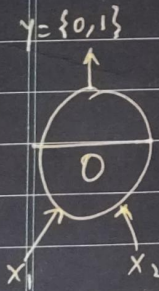
$x_1 = 2, \; x_2 = 0$

AND function.

$x_1 + x_2 = \sum_{i=1}^{2} x_i \geq 2$



(3)

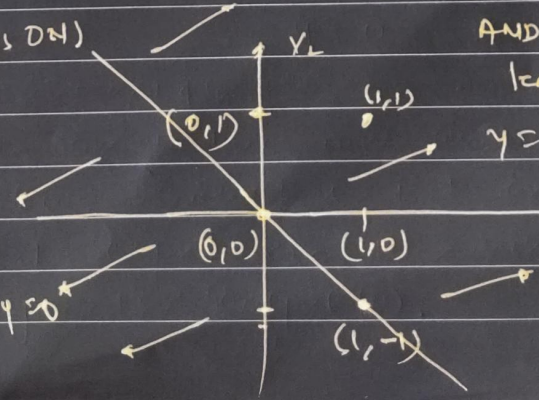


Tautology (always ON)

$x_1 + x_2 = 0$

$x_1 = 0, \; x_2 = 0$

$x_1 = 1, \; x_2 = 1$

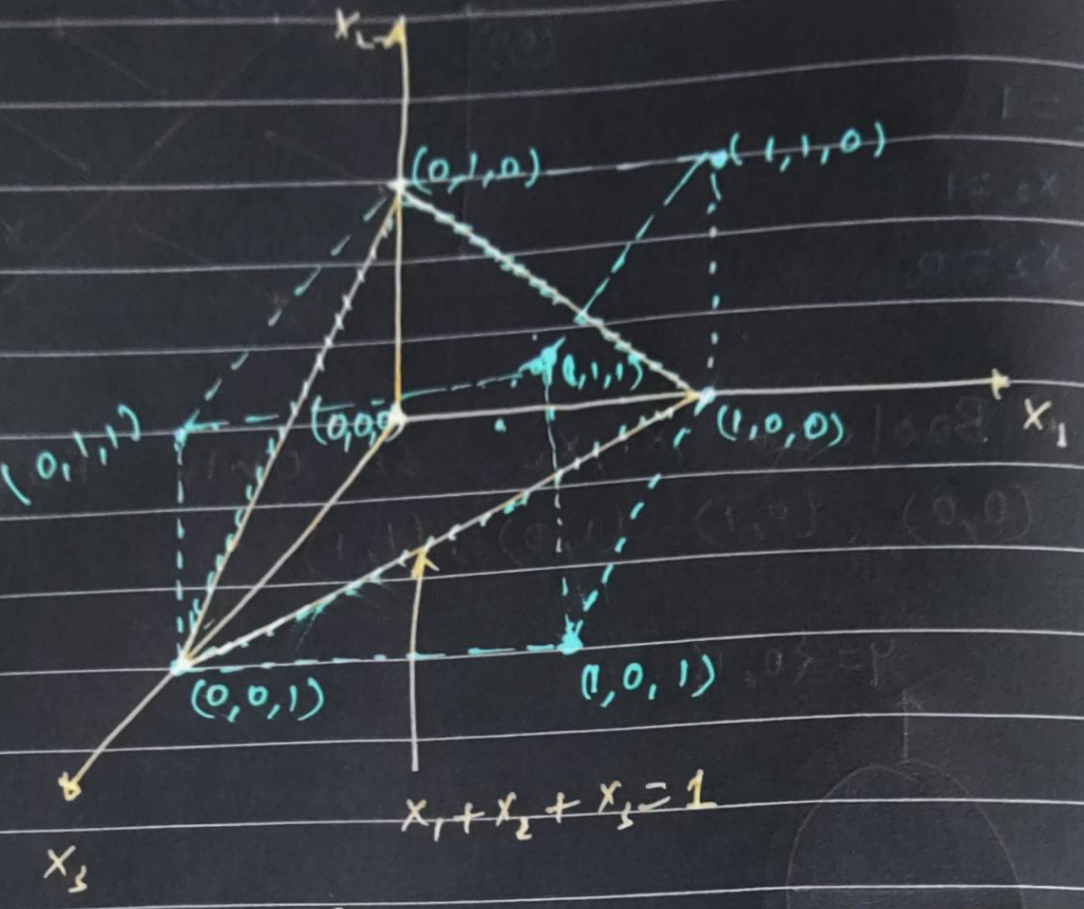


AND, function is linearly separable

- In case we have more than 2 inputs.
  - instead of line we will have a plane.

OR function

$x_1 + x_2 + x_3 = 0$



- Seperates the 7 points from (0,0,0) so linearly seperable.

- same for higher dimension.