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Question 1:

1. (a) $P(X, Y \mid Z) = P(X, Y, Z) / P(Z) = P(X, Y, Z) / (P(Y) P(X|Y)) * (P(Y) P(X|Y)) / P(Z) = P(Z \mid X, Y) P(Y) P(X|Y) / P(Z)$

   (b) $P(X, Y, Z) = P(X, Y, Z) / (P(Y) P(X|Y)) * (P(Y) P(X|Y)) = P(Z \mid X, Y) P(Y) P(X|Y)$
   However, $P(X|Y)$ is unable to calculate. Hence, there is no such expression.

   (c) $P(Z) = P(X|Y, Z) P(Z|Y) P(Y) / P(X, Y|Z) = P(X|Y, Z) P(Z|Y) P(Y) / (P(X \mid Z) P(Y \mid Z)) = P(X|Y, Z) P(Z|Y) / P(X)$

   (d) $P(Y \mid X, Z) = P(X, Y, Z) / P(X, Z) = P(Z \mid X, Y) P(Y \mid X) P(X) / P(X, Z)$. However, it is unable to get $P(X, Z)$. Hence, there is no such expression.

2. (a) 2; $X \perp\!\!\!\perp Y$ & $X \perp\!\!\!\perp Z$; $P(X \mid Z) P(Y) = P(X) P(Y) = P(X, Y)$

   (b) 0; $P(Y \mid X) P(X) / P(Y) = P(X, Y) / P(X) * P(X) / P(Y) = P(X, Y) / P(Y) = P(X \mid Y)$

   (c) 1; $X \perp\!\!\!\perp Y \mid Z$; $P(X \mid Z) P(Y \mid Z) P(Z) / ((P(X \mid Y) P(Y)) = P(X, Y \mid Z) P(Z) / (P(X \mid Y) P(Y)) = P(X, Y, Z) / P(Y) / P(X \mid Y) = P(X, Z \mid Y) / P(X \mid Y) = P(Z \mid X, Y)$

   (d) 0; $\sum_{z \in Z} P(X \mid Y) P(Y \mid z) P(z) = \sum_{z \in Z} P(X, Y) P(Y, z) / P(Y) = P(X, Y)$

3. (a) Yes. $\sum_{w \in W} P(X, Y, Z, w) / \sum_{w \in W} P(Y, Z, w) = P(X, Y, Z) / P(Y, Z) = P(X \mid Y, Z)$
   No. Unable to solve w in $P(Z, w)$ term.
   No. $P(Y \mid Z)$ does not equal to 1 without independence assumptions.
   No. w is in the given part
   Yes. w can be added together

   (b) Yes. $P(X, Y \mid Z) / P(Y \mid Z) = P(X \mid Z) P(Y \mid Z) / P(Y \mid Z) = P(X \mid Z)$
   No. Unable to solve $P(X \mid Y)$
   No. $P(X, Y, Z) / (P(Y, Z) P(Y)) = P(X \mid Y, Z) / P(Y) = P(X \mid Z) / P(Y) \neq P(X \mid Z)$
   Yes. $\sum_{y \in Y} P(X, y \mid Z) P(Z) / P(Z) = \sum_{y \in Y} P(X, y \mid Z) = P(X \mid Z)$
   Yes. $P(X, Z) P(Y) / (P(Z) P(Y|Z)) = P(X, Y, Z) / P(Y, Z) = P(X \mid Y, Z) = P(X \mid Z)$

Question 2:

1. A = -2; B = -2; C = 11; D = -11; E = -2; F = 11; G = 10

2. A(-inf, -2); B(-2, inf); C(-inf, -2); D(-inf, -11); E(-11, -2); F(-inf, -2); G(-inf, -2)
   No node is pruned. Since alpha value is always smaller than beta value.

3. I will use alpha-beta pruning over naïve minimax but not for larger trees. Since alpha-beta pruning is actually DFS algorithm and it may not finish the tree and time complexity could be extremely large.

4. P(F | C) = ¾; P(G | C) = ¼

5. A = 2.01; B = -1.92; C = 13.79; D = -7.17; E = -0.17; F = 13.83; G = 13.67

Question 3:

1. h_1 is not admissible while h_2 is admissible. Since heuristic much not larger than the real cost from current node to goal node, h1's heuristic value at node B is larger than the cost needed from node B to G (14 > 12).

2. Since h_1 is not admissible, we will only use h_2 heuristic values for A* search.
   Path: A -> B -> C -> D -> F -> G.
   It returns to the same path since A* search is always finding the optimal cost.

3. h_3 = [0, 10]

4. h_3 = [8, 10]