CS 540: Introduction to Artificial Intelligence
Homework # 8

Assigned: 4/09
Due: 4/16 before class

Question 1: Resolution [9 points]

Given the knowledge base

$$p \implies (q \implies r)$$

use resolution to prove the query

$$(p \wedge q) \implies (q \implies r).$$

Be sure to show what you convert to CNF (do not skip steps), and how you perform each resolution step.

Firstly convert $p \implies (q \implies r)$ to CNF:

$$\begin{aligned} &p \implies (q \implies r) \\ \equiv\ &p \implies (\neg q \vee r) \\ \equiv\ &\neg p \vee \neg q \vee r \end{aligned}$$

Then $(p \wedge q) \implies (q \implies r)$:

$$\begin{aligned} &(p \wedge q) \implies (q \implies r) \\ \equiv\ &\neg(p \wedge q) \vee (q \implies r) \\ \equiv\ &\neg(p \wedge q) \vee (\neg q \vee r) \\ \equiv\ &\neg p \vee \neg q \vee \neg q \vee r \\ \equiv\ &\neg p \vee \neg q \vee r \end{aligned}$$

Negated query will be $\neg(\neg p \vee \neg q \vee r) \equiv p \wedge q \wedge \neg r$. Our knowldge base will be

$$\begin{aligned} a_1 &: \neg p \vee \neg q \vee r \\ b_1 &: p \\ b_2 &: q \\ b_3 &: \neg r \end{aligned}$$

- Resolve a_1 and b_1: $\neg q \vee r$ (c_1)
- Resolve c_1 and b_2: r (c_2)
- Resolve c_2 and b_3: empty

Therefore, KB $| = (p \wedge q) \implies (q \implies r)$, the query is proved.
Grading Scale:

- **2** points for getting the correct CNF of $p \implies (q \implies r)$, three points for each step. There will be at least two steps in students' answers, if not, deduct three points for one missing step.
- **3** points for getting the correct CNF of $(p \wedge q) \implies (q \implies r)$, three points for each step. There will be at least three steps in students' answers, if not, three points for one missing step.

- **1** points for getting the correct negated query.
- **1** points for getting the correct knwoledge base.
- **2** points for resolution steps, **Two** for each step. There will be at least two steps in students' answer, if not, deduct two points for one missing step.

Please accept all equivalent answers. The order of students' steps may vary from the steps above.

Question 2: Translation from English to First-Order Logic [13 points]

1. (7 points) Consider the following axioms:

   - Every child loves Santa.
   - Everyone who loves Santa loves any reindeer.
   - Rudolph is a reindeer, and Rudolph has a red nose.
   - Anything which has a red nose is weird or is a clown.
   - No reindeer is a clown.
   - Scrooge does not love anything which is weird.
   - (Conclusion) Scrooge is not a child.

   Every child loves Santa.

   $\forall x(CHILD(x) \Rightarrow LOVES(x, Santa))$

   Everyone who loves Santa loves any reindeer.

   $\forall x(LOVES(x, Santa) \Rightarrow \forall y(REINDEER(y)LOVES(x, y)))$

   Rudolph is a reindeer, and Rudolph has a red nose.

   $REINDEER(Rudolph) \wedge REDNOSE(Rudolph)$

   Anything which has a red nose is weird or is a clown.

   $\forall x(REDNOSE(x) \Rightarrow WEIRD(x) \vee CLOWN(x))$

   No reindeer is a clown.

   $\neg\exists x(REINDEER(x) \wedge CLOWN(x))$ Scrooge does not love anything which is weird.

   $\forall x(WEIRD(x) \Rightarrow \neg LOVES(Scrooge, x))$

   Scrooge is not a child. $\neg CHILD(Scrooge)$

   1 point each

2. (6 points) Consider the following axioms:

   - Every Austinite who is not conservative loves some armadillo.
   - Anyone who wears maroon-and-white shirts is an Aggie.
   - Every Aggie loves every dog.
   - Nobody who loves every dog loves any armadillo.
   - Clem is an Austinite, and Clem wears maroon-and-white shirts.
   - There is a conservative Austinite?

Every Austinite who is not conservative loves some armadillo.

$\forall x(AUSTINITE(x) \wedge \neg CONSERVATIVE(x) \Rightarrow \exists y(ARMADILLO(y) \wedge LOVES(x, y)))$

Anyone who wears maroon-and-white shirts is an Aggie.

$\forall x(WEARS(x) \Rightarrow AGGIE(x))$

Every Aggie loves every dog.

$\forall x(AGGIE(x) \Rightarrow \forall y(DOG(y) \Rightarrow LOVES(x, y)))$

Nobody who loves every dog loves any armadillo.

$\neg\exists x((\forall y(DOG(y) \Rightarrow LOVES(x, y))) \wedge \exists z(ARMADILLO(z) \wedge LOVES(x, z)))$

Clem is an Austinite, and Clem wears maroon-and-white shirts.

$AUSTINITE(Clem) \wedge WEARS(Clem)$

There is a conservative Austinite.

$\exists x(AUSTINITE(x) \wedge CONSERVATIVE(x))$

1 point each

Question 3: First-Order Logic [8 points]

Victor has been murdered and Alice, Barney, and Caddy are the only suspects. As the chief detective on the case, you bring them in for questioning. Each one tells the truth except for the culprit, who may be lying. Here is what they told you:

- Alice says that she is innocent. She says that Barney and Victor were friends, and Caddy and Victor were not friends.
- Barney says that he is innocent, plus that he and Victor were not friends.
- Caddy says that she is innocent, and that Barney and Victor were friends.

As an astute detective you make the following assumptions about the world:

- Friends don't murder each other.
- There is no more than one murderer.
- If a person isn't a murderer, they don't lie.

Now you want to find the suspect.

(1) (1 point) Write a set of FOL sentences representing the information learned when interviewing the three suspects.

(2) (1 point) Write a set of FOL sentences representing the general knowledge assumptions you've made.

(3) (2 point) Convert all of your sentences in (1) and (2) to CNF.

(4) (1 point) State the goal to be solved as an FOL sentence.

(5) (1 point) Caddy later tells you that she was a friend of Victor. Write an FOL sentence that represents this new piece of information.

(6) (2 point) Is the knowledge base containing all of the sentences satisfiable? If so, give an interpretation that makes it true. If not, prove unsatisfiability using resolution.

Alice = A, Barney = B, Caddy = C.

$Murder(x)$ denotes that x murders Victor. $Friend(x)$ denotes that x is a friend of Victor. $Liar(x)$ denotes that x is a liar. x can be A, B, C.

Q1.

1. $Liar(A) \lor (\neg Murder(A) \land Friend(B) \land \neg Friend(C))$
2. $Liar(B) \lor (\neg Murder(B) \land \neg Friend(B))$
3. $Liar(C) \lor (\neg Murder(C) \land Friend(B))$

1 point

Q2

1. $\forall x(Friend(x) \Rightarrow \neg Murder(x)$
2. $\forall x \forall y((Murder(x) \land Murder(y)) \Rightarrow x = y)$
3. $\forall x(Liar(x) \Rightarrow Murder(x))$

1 point

Q3.

Based on Q1.1

$Liar(A) \lor \neg Murder(A)$

$Liar(A) \lor Friend(B)$

$Liar(A) \lor \neg Friend(C)$

Based on Q1.2

$Liar(B) \lor \neg Murder(B)$

$Liar(B) \lor \neg Murder(A)$

Based on Q1.3

$Liar(C) \lor \neg Murder(C)$

$Liar(C) \lor Friend(B)$

Based on Q2.1

$\neg Friend(A) \lor \neg Murder(A)$

$\neg Friend(B) \lor \neg Murder(B)$

$\neg Friend(C) \lor \neg Murder(C)$

Based on Q2.2

$\neg Murder(A) \lor \neg Murder(B)$

$\neg Murder(A) \lor \neg Murder(C)$

$\neg Murder(B) \lor \neg Murder(C)$

Based on Q2.3

$\neg Liar(A) \lor Murder(A)$

$\neg Liar(B) \lor Murder(B)$

$\neg Liar(C) \lor Murder(C)$

2 points

Q4

$\exists x Murder(x)$

1 point

Q5.

$Friend(C) \lor Liar(C)$

1 point

Q6.

The KB is not satisfiable.

Resolve $Liar(A) \lor Friend(B)$ and $Liar(B) \lor \neg Murder(A)$, $Liar(A) \lor Liar(B)$

Resolve $Liar(B) \lor \neg Murder(A)$ and $Liar(C) \lor Friend(B)$, $Liar(B) \lor Liar(C)$

Resolve $Liar(A) \lor Liar(B)$ and $\neg Liar(A) \lor Murder(A)$, $Murder(A) \lor Liar(B)$

Resolve $Liar(B) \lor Liar(C)$ and $\neg Liar(C) \lor Murder(C)$, $Murder(C) \lor Liar(B)$

Resolve $Murder(A) \lor Liar(B)$ and $\neg Liar(B) \lor Murder(B)$, $Murder(A) \lor Murder(B)$

Resolve $Murder(C) \lor Liar(B)$ and $\neg Liar(B) \lor Murder(B)$, $Murder(B) \lor Murder(C)$

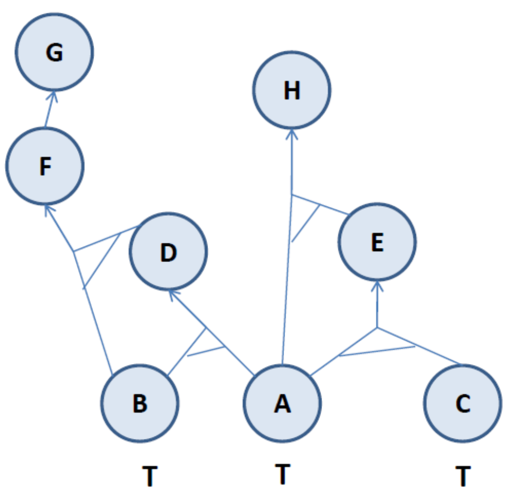
Resolve $Murder(B) \vee Murder(C)$ and $\neg Murder(A) \vee \neg Murder(C)$, $Murder(B) \vee \neg Murder(A)$
Resolve $Murder(A) \vee Murder(B)$ and $\neg Friend(B) \vee \neg Murder(B)$, $Murder(B)$
Resolve $Murder(B)$ and $\neg Friend(B) \vee \neg Murder(B)$, $\neg Friend(B)$
Resolve $Murder(B)$ and $\neg Murder(A) \vee \neg Murder(B)$, $\neg Murder(A)$
Resolve $\neg Murder(A)$ and $\neg Liar(A) \vee Murder(A)$, $\neg Liar(A)$
Resolve $\neg Liar(A)$ and $Liar(A) \vee Friend(B)$, $Friend(B)$
Resolve $Friend(B)$ and $\neg Friend(B)$, empty.
other answer may also be correct. 2 points

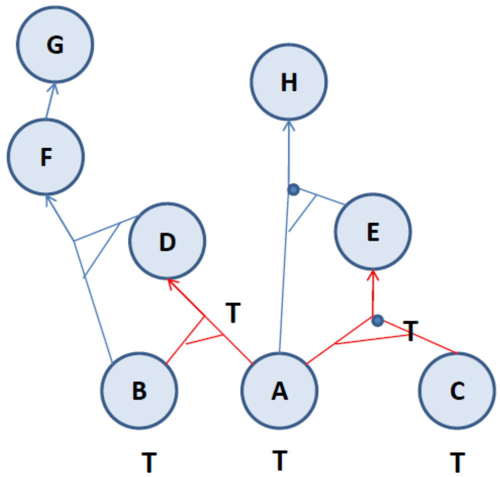
Question 4: Forward Chaining and backward Chaining [10 points]

1. (5 points) Use forward chaining to solve the following problem:

   Given:

   A
   B
   C
   $A \wedge B \Rightarrow D$
   $B \wedge D \Rightarrow F$
   $F \Rightarrow G$
   $A \wedge E \Rightarrow H$
   $A \wedge C \Rightarrow E$ Is H true? Draw a tree to illustrate the search for a proof.




G
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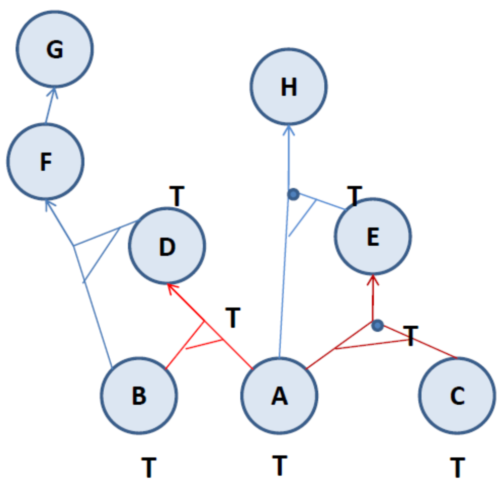
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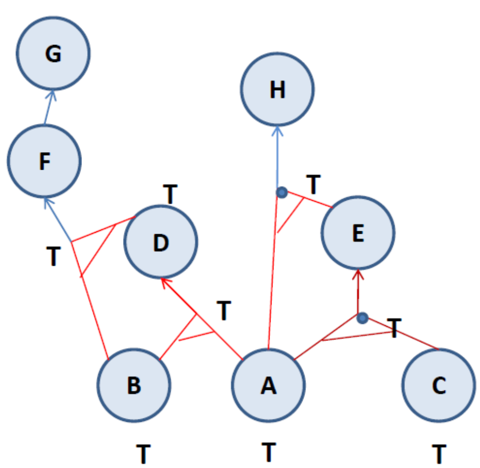
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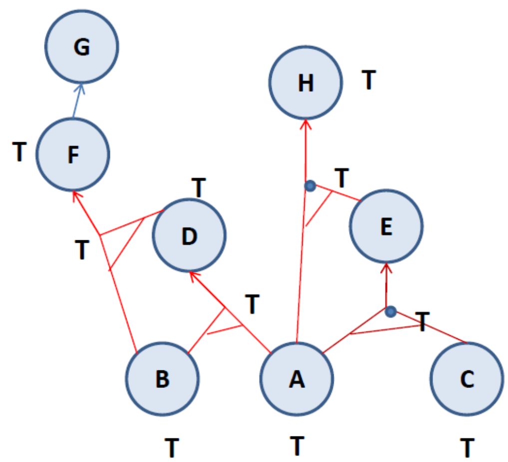


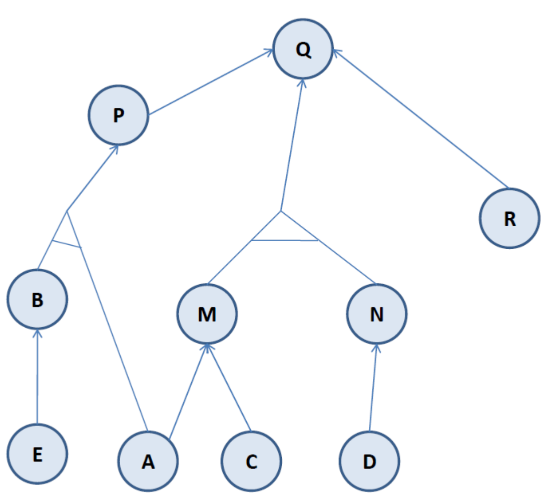
A ∧ B is true. A ∧ C is true. E is true. D is true. A ∧ E is true. B ∧ E is true. So H is true. F is true.

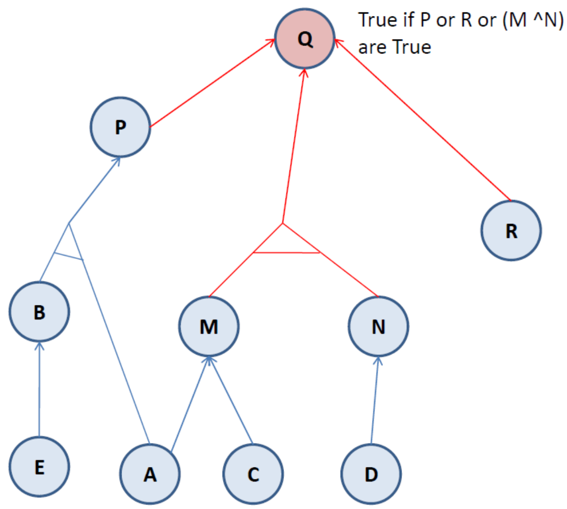
3 points for correct tree. 2 points for showing steps.

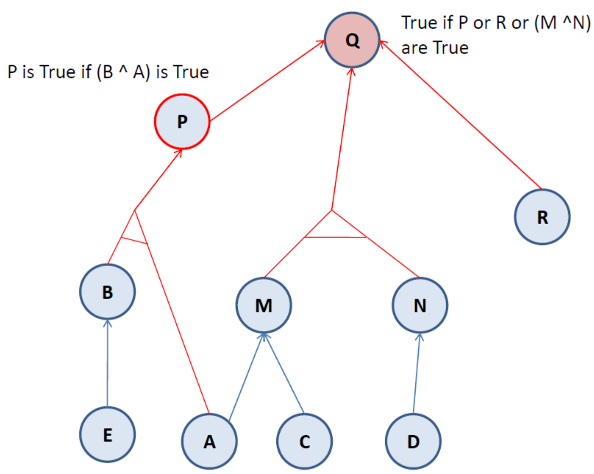
2. (5 points) Use backward chaining on the following KB to prove Q:

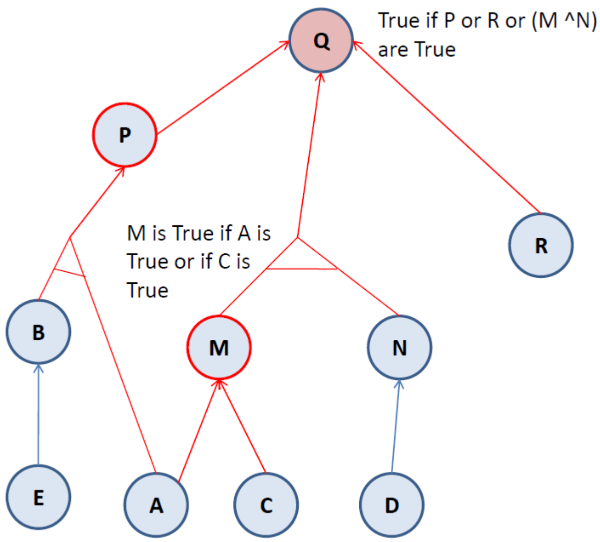
$P \Rightarrow Q$
$E \Rightarrow B$
$R \Rightarrow Q$
$M \wedge N \Rightarrow Q$
$A \wedge B \Rightarrow P$
$A \Rightarrow M$
$C \Rightarrow M$
$D \Rightarrow N$
D
A

Draw a tree to illustrate the search for a proof. Mark the nodes that are satisfied in this KB. What is the proof of Q? (write explicitly a sequence of steps to obtain Q)

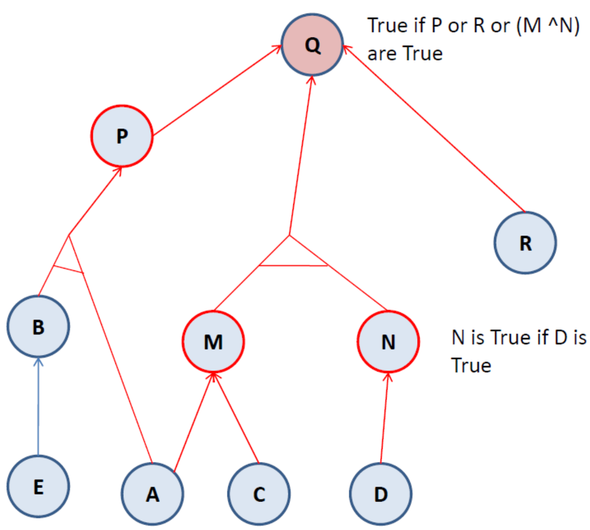



True if P or R or (M ^N)
are True
Q
P
R
B
M
N
E
A
C
D

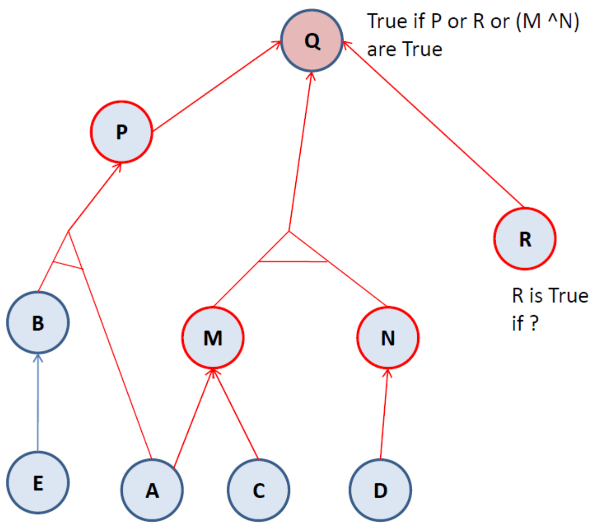

True if P or R or (M ^N)
are True
P is True if (B ^ A) is True
Q
P
R
B
M
N
E
A
C
D

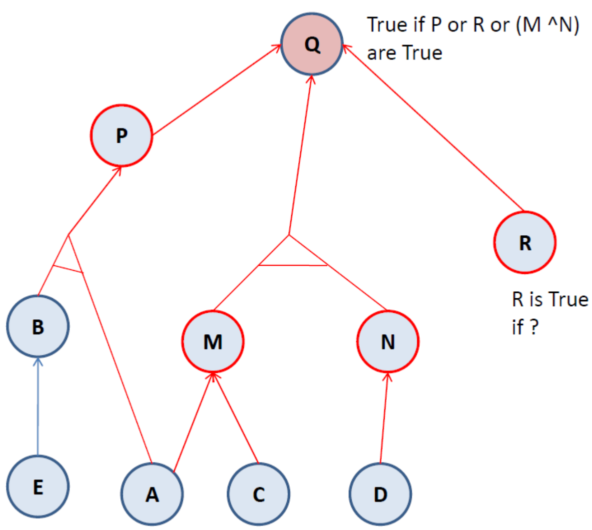


Q
True if P or R or (M ^N) are True
P
R
M is True if A is True or if C is True
B
M
N
E
A
C
D



Q
True if P or R or (M ^N) are True
P
R
B
M
N
N is True if D is True
E
A
C
D


True if P or R or (M ^N)
are True
Q
P
R
R is True
if ?
B
M
N
E
A
C
D


True if P or R or (M ^N)
are True
Q
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R
R is True
if ?
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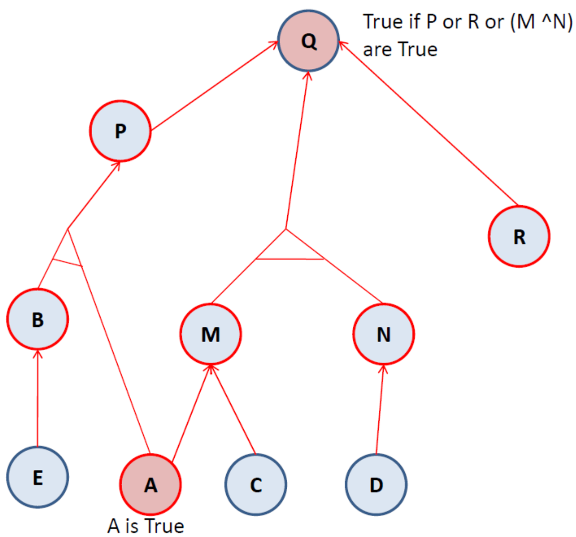
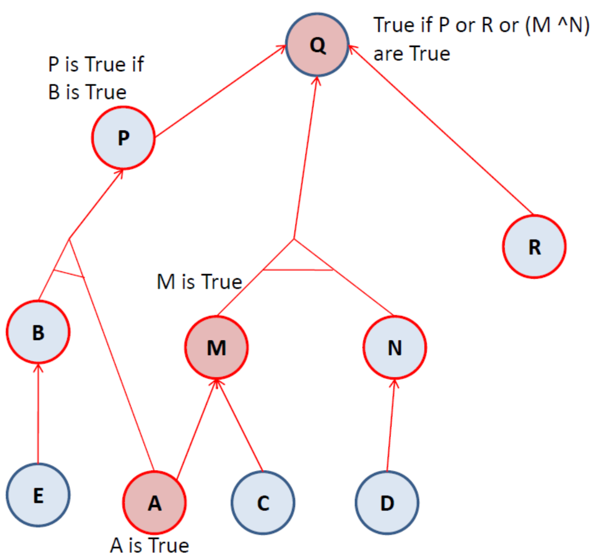
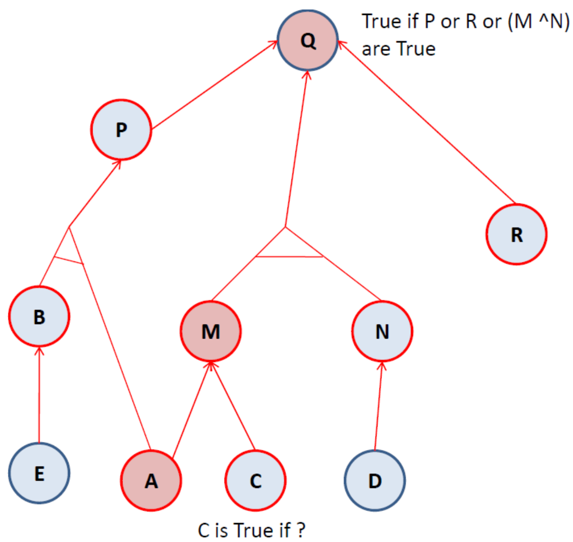
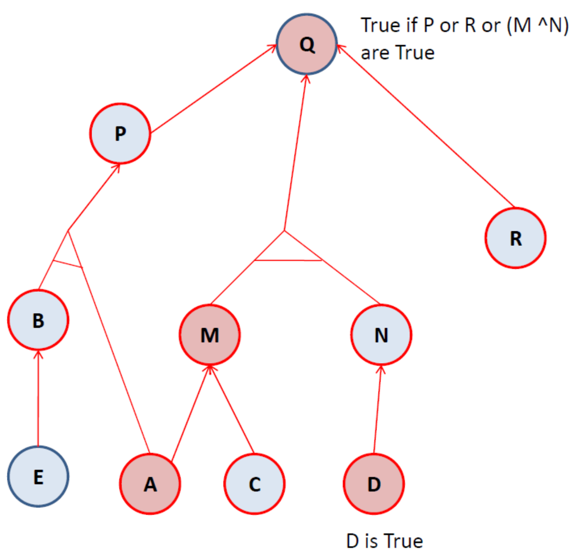
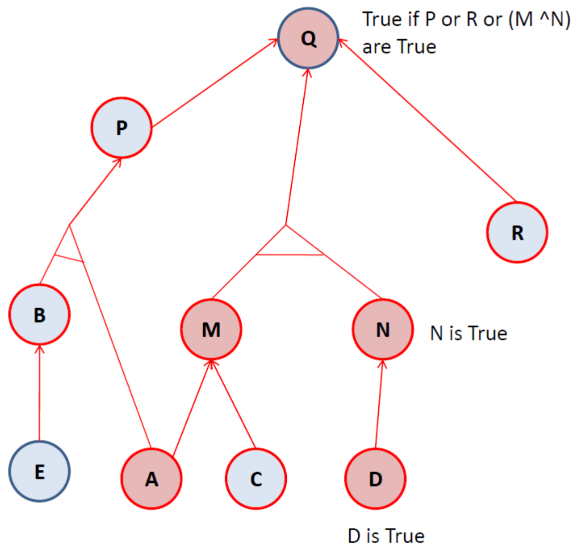
D

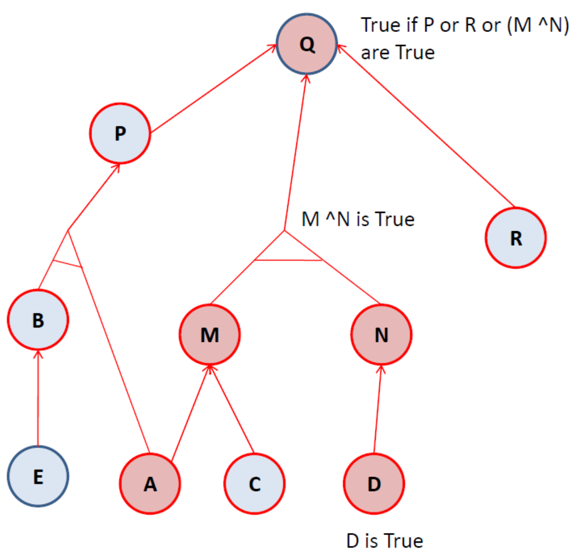

True if P or R or (M ^N)
are True
Q
P
R
B
M
N
E
A
C
D
A is True

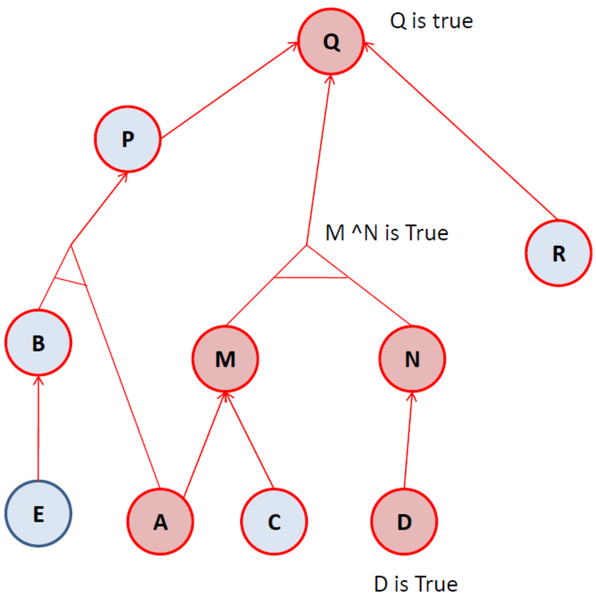

True if P or R or (M ^N)
are True
P is True if
B is True
Q
P
R
M is True
B
M
N
E
A
C
D
A is True


True if P or R or (M ^N)
are True
Q
P
R
B
M
N
E
A
C
D
C is True if ?


True if P or R or (M ^N)
are True
Q
P
R
B
M
N
E
A
C
D
D is True


Q
True if P or R or (M ^N)
are True
P
R
B
M
N
N is True
E
A
C
D
D is True


Q
True if P or R or (M ^N)
are True
P
M ^N is True
R
B
M
N
E
A
C
D
D is True



M is true N is true. M∧N is true. Than Q is true.

3 points for correct tree. 2 points for showing steps.

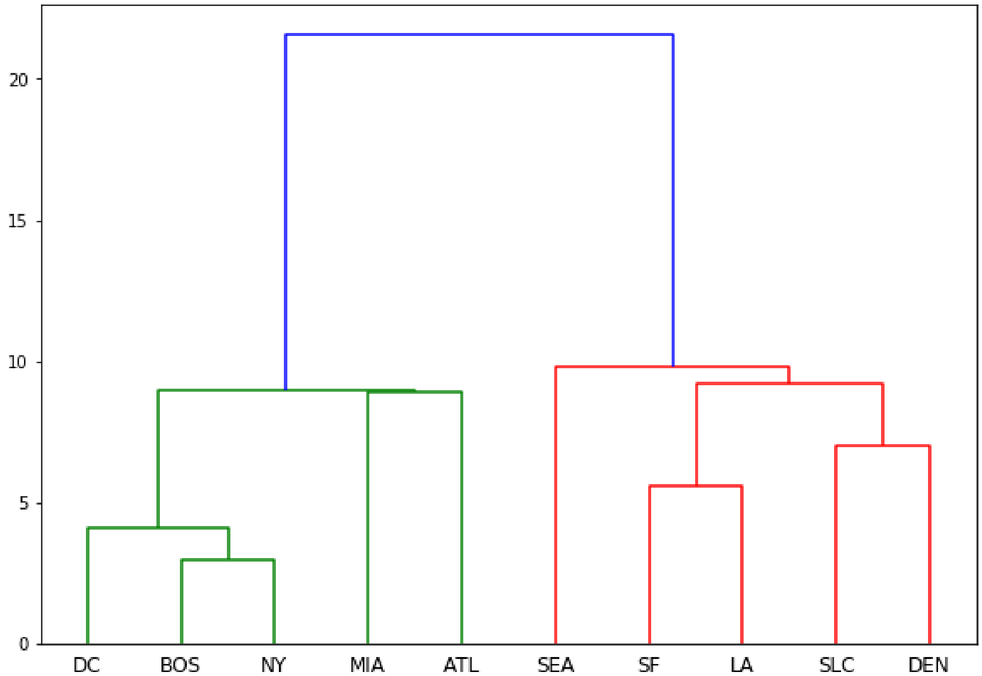
Question 5: Clustering [20 points]

Consider the following information about distances in miles between pairs of 10 U.S. cities:

The (latitude, longitude) locations of these cities are:
BOS (42.4, 71.1),
NY (41.7, 74.0),
DC (38.9, 77.0),
MIA (25.8, 80.2),
SLC (40.8, 111.9),
SEA (47.6, 122.3),
SF (37.8, 122.4),
LA (34.1, 118.2),
DEN (39.7, 105.0),
ATL (33.7, 84.3).

1. (10 points) Perform hierarchical clustering using single-linkage and the above data.
   - Show the resulting dendrogram.
   - What clusters of cities are created if you want 3 clusters?
2. (10 points) Show the results of one iteration of k-means clustering assuming k = 2 and the initial cluster centers are defined as c1 = (50, 90) and c2 = (30, 100)
   - Give the list of cities in the initial 2 clusters.
   - Give the coordinates of the new cluster centers.
   - Give the list of cities in the 2 clusters based on the new cluster centers computed in (ii).

Q1 Graph (5 points)



Three clusters:
1. DC BOS NY MIA ATL (2 points)
2. SEA (1 point)
3. SF LA SLC DEN (2 points)

Q2
initial:
[BOS NY DC] [MIA SLC SEA SF LA DEN ATL] (2 points)
center: first cluster (41.0, 74.0) second cluster (37.1, 106.2) (1 points)
New clusters: [BOS NY DC MIA ATL] [SLC SEA SF LA DEN]