

Constant Time

The functions in the implementation that operate in constant time are `append_element`, `__len__`, `__iter__`, and `__next__`.

`Append_element` is an $O(n)$ function since I implemented the function by creating a node called "tempnode" which meant I could insert the tempnode between the trailer node and the one before it. I accessed the nodes before the trailer and the trailer by using $O(1)$ ways (using `self.__trailer` and `.prev`) and used `.next` and `and.prev` to insert the new node, which are all $O(1)$ methods, which meant that worst case scenario is constant time.

The `__len__` function is $O(1)$ since the function just returns `self.__len`, which can be done in $O(1)$ time. The functions `__iter__` and `__next` are also $O(1)$ time complexity, since all `__iter__` does is create a pointer to the node right after the header, which is done in $O(1)$ time, and `__next__` just sets the pointer to the next node, which is also done in $O(1)$ time.

Linear Time

The functions which operate in linear time are `insert_element_at`, `remove_element_at`, `get_element_at`, `rotate_left`, `__str__`, and `__reversed__`.

`Insert_element_at` operates in linear time since the way it works is by creating a pointer to the header node, then using a for loop calling `.next` to take the pointer to the node before where the index where the function inserts the node, and then inserts the node at that location. Since I used a for-loop to get to the node right before where the `insert_element_at` function inserts the new element, that means that in a worst-case scenario, the method runs in $O(n)$ time.

The function `remove_element_at` is also linear since it runs similarly to `insert_element_at`, but insteads removes the node at the specified index, instead of adding a new node the way `insert_element_at` does.

The `rotate_left` function works by storing the value of first actual node in the `Linked_List`, (the one after the trailer). Next the function uses the iterator to go through the `Linked_List`, setting each nodes value to the value of the next node, stopping at the trailer node. To correct for the problem of the node before the trailer not having the right value, we set the value of the node before the trailer to the value that was stored at the beginning. Iterating through the `Linked_Lists` with $O(1)$ operations results in an $O(n)$ complexity, therefore the method operates in linear time.

The `__str__` function also operates at linear time since the way that it works is that first a string to store the string representation of the `Linked_List` is created. Next, the function iterates through the `Linked_List`, adding each value to the string as it goes through. Since the function iterates through the `Linked_Lists` while just accessing the value (which is $O(1)$), that is why the function operates in $O(n)$, or linear, time.

Finally the `__reversed__` function works by first creating a new `Linked_List`. Then starting with a pointer to the node before the trailer node, the function appends to the new `Linked_list` the value of the node currently being pointed at, and then changing the pointer to point at the node before the pointer until the pointer goes to the beginning of the list. The `__reversed__` method returns the new `Linked_List`. Since this function goes through the entire array, and the `appended_element` method runs at $O(1)$, that means `__reversed__` runs in linear time.