Comp 480/580: Assignment #3

Rice University — Due Date: Thrusday, 13/05/2025

1 Markov Chains: Simple Proofs

You can use this lecture notes `https://people.eecs.berkeley.edu/~sinclair/cs294/n6.pdf` for definitions if needed.

1. For a markov chain, prove that if the graph associated is undirected (i.e. P(x,y) = P(y,x)), then aperiodicity is equivalent to graph being non-bipartite.
2. Define the period of x as gcd {t : $P^t(x,x) > 0$}. Prove that for an irreducible Markov chain, the period of every $x \in \Omega$ is the same. [Hence, if the graph associated is undirected, the period is either 1 or 2.]
3. Suppose a markov chain with stochastic matrix P is irreducible. Then it is aperiodic if and only if there exists t such that $P(x,y)^t > 0$ for all $x, y \in \Omega$.
4. Suppose a markov chain with stochastic matrix P is irreducible and contains at least one self-loop (i.e., $P(x,x) > 0$ for some x). Then P is aperiodic.
5. Let P be an irreducible (but not necessarily aperiodic) transition matrix. For any $0 < \alpha < 1$, the transition matrix $P' = \alpha P + (1-\alpha)I$ leads to an irreducible and aperiodic markov chain, and has the same stationary distribution as P