1 PERCENTAGES

1.1. Understanding Percentages

The word percent can be understood as follows:

Per cent ⇒ for every 100.

So, when percentage is calculated for any value, it means that that you calculate the value for every 100 of the reference value.

Why Percentage?

Percentage is a concept evolved so that there can be a uniform platform for comparison of various things. (Since each value is taken to a common platform of 100.)

Example:

- To compare three different students depending on the marks they scored we cannot directly compare their marks until we know the maximum marks for which they took the test. But by calculating percentages they can directly be compared with one another.
- Before going deeper into the concept of percentage, let u have a look at some basics and tips for faster calculations:

1.2. Calculation of Percentage

$$\text{Percentage} = \left(\frac{\text{Value}}{\text{Total value}}\right) \times 100$$

Example: 50 is what % of 200?

Solution: $\text{Percentage} = \left(\frac{50}{200}\right) \times 100 = 25\%$.

1.2.1. Calculation of Value:

$$\text{Value} = \left(\frac{\text{Percentage}}{100}\right) \times \text{ total value}$$

Example: What is 20% of 200?

Solution: $\text{Value} = \left(\frac{20}{100}\right) \times 200$

Note: Percentage is denoted by "%", which means "/100".

Example: What is the decimal notation for 35%?

Solution: $35\% = \frac{35}{100} = 0.35$.

For faster calculations we can convert the percentages or decimal equivalents into their respective fraction notations.

1.3. Percentages – Fractions Conversions:

The following is a table showing the conversions of percentages and decimals into fractions:

Percentage	Decimal	Fraction
10%	0.1	$\frac{1}{10}$
12.5%	0.125	$\frac{1}{8}$
16.66%	0.1666	$\frac{1}{6}$
20%	0.2	$\frac{1}{5}$
25%	0.25	$\frac{1}{4}$
30%	0.3	$\frac{3}{10}$
33.33%	0.3333	$\frac{1}{3}$
40%	0.4	$\frac{2}{5}$
50%	0.5	$\frac{1}{2}$
60%	0.6	$\frac{3}{5}$
62.5%	0.625	$\frac{5}{8}$
66.66%	0.6666	$\frac{2}{3}$

Similarly we can go for converting decimals more than 1 from the knowledge of the above cited conversions as follows:

We know that $12.5\% = 0.125 = \frac{1}{8}$

Then, $1.125 = \frac{[8(1)+1]}{8} = \frac{9}{8}$ (i.e., the denominator will add to numerator once, denominator remaining the same.

Also, $2.125 = \frac{[8(2)+1]}{8} = \frac{17}{8}$ (here the denominator is added to numerator twice)

$3.125 = \frac{[8(3)+1]}{8} = \frac{25}{8}$ and so on.

Thus we can derive the fractions for decimals more than 1 by using those les than 1.

We will see how use of fractions will reduce the time for calculations:

Example: What is 62.5% of 320?

Solution: $\text{Value} = \left(\frac{5}{8}\right) \times 320 \left(\text{since } 62.5\% = \frac{5}{8}\right) = 200$.

1.4. Percentage Change

A change can be of two types – an increase or a decrease.

When a value is changed from initial value to a final value,

% change = (Difference between initial and final value/initial value) × 100

Example: If 20 changes to 40, what is the % increase?

Solution: $\% \; increase = \frac{(40-20)}{20} \times 100 = 100\%$.

Note:
1. If a value is doubled the percentage increase is 100.
2. If a value is tripled, the percentage change is 200 and so on.

1.5. Percentage Difference

% Difference = (Difference between values/value compared with) × 100.

Example: By what percent is 40 more than 30?

Solution: $\% \text{ difference} = \frac{(40-30)}{30} \times 100 = 33.33\%$

(Here 40 is compared with 30. So 30 is taken as denominator)

Example: By what % is 60 more than 30?

Solution: $\% \text{ difference} = \frac{(60-30)}{30} \times 100 = 100\%$.

(Here is 60 is compared with 30.)

Hint: To calculate percentage difference the value that occurs after the word "than" in the question can directly be used as the denominator in the formula.

1.6. Important Points to Note

1. When any value increases by

(a) 10%, it becomes 1.1 times of itself. (since 100+10 = 110% = 1.1)

(b) 20%, it becomes 1.2 times of itself.

(c) 36%, it becomes 1.36 times of itself.

(d) 4%, it becomes 1.04 times of itself.

Thus we can see the effects on the values due to various percentage increases.

2. When any value decreases by

(a) 10%, it becomes 0.9 times of itself. (Since 100 – 10 = 90% = 0.9)

(b) 20%, it becomes 0.8 times of itself

(c) 36%, it becomes 0.64 times of itself

(d) 4%, it becomes 0.96 times of itself.

Thus we can see the effects on a value due to various percentage decreases.

Note:
1. When a value is multiplied by a decimal more than 1 it will be increased and when multiplied by less than 1 it will be decreased.
2. The percentage increase or decrease depends on the decimal multiplied.

Example: 0.7 $\Rightarrow$ 30% decrease, 0.67 $\Rightarrow$ 33% decrease, 0. 956 $\Rightarrow$ 4.4% decrease and so on.

Example: When the actual value is x, find the value when it is 30% decreased.

Solution: 30% decrease $\Rightarrow$ 0.7 x.

Example: A value after an increase of 20% became 600. What is the value?

Solution: 1.2x = 600 (since 20% increase)

$\Rightarrow$ x = 500.

Example: If 600 is decrease by 20%, what is the new value?

Solution: New value = 0.8 × 600 = 480. (Since 20% decrease)

Thus depending on the decimal we can decide the % change and vice versa.

Example: When a value is increased by 20%, by what percent should it be reduced to get the actual value?

Solution: (It is equivalent to 1.2 reduced to 1 and we can use % decrease formula)

$$\% \text{ decrease} = \frac{(1.2 - 1)}{1.2} \times 100 = 16.66\% .$$

3. When a value is subjected multiple changes, the overall effect of all the changes can be obtained by multiplying all the individual factors of the changes.

Example: The population of a town increased by 10%, 20% and then decreased by 30%. The new population is what % of the original?

Solution: The overall effect = $1.1 \times 1.2 \times 0.7$ (Since 10%, 20% increase and 30% decrease)

= 0.924 = 92.4%.

Example: Two successive discounts of 10% and 20% are equal to a single discount of ___

Solution: Discount is same as decrease of price.

So, decrease = $0.9 \times 0.8 = 0.72 \Rightarrow$ 28% decrease (Since only 72% is remaining).

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2 AVERAGES & AGES

2.1. What is Average?

The concept of average is equal distribution of the overall value among all the things or persons present there. So the formula for finding the average is as follows:

Average, $A = \dfrac{\text{Total of all things, } T}{\text{Number of things, } N}$

Therefore, Total, $T = AN$

If any person joins a group with more value than the average of the group then the overall average increases. This is because the value in excess than the average will also be distributed equally among all the members.

Similarly when any value less than the average joins the group the overall group decreases as the deficit is divided equally among all the people present there.

Example:

Consider three people A, B and C with total of Rs. 30/-. Their average becomes Rs. 10/- for each. If another person D joins them with Rs. 50/- then he has Rs. 40/- more than actual average of Rs. 10/-.

So this Rs. 40/- will get distributed among those four and each gets Rs. 10/-. Thus the average becomes Rs. 20/- each.

Example:

The average age of a class of 30 students is 12. If the teacher is also included the average becomes 13 years. Find the teacher's age.

Solution:

- When the teacher is included there are totally 31 members in the class and the average is increased by 1 year. This means that everyone got 1 extra year after distributing the extra years of the teacher.
- So extra years of the teacher are as follow: $31 \times 1 = 31$ years.
- Age of the teacher = actual avg + extra years = $12 + 31 = 43$ years.

3 PROFIT AND LOSS

3.1. What is Profit?

When a person does a business transaction and gets more than what he had invested, then he is said to have profit. The profit he gets will be equal to the additional money he gets other than his investment.

So profit can be understood as the extra money one gets other than what he had invested.

Example: A person bought an article for Rs. 100 and sold it for Rs. 120. Then he got Rs. 20 extra and so his profit is Rs. 20.

3.2. What is Loss?

When a person gets an amount less than what he had invested, then he is said to have a loss. The loss will be equal to the deficit he got than the investment.

Example: A person bought an article at Rs. 100 and sold it for Rs. 90. Then he got a deficit of Rs. 10 and so his loss is Rs. 10.

3.3. Cost Price (CP)

- The money that the trader puts in his business is called Cost Price. The price at which the articles are bought is called Cost Price.
- In other words, Cost Price is nothing but the investment in the business.

3.4 Selling Price (SP)

- The price at which the articles are sold is called the Selling Price. The money that the trader gets from the business is called Selling Price.
- In other words, Selling Price is nothing but the returns from a business.

3.5. Marked/Market/List Price (MP):

- The price that a trader marks or lists his articles to is called the Marked Price.
- This is the only price known to the customer.

3.6. Discount

The waiver of cost from the Marked Price that the trader allows a customer is called Discount.

Note:

1. Profit or loss percentage is to be applied always to the Cost Price only.
2. Discount percentage is to be applied always to the Marked Price only.

3.7. Relationship Among CP, SP and MP:

A trader adds his profit to the investment and sells it at that increased price.
Also he allows a discount on Marked Price and sells at the discounted price.
So, we can say that,

- SP = CP + Profit. (CP applied with profit is SP)
- SP = MP – Discount. (MP applied with discount is SP)

3.8. Understanding Profit and Loss:

So, by now we came to know that if CP is increased and sold it would result in profit and vice versa.

Also whatever increase is applied to CP, that increase itself is the profit.

For Rs. 10 profit, CP is to be increased by RS. 10 and the increased price becomes SP.

For 10% profit, CP is to be increased by 10% and it is the SP.

(From previous chapter we know that any value increased by 10% becomes 1.1 times.)

So, for 10% profit, CP increased by 10% $\Rightarrow$ 1.1CP = SP.

- $SP = 1.1CP \Rightarrow \frac{SP}{CP} = 1.1 \Rightarrow$ 10% profit
- $SP = 1.07CP \Rightarrow \frac{SP}{CP} = 1.07 \Rightarrow$ 7% profit
- $SP = 1.545CP \Rightarrow \frac{SP}{CP} = 1.545 \Rightarrow$ 54.5% profit and so on.

Similarly,

- $SP = 0.9CP \Rightarrow \frac{SP}{CP} = 0.9 \Rightarrow$ 10% loss (Since 10% decrease)
- $SP = 0.7CP \Rightarrow \frac{SP}{CP} = 0.76 \Rightarrow$ 24% loss and so on.

So, to calculate profit % or loss %, it is enough for us to find the ratio of SP to CP.

Note:

1. If SP/CP > 1, it indicates profit.
2. If SP/CP < 1, it indicates loss.

3.9. Multiple Profits or Losses

A trader may sometimes have multiple profits or losses simultaneously. This is equivalent to having multiple changes and so all individual changes are to be multiplied to get the overall effect.

Examples: A trader uses a 800gm weight instead of 1 kg. Find his profit %.

Solution: (He is buying 800 gm but selling 1000 gm.

So, CP is for 800 gm and SP is for 1000 gm.)

$$\frac{SP}{CP} = \frac{1000}{800} = 1.25 \Rightarrow 25\% \text{ profit.}$$

Examples: A trader uses 1 kg weight for 800 gm and increases the price by 20%. Find his profit/loss %.

Solution: 1 kg weight for 800 gm $\Rightarrow$ loss (decrease) $\Rightarrow$ 800/1000 = 0.8

20% increase in price $\Rightarrow$ profit (increase) $\Rightarrow$ 1.2

So, net effect = $(0.8) \times (1.2) = 0.96 \Rightarrow 4\%$ loss.

Examples: A milk vendor mixes water to milk such that he gains 25%. Find the percentage of water in the mixture.

Solution: To gain 25%, the volume has to be increased by 25%.

So, for 1 lt of milk, 0.25 lt of water is added $\Rightarrow$ total volume = 1.25 lt

$$\% \text{ of water} = \frac{0.25}{1.25} \times 100 = 20\%.$$

Examples: A trader bought an item for Rs. 200. If he wants a profit of 22%, at what price must he sell it?

Solution: CP=200, Profit = 22%.

So, SP = 1.22CP = 1.22 × 200 = 244/-.

Examples: A person buys an item at Rs. 120 and sells to another at a profit of 25%. If the second person sells the I tem to another at Rs. 180, what is the profit % of the second person?

Solution: SP of 1st person = CP of 2nd person = 1.25 × 120 = 150.

SP of 2nd person = 180.

$$\text{Profit \%} = \frac{SP}{CP} = \frac{180}{150} = 1.2 \Rightarrow 20\%.$$

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4 RATIOS AND PROPORTIONS

4.1. What is a Ratio?

A ratio is a representation of distribution of a value present among the persons present and is shown as follows:

If a total is divided among A, B and C such that A got 4 parts, B got 5 parts and C got 6 parts then it is represented in ratio as A:B:C = 4:5:6.

So, 4:5:6 means that the total value is divided into 4+5+6 = 15 equal parts and then distributed as per the ratio.

Example 1: Divide Rs. 580 between A and B in the ratio of 14:15.

Solution: A:B = 14:15 $\Rightarrow$ 580 is divided into 29 equal parts $\Rightarrow$ each part = Rs. 20.

So A's share = 14 parts = 14 × 20 = Rs. 280

B's share = 15 parts = Rs. 300.

Example 2: If A:B = 2:3 and B:C = 4:5 then find A:B:C.

Solution: To combine two ratios the proportions common for them shall be in equal parts. Here the common proportion is B for the given ratios.

Making B equal in both ratios they become 8:12 and 12:15 $\Rightarrow$ A:B:C = 8:12:15.

Example 3: Three numbers are in the ratio of 3: 4 : 8 and the sum of these numbers is 975. Find the three numbers.

Solution: Let the numbers be $3x$, $4x$ and $8x$. Then their sum = $3x + 4x + 8x = 15x = 975 \Rightarrow x = 65$.

So the numbers are $3x = 195$, $4x = 260$ and $8x = 520$.

Example 4: Two numbers are in the ratio of 4 : 5. If the difference between these numbers is 24, then find the numbers.

Solution: Let the numbers be $4x$ and $5x$. Their difference = $5x - 4x = x = 24$ (given).

So the numbers are $4x = 96$ and $5x = 120$.

Example 5: Given two numbers are in the ratio of 3 : 4. If 8 is added to each of them, their ratio is changed to 5 : 6. Find two numbers.

Solution: Let the numbers be a and b.

$A:B = 3:4 \Rightarrow \frac{A}{B} = \frac{3}{4}$. Also, $\frac{(A+8)}{(B+8)} = \frac{5}{6}$.

Solving we get, A=12 and B = 16.

5 TIME AND DISTANCE

5.1. Speed

We have the relation between speed, time and distance as follows:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

So the distance covered in unit time is called speed.

This forms the basis for Time and Distance. It can be re-written as Distance = Speed X Time or

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}.$$

5.1.1. Units of Speed

The units of speed are kmph (km per hour) or m / s.

$$1 \text{ kmph} = \frac{5}{18} \text{ m/s}$$

$$1 \text{ m/s} = \frac{18}{5} \text{ kmph}$$

5.1.2. Average Speed

When the travel comprises of various speeds then the concept of average speed is to be applied.

$$\text{a Average Speed} = \frac{\text{Total distance covered}}{\text{Total time of travel}}$$

Note: In the total time above, the time of rest is not considered.

Example 1: If a car travels along four sides of a square at 100 kmph, 200 kmph, 300 kmph and 400 kmph find its average speed.

Solution: $\text{Average Speed} = \dfrac{\text{Total distance}}{\text{Total time}}.$

Let each side of square be x km. Then the total distance = $4x$ km.

The total time is sum of individual times taken to cover each side.

To cover x km at 100 kmph, time $= \frac{x}{100}$.

For the second side time $= \frac{x}{200}$.

Using this we can write average speed $= \frac{4x}{\left(\frac{x}{100}+\frac{x}{200}+\frac{x}{300}+\frac{x}{400}\right)} = 192$ kmph.

Example 2: A man if travels at $\frac{5}{6}$ th of his actual speed takes 10 min more to travel a distance. Find his usual time.

Solution: Let s be the actual speed and t be the actual time of the man.

Now the speed is $\left(\frac{5}{6}\right)$s and time is (t +10) min. But the distance remains the same.

So distance 1 = distance 2 $\Rightarrow s \times t = \left(\frac{5}{6}\right)s \times (t+10) \Rightarrow t = 50$ min.

Example 3: If a person walks at 30 kmph he is 10 min late to his office. If he travels at 40 kmph then he reaches to his office 5 min early. Find the distance to his office.

Solution: Let the distance to his office be d. The difference between the two timings is given as 15 min $= \frac{1}{4}$ hr.

Now if d km are covered at 30 kmph then time = d/30. Similarly second time = d/40.

So, $\frac{d}{30} - \frac{d}{40} = \frac{1}{4} \Rightarrow d = 30$ km.

Note: When two objects move with speeds s_1 and s_2
- In opposite directions their combined speed = $s_1 + s_2$
- In same direction their combined speed = $s_1 \sim s_2$.

Example 4: Two people start moving from the same point at the same time at 30 kmph and 40 kmph in opposite directions. Find the distance between them after 3 hrs.

Solution: Speed = 30 + 40 = 70 kmph (since in opposite directions)

Time = 3 hrs

So distance = speed × time = 70 × 3 = 210 km.

Example 5: A starts from X to Y at 6 am at 40 kmph and at the same time B starts from Y to X at 50 kmph. When will they meet if X and Y are 360 km apart?

Solution: Distance = 360 km, Speed = 40 + 50 = 90 kmph.

Time $= \frac{\text{distance}}{\text{speed}} = \frac{360}{90} = 4$hrs from 6 am $\Rightarrow 10$ am.

6 TIME AND WORK

6.1. Introduction

If a person can complete a work in ‘n’ days then he can do $\frac{1}{n}$ part of the work in one day.

The amount of work done be a person in 1 day is called his efficiency.

Example: A can do a work in 10 days. Then the efficiency of A is given by $A = \frac{1}{10}$.

Note: Number of days required to do a work = work to be done/work per day.

Example 1: If A can do a work in 10 days, B can do it in 20 days and C in 30 days in how many days will the three together do it?

Solution: The efficiencies are $A = \frac{1}{10}, B = \frac{1}{20}$ and $C = \frac{1}{30}$

So work done per day by the three = $\frac{1}{10} + \frac{1}{20} + \frac{1}{30} = \frac{11}{60} \Rightarrow No\ of\ days = \frac{60}{11} = 5.45\ days$.

Example 2: If A and B can do a work in 10 days, B and C can do it in 20 days and C and A can do it in 40 days in what time all the three can do it?

Solution:

$$A + B = \frac{1}{10}$$

$$B + C = \frac{1}{20}$$

$$C + A = \frac{1}{40}$$

Adding all the three we get $2(A + B + C) = \frac{7}{40} \Rightarrow A + B + C = \frac{7}{80} \Rightarrow No\ of\ days = \frac{80}{7}\ days$.

Note: If all the people do not work for all the time then the principle below can be used:

$$mA + nB + \boldsymbol{oC} = \mathbf{1}. \quad \text{(1 is the total work)}$$

Here, m = no of days A worked

n = no of days B worked

o = no of days C worked

A, B, C = efficiencies

Example 3: If A can do a work in 12 days, B can do it in 18 days and C in 24 days. All the three started the work. A left after two days and C left three days before the completion of the work. How many days are required to complete the work?

Solution: Let the total no of days be x.

A worked only for 2 days, B worked for x days and C worked for $x - 3$ days.

So, mA + nB + oC = 1

$$\Rightarrow 2\left(\frac{1}{12}\right) + x\left(\frac{1}{18}\right) + (x-3)\left(\frac{1}{24}\right) = 1$$

$$\Rightarrow 12 + 4x + 3(x\text{-}3) = 72$$

$$\Rightarrow x = \frac{69}{7}\ days.$$

Note: The ratio of dividing wages = ratio of efficiencies = ratio of parts of work done

Example 4: A can do a work in 10 days and B can do it in 30 days and C in 60 days. If the total wages for the work is Rs. 1800 what is the share of A?

Solution: Ratio of wages = $\frac{1}{10} : \frac{1}{30} : \frac{1}{60} = 6 : 2 : 1$ (Multiplying each term by LCM 60)

So total 9 equal parts in Rs. 1800 $\Rightarrow$ each part = Rs. 200 $\Rightarrow$ share of A = 6 parts = Rs. 1200.

Note: When pipes are used filling the tank they are treated similar to the men working but some outlet pipes emptying the tank are present whose work will be considered negative.

Example 5: A pipe can fill a tank in 5 hrs but because of a leak a the bottom it takes 1 hr extra. In what time can the leak alone empty the tank?

Solution: Let the filling pipe be A.

$$A = \frac{1}{5}$$

But with the leak L, $A - L = \frac{1}{6}$ (A-L because leak is outlet)

So, $\frac{1}{L} = \frac{1}{5} - \frac{1}{6} = \frac{1}{30} \Rightarrow$ Leak can empty the tank in 30 hrs.

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7 CLOCKS

7.1. Introduction

In a clock the most important hands are the minutes hand and the hours hand. Whatever may be the shape of the dial they move in a circular track.

The total angle of 360 degrees in a watch is divided into 1 sectors, one for each hour.

So one hour sector = $\frac{360}{12} = 30$ degrees.

For every one hour (60 min),

- The minutes hand moves through 360 deg.
- The hours hand moves through 30 deg.

So for every minute,

- The minutes hand moves through 6 deg
- The hours hand moves through 0.5 deg.

They move in same direction. So their relative displacement for every minute is 5.5 deg.

This 5.5 deg movement constitutes the movements of both the hands.

So for every minute both the hands give a displacement of 5.5 deg.

Note:

1. Between every two hours i.e., between 1 and 2, 2 and 3 and so on the hands of the clock coincide with each other for one time except between 11, 12 and 12, 1.
   In a day they coincide for 22 times.
2. Between every two hours they are perpendicular to each other two times except between 2, 3 and 3, 4 and 8, 9 and 9, 10.
   In a day they will be perpendicular for 44 times.
3. Between every two hours they will be opposite to each other one time except between 5, 6 and 6, 7.
   In a day they will be opposite for 22 times.

Examples: At what time between 5 and 6 will the hands of the clock coincide?

Solution: At 5 the angle between the hands is 150 deg.

To coincide, they collectively have to travel this distance. Every minute they travel 5.5 deg.

So no. of minutes required to coincide = $\frac{150}{5.5} = \frac{300}{11} = 27\frac{3}{11}\ min$.

Examples: At what time between 6 and 7 will the hands be perpendicular?

Solution: At 6 the angle between the hands is 180 deg.

To form 90 deg they have to cover 90 deg (out of 180 if 90 is covered 90 will remain)

So no. of minutes required = $\frac{90}{5.5} = \frac{180}{11} = \frac{164}{11} min$.

But they will be perpendicular for two times. The second one will happen after the minutes hand crosses the hours hand and then for 90 deg.

So it has to travel 180 + 90 = 270 deg.

So time = $\frac{270}{5.5} = \frac{540}{11} = 49\frac{1}{11} min$.

Examples: What is the angle between the hands of the clock at 3.45?

Solution: At 3, the angle between the hands = A = 90 deg.

In 45 min the hands will move angle of B = 45 × 5.5 deg (since 5.5 deg for 1 min)

B = 247.5 deg.

Required angle = A ~ B = 157.5 deg.

Examples: What is the angle between the hands at 4.40?

Solution: At 4 the angle between the hands, A = 120 deg.

In 40 min, B = 40 × 5.5 = 220 deg.

The required angle = A ~ B = 100 deg.

Examples: A clock loses 5 min for every hour and another gains 5 min for every hour. If they are set correct at 10 am on Monday then when will they be 12 hrs apart?

Solution: For every hour watch A loses 5 min and watch B gains 5 min.

So for every hour they will differ by 10 min.

For 12 hrs (720 min) difference between them the time required = $\frac{720}{10} = 72\ hrs$

So they will be 12 hrs apart after 3 days i.e., at 10 am on Thursday.

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8 CALENDARS

8.1. Calendars

Here you mainly deal in finding the day of the week on a particular given date.

The process of finding this depends on the number of odd days.

Odd days are quite different from the odd numbers.

- **Odd Days:** The days more than the complete number of weeks in a given period are called odd days.
- **Ordinary Year:** An year that has 365 days is called Ordinary Year.
- **Leap Year:** The year which is exactly divisible by 4 (except century) is called a leap year.

Example: 1968, 1972, 1984, 1988 and so on are the examples of Leap Years.

1986, 1990, 1994, 1998, and so on are the examples of non leap years.

Note: The Centuries divisible by 400 are leap years.

Important Points:

- An ordinary year has 365 days = 52 weeks and 1 odd day.
- A leap year has 366 days = 52 weeks and 2 odd days.
- Century = 76 Ordinary years + 24 Leap years.
- Century contain 5 odd days.
- 200 years contain 3 odd days.
- 300 years contain 1 odd day.
- 400 years contain 0 odd days.
- Last day of a century cannot be Tuesday, Thursday or Saturday.
- First day of a century must be Monday, Tuesday, Thursday or Saturday.

Explanation:

100 years = 76 ordinary years + 24 leap years

= 76 odd days + 24 × 2 odd days

= 124 odd days = 17 weeks + 5 days

∴ 100 years contain 5 odd days.

No. of odd days in first century = 5

∴ Last day of first century is Friday.

No. of odd days in two centuries = 3

∴ Wednesday is the last day.

No. of odd days in three centuries = 1

∴ Monday is the last day.

No. of odd days in four centuries = 0

∴ Sunday is the last day.

Since the order is continually kept in successive cycles, the last day of a century cannot be Tuesday, Thursday or Saturday.

So, the last day of a century should be Sunday, Monday, Wednesday or Friday.

Therefore, the first day of a century must be Monday, Tuesday, Thursday or Saturday.

8.2. Working Rules

Working rule to find the day of the week on a particular date when reference day is given:

Step 1: Find the net number of odd days for the period between the reference date and the given date (exclude the reference day but count the given date for counting the number of net odd days).

Step 2: The day of the week on the particular date is equal to the number of net odd days ahead of the reference day (if the reference day was before this date) but behind the reference day (if this date was behind the reference day).

Working rule to find the day of the week on a particular date when no reference day is given

Step 1: Count the net number of odd days on the given date

Step 2: Write:

For 0 odd days – Sunday

For 1 odd day – Monday

For 2 odd days – Tuesday

⋮ ⋮ ⋮

For 6 odd days - Saturday

Examples: If 11^{th} January 1997 was a Sunday then what day of the week was on 10^{th} January 2000?

Solution: Total number of days between 11^{th} January 1997 and 10^{th} January 2000

= (365 – 11) in 1997 + 365 in 1998 + 365 in 1999 + 10 days in 2000

= (50 weeks + 4 odd days) + (52 weeks + 1 odd day) + (52 weeks + 1 odd day) + (1 week + 3 odd days)

Total number of odd days = 4 + 1 + 1 + 3 = 9 days = 1 week + 2 days

Hence, 10^{th} January, 2000 would be 2 days ahead of Sunday i.e. it was on Tuesday.

Examples: What day of the week was on 10^{th} June 2008?

Solution: 10^{th} June 2008 = 2007 years + First 5 months up to May 2008 + 10 days of June

2000 years have 0 odd days.

Remaining 7 years has 1 leap year and 6 ordinary years $\Rightarrow$ 2 + 6 = 8 odd days

So, 2007 years have 8 odd days.

No. of odd days from 1^{st} January 2008 to 31^{st} May 2008 = 3+1+3+2+3 = 12

10 days of June has 3 odd days.

Total number of odd days = 8+12+3 = 23

23 odd days = 3 weeks + 2 odd days.

Hence, 10^{th} June, 2008 was Tuesday.

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9 BLOOD RELATIONS

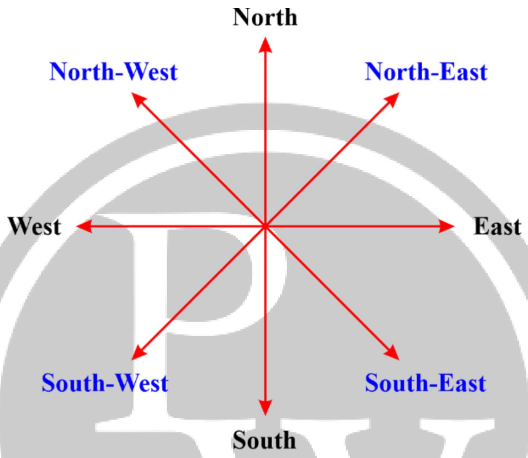
9.1. Introduction

The standard definitions of relations are given below

A/An ⇓	is related to a PERSON as ⇓
Grandfather	The father of his/her mother or father
Grandmother	The mother of his/her mother or father
Grandson	The son of his/her daughter/son
Granddaughter	The daughter of his/her daughters/son
Uncle	The brother of his/her mother or father
Aunt	The sister of his/her mother or father
Nephew	The son of his/her brother or sister
Cousin	The son or daughter of his/her aunt of uncle
Niece	The daughter of his/her brother or sister
Spouse	as her husband or his wife
Father-in-law	the father of his/her spouse
Mother-in-law	the mother of his/her spouse
Sister-in-law	the sister of his/her spouse
Brother-in-law	the brother of his/her spouse
Son-in-law	the spouse of his/her daughter
Daughter-in-law	the spouse of his/her son

10 DIRECTIONS

10.1. Introduction

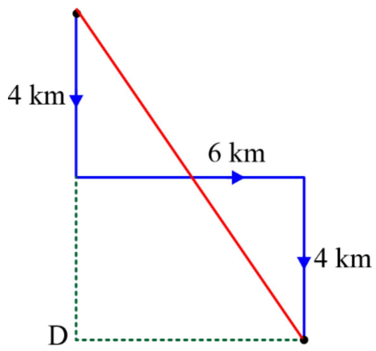


Examples:

Example 1: Ravi traveled 4 km straight towards south. He turned left and traveled 6 km straight, then turned right and traveled 4 km straight. How far is he from the starting point?

(a) 8 km (b) 10 km (c) 18 km (d) 12 km

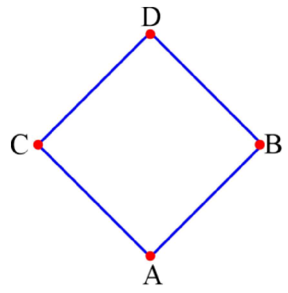
Solution. 10 km. B is the finishing point and A is starting point. The distance of A from B is



Example 2. A is to the South-East of C, B is to the East of C and North-East of A. If D is to the North of A and North-West of B. In which direction of C is D located?

(a) North-West (b) South-West (c) North-East (d) South-East

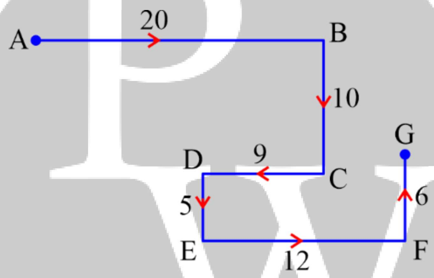
Solution. North-East D is located to the North-East of C.



Example 3. A rat runs 20' towards East and turns to right, runs 10' and turns to right, runs 9' and again turns to left, runs 5' and then runs to left, runs 12' and finally turns to left and runs 6'. Now, which direction is the rat facing?

(a) East (b) West
(c) North (d) South

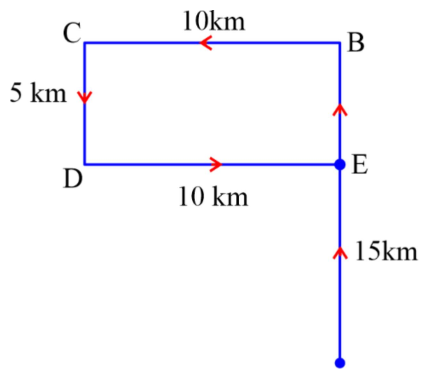
Solution. North. The movements of the rat from A to G are as shown in the fig. It is clear, rat is walking in one direction FG, *i.e.*, North.



Example 4. From his house, Lokesh went 15 km to the North. Then he turned west and covered 10 km, then he turned South and covered 5 km. Finally, turning to East, he covered 10 km. In which direction is he from his house?

(a) East (b) South
(c) North (d) West

Solution. North. Starting point is A and ending point is E. E is to the north of his house at A.



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11 DATA INTERPRETATION

11.1. Data Interpretation

- It deals with careful reading, understanding, organizing and interpreting the data provided so as to derive meaningful conclusions.
- Mostly used tools for interpretation of a data are
  - Ratio
  - Percentage
  - Rate
  - Average

11.2. Types of Data Interpretation:

The numerical data pertaining to any event can be presented by any one or more of the following methods.

1. Tables
2. Line Graphs
3. Bar Graphs or Bar Charts
4. Pie Charts or Circle Graphs

11.2.1. Tables

It is the systematic presentation of data in tabular form to understand the given information and to make clear the problem in a certain field of study. It has six elements namely:

- **Title:** It is the heading of the table.
- **Stule:** It is the section of the table containing row headings
- **Column Captions:** It is the heading of each column
- **Body:** It consists the numerical figures
- **Footnotes:** It is for further explanation of the table
- **Source:** It is the authority of the data

Example: Study the following table and answer the questions given below it.

Annual Income of Five Schools

Figures in '00 rupees

Sources of Income	School A	School B	School C	School D	School E
Tuition Fee	120	60	210	90	120
Term Fee	24	12	45	24	30
Donations	54	21	60	51	60
Funds	60	54	120	42	55
Miscellaneous	12	3	15	3	15
Total	270	150	450	210	280

Example: The income by way of donation to school D is what per-cent of its miscellaneous?

Solution: Required percentage = $\frac{5100}{300} = 27\%$

11.2.2. Line Graph

A line graph indicates the variation of a quantity w.r.t two parameters calibrated on X and Y-axis respectively.

Note:

1. Any part of the line graph parallel to X-axis represents no change in the value of Y parameter w.r.t the value of X parameter.
2. The steepest or maximum part of the line graph indicates maximum percentage change of the value during the two consecutive period in which the related part lies.
3. If the steepest part is a rise slope, then it is the highest percentage growth.
4. If the steepest part is a decline slope, it will represent a maximum percentage fall of the value calibrated in the other axis.

11.2.3. Bar Graph

Bar graphs are diagrammatic representation of a discrete data.

Types of Bar Graphs:

- **Simple Bar Graphs:** A simple bar graph relates to only one variable. The values of the variables may relate to different years or different terms.
- **Sub-divided Bar Graph:** It is used to represent various parts of sub-classes of total magnitude of the given variable.
- **Multiple Bar Graphs:** In this type, two or more bars are constructed adjoining each other, to represent either different components of a total or to show multiple variables.

11.2.4. Pie Chart

In this method of representation, the total quantity is distributed over a total angle of 360° which is one complete circle or pie.

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12 DATA SUFFICIENCY

12.1. Introduction

Data sufficiency questions are designed to measure your ability to analyze a quantities problem, recognize which given information is relevant, and determine at what point there is sufficient information to solve a problem. In these questions, you are to classify each problem according to the five or four fixed answer choice, rather than find a solution to the problem.

Each Data sufficiency question consists of a question, often accompanied by some initial information, and two statements, labeled (1) and (2), which contain additional information. You must decide whether the information in each statement is sufficient to answer the question or- if neither statement provides enough information –whether the information in the two statements together is sufficient. It is also possible that the statements in combination do not give enough information answer the question.

Begin by reading the initial information and the question carefully. Next, consider the first statement. Does the information provided by the first statement is sufficient to answer the question? Go on the statement. Try to ignore the information given in the first statement when you consider the second statement. Now you should be able to say, for each statement, whether it is sufficient to determine the answer.

Next, consider the two statements in tandem. Do they, together, enable you to answer the question?

Give our answers as per the following statements

A Statement (1) alone is sufficient but
Statement (2) alone is not sufficient

B Statement (2) alone is sufficient but
Statement (1) alone is not sufficient

C Both statements together are sufficient
but neither statement alone is sufficient

D Each statement alone is sufficient

E Both statement together are still not sufficient.

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