Branch : CSE/IT **Batch : Hinglish**

Discrete Mathematics II

Set Theory

DPP-07

[MCQ]

1. Which of the following pairs of elements are comparable in the poset (N, |) where N is set of all integers.

(a) 6, 7 (b) 3, 5

(c) 5, 15 (d) 4, 6

[MCQ]

2. Which of the following pairs of sets are not comparable in the poset [$P(A)$, $\subseteq$] where

A = {0, 1, 2}

(a) {0}, {0, 1} (b) ϕ, {0, 1, 2}

(c) {1, 2} {0, 1, 2} (d) {0}, {1}

[NAT]

3. Let $U = \{1, 2, 3, 4, 5, 6, 7\}$, with $A = P(U)$, and let R be the subset relation on A. For $B = \{\{1\}, \{2\}, \{2, 3\}\} \subseteq A$, then the number of upper bounds that exist for B is X and the number of lower bounds that exist for B is Y, then value of $X + Y$ is?

[MCQ]

4. Let $(A, R_1), (B, R_2)$ be two posets. On $A \times B$, define relation R by $(a, b) R (x, y)$ if $a\ R_1 x$ and $b\ R_2 y$.

Consider the following statement:

I. R is a partial order.

II. R is not a partial order.

(a) Only **I** is true.

(b) Only **II** is true.

(c) Both **I** and **II** are true.

(d) Neither **I** nor **II** is true.

[NAT]

5. Define the relation R on the set Z by a Rb if $a - b$ is a nonnegative even integer. Then consider the following statement regarding relation R:

I. R is a partial order.

II. R is a total order.

The number of correct statements for R is/are ?

Answer Key

1. (c)
2. (d)
3. (17)
4. (a)
5. (1)

Hints and Solutions

1. (c)

Two elements an and b are said to be comparable with respect to R, if (aRb) or (bRa).

$\therefore$ Only 5 and 15 are comparable

2. (d)

The sets {0} and {1} are not comparable because neither of the two is a subset of the other.

3. (17)

The number of upper bounds that exist for B

$$\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}=2^4=16$$

X = 16

The number of lower bounds that exist for B

One – namely ϕ

Y = 1, therefore X + Y = 16 + 1 = 17.

4. (a)

For all $a \in A, b \in B, aR_1a$ and bR_2b so $(a,b)R(a,b)$, and R is reflexive. Next $(a,b)R(c,d),(c,d)R(a,b) \Rightarrow aR_1c, cR_1a$ and $bR_2d, dR_2b \Rightarrow a=c, b=d \Rightarrow (a,b)=(c,d)$, so R is antisymmetric. Finally, $(a,b)R(c,d),(c,d)R(e,f) \Rightarrow aR_1c, cR_1e$ and $bR_2d, dR_2f \Rightarrow aR_1e, bR_2f \Rightarrow (a,b)R(e,f)$, and R is transitive. Consequently, R is a partial order.

5. (1)

For each a$\in$Z it follows that aRa because $a-a=0$, an even nonnegative integer. Hence R is reflexive. If $a, b, c, \in Z$ with aRb and bRc then

$$a-b=2m, \text{ for some } m \in N$$
$$b-c=2n, \text{ for some } n \in N,$$

and $a-c=(a-b)+(b-c)=2(m+n)$, where $m+n \in N$. Therefore, aRc and R is transitive. Finally, suppose that aRb and bRa for some $a,b \in Z$. Then $a-b$ and $b-a$ are both nonnegative integers. Since this can only occur for $a-b=b-a$, we find that $[aRb \wedge bRa] \Rightarrow a=b$, so R is antisymmetric.

Consequently, the relation R is a partial order for Z. But it is not a total order. For example, 2, 3 $\in$ Z and we have neither 2R3 nor 3R2, because neither – 1 not 1, respectively, is a non-negative even integer.