# All Branches Hinglish

## Subject: Engineering Mathematics
## Chapter: Linear Algebra

**DPP-01**

**1.** Consider $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

The eigenvalues of M are

(a) 0, 1, 2 (b) 0, 0, 3
(c) 1, 1, 1 (d) –1, 1, 3

**2.** A 3 × 3 matrix M has Tr[M] = 6, Tr[$M^2$] = 26, Tr [$M^3$]= 90. Which of the following can be possible set of eigenvalues of M?

(a) {1,1,4} (b) {–1,0,7}
(c) {–1,3,4} (d) {2,2,2}

**3.** The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

(a) (1, 4, 9) (b) (0, 7, 7)
(c) (0, 1, 13) (d) (0, 0, 14)

**4.** The eigenvalue of the anti-symmetric $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where $n_1$, $n_2$, $n_3$ are components of an unit vector, are

(a) 0, –i, i (b) 0, 1, –1
(c) 0, 1 + i, –1 –i (d) 0, 0, 0

**5.** A 2 × 2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of' n such that $A^n = I$

(a) 20 (b) 30
(c) 60 (d) 120

**6.** Given a 2 × 2 unitary matrix satisfying U'U = UU' = I with det U = $e^{i\varphi}$ , one can construct a unitary matrix V (V'V = VV' = $I$) with det V= 1 from it by

(a) Multiplying U by $e^{-i\varphi/2}$
(b) Multiplying a single element of U by $e^{-i\varphi}$
(c) Multiplying any row or column by $e^{-i\varphi/2}$
(d) Multiplying U by $e^{-i\varphi}$

**7.** Consider *a n x n* (*n*> 1) matrix A, in which $A_{ij}$ is the product of the indices *i* and *j* (namely $A_{ij}$ = ij) The matrixA

(a) has one degenerate eigenvalue with degeneracy ($n$ – 1)
(b) has two degenerate eigenvalues with degeneracies 2 and ($n$ – 2)
(c) has one degenerate eigenvalues with degeneracy $n$
(d) does not have any degenerate eigenvalues

**8.** Consider the matrix $M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$

The eigenvalues of M are

(a) –5, –2, 7 (b) –7, 0, 7
(c) –4i, 2i, 2i (d) 2, 3, 6

**9.** The matrices $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

and $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ satisfy the commutation relations

(a) [A, B] = B + C, [B, C] = 0, [C, A] = B + C
(b) [A, B] = C, [B, C] = A, [C, A] = B
(c) [A, B] = B, [B, C] = 0, [C, A] = A
(d) [A, B] = C, [B, C] = 0, [C, A] = B

**10.** The column vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is a simultaneous eigenvector of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ if

(a) $b = 0$ or $a = 0$ (b) $b = a$ or $b = -2a$

(c) $b = 2a$ or $b = -a$ (d) $b = a/2$ or $b = -a/2$

**11.** For any operator $A, i\left(A^{+} - A\right)$ is:

(a) Hermitian (b) Anti-Hermitian

(c) Unitary (d) Orthogonal

**12.** If two matrices A and B can be diagonalized simultaneously, which of the following is true?

(a) $A^2 B = B^2 A$ (b) $A^2 B^2 = B^2 A$

(c) $AB = BA$ (d) $AB^2 AB = BABA^2$

**13.** Which one of the following is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?

(a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

**14.** A $3 \times 3$ matrix has eigenvalues 0, 2 + i and 2 – i. Which one of the following statements is correct?

(a) The matrix is hermitian

(b) The matrix is unitary

(c) The inverse of the matrix exist

(d) The determinant of the matrix is zero

**15.** A real traceless $4 \times 4$ unitary matrix has two eigen values – 1 and 1. The other eigenvalues are

(a) Zero and +2 (b) –1 and + 1

(c) Zero and +1 (d) +1 and +1

**16.** The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are

(a) +1 and +1 (b) Zero and +1

(c) Zero and +2 (d) –1 and +1

**17.** If A is $2 \times 2$ matrix with determinant 2, then the determinant of adj.[adj.[adj($A^{-1}$)]] is equal to

(a) 1/512 (b) 1/1024

(c) 1/128 (d) 1/256

**18.** The eigenvalues of ($A^4$ + 3A -2I), where A is

$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{pmatrix}$, are

(a) 2,20,88 (b) 1,2,3

(c) 2,20,3 (d) 1,20,88

**19.** Eigen value of matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are

(a) –2, –1, 1, 2 (b) –1, 1, 0, 2

(c) 1, 0, 2, 3 (d) –1, 1, 0, 3

**20.** A linear transformation T, defined as

$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\vec{x}$ from a three-dimensional space to a two-dimensional real space. The transformation matrix T is

(a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

**21.** One of the eigenvalues of the matrix

$\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5

The other two eigen values are

(a) 0 and 0 (b) 1 and 1

(c) 1 and –1 (d) –1 and –1

**22.** The normalized eigen vector corresponding to the eigen value 5 is:

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\\0\end{pmatrix}$

**23.** The eigenvalues of a matrix are i, –2i and 3i. The matrix is

(a) Unitary (b) Anti-Unitary

(c) Hermitian (d) Anti-Hermitian

**24.** The eigenvalues and eigenvectors of the matrix $\begin{bmatrix}5 & 4\\1 & 2\end{bmatrix}$ are

(a) $6,1 \text{and} \begin{bmatrix}4\\1\end{bmatrix},\begin{bmatrix}1\\-1\end{bmatrix}$

(b) 2, 5 and $\begin{bmatrix}4\\1\end{bmatrix},\begin{bmatrix}1\\-1\end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix}4\\1\end{bmatrix},\begin{bmatrix}1\\-1\end{bmatrix}$

(d) 2, 5 and $\begin{bmatrix}4\\1\end{bmatrix},\begin{bmatrix}1\\-1\end{bmatrix}$

**25.** Consider a vector $\vec{p}=2\hat{i}+3\hat{j}+2\hat{k}$ in the coordinate system $(\hat{i},\hat{j},\hat{k})$. The axes are rotated anti clockwise about Y axis by an angle of 60°. The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}',\hat{j}',\hat{k}')$ is

(a) $(1-\sqrt{3})\hat{i}'+3\hat{j}'+(1+\sqrt{3})\hat{k}'$

(b) $(1+\sqrt{3})\hat{i}'+3\hat{j}'+(1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{i}'+(3+\sqrt{3})\hat{j}'+2\hat{k}'$

(d) $(1-\sqrt{3})\hat{i}'+(3-\sqrt{3})\hat{j}'+2\hat{k}'$

**26.** For arbitrary matrices E, F, G and H, if EF – FE = 0, then Trace (EFGH) is equal

(a) Trace (HFEG)

(b) Trace (E) , Trace (F), Trace (G), Trace (H)

(c) Trace (GFEH)

(d) Trace (EGHF)

**27.** An unitary matrix $\begin{bmatrix}ae^{i\alpha} & b\\ce^{i\beta} & d\end{bmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is

(a) $\begin{bmatrix}ae^{i\alpha} & -ce^{i\beta}\\b & d\end{bmatrix}$ (b) $\begin{bmatrix}ae^{i\alpha} & ce^{i\beta}\\b & d\end{bmatrix}$

(c) $\begin{bmatrix}ae^{i\alpha} & b\\ce^{i\beta} & d\end{bmatrix}$ (d) $\begin{bmatrix}ae^{-i\alpha} & ce^{-i\beta}\\b & d\end{bmatrix}$

**28.** The eigenvalue of the matrix $A=\begin{bmatrix}0 & i\\i & 0\end{bmatrix}$ are

(a) Real and Distinct

(b) Complex and Distinct

(c) Complex and Coinciding

(d) Real and Coinciding

**29.** The eigen values of the matrix $\begin{pmatrix}2 & 3 & 0\\3 & 2 & 0\\0 & 0 & 1\end{pmatrix}$ are

(a) 5, 2, –2 (b) –5, –1, 1

(c) 5, 1, –1 (d) –5, 1, 1

**30.** Two matrices A and B are said to be similar if B = $P^{-1}$ AP for some invertible matrix P. Which of the following statements is NOT TRUE?

(a) Det A = Det B

(b) Trace of A = Trace of B

(c) A and B have the same eigenvectors

(d) A and B have the same eigenvalues

**31.** A 3 × 3 matrix has element such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is:

(a) 18 (b) 12

(c) 9 (d) 6

**32.** The eigenvalues of the matrix $\begin{pmatrix}0 & 1 & 0\\1 & 0 & 1\\0 & 1 & 0\end{pmatrix}$ are

(a) 0, 1, 1 (b) 0, $-\sqrt{2},\sqrt{2}$

(c) $\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0$ (d) $\sqrt{2},\sqrt{2},0$

**33.** The degenerate eigenvalue of the matrix $M=\begin{pmatrix}4 & -1 & -1\\-1 & 4 & -1\\-1 & -1 & 4\end{pmatrix}$ is …….

**34.** The matrix $A=\frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1+i\\ 1-i & -1\end{bmatrix}$ is

(a) Orthogonal (b) Symmetric
(c) Anti-Symmetric (d) Unitary

**35.** Which of the following is INCORRECT for the matrix $M=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

(a) It is its own inverse
(b) It is its own transpose
(c) It is non-orthogonal
(d) it has eigen values ±1

**36.** The symmetric pair of $P=\begin{pmatrix}a\\ b\end{pmatrix}(a-2b)$ is:

(a) $\begin{pmatrix}a^2-2 & ba-1\\ ba-1 & b^2-2\end{pmatrix}$

(b) $\begin{pmatrix}a(a-1) & b\\ b & b^2\end{pmatrix}$

(c) $\begin{pmatrix}a(a-1) & b(a-1)\\ b(a-1) & b^2\end{pmatrix}$

(d) $\begin{pmatrix}a(a-2) & b(a-1)\\ b(a-1) & b^2\end{pmatrix}$

**37.** $(x\ \ y)\begin{pmatrix}5 & -7\\ 7 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=15$

The matrix equation above represents.

(a) A circle of radius $\sqrt{15}$

(b) An ellipse of semi major axis $\sqrt{5}$

(c) An ellipse of semi major axis 5

(d) A hyperbola

**38.** The product $PQ$ of any two real, symmetric matrices $P$ and $Q$ is:

(a) Symmetric for all $P$ and $Q$

(b) Never symmetric

(c) Symmetric if $PQ = QP$

(d) Antisymmetric for all $P$ and $Q$

**39.** A matrix is given by $M=\frac{1}{\sqrt{2}}\begin{pmatrix}i & 1\\ 1 & i\end{pmatrix}$. The eigenvalues of the $M$ are

(a) Real and positive
(b) Purely imaginary with modulus 1
(c) Complex with modulus 1
(d) Real and negative.

**40.** Given two $(n \times n)$ matrices $\hat{P}$ and $\hat{Q}$ such that $\hat{P}$ is hermitian and $\hat{Q}$ is skew (anti)-hermitian. Which one of the following combinations of $\hat{P}$ and $\hat{Q}$ is necessarily a Hermitian matrix?

(a) $\hat{P}\hat{Q}$ (b) $i\hat{P}\hat{Q}$
(c) $\hat{P}+i\hat{Q}$ (d) $\hat{P}+i\hat{Q}$

**41.** The inverse of the matrix $M=\begin{pmatrix}0 & 1 & 1\\ 0 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}$ is

(a) $M-1$ (b) $M^2-1$
(c) $1-M^2$ (d) $1-M$

where $I$ is the identity matrix.

**42.** The normalized eigenvectors of the matrix

$N=\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$ are $\beta_1$ and $\beta_2$

with the eigenvalues $\lambda_1$ and $\lambda_2$ respectively and $\lambda_1 > \lambda_2$. If the eigenvector $\alpha=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$ expressed as $\alpha = P\beta_1 + Q\beta_2$. Find the constants $P$ and $Q$.

(a) $\frac{1+i}{2},\frac{1-i}{2}$ (b) $\frac{2+i}{2},\frac{1+i}{2}$
(c) $\frac{1+i}{3},\frac{1-i}{4}$ (d) $\frac{i}{2},\frac{1+i}{2}$

**43.** The trace of a 2 × 2 matrix is 4 and its determinant is 8. If one of the eigenvalue is $2(1+i)$, the other eigenvalue is

(a) $2(1-i)$ (b) $2(1+i)$
(c) $(1+2i)$ (d) $(1-2i)$

**44.** The eigenvalues of the matrix representing the following pair of linear equations

$x + iy = 0$
$ix + y = 0$

are

(a) $1 + i, 1 + i$ (b) $1 - i, 1 - i$
(c) $1, i$ (d) $1 + i, 1 - i$

**45.** For the given set of equations:
$x + y = 1$
$y + z = 1$
$x + z = 1,$
Which one of the following statements is correct?
(a) Equations are inconsistent
(b) Equations are consistent and a single non-trivial solution exists
(c) Equations are consistent and many solutions
(d) Equations are consistent and only a trivial solution exists

□□□

# Answer Key

1. (b)
2. (c)
3. (d)
4. (a)
5. (c)
6. (a)
7. (a)
8. (b)
9. (d)
10. (b)
11. (a)
12. (c)
13. (c)
14. (d)
15. (b)
16. (c)
17. (c)
18. (a)
19. (a)
20. (a)
21. (c)
22. (d)
23. (d)
24. (a)
25. (a)
26. (a)
27. (d)
28. (b)
29. (c)
30. (c)
31. (d)
32. (b)
33. (5)
34. (d)
35. (c)
36. (d)
37. (b)
38. (c)
39. (c)
40. (c)
41. (b)
42. (a)
43. (a)
44. (d)
45. (b)

□□□