Branch : CSE/IT Batch : Hinglish

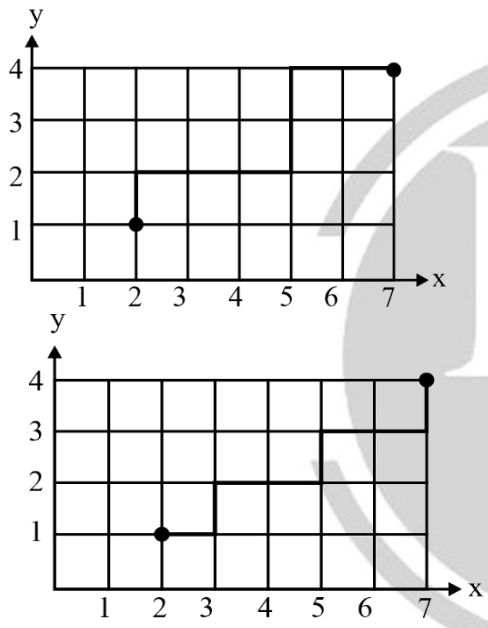
Discrete Mathematics II

Combinatorics

DPP-01

[NAT]

1. Determine the number of (staircase) paths in the xy – plane from (2, 1) to (7, 4), where each such path is made up of individual steps going one unit to the right (R) or one unit upward (U). The bold lines in Figure. show two of these paths.



[MCQ]

2. How many nonnegative integer solution are there to the pair of equations $x_1 + x_2 + x_3 + \ldots + x_7 = 37$, $x_1 + x_2 + x_3 = 6$?

(a) $\binom{42}{12}\binom{34}{31}$ (b) $\binom{5}{3}\binom{34}{31}$

(c) $\binom{8}{6}\binom{34}{31}$ (d) None of these

[MCQ]

3. Determine the number of integer solution of
$x_1 + x_2 + x_3 + x_4 + x_5 < 40$,
where $x_i \geq 0$, $1 \leq i \leq 5$.

(a) $\binom{44}{39}$ (b) $\binom{39}{38}$

(c) $\binom{41}{11}$ (d) None of these

[NAT]

4. Determine the number of integer solution of
$x_1 + x_2 + x_3 + x_4 + x_5 < 40$,
where
$x_i \geq -3$, $1 \leq i \leq 5$

[NAT]

5. How many ways are there to place 12 marbles of the same size in five distinct jars if the marbles are all black?

[MCQ]

6. Determine the number of integer solutions of
$x_1 + x_2 + x_3 + x_4 = 32$,
where
(I) $x_1 \geq 0$, $1 \leq i \leq 4$
(II) $x_1 > 0$, $1 \leq i \leq 4$

(a) $\binom{11}{8}, \binom{35}{32}$ (b) $\binom{35}{32}, \binom{31}{28}$

(c) $\binom{35}{32}, \binom{11}{8}$ (d) None of these

[MCQ]

7. A certain ice cream store has 31 flavors of ice cream available. In how many ways can we order a dozen ice cream cones if a flavor may be ordered as many as 12 times?

(a) $\binom{42}{12}$ (b) $\binom{31}{12}$

(c) $\binom{42}{11}$ (d) None of these

[NAT]

8. In how many ways can 10 (identical) dimes be distributed among five children if there are no restrictions?

(a) $\binom{12}{8}$ (b) $\binom{14}{10}$

(c) $\binom{23}{11}$ (d) $\binom{14}{9}$

[NAT]

9. Find the coefficient of x^{16} in the expansion of $\left(2x^2 - \frac{x}{2}\right)^{12}$.

[MCQ]

10. How many ways are there to place 12 pebbles of the same size in five distinct jars if each pebble is a different color?

(a) $\binom{16}{12}$ (b) 5^{12}

(c) 12^5 (d) $\binom{14}{12}$

Answer Key

1. ()
2. (c)
3. (a)
4. ()
5. ()
6. (b)
7. (a)
8. (b)
9. ()
10. (b)

Hints and Solutions

1. (126)

${}^{9}C_{5}$.

2. (c)

There are $\binom{3+6-1}{6}=\binom{8}{6}$ solution for $x_1 + x_2 + x_3 = 6$ and $\binom{4+31-1}{31}=\binom{34}{31}$ solution for $x_4 + x_5 + x_6 + x_7 = 31$, where $x_i \geq 0$, $1 \leq i \leq 7$ by the rule of product the pair of equation has $\binom{8}{6}\binom{34}{31}$ solutions.

3. (a)

(a) The number of solution for $x_1 + x_2 + ... + x_5 < 40$, $x_i \geq 0$, $1 \leq i \leq 5$, is the same as the number for $x_1 + x_2 + ...x_5 \leq 39$, $x_i \geq 0$ $i \leq 5$, and this equal the number of solutions for $x_1 + x_2 + ...+ x_5 + x_6 = 39$, $x_i \geq 0$, $1 \leq i \leq 6$. There are $\binom{6+39-1}{39}\binom{44}{39}$ such solutions

4. ()

Let $y_i = x_i + 3$, $1 \leq i \leq 5$, and consider the inequality $y_1 + y_2 + \ldots y_5 \leq 54$, $y_i \geq 0$. There are $\binom{6+54-1}{54}\binom{59}{54}$ solutions.

5. $\binom{16}{12}$

$\binom{5+12-1}{12}=\binom{16}{12}$

6. (b)

(I) $\binom{4+32-1}{32}=\binom{35}{32}$

(II) $\binom{4+28-1}{26}=\binom{31}{28}$

7. (a)

$\binom{31}{12}$

$\binom{31+12-1}{12}=\binom{42}{12}$

8. (b)

x_i, $1 \leq i \leq 5$, denotes the amount given to the five children.

The number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 10$, $0 \leq x_i$, $1 \leq i \leq 5$, is $\binom{5+10-1}{10}=\binom{14}{10}$.

Here n = 5, r = 10.

9. ()

The general term in the expansion of this expression by the Binomial theorem is

$$\binom{12}{k}\left(2x^2\right)^{12-k}\left(-\frac{x}{2}\right)^{k}=\binom{12}{k}2^{12-k}\left(-\frac{1}{2}\right)^{k}x^{24-k}$$

We want 24 – k = 16 thus k = 8. The coefficient is

$$\binom{12}{8}2^{4}\left(-\frac{1}{2}\right)^{8}=\frac{1}{16}\binom{12}{8}=\frac{495}{16}.$$

10. (b)

5^{12}