All Branches Hinglish

Subject: Engineering Mathematics
Chapter: Probability & Statistics
Topic : Random Variable

DPP-03

1. A fair coin is tossed 3 times. Let the random variable X denote the number of heads in 3 tosses of the coin. Find the probability density function of X.

(a) $\left(\frac{3}{x}\right)\left(\frac{1}{2}\right)^{2x}\left(\frac{1}{2}\right)^{2-x}$

(b) $\left(\frac{3}{2x}\right)\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{1-x}$

(c) $\left(\frac{3}{x}\right)\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{3-x}$

(d) $\left(\frac{3}{x}\right)\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{4-x}$

2. If the probability of a random variable X is given by $f(x) = k(2x - 1)$, $x = 1, 2, 3\ldots, 12$. Find k.

3. The density function for the continuous random variable X is

$$f_x(x) = \begin{cases} e^{-x} \text{ for } x > 0 \\ 0 \text{ for } x \le 0 \end{cases}$$

Find the Probability P [X ≤ 2 | X > 1].

4. A continuous random variable X has density function

$$f(x) = \begin{cases} 2x & 0 < x < \frac{1}{2} \\ \frac{4-2x}{3} & \frac{1}{2} \le x < 2 \\ 0 & \textit{elsewhere} \end{cases}$$

Find P [0.25 < x ≤ 1.25]

5. Let X be a continuous random variable with probability density function

$$f(x) = \frac{1}{2}e^{-|x-1|}, -\infty < x < \infty$$

Find the value of P(1 < | X | < 2)

6. The probability function of a random variable X is given by

$$f(x) = \begin{cases} \frac{1}{4} & |x| \le 1 \\ \frac{1}{4x^2} & \textit{otherwise} \end{cases}$$

Then $P\left(-\frac{1}{2} \le X \le 2\right)$ = _____.

7. Let X be a continuous random variable with the probability density function

$$f(x) = \begin{cases} \frac{x}{8} & 0 < x < 2 \\ \frac{k}{8} & 2 \le x \le 4 \\ \frac{6-x}{8} & 4 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a real constant. Then P (1 < X < 5) equals _____.

8. Suppose the random variable X has a probability density function

$$f(x) = \begin{cases} \frac{|x|}{4}, & -c \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

The value of c is

(a) 0.5 (b) 1

(c) 2 (d) 4

9. A random variable X has probability density function

$$f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

The value of k is

(a) 2 (b) 6

(c) 5 (d) 4

10. The probability distribution of a discrete random variable X is given in the table below.

x	0	1	2	3	4	5
P (X = x)	0.1	0.3	0.15	0.25	0.15	0.05

The P $(1 < X \le 4)$ is

(a) 0.55 (b) 0.85
(c) 0.70 (d) 0.40

11. Suppose the random variable X has a probability density function

$$f(x) = \begin{cases} kx^3e^{-x/2}, & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

The vale of k is
(a) 1/96 (b) 96
(c) 8/3 (d) 1/4

12. Let X be a continuous random variable with pdf

$$f_x(x) = \begin{cases} cx^2, & \text{for } 0 < x \le 1, \\ 0 & \text{otherwise} \end{cases}$$

For some positive constant c. The value of P $\left(X \le \frac{2}{3} | X > \frac{1}{3}\right)$ is

(a) 3/26 (b) 5/26
(c) 7/26 (d) 11/26

13. Suppose the random variable X has the probability density function

$$f(x) = \begin{cases} ce^{x/3}, & x \le 0, \\ ce^{-x/3}, & x > 0, \end{cases}$$

For some positive constant c. The value of P [X > 6/X > 0] is

(a) e^{-2} (b) ce^{-2}
(c) 0 (d) $1 - e^{-2}$

14. Let X be a discrete random variable with probability function $P(X = x) = \frac{2}{3^x}$, for x = 1, 2, 3, ……. What is the probability that X is even?

(a) $\frac{1}{4}$ (b) $\frac{2}{7}$
(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

15. Let $f(x) = \frac{k|x|}{(1+|x|)^4}$, $-\infty < x < \infty$

Then the value of k for which $f(x)$ is a probability density function is

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$
(c) 3 (d) 6

16. A random variable X has a probability mass of 0.2 at X = 0 and a probability mass of 0.1 at X = 1. For all other values, X has the following density function

$$f(x) = \begin{cases} 0 & x < 0 \\ x & 0 < x < 1 \\ 2x & 1 < x < c, \text{ where c is constant} \\ 0 & x \ge c \end{cases}$$

Find P (X < 1/X > 0.5)
(a) (0, 0.6) (b) (0.6, 0.7)
(c) (0.7, 0.8) (d) (0.8, 0.9)

17. The distribution function of a random variable X is given by

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4} & 0 \le x < \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \le x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \le x < \frac{3}{4} \\ \frac{x+3}{5} & \frac{3}{4} \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

Then P $\left(\frac{1}{4} \le X \le 1\right)$ is

(a) $\frac{1}{20}$ (b) $\frac{11}{20}$

(c) $\frac{7}{20}$ (d) $\frac{13}{20}$

18. Let X be a random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & for\ x \le 0 \\ 1-e^{-x} & for\ x > 0 \end{cases},$$

What is P ($0 \le e^x \le 4$) ?

(a) e^{-4} (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

19. Let X is a random variable with density

$$f(x) = \frac{1}{4} e^{-\frac{|x|}{2}},\ -\infty < x < \infty$$

Then E(| X |) = _____ .

20. If X is a random variable with density function

$$f(x) = \begin{cases} 1.4e^{-2x} + 0.9e^{-3x}, & x > 0, \\ 0 & \text{elsewhere} \end{cases}$$

Then E [X] =

(a) $\frac{9}{20}$ (b) $\frac{5}{6}$

(c) 1 (d) $\frac{230}{126}$

21. You are given a random variable X such that its density is

$$f_x(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

A square with diagonal of length X is constructed. Find the expected value of the area of that square.

(a) 0.1 (b) 0.25

(c) $\frac{4}{7}$ (d) 0.3

22. X has a distribution which is partly continuous and partly discrete

$$f(x) = \begin{cases} \frac{1-p}{2}, & 0 < x < 1 \\ p & x = 1 \\ \frac{1-p}{2}, & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the variance of X in terms of p

(a) $\frac{1-p}{3}$ (b) $\frac{2-p}{3}$

(c) $\frac{1-p}{2}$ (d) $\frac{2-p}{2}$

23. X has a mean of 2 and a variance of 4. Y = aX + b has a mean of 5 and a variance of 1. What is ab assuming that a > 0?

(a) 1 (b) 2
(c) 3 (d) 4

24. Let X be a random variable with E(X) = 5 and $E(X^2)$= 25. Then $E(X + E(X))^3$ is

(a) 0 (b) 125
(c) 1000 (d) 250

25. Let X be a continuous variable with the probability density function symmetric about 0.
If V(X) < ∞. Then which of the following statement is true?

(a) E(| X |) = E(X)
(b) V(| X |) = V(X)
(c) V(|X|) < V(X)
(d) V(|X|) > V(X)

Answer Key

1. **(c)**
2. **(0.0069)**
3. **(0.63)**
4. **(0.75)**
5. **(0.78)**
6. **(0.5)**
7. **(0.875)**
8. **(c)**
9. **(b)**
10. (a)
11. **(a)**
12. **(c)**
13. **(a)**
14. **(a)**
15. **(c)**
16. **(a)**
17. **(b)**
18. **(d)**
19. **(2)**
20. **(a)**
21. **(d)**
22. **(a)**
23. **(b)**
24. **(c)**
25. **(c)**

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