2

October 2, 2023

[ ]:
```
using JuMP
using CPLEX
```

[ ]:
```
m = 10
```

```
10
```

[ ]:
```
n = 30
```

```
30
```

[ ]:
```
p = 20
```

```
20
```

[ ]:
```
Q = rand(10:50, m, n)
```

```
10×30 Matrix{Int64}:
 44  11  40  46  33  40  43  30  32  …  49  22  50  40  36  30  48  37  12
 37  36  27  38  30  27  39  15  24     12  10  16  19  27  22  47  45  11
 49  21  15  45  14  39  35  45  14     12  12  49  48  48  44  25  22  17
 18  34  39  48  11  27  23  30  43     46  47  25  22  47  40  44  39  37
 13  32  27  14  12  30  40  27  25     41  39  13  38  30  34  41  30  30
 32  48  31  32  22  39  40  21  19  …  50  26  33  30  15  49  35  24  49
 48  35  26  44  21  35  27  46  46     12  32  29  41  40  38  34  36  46
 20  26  11  45  13  11  49  35  12     11  16  28  33  23  30  35  24  34
 36  19  41  45  44  23  46  15  34     43  15  10  26  41  15  16  44  32
 17  27  46  45  21  16  42  32  10     17  21  34  23  10  32  34  41  45
```

[ ]:
```
C = rand(60:100, p, n)
```

```
20×30 Matrix{Int64}:
 63  100  83  76  64   93  100  75  75  …  90   67   64   74  96  87   86  87
 81   71  82  71  89   80   62  94  60     61   69   70   95  97  91   62  82
 80   81  89  96  80   97   68  62  92     88   61   95   86  63  68   95  67
 94   74  99  69  73   96   78  77  82     75  100   87   63  97  71   76  96
 68   88  97  75  68   81   61  95  80     72   80  100   68  91  66   65  68
 63   68  73  72  64   87  100  65  82  …  83   77   75   72  89  63   72  71
 97   81  68  75  61   62   69  92  84     74   61   99   85  93  68   70  63
 92   61  97  74  91   97   78  80  62     87   97   89   61  94  86   95  64
 97   64  71  83  78   73   64  91  62     70   92   89   96  95  76   74  84
 97   74  80  66  98   77   62  69  94     92   98   83   95  98  74   62  63
 70   88  99  93  79   85   88  93  93  …  98   91   82   91  85  65  100  96
 62   88  90  80  68   86   98  72  88     78   74   97   95  60  73   96  85
 60   65  80  63  92   82   97  98  99     69   71   80   78  78  99  100  98
 75   63  65  99  88   99   75  86  87     81   96   73   91  71  90   92  94
 88   66  98  72  75   85   70  75  88     94   95   76   61  82  96   86  95
 65   87  71  94  95   91   86  89  91  …  83   98   86   99  95  92   67  85
 76   88  63  62  98   60   69  92  83     61   99   67  100  89  77   61  96
 61   90  88  87  69   99   69  85  85     65   79   95   64  73  98   92  63
 76   80  65  75  74  100   78  78  79     74   95   90   88  63  80   86  97
 83   81  98  94  65   74   83  87  69     71  100   65   78  99  71   69  63
```

[ ]:
```
model = Model(CPLEX.Optimizer)
```

```
A JuMP Model
Feasibility problem with:
Variables: 0
Model mode: AUTOMATIC
CachingOptimizer state: EMPTY_OPTIMIZER
Solver name: CPLEX
```

[ ]:
```
@variable(model, x[i=1:m, j =1:p], lower_bound = 0, Bin) # Binary variable
```

```
10×20 Matrix{VariableRef}:
 x[1,1]   x[1,2]   x[1,3]   x[1,4]   x[1,5]   …  x[1,18]   x[1,19]   x[1,20]
 x[2,1]   x[2,2]   x[2,3]   x[2,4]   x[2,5]      x[2,18]   x[2,19]   x[2,20]
 x[3,1]   x[3,2]   x[3,3]   x[3,4]   x[3,5]      x[3,18]   x[3,19]   x[3,20]
 x[4,1]   x[4,2]   x[4,3]   x[4,4]   x[4,5]      x[4,18]   x[4,19]   x[4,20]
 x[5,1]   x[5,2]   x[5,3]   x[5,4]   x[5,5]      x[5,18]   x[5,19]   x[5,20]
 x[6,1]   x[6,2]   x[6,3]   x[6,4]   x[6,5]   …  x[6,18]   x[6,19]   x[6,20]
 x[7,1]   x[7,2]   x[7,3]   x[7,4]   x[7,5]      x[7,18]   x[7,19]   x[7,20]
 x[8,1]   x[8,2]   x[8,3]   x[8,4]   x[8,5]      x[8,18]   x[8,19]   x[8,20]
 x[9,1]   x[9,2]   x[9,3]   x[9,4]   x[9,5]      x[9,18]   x[9,19]   x[9,20]
 x[10,1]  x[10,2]  x[10,3]  x[10,4]  x[10,5]     x[10,18]  x[10,19]  x[10,20]
```

[ ]:
```
@constraint(model, sum(x[:, j] for j in 1:p) .== 1) # Ensuring 10 plants be␣
 ↪established
```

```
10-element Vector{ConstraintRef{Model, MathOptInterface.
 ↪ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64},␣
 ↪MathOptInterface.EqualTo{Float64}}, ScalarShape}}:
 x[1,1] + x[1,2] + x[1,3] + x[1,4] + x[1,5] + x[1,6] + x[1,7] + x[1,8] + x[1,9]␣
 ↪+ x[1,10] + x[1,11] + x[1,12] + x[1,13] + x[1,14] + x[1,15] + x[1,16] +␣
 ↪x[1,17] + x[1,18] + x[1,19] + x[1,20] == 1
 x[2,1] + x[2,2] + x[2,3] + x[2,4] + x[2,5] + x[2,6] + x[2,7] + x[2,8] + x[2,9]␣
 ↪+ x[2,10] + x[2,11] + x[2,12] + x[2,13] + x[2,14] + x[2,15] + x[2,16] +␣
 ↪x[2,17] + x[2,18] + x[2,19] + x[2,20] == 1
 x[3,1] + x[3,2] + x[3,3] + x[3,4] + x[3,5] + x[3,6] + x[3,7] + x[3,8] + x[3,9]␣
 ↪+ x[3,10] + x[3,11] + x[3,12] + x[3,13] + x[3,14] + x[3,15] + x[3,16] +␣
 ↪x[3,17] + x[3,18] + x[3,19] + x[3,20] == 1
 x[4,1] + x[4,2] + x[4,3] + x[4,4] + x[4,5] + x[4,6] + x[4,7] + x[4,8] + x[4,9]␣
 ↪+ x[4,10] + x[4,11] + x[4,12] + x[4,13] + x[4,14] + x[4,15] + x[4,16] +␣
 ↪x[4,17] + x[4,18] + x[4,19] + x[4,20] == 1
 x[5,1] + x[5,2] + x[5,3] + x[5,4] + x[5,5] + x[5,6] + x[5,7] + x[5,8] + x[5,9]␣
 ↪+ x[5,10] + x[5,11] + x[5,12] + x[5,13] + x[5,14] + x[5,15] + x[5,16] +␣
 ↪x[5,17] + x[5,18] + x[5,19] + x[5,20] == 1
 x[6,1] + x[6,2] + x[6,3] + x[6,4] + x[6,5] + x[6,6] + x[6,7] + x[6,8] + x[6,9]␣
 ↪+ x[6,10] + x[6,11] + x[6,12] + x[6,13] + x[6,14] + x[6,15] + x[6,16] +␣
 ↪x[6,17] + x[6,18] + x[6,19] + x[6,20] == 1
 x[7,1] + x[7,2] + x[7,3] + x[7,4] + x[7,5] + x[7,6] + x[7,7] + x[7,8] + x[7,9]␣
 ↪+ x[7,10] + x[7,11] + x[7,12] + x[7,13] + x[7,14] + x[7,15] + x[7,16] +␣
 ↪x[7,17] + x[7,18] + x[7,19] + x[7,20] == 1
 x[8,1] + x[8,2] + x[8,3] + x[8,4] + x[8,5] + x[8,6] + x[8,7] + x[8,8] + x[8,9]␣
 ↪+ x[8,10] + x[8,11] + x[8,12] + x[8,13] + x[8,14] + x[8,15] + x[8,16] +␣
 ↪x[8,17] + x[8,18] + x[8,19] + x[8,20] == 1
 x[9,1] + x[9,2] + x[9,3] + x[9,4] + x[9,5] + x[9,6] + x[9,7] + x[9,8] + x[9,9]␣
 ↪+ x[9,10] + x[9,11] + x[9,12] + x[9,13] + x[9,14] + x[9,15] + x[9,16] +␣
 ↪x[9,17] + x[9,18] + x[9,19] + x[9,20] == 1
 x[10,1] + x[10,2] + x[10,3] + x[10,4] + x[10,5] + x[10,6] + x[10,7] + x[10,8] +␣
 ↪x[10,9] + x[10,10] + x[10,11] + x[10,12] + x[10,13] + x[10,14] + x[10,15] +␣
 ↪x[10,16] + x[10,17] + x[10,18] + x[10,19] + x[10,20] == 1
```

```
[ ]: @constraint(model, sum(x[i, :] for i in 1:m) .<= 1) # One place can't have more␣
      ↪than 1 plant
```

```
20-element Vector{ConstraintRef{Model, MathOptInterface.
 ↪ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64},␣
 ↪MathOptInterface.LessThan{Float64}}, ScalarShape}}:
 x[1,1] + x[2,1] + x[3,1] + x[4,1] + x[5,1] + x[6,1] + x[7,1] + x[8,1] + x[9,1]␣
 ↪+ x[10,1] <= 1
 x[1,2] + x[2,2] + x[3,2] + x[4,2] + x[5,2] + x[6,2] + x[7,2] + x[8,2] + x[9,2]␣
 ↪+ x[10,2] <= 1
 x[1,3] + x[2,3] + x[3,3] + x[4,3] + x[5,3] + x[6,3] + x[7,3] + x[8,3] + x[9,3]␣
 ↪+ x[10,3] <= 1
 x[1,4] + x[2,4] + x[3,4] + x[4,4] + x[5,4] + x[6,4] + x[7,4] + x[8,4] + x[9,4]␣
 ↪+ x[10,4] <= 1
 x[1,5] + x[2,5] + x[3,5] + x[4,5] + x[5,5] + x[6,5] + x[7,5] + x[8,5] + x[9,5]␣
 ↪+ x[10,5] <= 1
 x[1,6] + x[2,6] + x[3,6] + x[4,6] + x[5,6] + x[6,6] + x[7,6] + x[8,6] + x[9,6]␣
 ↪+ x[10,6] <= 1
 x[1,7] + x[2,7] + x[3,7] + x[4,7] + x[5,7] + x[6,7] + x[7,7] + x[8,7] + x[9,7]␣
 ↪+ x[10,7] <= 1
 x[1,8] + x[2,8] + x[3,8] + x[4,8] + x[5,8] + x[6,8] + x[7,8] + x[8,8] + x[9,8]␣
 ↪+ x[10,8] <= 1
 x[1,9] + x[2,9] + x[3,9] + x[4,9] + x[5,9] + x[6,9] + x[7,9] + x[8,9] + x[9,9]␣
 ↪+ x[10,9] <= 1
 x[1,10] + x[2,10] + x[3,10] + x[4,10] + x[5,10] + x[6,10] + x[7,10] + x[8,10] +␣
 ↪x[9,10] + x[10,10] <= 1
 x[1,11] + x[2,11] + x[3,11] + x[4,11] + x[5,11] + x[6,11] + x[7,11] + x[8,11] +␣
 ↪x[9,11] + x[10,11] <= 1
 x[1,12] + x[2,12] + x[3,12] + x[4,12] + x[5,12] + x[6,12] + x[7,12] + x[8,12] +␣
 ↪x[9,12] + x[10,12] <= 1
 x[1,13] + x[2,13] + x[3,13] + x[4,13] + x[5,13] + x[6,13] + x[7,13] + x[8,13] +␣
 ↪x[9,13] + x[10,13] <= 1
 x[1,14] + x[2,14] + x[3,14] + x[4,14] + x[5,14] + x[6,14] + x[7,14] + x[8,14] +␣
 ↪x[9,14] + x[10,14] <= 1
 x[1,15] + x[2,15] + x[3,15] + x[4,15] + x[5,15] + x[6,15] + x[7,15] + x[8,15] +␣
 ↪x[9,15] + x[10,15] <= 1
 x[1,16] + x[2,16] + x[3,16] + x[4,16] + x[5,16] + x[6,16] + x[7,16] + x[8,16] +␣
 ↪x[9,16] + x[10,16] <= 1
 x[1,17] + x[2,17] + x[3,17] + x[4,17] + x[5,17] + x[6,17] + x[7,17] + x[8,17] +␣
 ↪x[9,17] + x[10,17] <= 1
 x[1,18] + x[2,18] + x[3,18] + x[4,18] + x[5,18] + x[6,18] + x[7,18] + x[8,18] +␣
 ↪x[9,18] + x[10,18] <= 1
 x[1,19] + x[2,19] + x[3,19] + x[4,19] + x[5,19] + x[6,19] + x[7,19] + x[8,19] +␣
 ↪x[9,19] + x[10,19] <= 1
 x[1,20] + x[2,20] + x[3,20] + x[4,20] + x[5,20] + x[6,20] + x[7,20] + x[8,20] +␣
 ↪x[9,20] + x[10,20] <= 1
```

```
[ ]: cost = sum(x[i, k] * C[k, j] * Q[i, j] for i in 1:m for j in 1:n for k in 1:p)
```

$79025x_{1,1}+75279x_{1,2}+79614x_{1,3}+80530x_{1,4}+76856x_{1,5}+74016x_{1,6}+75501x_{1,7}+78629x_{1,8}+78341x_{1,9}+78042x_{1,10}+$

```
[ ]: @objective(model, Min, cost)
```

$79025x_{1,1}+75279x_{1,2}+79614x_{1,3}+80530x_{1,4}+76856x_{1,5}+74016x_{1,6}+75501x_{1,7}+78629x_{1,8}+78341x_{1,9}+78042x_{1,10}+$

```
[ ]: optimize!(model)
```

```
Version identifier: 22.1.1.0 | 2022-11-26 | 9160aff4d
Found incumbent of value 0.000000 after 0.02 sec. (0.01 ticks)

Root node processing (before b&c):
  Real time             =    0.02 sec. (0.01 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
```

```
                              ------------
Total (root+branch&cut) =    0.02 sec. (0.01 ticks)
Version identifier: 22.1.1.0 | 2022-11-26 | 9160aff4d
Found incumbent of value 716973.000000 after 0.00 sec. (0.01 ticks)
Tried aggregator 1 time.
Reduced MIP has 30 rows, 200 columns, and 400 nonzeros.
Reduced MIP has 200 binaries, 0 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.03 sec. (0.23 ticks)
Probing time = 0.00 sec. (0.27 ticks)
Tried aggregator 1 time.
Reduced MIP has 30 rows, 200 columns, and 400 nonzeros.
Reduced MIP has 200 binaries, 0 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.00 sec. (0.23 ticks)
Probing time = 0.00 sec. (0.27 ticks)
Clique table members: 30.
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 8 threads.
Root relaxation solution time = 0.02 sec. (0.16 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt     Gap

*     0+    0                       716973.0000        0.0000           100.00%
*     0+    0                       706224.0000        0.0000           100.00%
*     0     0      integral     0   698289.0000   698289.0000       23    0.00%
Elapsed time = 0.08 sec. (1.46 ticks, tree = 0.00 MB, solutions = 3)
```

[ ]:
```
@show value.(x)
```

```
value.(x) = [-0.0 -0.0 -0.0 -0.0 -0.0 0.0 0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0
-0.0 -0.0 1.0 -0.0 -0.0 -0.0; -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0
-0.0 -0.0 -0.0 -0.0 -0.0 -0.0 0.0 -0.0 1.0 -0.0; -0.0 -0.0 -0.0 -0.0 1.0 -0.0
-0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0; -0.0 1.0
-0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0
-0.0 -0.0; -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0
-0.0 -0.0 -0.0 0.0 -0.0 1.0; -0.0 0.0 -0.0 -0.0 -0.0 -0.0 1.0 -0.0 -0.0 -0.0
-0.0 -0.0 -0.0 -0.0 -0.0 -0.0 0.0 -0.0 -0.0 -0.0; -0.0 -0.0 -0.0 -0.0 0.0 1.0
-0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0; -0.0 -0.0
-0.0 -0.0 -0.0 -0.0 -0.0 1.0 -0.0 0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0
-0.0 -0.0; -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0
-0.0 -0.0 -0.0 1.0 -0.0 -0.0; -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 1.0
-0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 -0.0 0.0 -0.0]

10×20 Matrix{Float64}:
 -0.0  -0.0  -0.0  -0.0  -0.0   0.0  …  -0.0  -0.0   1.0  -0.0  -0.0  -0.0
 -0.0  -0.0  -0.0  -0.0  -0.0  -0.0     -0.0  -0.0   0.0  -0.0   1.0  -0.0
 -0.0  -0.0  -0.0  -0.0   1.0  -0.0     -0.0  -0.0  -0.0  -0.0  -0.0  -0.0
 -0.0   1.0  -0.0  -0.0  -0.0  -0.0     -0.0  -0.0  -0.0  -0.0  -0.0  -0.0
 -0.0  -0.0  -0.0  -0.0  -0.0  -0.0     -0.0  -0.0  -0.0   0.0  -0.0   1.0
 -0.0   0.0  -0.0  -0.0  -0.0  -0.0  …  -0.0  -0.0   0.0  -0.0  -0.0  -0.0
 -0.0  -0.0  -0.0  -0.0   0.0   1.0     -0.0  -0.0  -0.0  -0.0  -0.0  -0.0
 -0.0  -0.0  -0.0  -0.0  -0.0  -0.0     -0.0  -0.0  -0.0  -0.0  -0.0  -0.0
 -0.0  -0.0  -0.0  -0.0  -0.0  -0.0     -0.0  -0.0  -0.0   1.0  -0.0  -0.0
 -0.0  -0.0  -0.0  -0.0  -0.0  -0.0     -0.0  -0.0  -0.0  -0.0   0.0  -0.0
```

[ ]:

```
@show objective_value(model)
```

```
objective_value(model) = 698289.0
```

```
698289.0
```