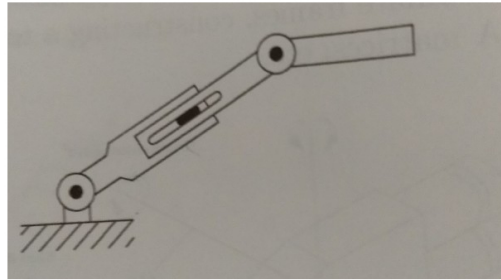


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## Question: Derive the Euler-Lagrange equations for the planar RP robot b...



Derive the Euler-Lagrange equations for the planar RP robot below



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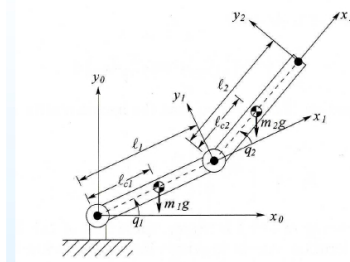
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2

The planar RP robot consists of a prismatic joint axed to the end of a link that is attached to a rotational joint. The only challenge here, other than the tediousness of



### Euler-Lagrange Equations for 2-Link Cartesian Manipulator

Given the kinetic  $\mathcal{K}$  and potential  $\mathcal{P}$  energies, the dynamics are

$$\frac{d}{dt} \left[ \frac{\partial (\mathcal{K} - \mathcal{P})}{\partial \dot{q}} \right] - \frac{\partial (\mathcal{K} - \mathcal{P})}{\partial q} = \tau$$

With kinetic and potential energies

$$\mathcal{K} = \frac{1}{2} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^T \begin{bmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, \quad \mathcal{P} = g(m_1 + m_2)q_1 + C$$

the Euler-Lagrange equations are

$$\begin{aligned} (m_1 + m_2)\ddot{q}_1 + g(m_1 + m_2) &= \tau_1 \\ m_2\ddot{q}_2 &= \tau_2 \end{aligned}$$

### Forward Kinematics and Jacobian

DH parameters for computing homogeneous transformations

$$T(q_i) = \text{Rot}_{z, \theta} \cdot \text{Trans}_{z, d} \cdot \text{Trans}_{x, a} \cdot \text{Rot}_{x, \alpha}$$

are

$$\begin{aligned} T_1^0: \quad \theta &= q_1, \quad d = 0, \quad a = l_1, \quad \alpha = 0 \\ T_2^1: \quad \theta &= q_2, \quad d = 0, \quad a = l_2, \quad \alpha = 0 \end{aligned}$$

The kinetic energy of the system is

$$\mathcal{K} = \frac{1}{2} [m_1 v_{c1}^2 + \omega_1^T I_1 \omega_1] + \frac{1}{2} [m_2 v_{c2}^2 + \omega_2^T I_2 \omega_2]$$

and

$$v_{c1} = \begin{bmatrix} J_{v1}^{(1)} & J_{v1}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J_{v1}^{(1)} \dot{q}_1 + J_{v1}^{(2)} \dot{q}_2$$

$$v_{c2} = \begin{bmatrix} J_{v2}^{(1)} & J_{v2}^{(2)} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J_{v2}^{(1)} \dot{q}_1 + J_{v2}^{(2)} \dot{q}_2$$

DH parameters for computing homogeneous transformations

$$T(q_i) = \text{Rot}_{z, \theta} \cdot \text{Trans}_{z, d} \cdot \text{Trans}_{x, a} \cdot \text{Rot}_{x, \alpha}$$

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To compute the Jacobian we can use the DH-frames, i.e

$$\begin{aligned} J_v^{(i)} &= \begin{cases} z_{i-1}^0, & \text{for prismatic joint} \\ z_{i-1}^0 \times [o_i^0 - o_{i-1}^0], & \text{for revolute joint} \end{cases} \\ J_\omega^{(i)} &= \begin{cases} 0, & \text{for prismatic joint} \\ z_{i-1}^0, & \text{for revolute joint} \end{cases} \end{aligned}$$

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The formula

$$\mathbf{J}_v^{(i)} = \begin{cases} \mathbf{z}_{i-1}^0, & \text{for prismatic joint} \\ \mathbf{z}_{i-1}^0 \times [\mathbf{o}_c^0 - \mathbf{o}_{i-1}^0], & \text{for revolute joint} \end{cases}$$

gives

$$\mathbf{J}_{v1}^{(1)} = \mathbf{z}_0 \times (\vec{o}_{c1} - \vec{o}_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c1} \cos q_1 \\ l_{c1} \sin q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_{c1} \sin q_1 \\ l_{c1} \cos q_1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \mathbf{J}_{v2}^{(1)} &= \mathbf{z}_0 \times (\vec{o}_{c2} - \vec{o}_0) \\ &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left( \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_{c2} \cos(q_1 + q_2) \\ l_{c2} \sin(q_1 + q_2) \\ 0 \end{bmatrix} \right) \end{aligned}$$

The formula

$$\mathbf{J}_v^{(i)} = \begin{cases} \mathbf{z}_{i-1}^0, & \text{for prismatic joint} \\ \mathbf{z}_{i-1}^0 \times [\mathbf{o}_c^0 - \mathbf{o}_{i-1}^0], & \text{for revolute joint} \end{cases}$$

gives

$$\mathbf{J}_{v1}^{(1)} = \mathbf{z}_0 \times (\vec{o}_{c1} - \vec{o}_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c1} \cos q_1 \\ l_{c1} \sin q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_{c1} \sin q_1 \\ l_{c1} \cos q_1 \\ 0 \end{bmatrix}$$

$$\mathbf{J}_{v2}^{(1)} = \mathbf{z}_0 \times (\vec{o}_{c2} - \vec{o}_0) = \begin{bmatrix} -l_1 \sin q_1 - l_{c2} \sin(q_1 + q_2) \\ l_1 \cos q_1 + l_{c2} \cos(q_1 + q_2) \\ 0 \end{bmatrix}$$

$$\mathbf{J}_{v2}^{(2)} = \mathbf{z}_1 \times (\vec{o}_{c2} - \vec{o}_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c2} \cos(q_1 + q_2) \\ l_{c2} \sin(q_1 + q_2) \\ 0 \end{bmatrix}$$

The formula

$$\mathbf{J}_v^{(i)} = \begin{cases} \mathbf{z}_{i-1}^0, & \text{for prismatic joint} \\ \mathbf{z}_{i-1}^0 \times [\mathbf{o}_c^0 - \mathbf{o}_{i-1}^0], & \text{for revolute joint} \end{cases}$$

gives

$$\mathbf{J}_{v1}^{(1)} = \mathbf{z}_0 \times (\vec{o}_{c1} - \vec{o}_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c1} \cos q_1 \\ l_{c1} \sin q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_{c1} \sin q_1 \\ l_{c1} \cos q_1 \\ 0 \end{bmatrix}$$

$$\mathbf{J}_{v2}^{(1)} = \mathbf{z}_0 \times (\vec{o}_{c2} - \vec{o}_0) = \begin{bmatrix} -l_1 \sin q_1 - l_{c2} \sin(q_1 + q_2) \\ l_1 \cos q_1 + l_{c2} \cos(q_1 + q_2) \\ 0 \end{bmatrix}$$

$$\mathbf{J}_{v2}^{(2)} = \mathbf{z}_1 \times (\vec{o}_{c2} - \vec{o}_1) = \begin{bmatrix} -l_{c2} \sin(q_1 + q_2) \\ l_{c2} \cos(q_1 + q_2) \\ 0 \end{bmatrix}$$

#### Forward Kinematics and Jacobian (Cont'd)

The formula

$$\mathbf{J}_\omega^{(i)} = \begin{cases} 0, & \text{for prismatic joint} \\ \mathbf{z}_{i-1}^0, & \text{for revolute joint} \end{cases}$$

gives

$$\mathbf{J}_{\omega_1}^{(1)} = \mathbf{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{J}_{\omega_2}^{(1)} = \mathbf{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{J}_{\omega_2}^{(2)} = \mathbf{z}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

To sum up, the kinetic energy  $\mathcal{K}$  is

$$\begin{aligned} \mathcal{K} &= \frac{1}{2} [m_1 v_{c1}^2 + \omega_1^T \mathcal{I}_1 \omega_1] + \frac{1}{2} [m_2 v_{c2}^2 + \omega_2^T \mathcal{I}_2 \omega_2] \\ &= \frac{1}{2} \left[ m_1 \left( \mathbf{J}_{v1}^{(1)} \dot{q}_1 \right)^2 + \mathcal{I}_1 \left( \mathbf{J}_{\omega_1}^{(1)} \dot{q}_1 \right)^2 \right] + \\ &\quad + \frac{1}{2} \left[ m_2 \left( \mathbf{J}_{v2}^{(1)} \dot{q}_1 + \mathbf{J}_{v2}^{(2)} \dot{q}_2 \right)^2 + \mathcal{I}_2 \left( \mathbf{J}_{\omega_2}^{(1)} \dot{q}_1 + \mathbf{J}_{\omega_2}^{(2)} \dot{q}_2 \right)^2 \right] \\ &= \frac{1}{2} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^T \begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \end{aligned}$$

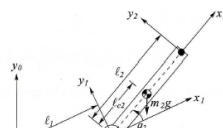
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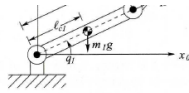
$$d_{11} = m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos q_2) + \mathcal{I}_1 + \mathcal{I}_2$$

$$d_{12} = m_2 (l_{c2}^2 + l_1 l_{c2} \cos q_2) + \mathcal{I}_2$$

$$d_{22} = m_2 l_{c2}^2 + \mathcal{I}_2$$

#### Potential Energy (PE) for Two-Link Elbow Manipulator





- PE of the 1st link is  $\mathcal{P}_1 = m_1 g y_{c_1} = m_1 g l_{c_1} \sin q_1$
- PE of the 2nd link is  $\mathcal{P}_2 = m_1 g y_{c_2} = m_2 g (l_1 \sin q_1 + l_{c_2} \sin(q_1 + q_2))$
- Total PE is  $\mathcal{P}_1 + \mathcal{P}_2$

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