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Question: 7-13 There is another formulation of the equations of motion of...



7-13 There is another formulation of the equations of motion of a mechanical system that is useful, the so-called **Hamiltonian** formulation. Define the Hamiltonian function H by

$$H = \sum_{k=1}^n \dot{q}_k p_k - L$$

- (a) Show that $H = K + V$.
 (b) Using the Euler-Lagrange equations, derive Hamilton's equations

$$\dot{q}_k = \frac{\partial H}{\partial p_k}$$

$$\dot{p}_k = -\frac{\partial H}{\partial q_k} + \tau_k$$

where τ_k is the input generalized force.

- (c) For two-link manipulator of Figure 7.8 compute Hamiltonian equations in matrix form. Note that Hamilton's equations are a system of first order differential equations as opposed to a second order system given by Lagrange's equations.

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Expert Answer ①

Anonymous answered this
6 answers

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③ $H = \sum_{k=1}^n \dot{q}_k p_k - L$

Again, Lagrangian of a system is defined by $L = K - V$

$$K = \sum_i \frac{1}{2} m_i \dot{q}_i^2 = \sum_i \frac{1}{2} (m_i \dot{q}_i) \dot{q}_i = \sum_i \frac{p_i \dot{q}_i}{2}$$

$$\Rightarrow \sum p_i \dot{q}_i = 2K$$

$$\Rightarrow H = 2K - K + V = K + V \quad \text{Proved}$$

④ $H = \sum_{k=1}^n \dot{q}_k p_k - L$

$$\frac{\partial H}{\partial p_k} = \sum \dot{q}_k - \frac{\partial L}{\partial p_k}$$

Since, L is only function of q & \dot{q} , $\frac{\partial L}{\partial p_k} = 0$

$$\Rightarrow \dot{q}_k = \frac{\partial H}{\partial p_k}$$

$$\frac{\partial H}{\partial q_k} = \sum \frac{\partial}{\partial q_k} \dot{q}_k p_k - \frac{\partial L}{\partial q_k}$$

Generalized input force

$$= -\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) + \frac{\partial}{\partial q_k} \sum \dot{q}_i p_i$$

From Euler-Lagrange eqn. $\frac{\partial L}{\partial \dot{q}_k} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right)$

$$\therefore \frac{\partial H}{\partial q_k} = 0$$

$$\Rightarrow \dot{p}_k = \frac{\partial H}{\partial q_k}$$

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