Question: 7-13 There is another formulation of the equations of motion of...

7-13 There is another formulation of the equations of motion of a mechanical system that is useful, the so-called **Hamiltonian** formulation. Define the Hamiltonian function H by

$$H = \sum_{k=1}^{n} \dot{q}_k p_k - L$$

(a) Show that $H = K + V$.

(b) Using the Euler-Lagrange equations, derive Hamilton's equations

$$\dot{q}_k = \frac{\partial H}{\partial p_k}$$

$$\dot{p}_k = -\frac{\partial H}{\partial q_k} + \tau_k$$

where τ_k is the input generalized force.

(c) For two-link manipulator of Figure 7.8 compute Hamiltonian equations in matrix form. Note that Hamilton's equations are a system of first order differential equations as opposed to a second order system given by Lagrange's equations.

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Expert Answer

Anonymous answered this
6 answers

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(a) $H = \sum_{k=1}^{n} \dot{q}_k p_k - L$

Again, Lagrangian of a system is defined by

$L = K - V$

$= \cancel{\frac{1}{2} m \dot{q}^2}$

$K = \sum_i \frac{1}{2} m_i \dot{q}_i^2$

$= \sum_i \frac{1}{2} (m_i \dot{q}_i) \dot{q}_i = \sum \frac{p_i \dot{q}_i}{2}$

$\Rightarrow \sum p_i \dot{q}_i = 2K$

$\Rightarrow H = 2K - K + V = K + V$ Proved

(b) $H = \sum_{k=1}^{n} \dot{q}_k p_k - L$

$\frac{\partial H}{\partial p_k} = \sum \dot{q}_k - \frac{\partial L}{\partial p_k}$

Since, L is only function of q & $\dot{q}$, $\frac{\partial L}{\partial p_k} = 0$

$\Rightarrow \dot{q}_k = \frac{\partial H}{\partial p_k}$

$\frac{\partial H}{\partial q_k} = \frac{\partial}{\partial q_k} \sum_{k=1}^{n} \dot{q}_k p_k - \frac{\partial L}{\partial q_k}$

$= -\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) + \underbrace{\frac{\partial}{\partial q_k} \sum_{i=1}^{n} \dot{q}_i p_k}_{\text{Generalized input force}}$

$= -\dot{p}_k + \tau_k$

[From Euler-Lagrange eqn. $\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$

$\because \frac{\partial H}{\partial \dot{q}} = 0$

$\Rightarrow p_k = \frac{\partial L}{\partial \dot{q}}$

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