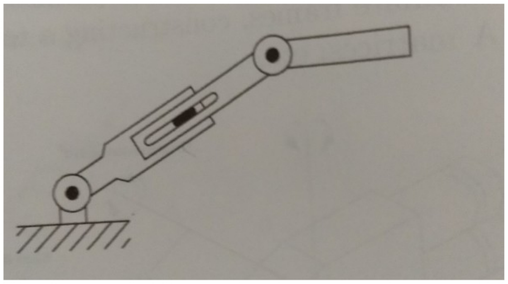
Question: Derive the Euler-Lagrange equations for the planar RP robot b...

Derive the Euler-Lagrange equations for the planar RP robot below



Expert Answer

Anonymous answered this
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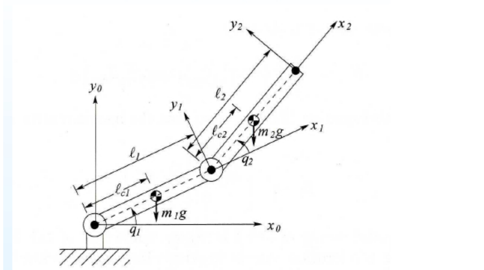
The planar RP robot consists of a prismatic joint axed to the end of a link that is attached to a rotational joint. The only challenge here, other than the tediousness of



Euler-Lagrange Equations for 2-Link Cartesian Manipulator

Given the kinetic $\mathcal{K}$ and potential $\mathcal{P}$ energies, the dynamics are

$$\frac{d}{dt}\left[\frac{\partial(\mathcal{K}-\mathcal{P})}{\partial \dot{q}}\right] - \frac{\partial(\mathcal{K}-\mathcal{P})}{\partial q} = \tau$$

With kinetic and potential energies

$$\mathcal{K} = \frac{1}{2}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}^T \begin{bmatrix} m_1+m_2 & 0 \\ 0 & m_2\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}, \quad \mathcal{P} = g\,(m_1+m_2)\,q_1 + C$$

the Euler-Lagrange equations are

$$\begin{aligned}(m_1+m_2)\ddot{q}_1 + g(m_1+m_2) &= \tau_1 \\ m_2\ddot{q}_2 &= \tau_2\end{aligned}$$

Forward Kinematics and Jacobian

DH parameters for computing homogeneous transformations

$$T(q_i) = \text{Rot}_{z,\theta}\cdot\text{Trans}_{z,d}\cdot\text{Trans}_{x,a}\cdot\text{Rot}_{x,\alpha}$$

are

$$\begin{aligned} T_1^0: &\quad \theta = q_1, \quad d = 0, \quad a = l_1, \quad \alpha = 0 \\ T_2^1: &\quad \theta = q_2, \quad d = 0, \quad a = l_2, \quad \alpha = 0 \end{aligned}$$

The kinetic energy of the system is

$$\mathcal{K} = \frac{1}{2}\left[m_1 v_{c1}^2 + \omega_1^T \mathcal{I}_1 \omega_1\right] + \frac{1}{2}\left[m_2 v_{c2}^2 + \omega_2^T \mathcal{I}_2 \omega_2\right]$$

and

$$\begin{aligned} v_{c1} &= \left[J_{v1}^{(1)}\; J_{v1}^{(2)}\right]\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = J_{v1}^{(1)}\dot{q}_1 + J_{v1}^{(2)}\dot{q}_2 \\ v_{c2} &= \left[J_{v2}^{(1)}\; J_{v2}^{(2)}\right]\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = J_{v2}^{(1)}\dot{q}_1 + J_{v2}^{(2)}\dot{q}_2 \end{aligned}$$

DH parameters for computing homogeneous transformations

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To compute the Jacobian we can use the DH-frames, i.e

$$J_v^{(i)} = \begin{cases} z_{i-1}^0, & \text{for prismatic joint} \\ z_{i-1}^0 \times \left[o_c^0 - o_{i-1}^0\right], & \text{for revolute joint}\end{cases}$$

$$J_\omega^{(i)} = \begin{cases} 0, & \text{for prismatic joint} \\ z_{i-1}^0, & \text{for revolute joint}\end{cases}$$

The formula

$$J_v^{(i)} = \begin{cases} z_{i-1}^0, & \text{for prismatic joint} \\ z_{i-1}^0 \times \left[o_c^0 - o_{i-1}^0\right], & \text{for revolute joint} \end{cases}$$

gives

$$J_{v1}^{(1)} = \vec{z}_0 \times (\vec{o}_{c_1} - \vec{o}_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c_1}\cos q_1 \\ l_{c_1}\sin q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_{c_1}\sin q_1 \\ l_{c_1}\cos q_1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} J_{v2}^{(1)} &= \vec{z}_0 \times (\vec{o}_{c2} - \vec{o}_0) \\ &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(\begin{bmatrix} l_1\cos q_1 \\ l_1\sin q_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_{c_2}\cos(q_1+q_2) \\ l_{c_2}\sin(q_1+q_2) \\ 0 \end{bmatrix} \right) \end{aligned}$$

The formula

$$J_v^{(i)} = \begin{cases} z_{i-1}^0, & \text{for prismatic joint} \\ z_{i-1}^0 \times \left[o_c^0 - o_{i-1}^0\right], & \text{for revolute joint} \end{cases}$$

gives

$$J_{v1}^{(1)} = \vec{z}_0 \times (\vec{o}_{c_1} - \vec{o}_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c_1}\cos q_1 \\ l_{c_1}\sin q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_{c_1}\sin q_1 \\ l_{c_1}\cos q_1 \\ 0 \end{bmatrix}$$

$$J_{v2}^{(1)} = \vec{z}_0 \times (\vec{o}_{c2} - \vec{o}_0) = \begin{bmatrix} -l_1\sin q_1 - l_{c_2}\sin(q_1+q_2) \\ l_1\cos q_1 + l_{c_2}\cos(q_1+q_2) \\ 0 \end{bmatrix}$$

$$J_{v2}^{(2)} = \vec{z}_1 \times (\vec{o}_{c2} - \vec{o}_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c_2}\cos(q_1+q_2) \\ l_{c_2}\sin(q_1+q_2) \\ 0 \end{bmatrix}$$

The formula

$$J_v^{(i)} = \begin{cases} z_{i-1}^0, & \text{for prismatic joint} \\ z_{i-1}^0 \times \left[o_c^0 - o_{i-1}^0\right], & \text{for revolute joint} \end{cases}$$

gives

$$J_{v1}^{(1)} = \vec{z}_0 \times (\vec{o}_{c_1} - \vec{o}_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_{c_1}\cos q_1 \\ l_{c_1}\sin q_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_{c_1}\sin q_1 \\ l_{c_1}\cos q_1 \\ 0 \end{bmatrix}$$

$$J_{v2}^{(1)} = \vec{z}_0 \times (\vec{o}_{c2} - \vec{o}_0) = \begin{bmatrix} -l_1\sin q_1 - l_{c_2}\sin(q_1+q_2) \\ l_1\cos q_1 + l_{c_2}\cos(q_1+q_2) \\ 0 \end{bmatrix}$$

$$J_{v2}^{(2)} = \vec{z}_1 \times (\vec{o}_{c2} - \vec{o}_1) = \begin{bmatrix} -l_{c_2}\sin(q_1+q_2) \\ l_{c_2}\cos(q_1+q_2) \\ 0 \end{bmatrix}$$

Forward Kinematics and Jacobian (Cont'd)

The formula

$$J_\omega^{(i)} = \begin{cases} 0, & \text{for prismatic joint} \\ z_{i-1}^0, & \text{for revolute joint} \end{cases}$$

gives

$$J_{\omega_1}^{(1)} = \vec{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{\omega_2}^{(1)} = \vec{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{\omega_2}^{(2)} = \vec{z}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

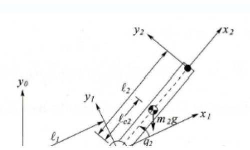
To sum up, the kinetic energy $\mathcal{K}$ is

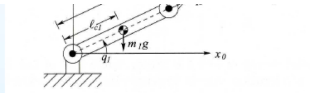
$$\begin{aligned} \mathcal{K} &= \tfrac{1}{2}\left[m_1 v_{c1}^2 + \omega_1^T \mathcal{I}_1 \omega_1\right] + \tfrac{1}{2}\left[m_2 v_{c2}^2 + \omega_2^T \mathcal{I}_2 \omega_2\right] \\ &= \tfrac{1}{2}\left[m_1\left(J_{v_1}^{(1)}\dot{q}_1\right)^2 + I_1\left(J_{\omega_1}^{(1)}\dot{q}_1\right)^2\right] + \\ &\quad + \tfrac{1}{2}\left[m_2\left(J_{v_2}^{(1)}\dot{q}_1 + J_{v_2}^{(2)}\dot{q}_2\right)^2 + I_2\left(J_{\omega_2}^{(1)}\dot{q}_1 + J_{\omega_2}^{(2)}\dot{q}_2\right)^2\right] \\ &= \tfrac{1}{2}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^T \begin{bmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \end{aligned}$$

with

$$\begin{aligned} d_{11} &= m_1 l_{c_1}^2 + m_2\left(l_1^2 + l_{c_2}^2 + 2 l_1 l_{c_2}\cos q_2\right) + I_1 + I_2 \\ d_{12} &= m_2\left(l_{c_2}^2 + l_1 l_{c_2}\cos q_2\right) + I_2 \\ d_{22} &= m_2 l_{c_2}^2 + I_2 \end{aligned}$$

Potential Energy (PE) for Two-Link Elbow Manipulator





- PE of the 1st link is $\mathcal{P}_1 = m_1 g y_{c_1} = m_1 g l_{c_1} \sin q_1$
- PE of the 2nd link is $\mathcal{P}_2 = m_1 g y_{c_2} = m_2 g \left(l_1 \sin q_1 + l_{c_2} \sin(q_1 + q_2)\right)$
- Total PE is $\mathcal{P}_1 + \mathcal{P}_2$

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