

Intensity-based Registration



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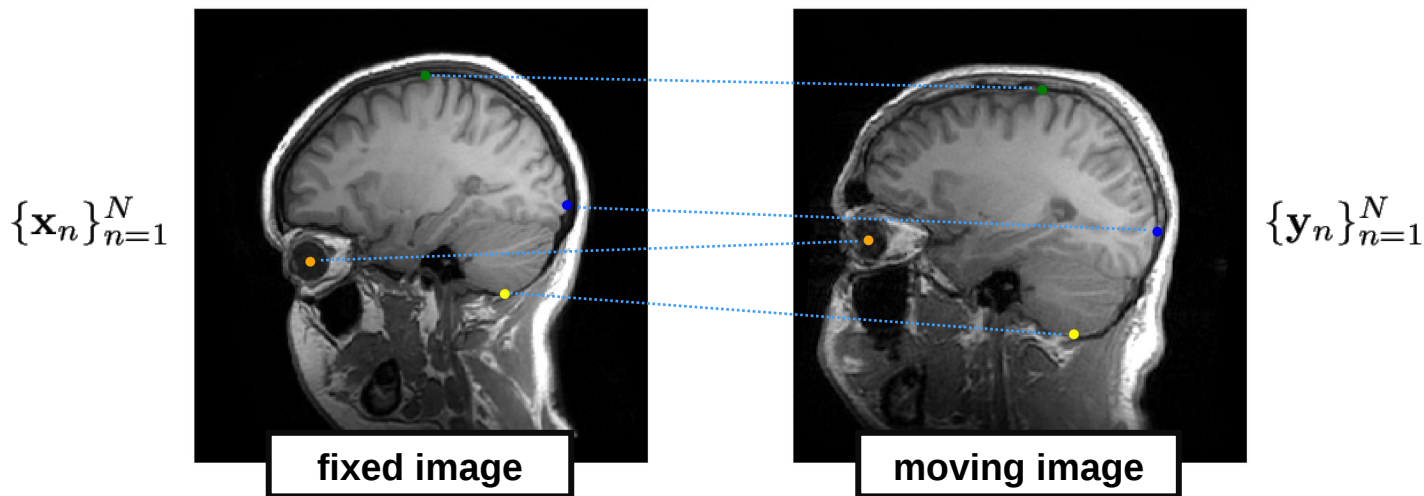
Medical Image Analysis

Koen Van Leemput

Fall 2023

Recall landmark-based registration

- ✓ Manually annotate N corresponding points in two images:



- ✓ Register the images by minimizing the distance between matching point pairs:

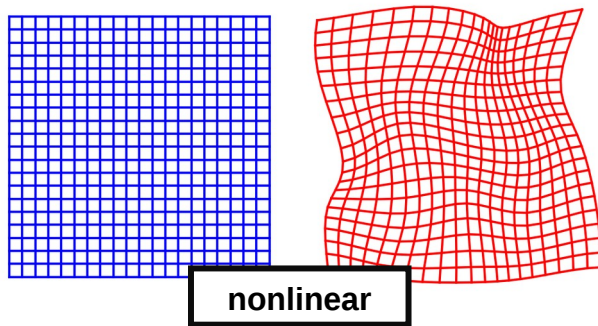
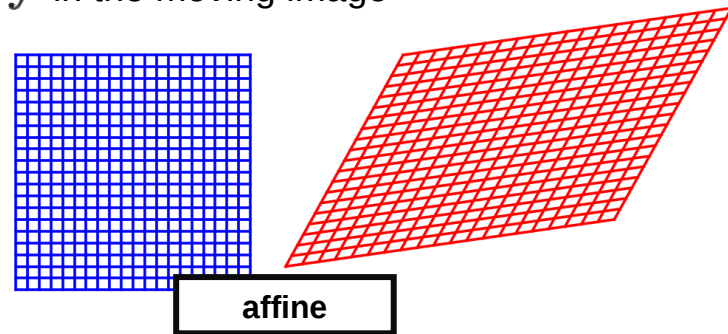
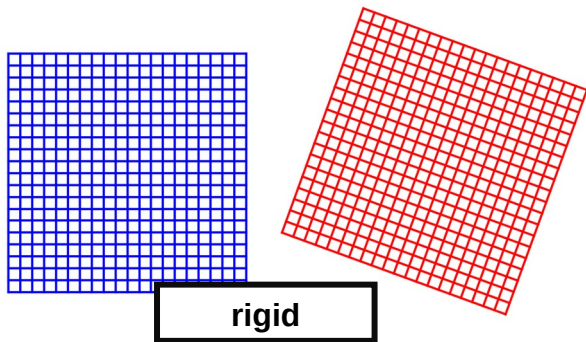
$$E(\mathbf{w}) = \sum_{n=1}^N \|\mathbf{y}_n - \mathbf{y}(\mathbf{x}_n, \mathbf{w})\|^2$$

Spatial transformation
model

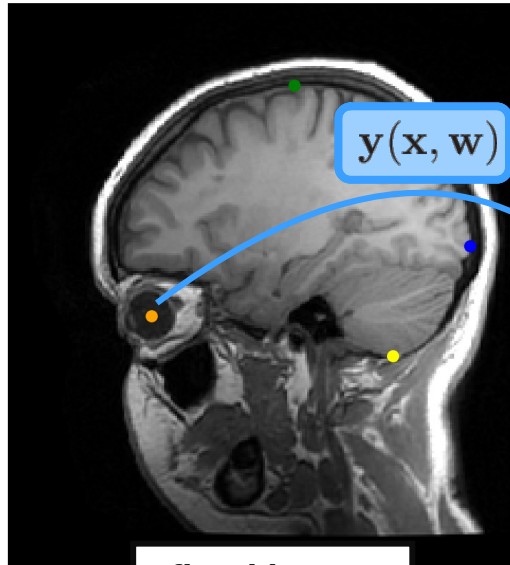
Spatial transformation models

Spatial transformation $\mathbf{y}(\mathbf{x}, \mathbf{w})$:

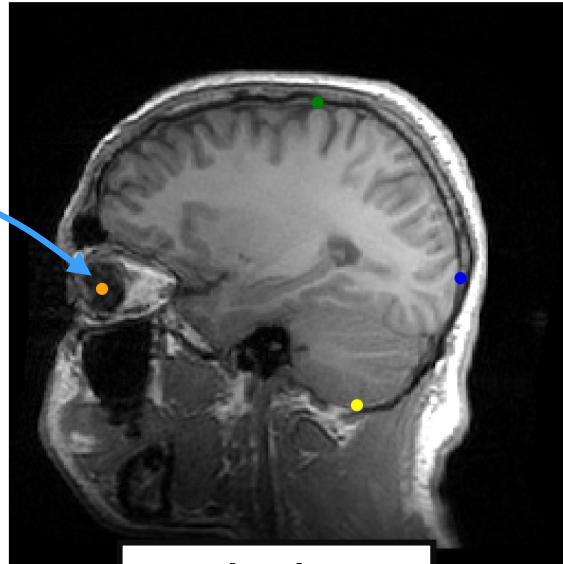
- ✓ maps world positions \mathbf{x} in the fixed image to world positions \mathbf{y} in the moving image
- ✓ controlled by parameters \mathbf{w}



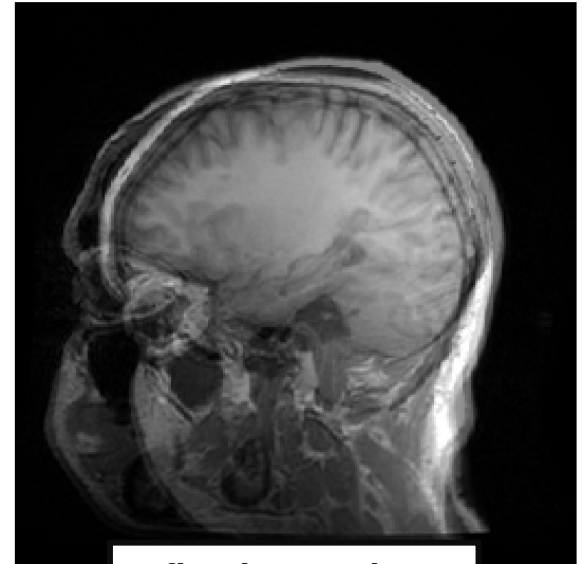
Landmark-based registration



fixed image
 $\mathcal{F}(x)$

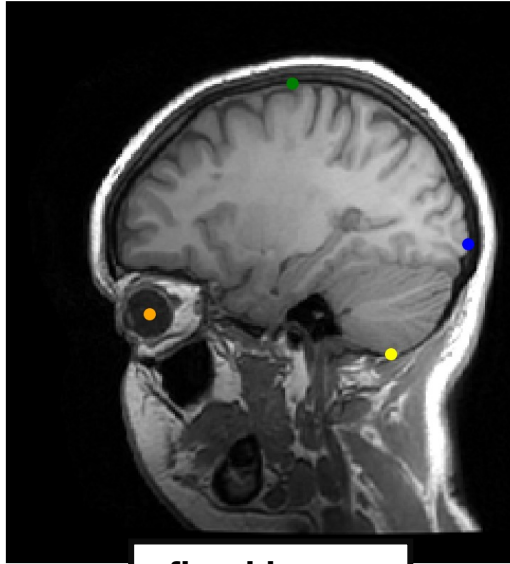


moving image
 $\mathcal{M}(y)$

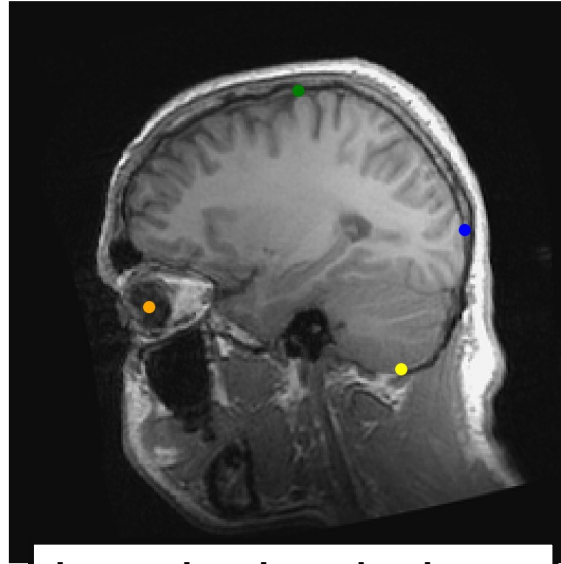


fixed + moving

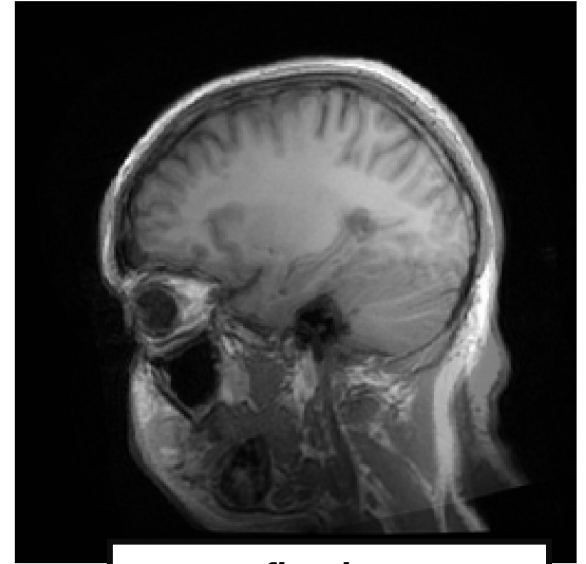
Landmark-based registration



fixed image
 $\mathcal{F}(\mathbf{x})$



interpolated moving image
 $\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



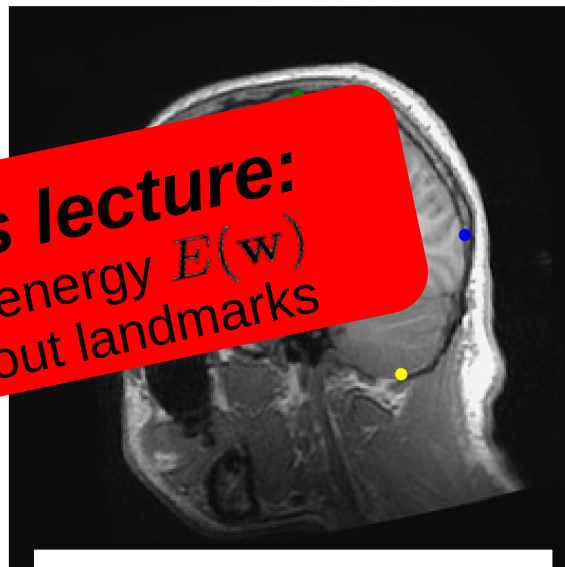
fixed +
interpolated moving

After registration

Landmark-based registration

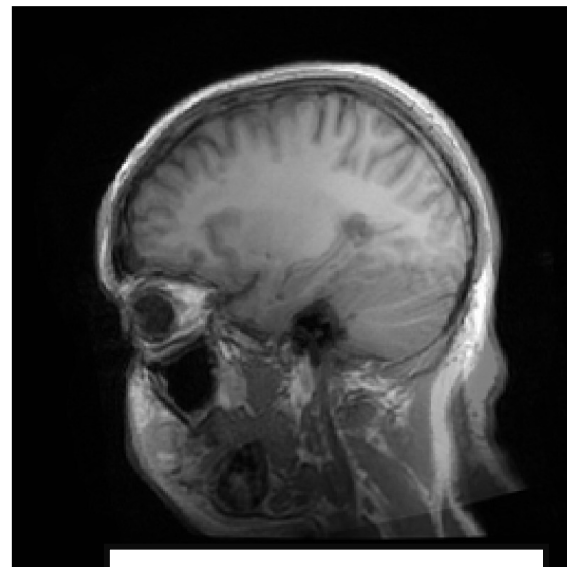


fixed image
 $\mathcal{F}(\mathbf{x})$



This lecture:
define energy $E(\mathbf{w})$
without landmarks

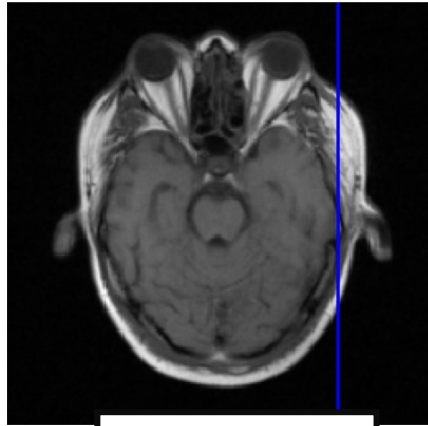
interpolated moving image
 $\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



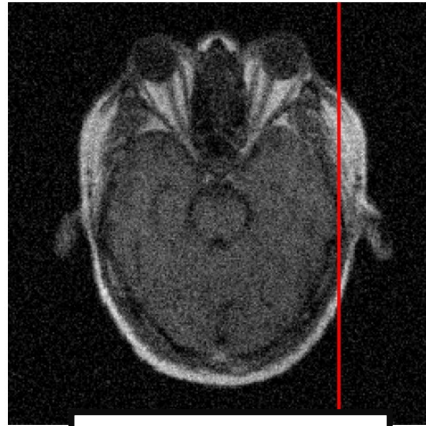
fixed +
interpolated moving

Intra-modal registration

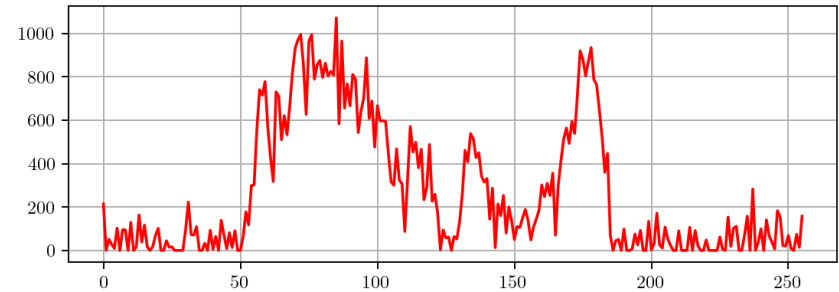
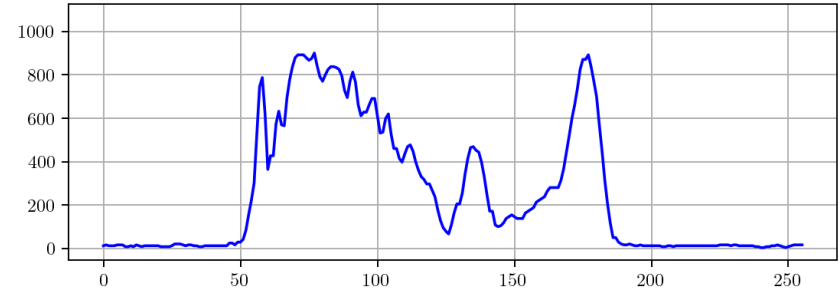
Images have similar intensity characteristics



$\mathcal{F}(\mathbf{x})$



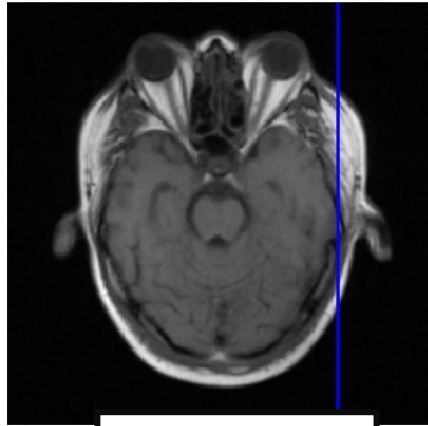
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



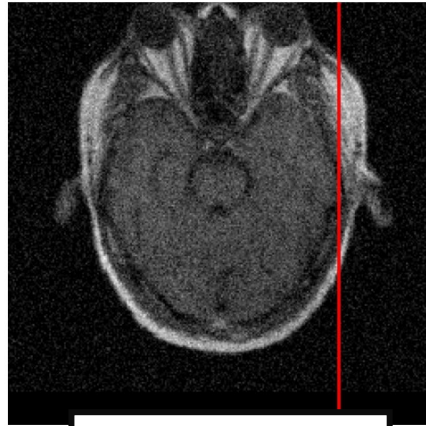
Task: what's a good energy function $E(\mathbf{w})$?

Intra-modal registration

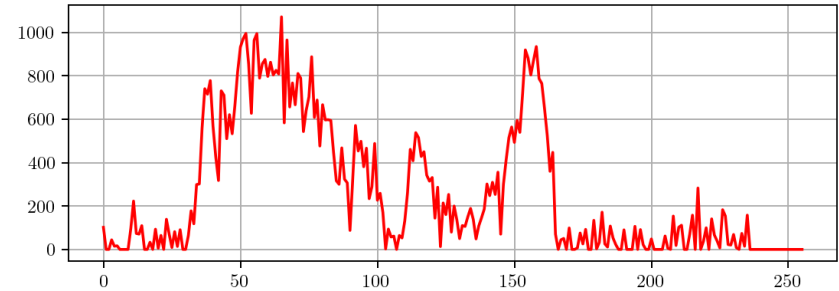
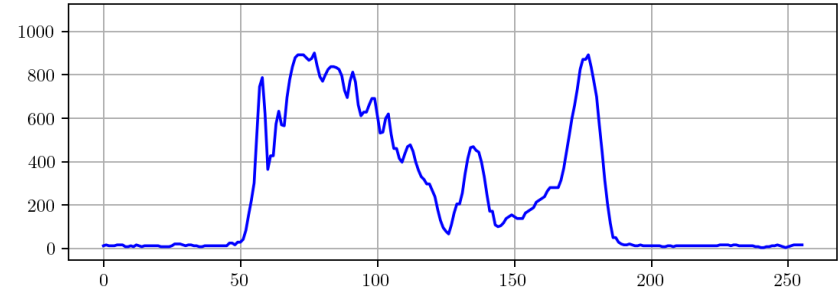
Images have similar intensity characteristics



$\mathcal{F}(\mathbf{x})$



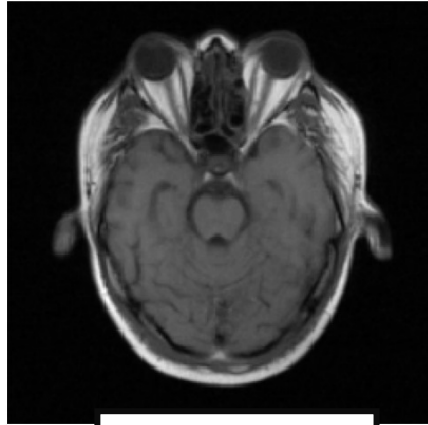
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



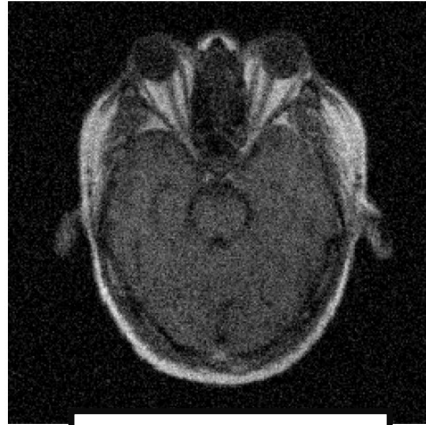
Task: what's a good energy function $E(\mathbf{w})$?

Intra-modal registration

Images have similar intensity characteristics



$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

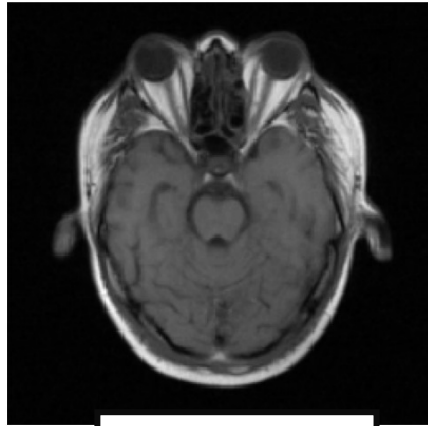


$[\mathcal{F}(\mathbf{x}) - \mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))]^2$

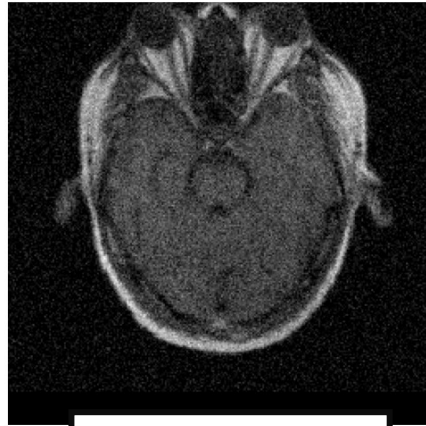
$$E(\mathbf{w}) = \sum_{n=1}^N [\mathcal{F}(\mathbf{x}_n) - \mathcal{M}(\mathbf{y}(\mathbf{x}_n, \mathbf{w}))]^2$$

Intra-modal registration

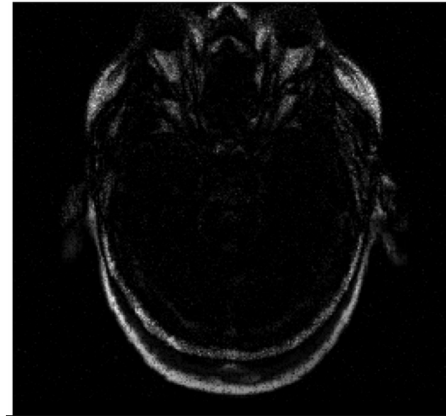
Images have similar intensity characteristics



$\mathcal{F}(\mathbf{x})$



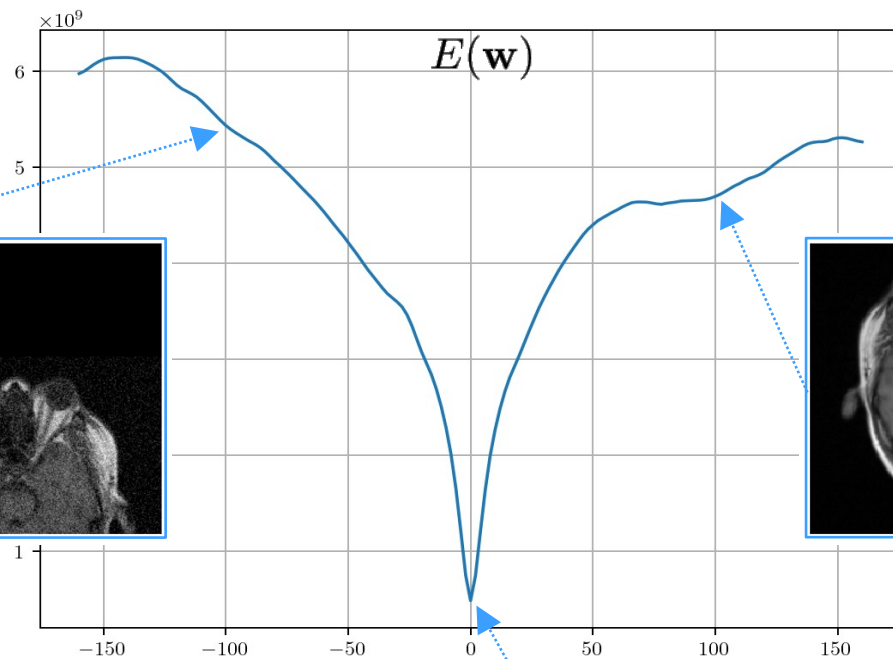
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



$[\mathcal{F}(\mathbf{x}) - \mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))]^2$

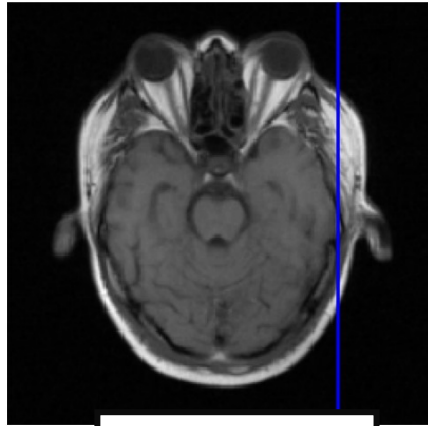
$$E(\mathbf{w}) = \sum_{n=1}^N [\mathcal{F}(\mathbf{x}_n) - \mathcal{M}(\mathbf{y}(\mathbf{x}_n, \mathbf{w}))]^2$$

sum over all voxels

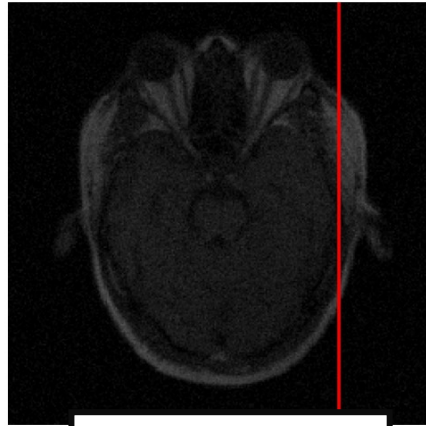


Intra-modal registration

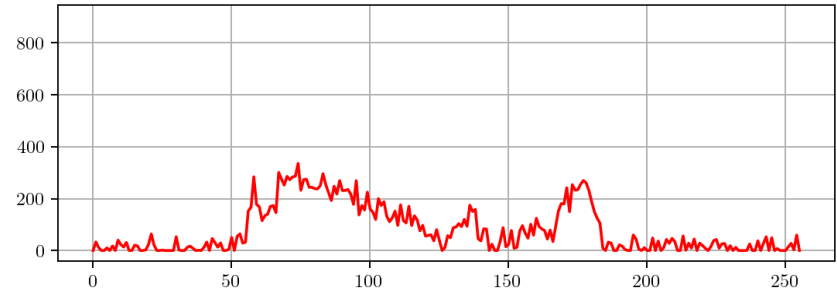
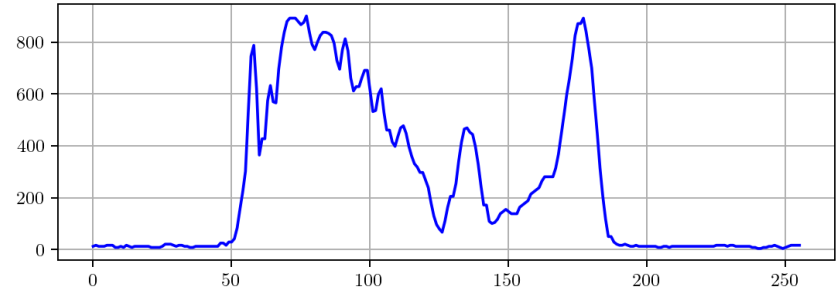
Same but images are scaled differently



$\mathcal{F}(\mathbf{x})$



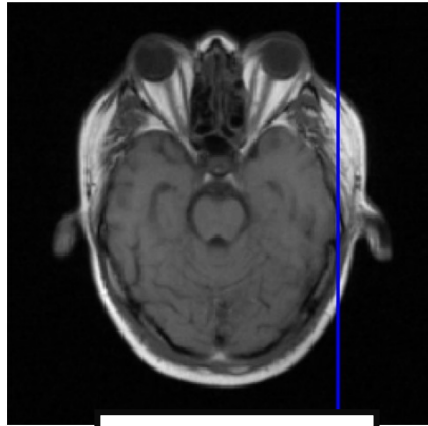
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



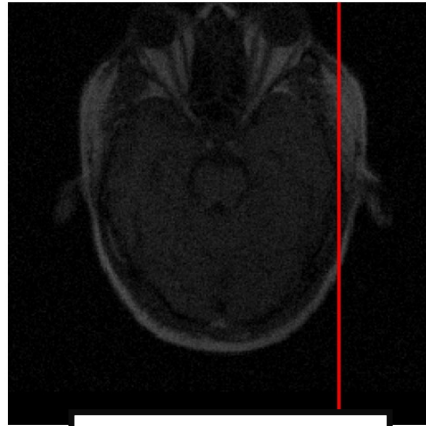
Task: what's a good energy function $E(\mathbf{w})$?

Intra-modal registration

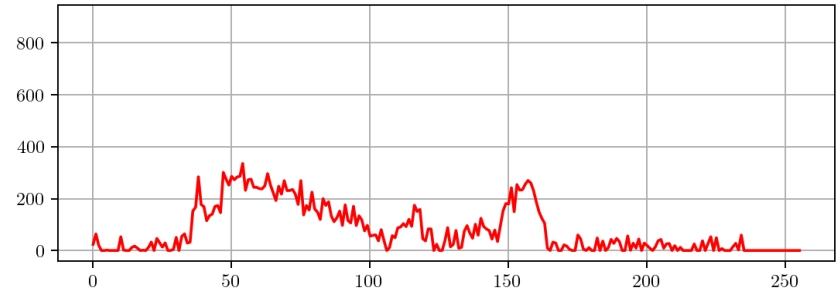
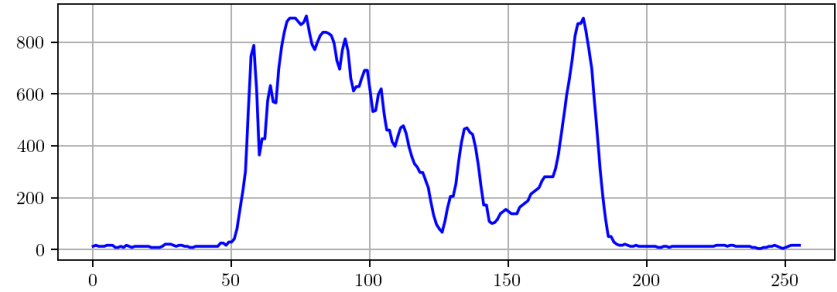
Same but images are scaled differently



$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



Task: what's a good energy function $E(\mathbf{w})$?

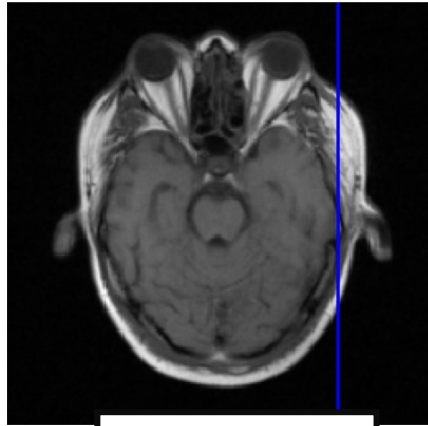
Intra-modal registration

Same but images are scaled differently

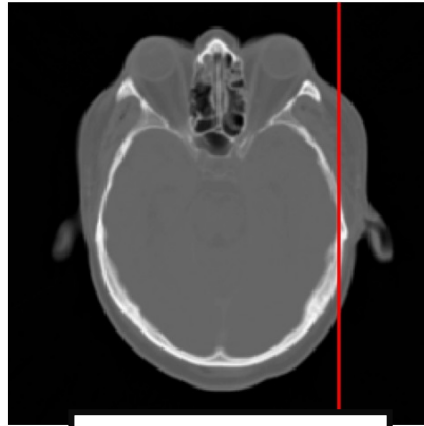
TODO: robust min/max histogram from cumulative histogram

Inter-modal registration

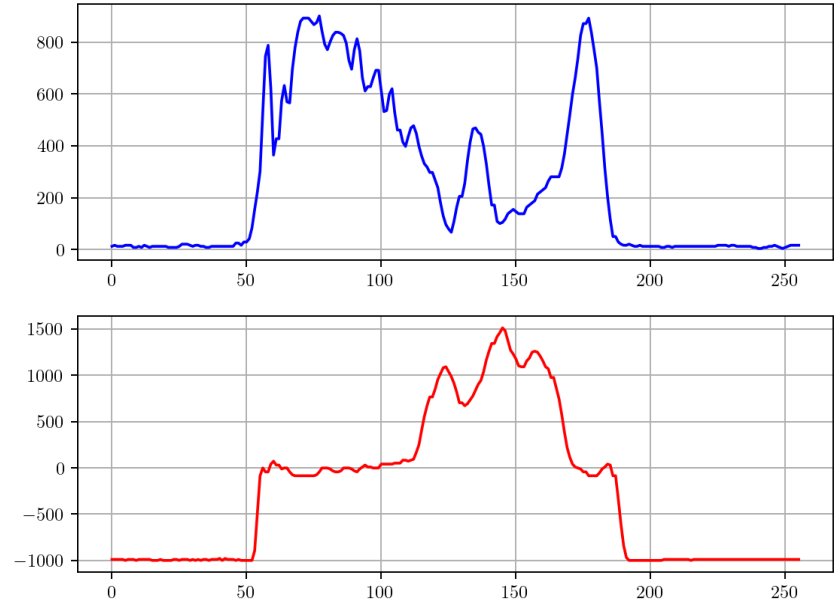
Images have different intensity characteristics



$\mathcal{F}(\mathbf{x})$



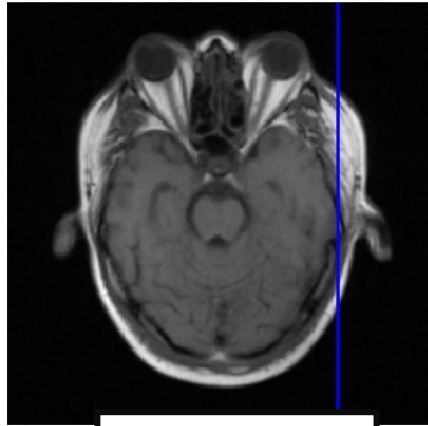
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



Task: what's a good energy function $E(\mathbf{w})$?

Inter-modal registration

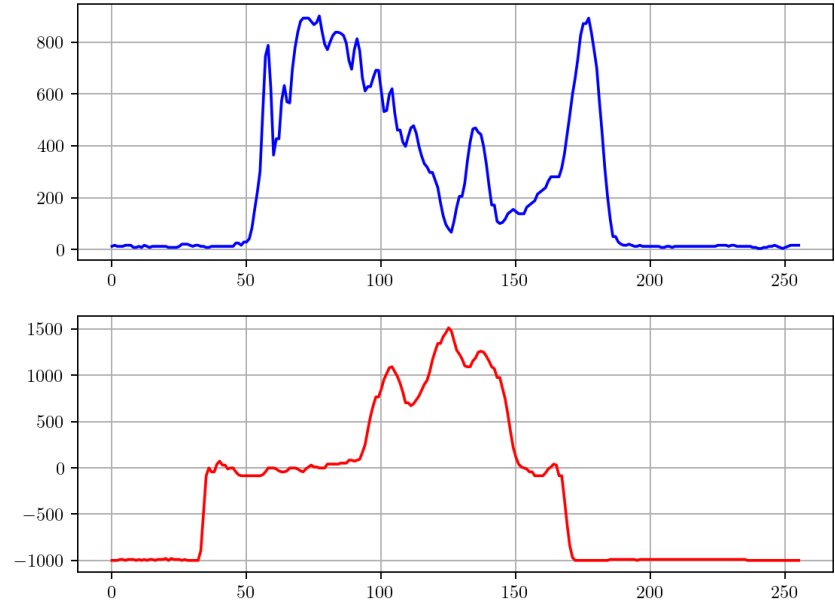
Images have different intensity characteristics



$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



Task: what's a good energy function $E(\mathbf{w})$?

Inter-modal registration

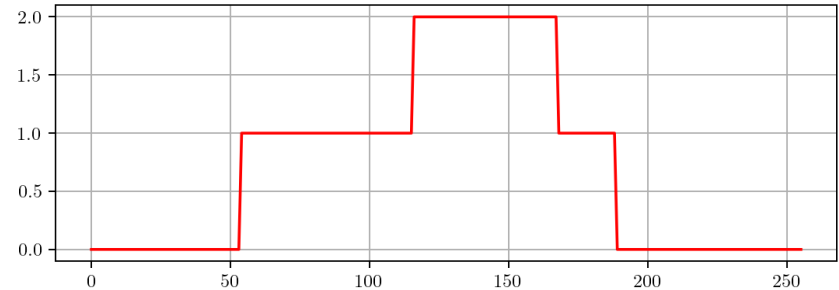
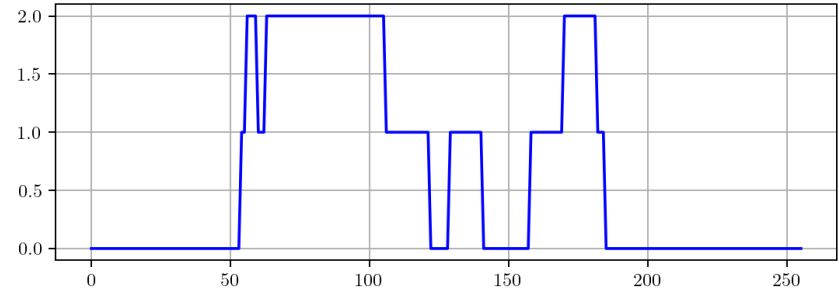
Images have different intensity characteristics



$\mathcal{F}(x)$



$\mathcal{M}(y(x, w))$



Easier task: what's a good energy function $E(w)$ now?

Inter-modal registration

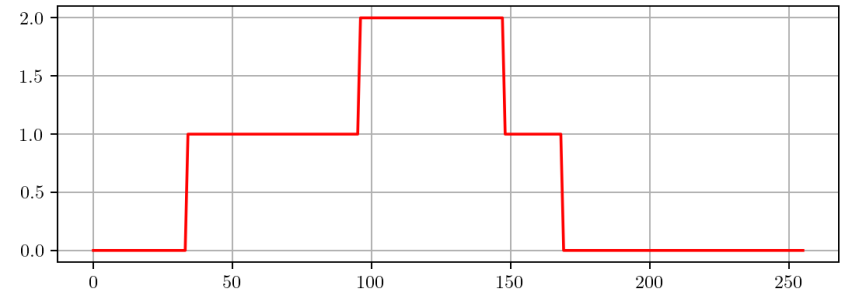
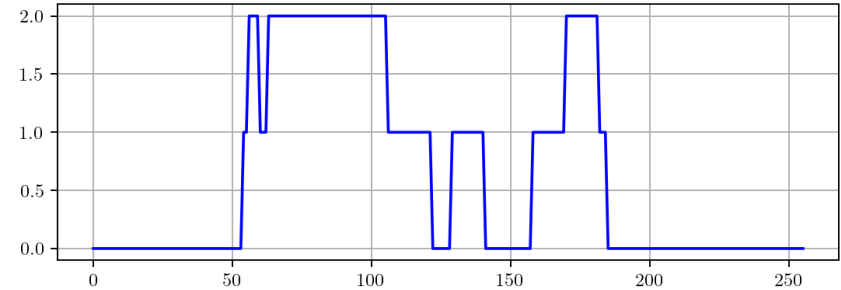
Images have different intensity characteristics



$\mathcal{F}(\mathbf{x})$



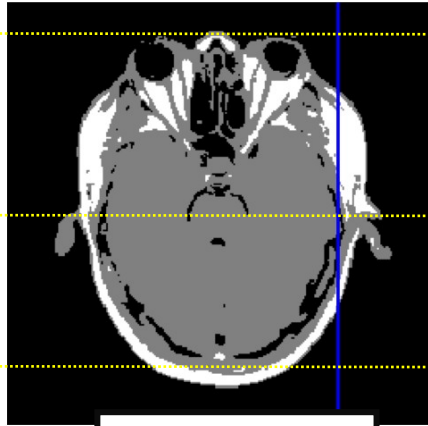
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



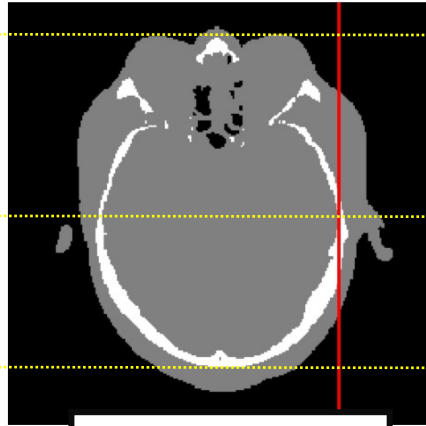
Easier task: what's a good energy function $E(\mathbf{w})$ now?

Inter-modal registration

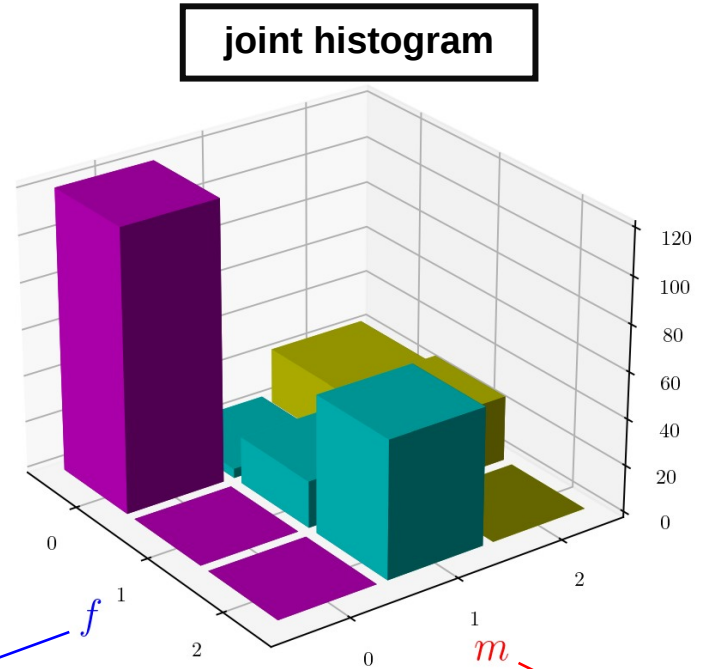
Images have different intensity characteristics



$\mathcal{F}(x)$



$\mathcal{M}(y(x, w))$

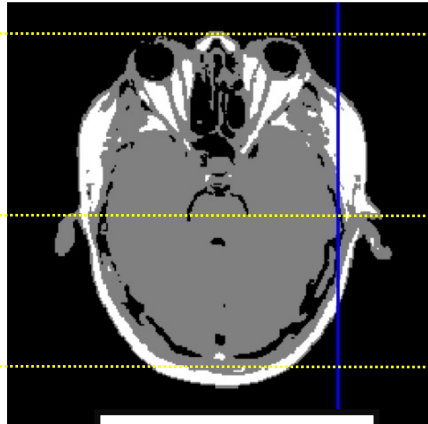


intensity in
fixed image

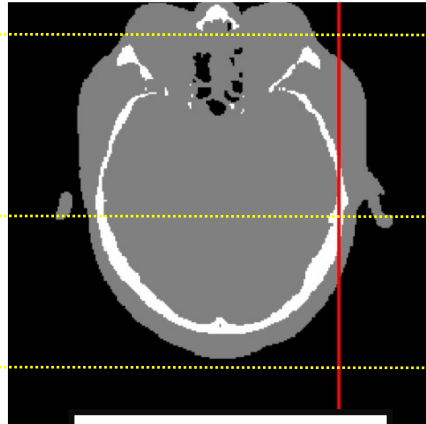
intensity in
moving image

Inter-modal registration

Images have different intensity characteristics

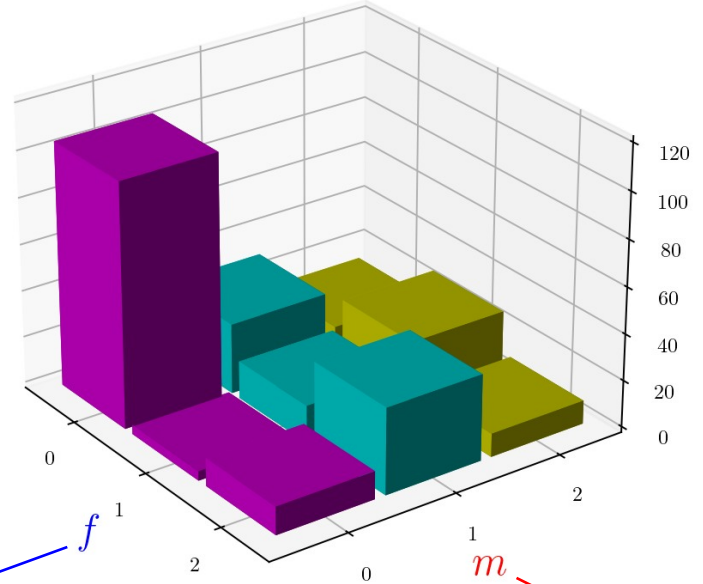


$\mathcal{F}(x)$



$\mathcal{M}(y(x, w))$

joint histogram

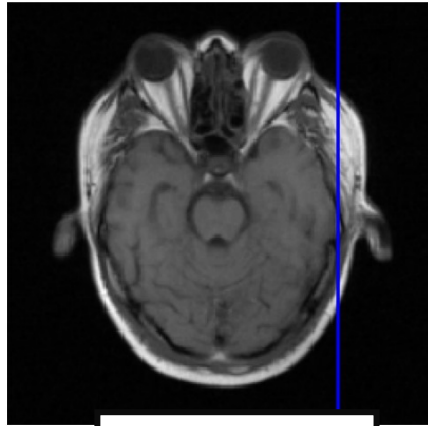


intensity in
fixed image

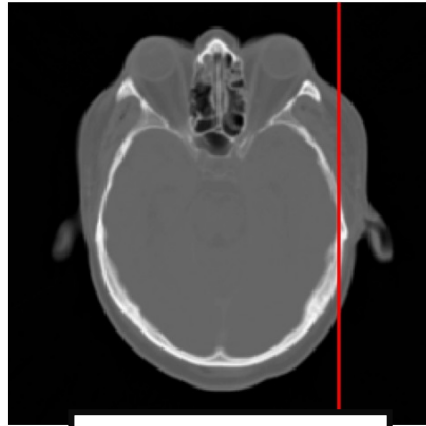
intensity in
moving image

Inter-modal registration

Images have different intensity characteristics

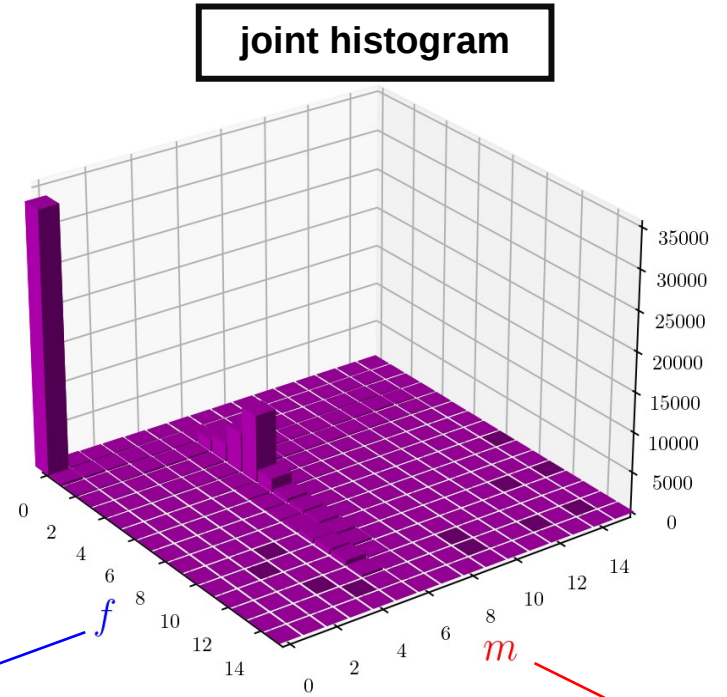


$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

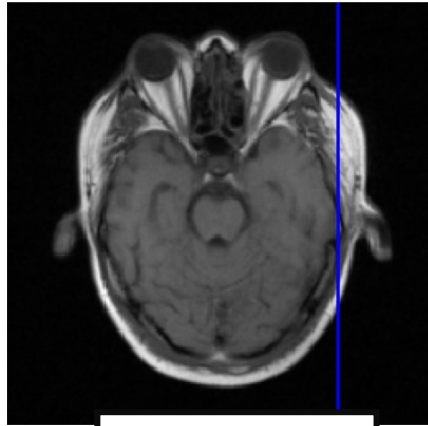
intensity in
fixed image



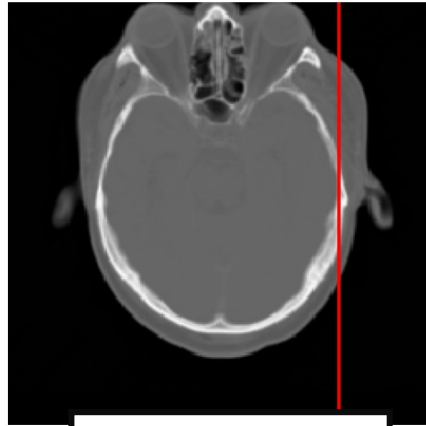
intensity in
moving image

Inter-modal registration

Images have different intensity characteristics

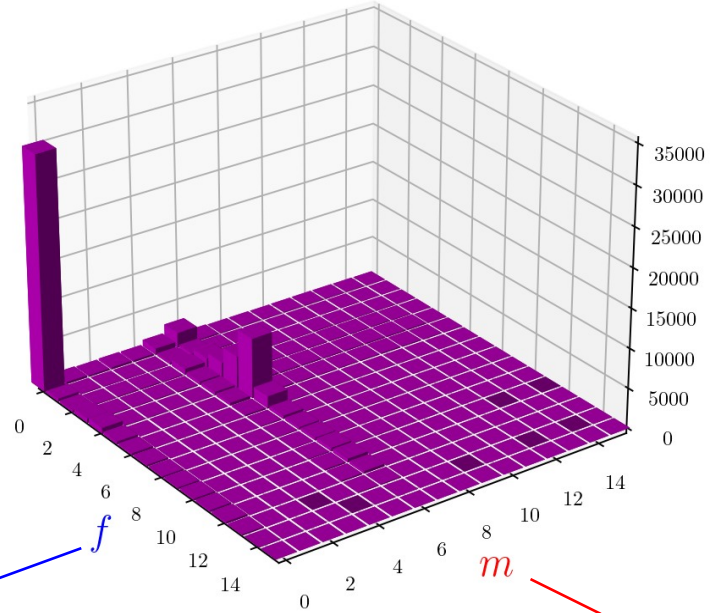


$\mathcal{F}(x)$



$\mathcal{M}(y(x, w))$

joint histogram



intensity in
fixed image

intensity in
moving image

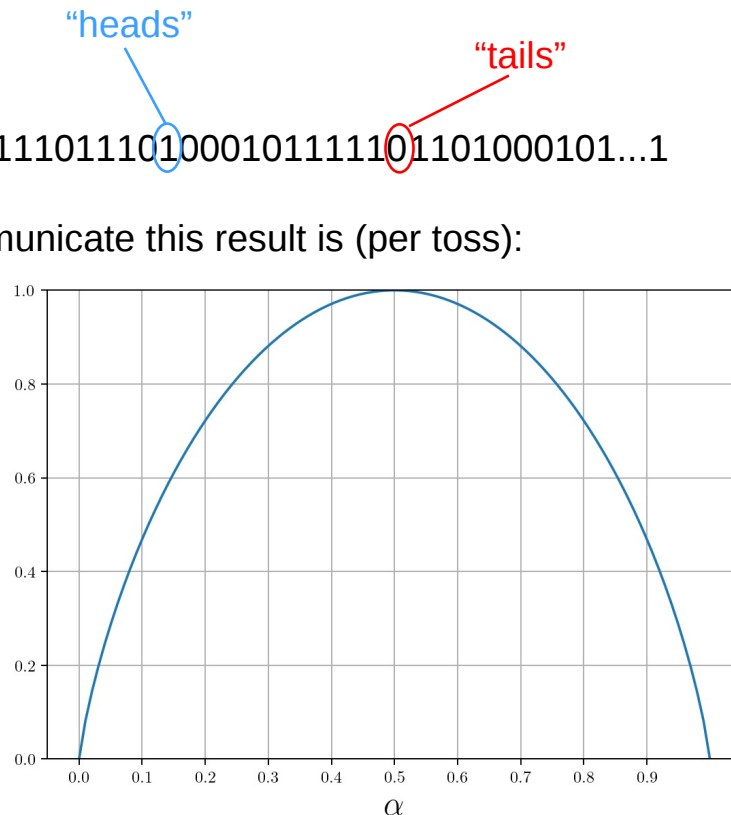
A bit of information theory...

Imagine that a coin is “rigged”:

- ✓ lands on heads with probability $0 \leq \alpha \leq 1$
- ✓ I toss it many times, and the result is 11010001011111011101000101111011101000101...1
- ✓ The minimum number of bits required to store/communicate this result is (per toss):

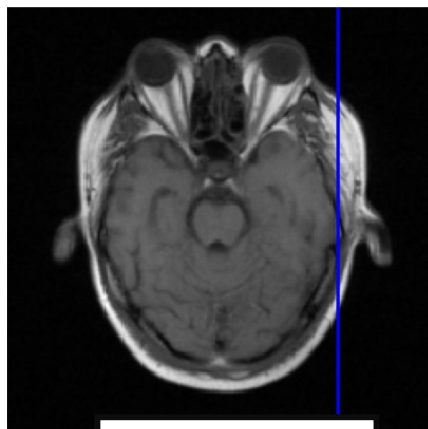
$$-\alpha \log_2(\alpha) - (1 - \alpha) \log_2(1 - \alpha)$$

“entropy”

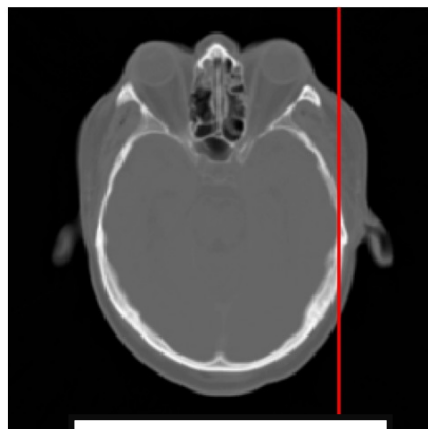


Inter-modal registration

Images have different intensity characteristics

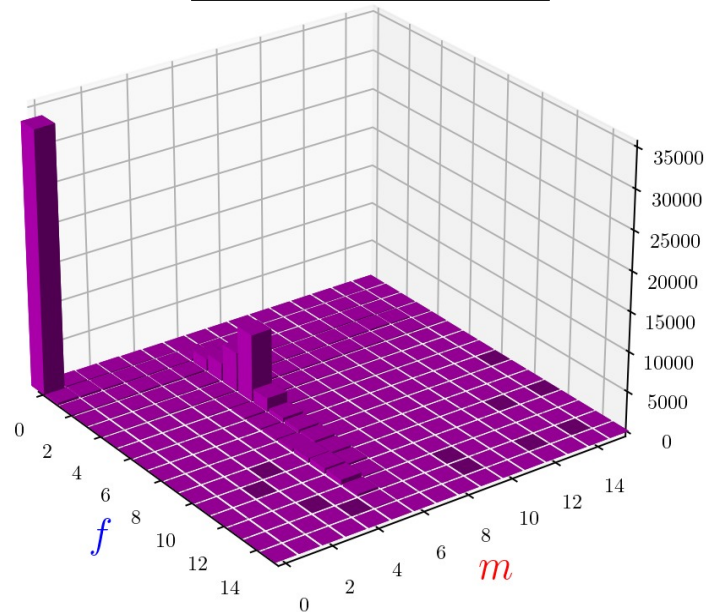


$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram

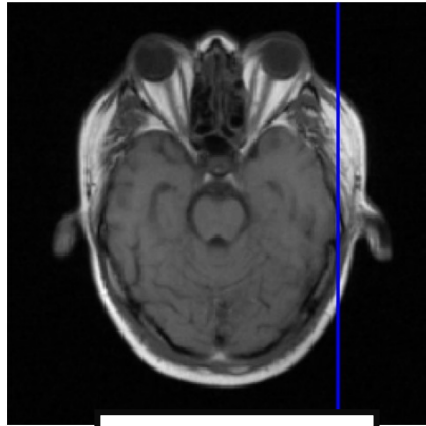


$$E(\mathbf{w}) = H_{F,M} \quad \text{where} \quad H_{F,M} = - \sum_{f=1}^B \sum_{m=1}^B p_{f,m} \log(p_{f,m})$$

normalized
histogram counts

Inter-modal registration

Images have different intensity characteristics

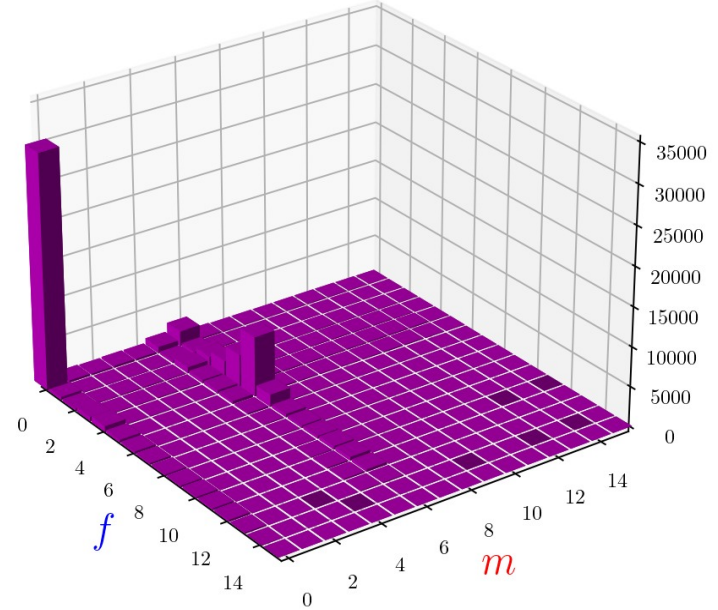


$\mathcal{F}(\mathbf{x})$



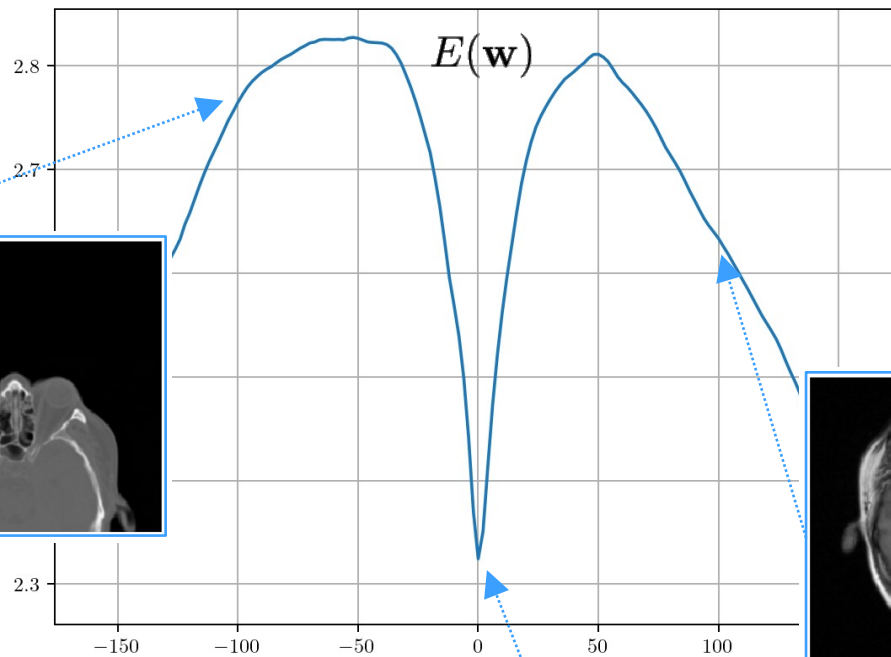
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram



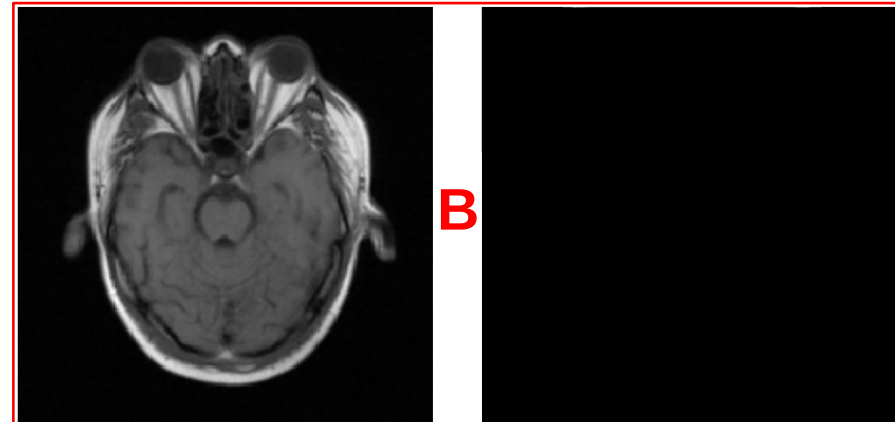
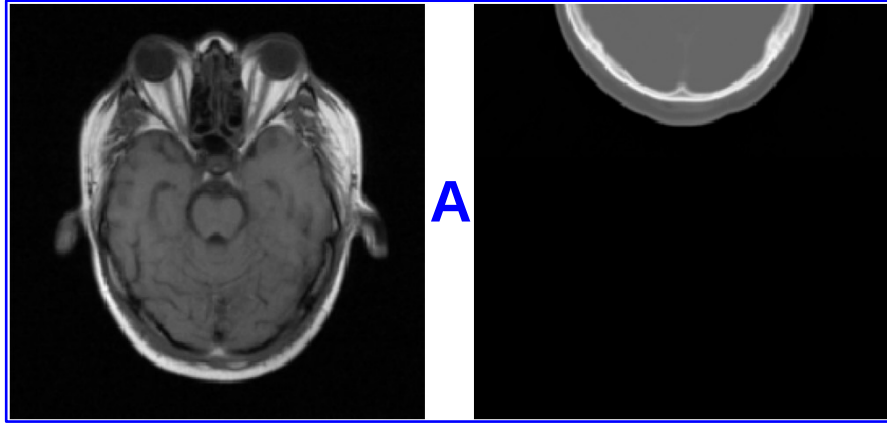
$$E(\mathbf{w}) = H_{F,M} \quad \text{where} \quad H_{F,M} = - \sum_{f=1}^B \sum_{m=1}^B p_{f,m} \log(p_{f,m})$$

normalized
histogram counts



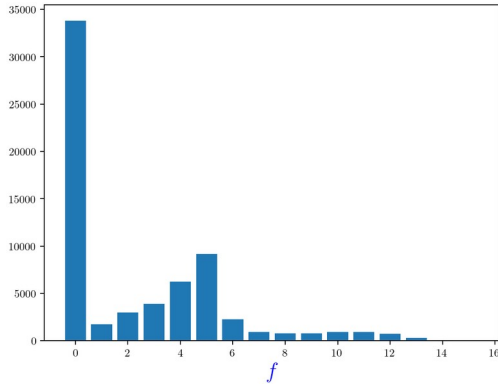
Diagnosing the problem

Question: which image pair takes more bits to encode?



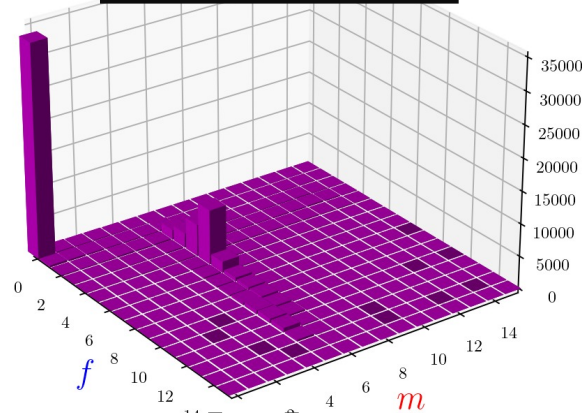
Solution

histogram fixed image



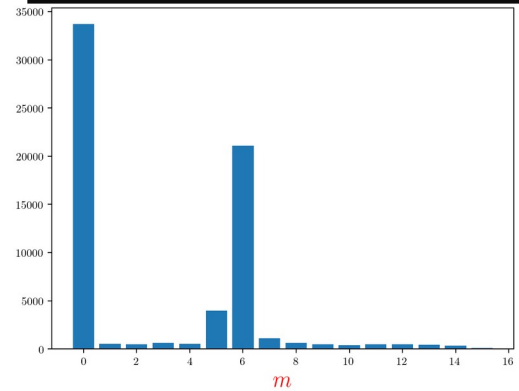
$$H_F = - \sum_{f=1}^B p_f \log(p_f)$$

joint histogram

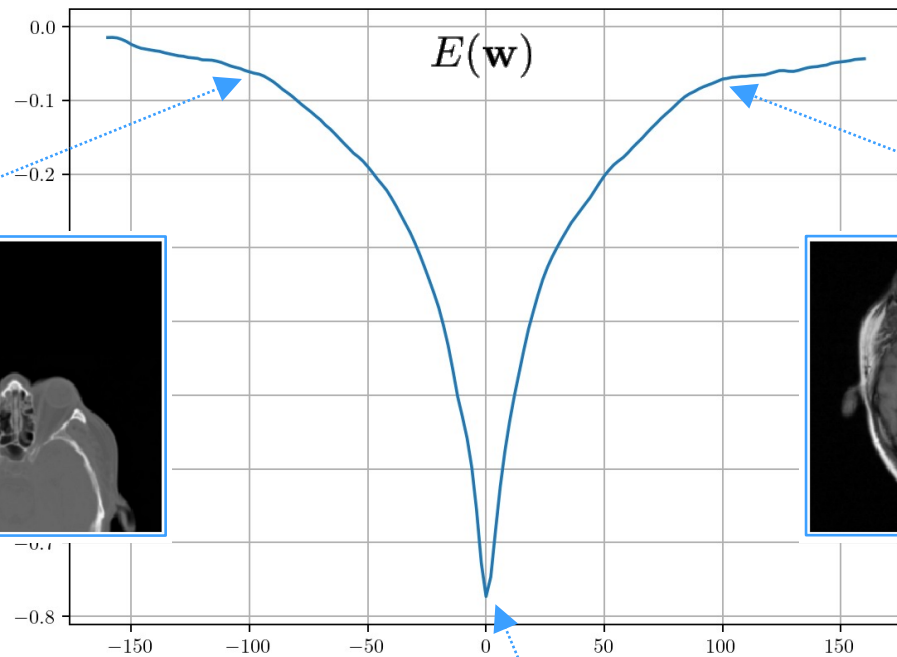


$$H_{F,M} = - \sum_{f=1}^B \sum_{m=1}^B p_{f,m} \log(p_{f,m})$$

histogram moving image



$$H_M = - \sum_{m=1}^B p_m \log(p_m)$$



Numerical optimization

Find transformation parameters \mathbf{w} that minimize $E(\mathbf{w})$

