Intensity-based Registration

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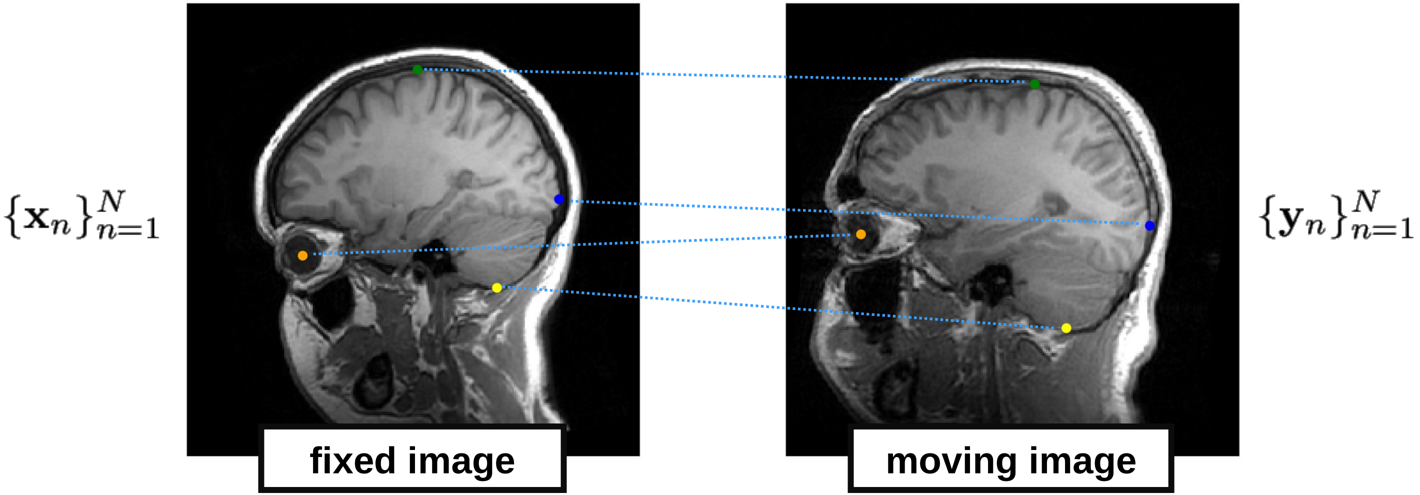
Medical Image Analysis

Koen Van Leemput

Fall 2023

Recall landmark-based registration

- ✔ Manually annotate N corresponding points in two images:



- ✔ Register the images by minimizing the distance between matching point pairs:

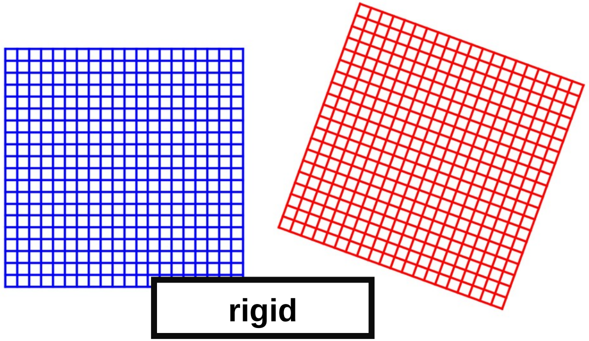
$$E(\mathbf{w}) = \sum_{n=1}^{N} \|\mathbf{y}_n - \mathbf{y}(\mathbf{x}_n, \mathbf{w})\|^2$$

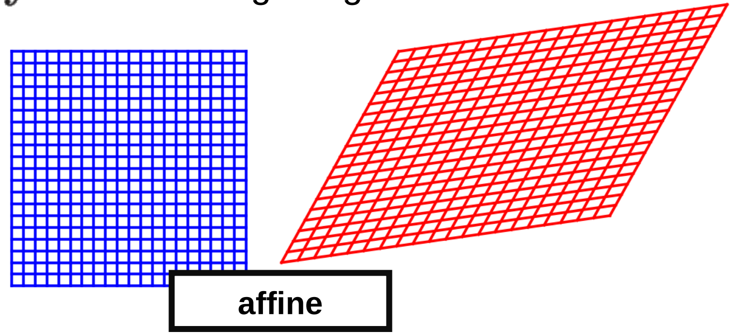
Spatial transformation model

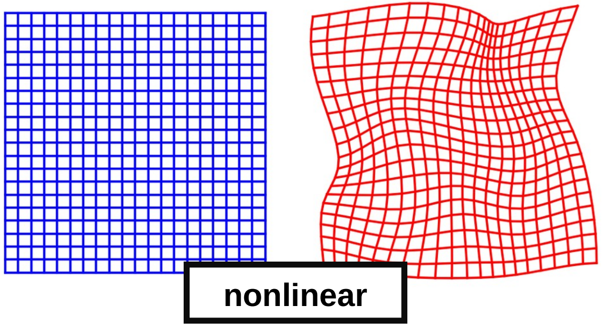
Spatial transformation models

Spatial transformation $\mathbf{y}(\mathbf{x}, \mathbf{w})$:

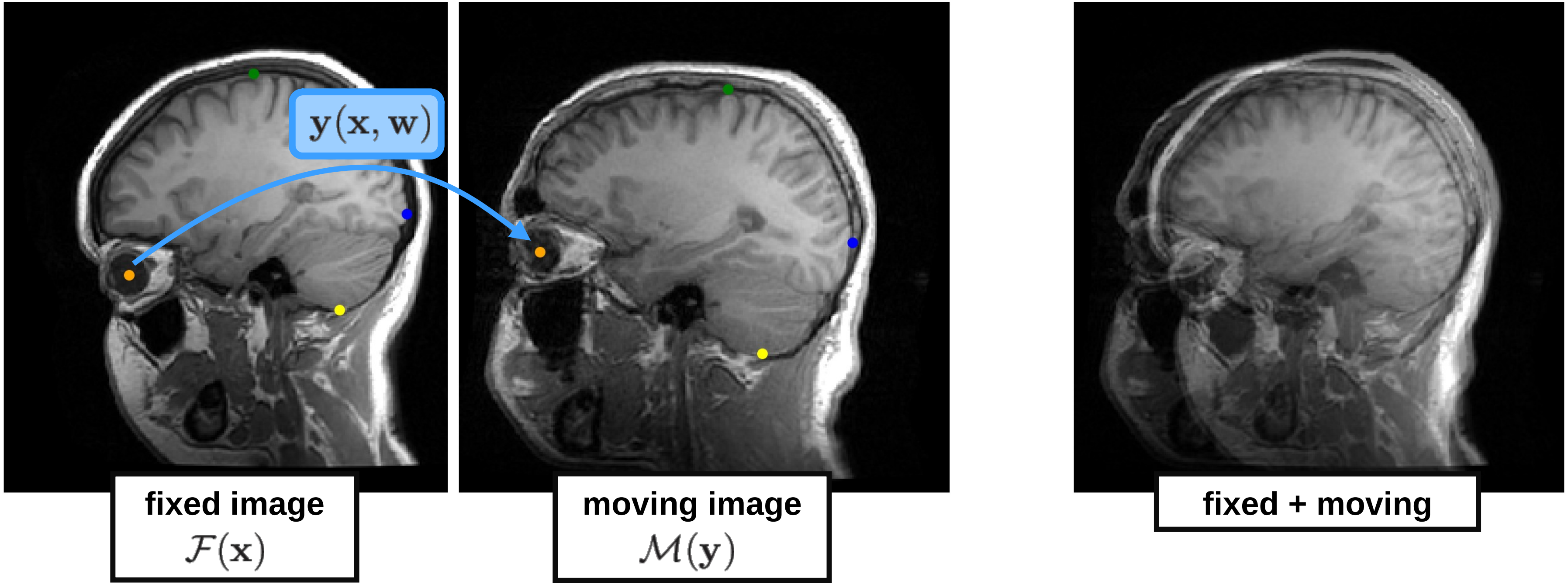
- ✔ maps world positions $\mathbf{x}$ in the fixed image to world positions $\mathbf{y}$ in the moving image
- ✔ controlled by parameters $\mathbf{w}$





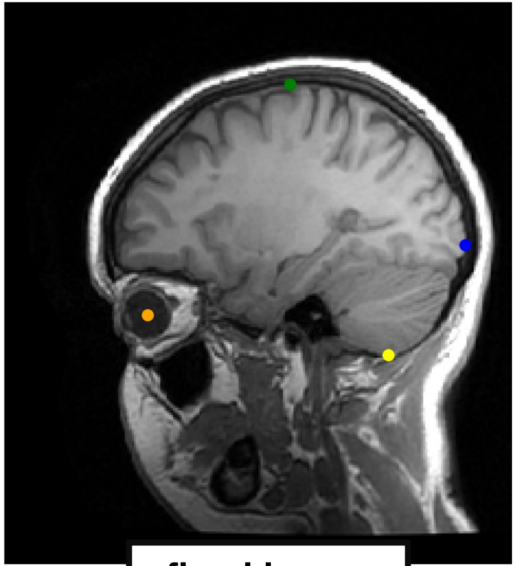


Landmark-based registration

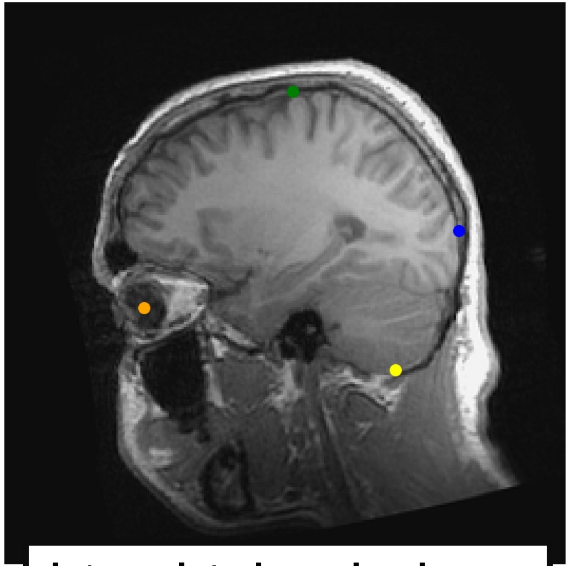


Before registration

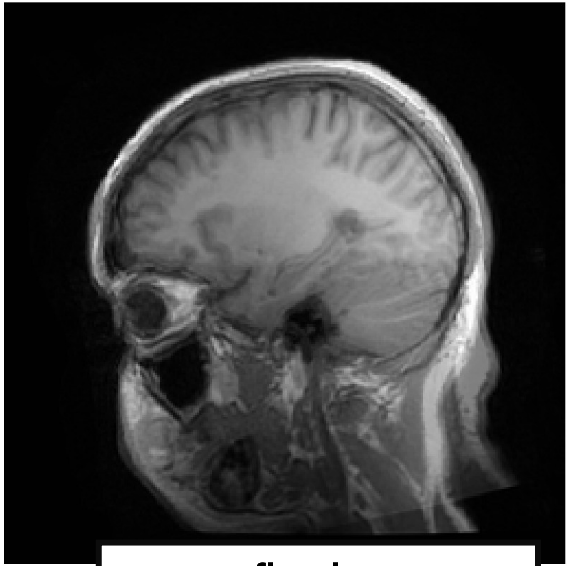
Landmark-based registration



fixed image
$\mathcal{F}(\mathbf{x})$



interpolated moving image
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

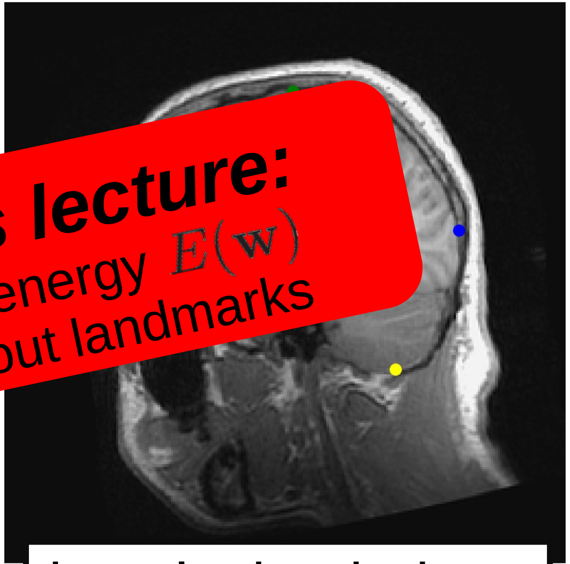


fixed +
interpolated moving

After registration

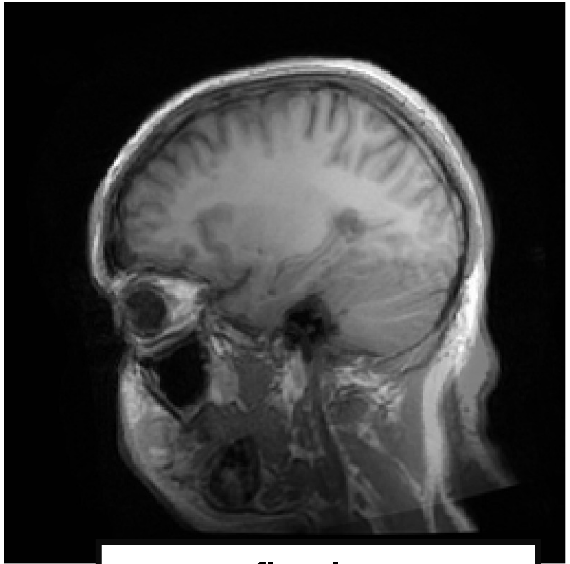
Landmark-based registration

fixed image
$\mathcal{F}(\mathbf{x})$



interpolated moving image
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

This lecture:
define energy $E(\mathbf{w})$
without landmarks

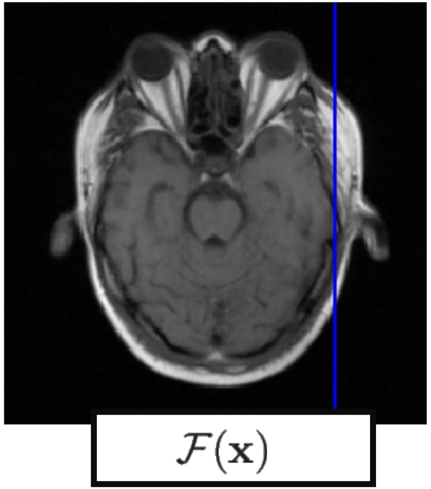


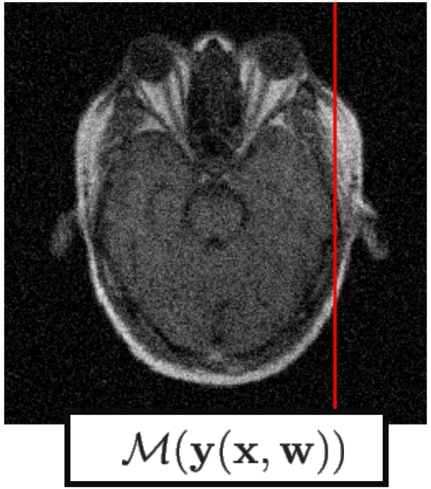
fixed +
interpolated moving

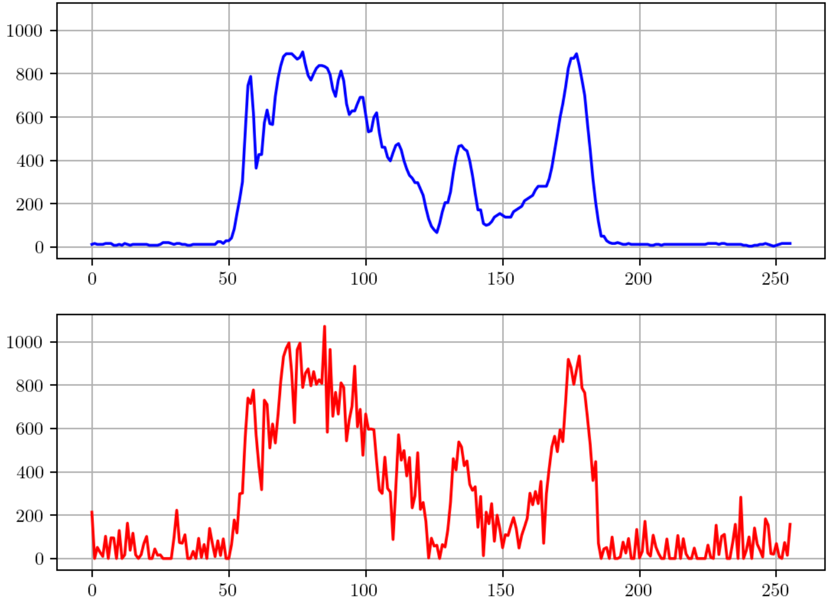
After registration

Intra-modal registration

Images have similar intensity characteristics



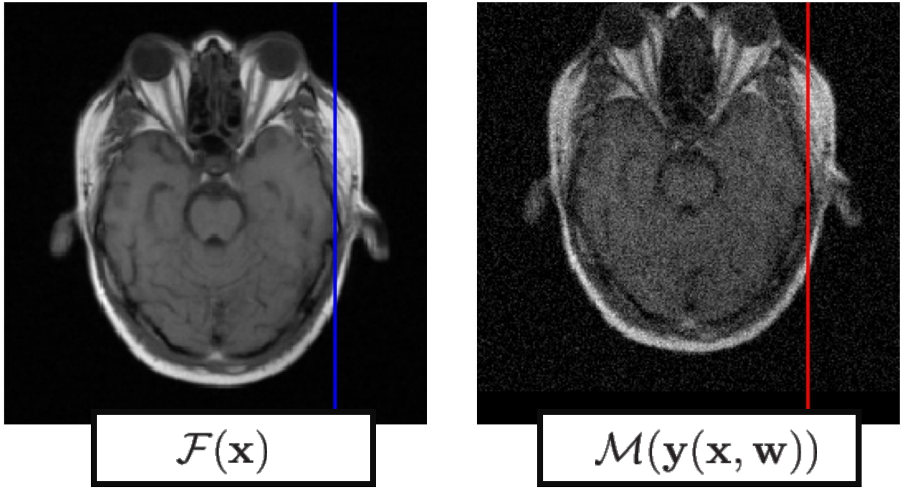


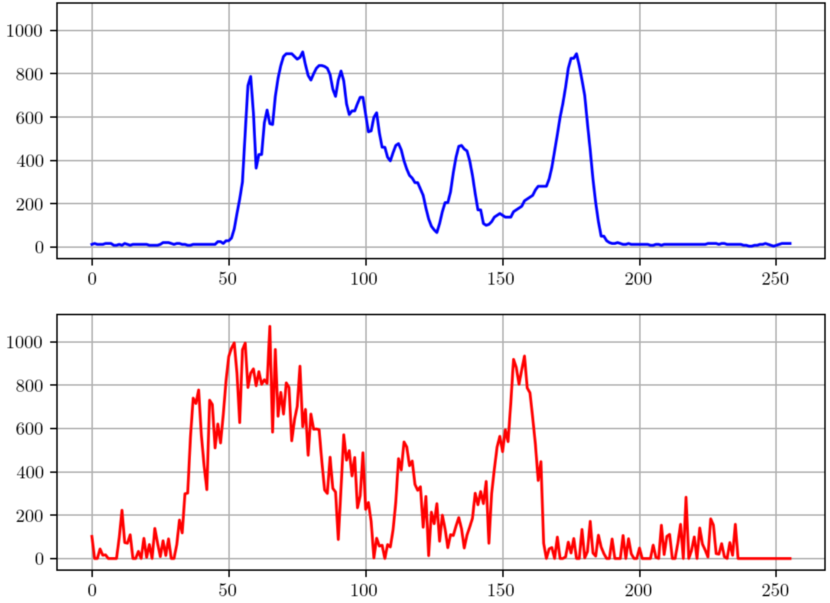


Task: what's a good energy function $E(\mathbf{w})$?

Intra-modal registration

Images have similar intensity characteristics





Task: what's a good energy function $E(\mathbf{w})$?

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Intra-modal registration

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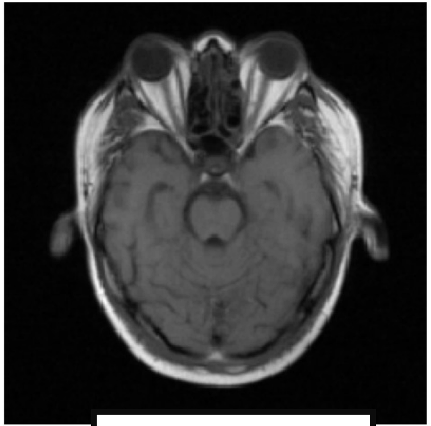
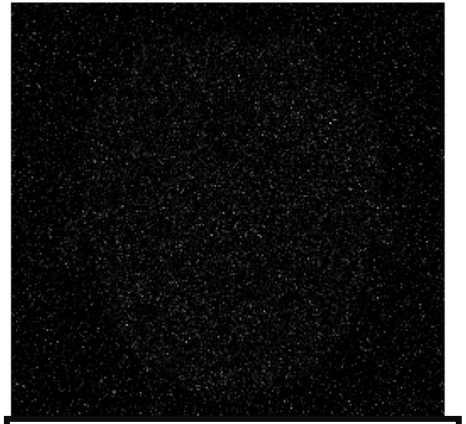


$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



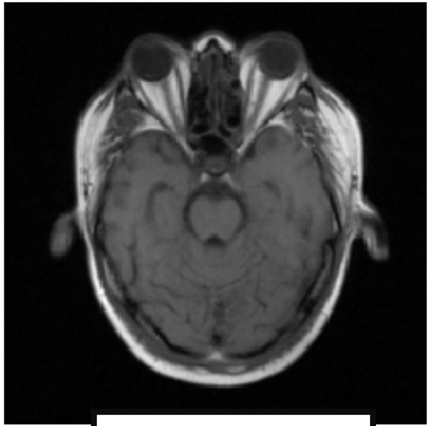
$[\mathcal{F}(\mathbf{x}) - \mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))]^2$

$$\boxed{E(\mathbf{w}) = \sum_{n=1}^{N} [\mathcal{F}(\mathbf{x}_n) - \mathcal{M}(\mathbf{y}(\mathbf{x}_n, \mathbf{w}))]^2}$$

sum over all voxels

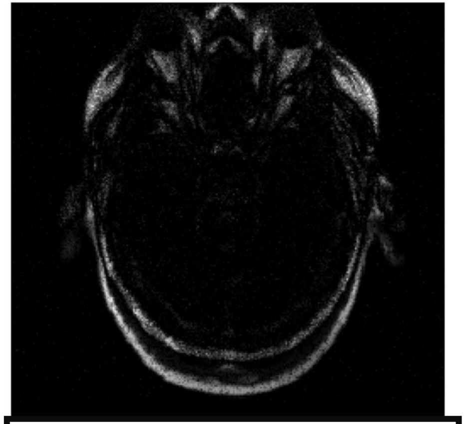
Intra-modal registration

Images have similar intensity characteristics



$\mathcal{F}(\mathbf{x})$

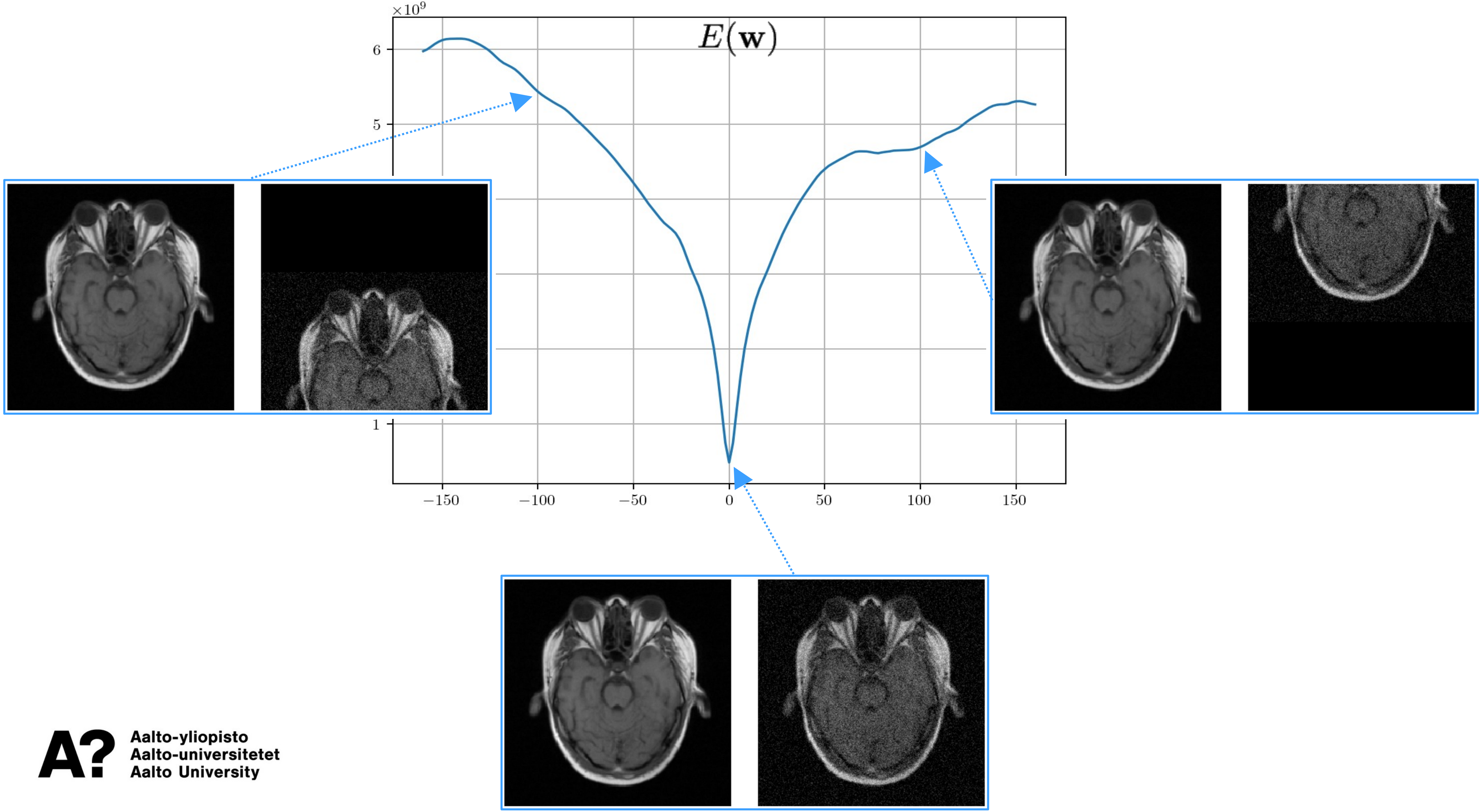
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



$[\mathcal{F}(\mathbf{x}) - \mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))]^2$

$$E(\mathbf{w}) = \sum_{n=1}^{N} [\mathcal{F}(\mathbf{x}_n) - \mathcal{M}(\mathbf{y}(\mathbf{x}_n, \mathbf{w}))]^2$$

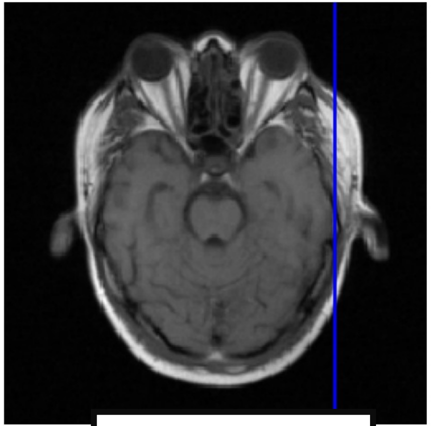
sum over all voxels



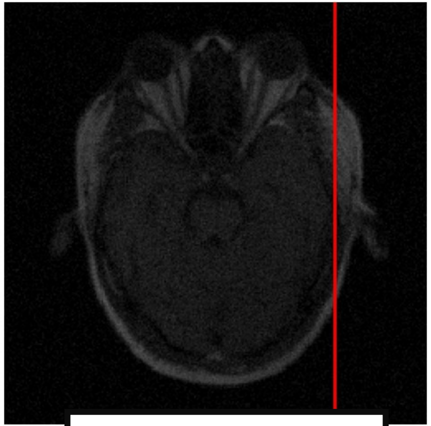
$\times 10^9$
$E(\mathbf{w})$
6
5
1
−150
−100
−50
0
50
100
150

Intra-modal registration

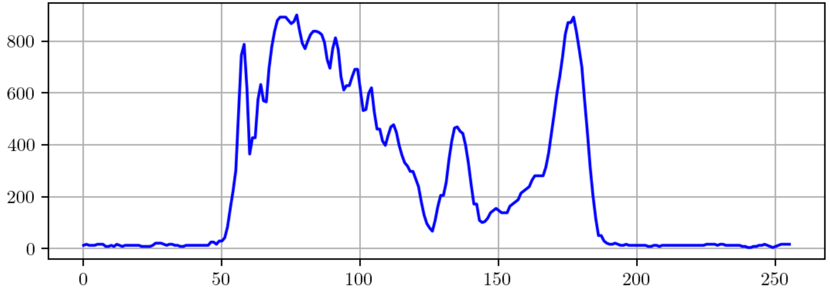
Same but images are scaled differently

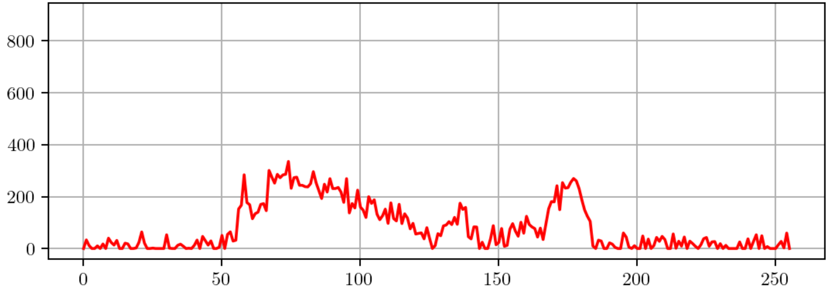


$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$





Task: what's a good energy function $E(\mathbf{w})$?

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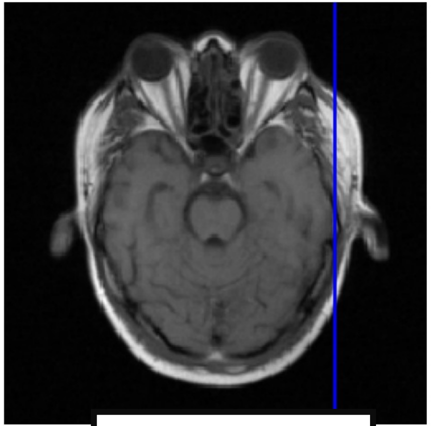
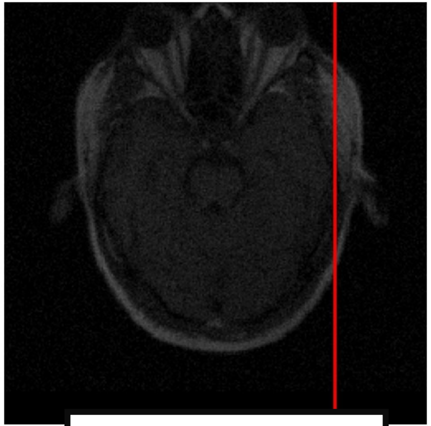
Intra-modal registration

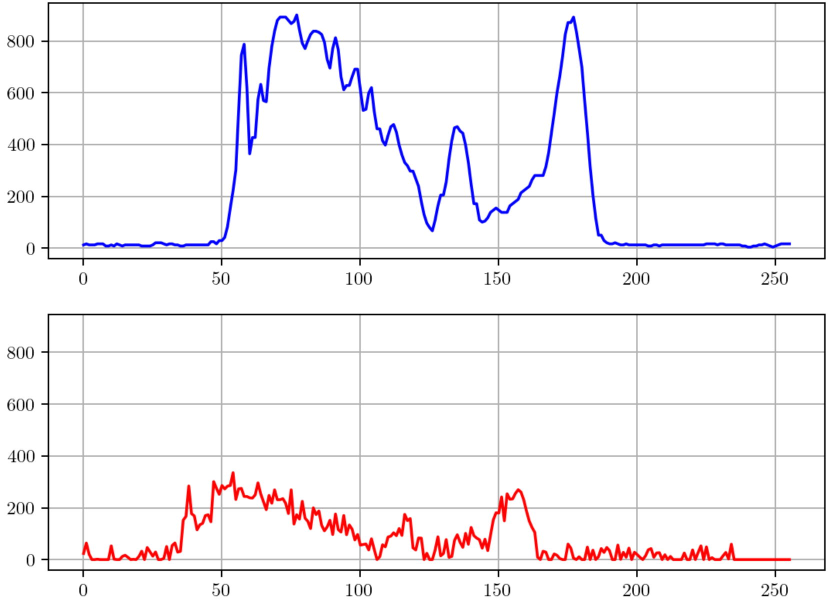
Same but images are scaled differently



$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$



Task: what's a good energy function $E(\mathbf{w})$?

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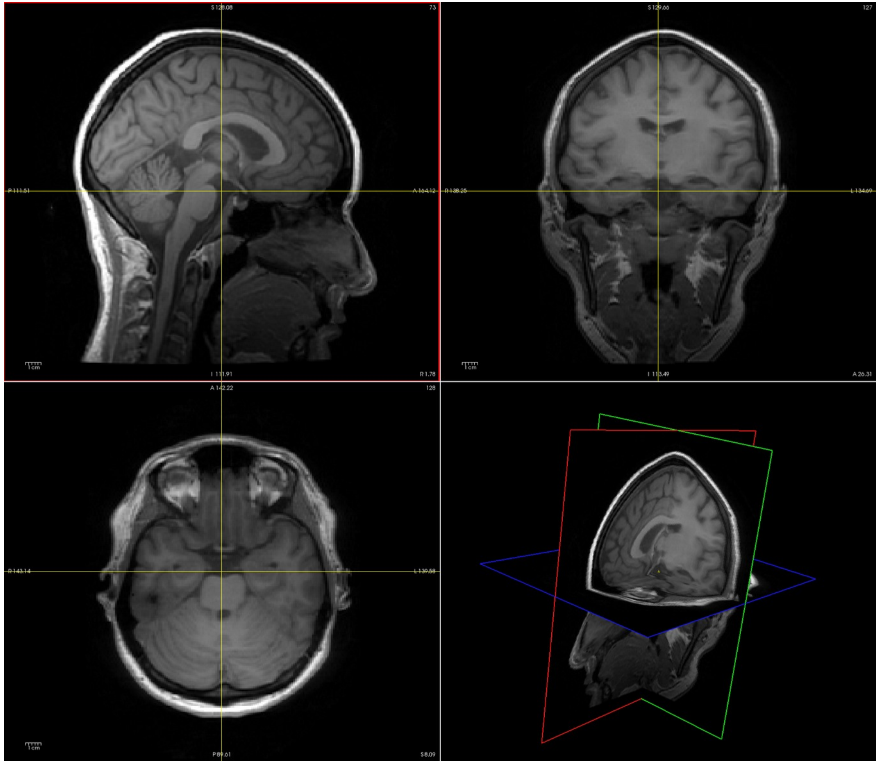
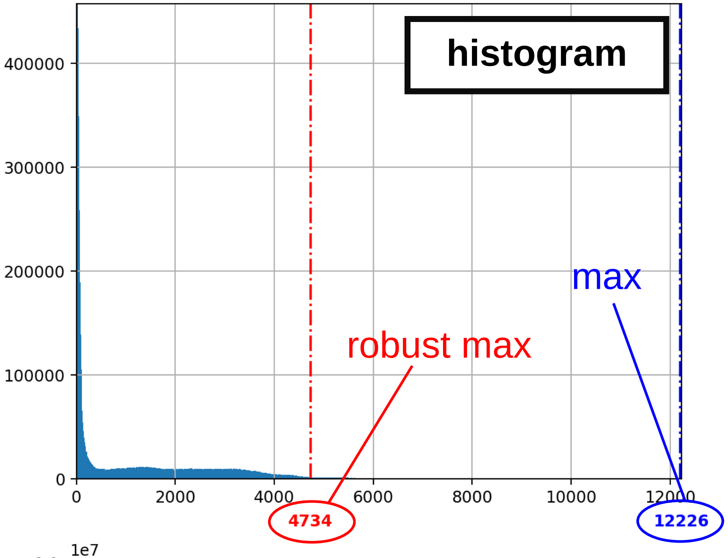
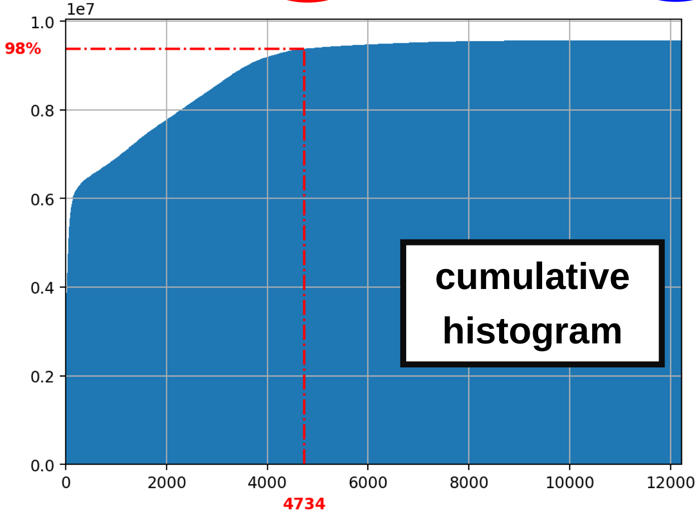
“Maximum” intensity







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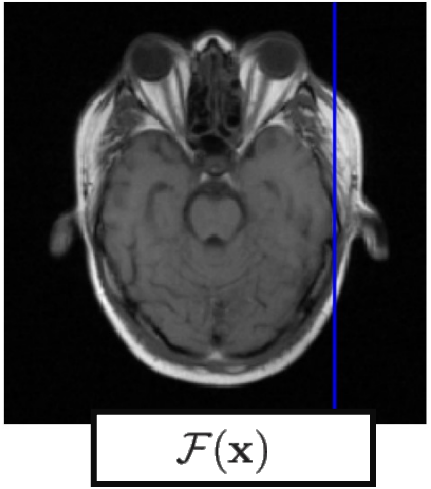
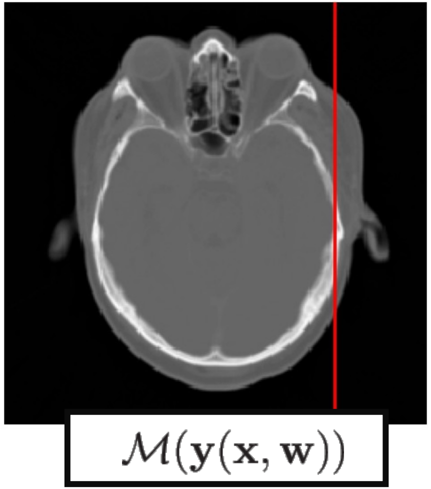
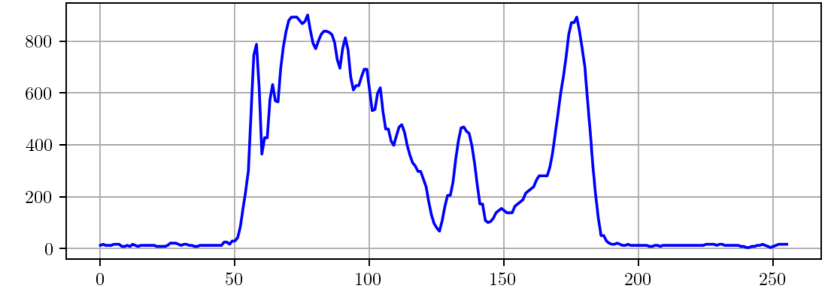
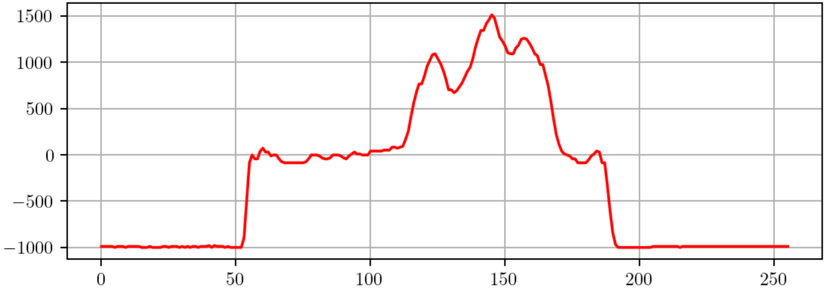
Inter-modal registration

Images have different intensity characteristics









Task: what's a good energy function $E(\mathbf{w})$?

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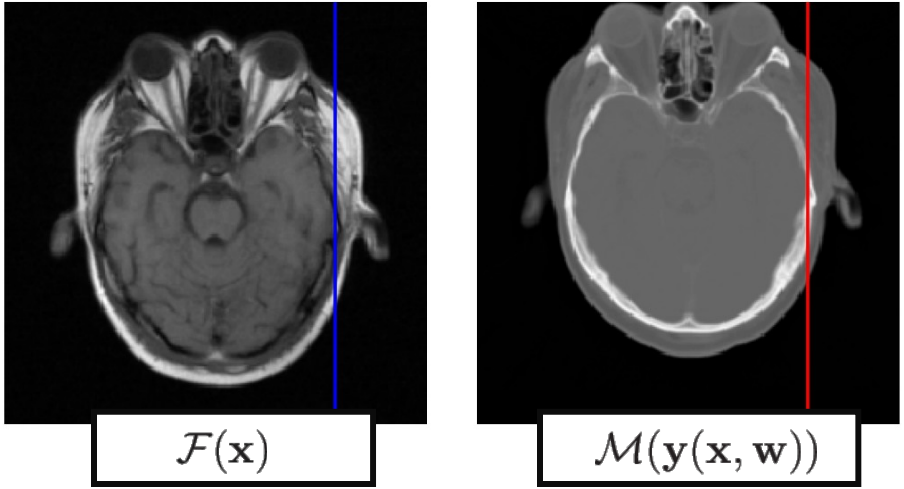

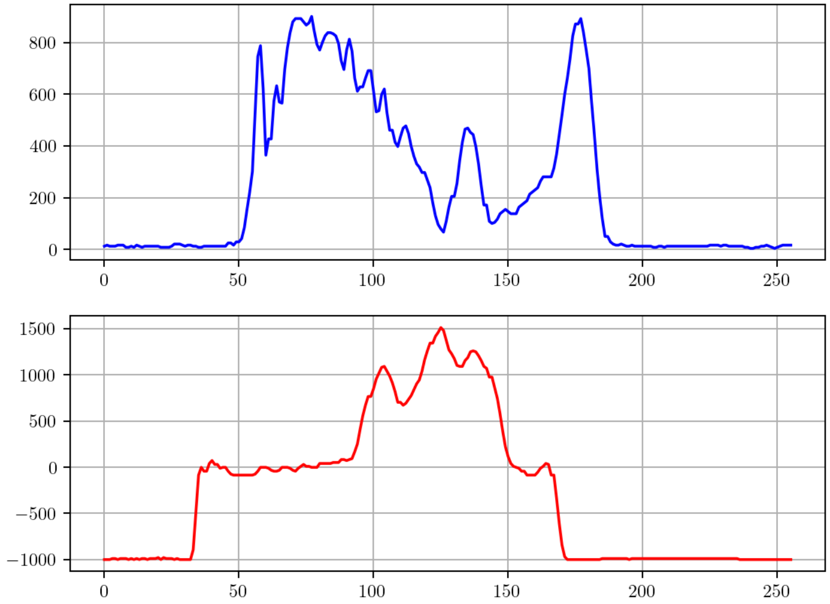

Task: what's a good energy function $E(\mathbf{w})$?

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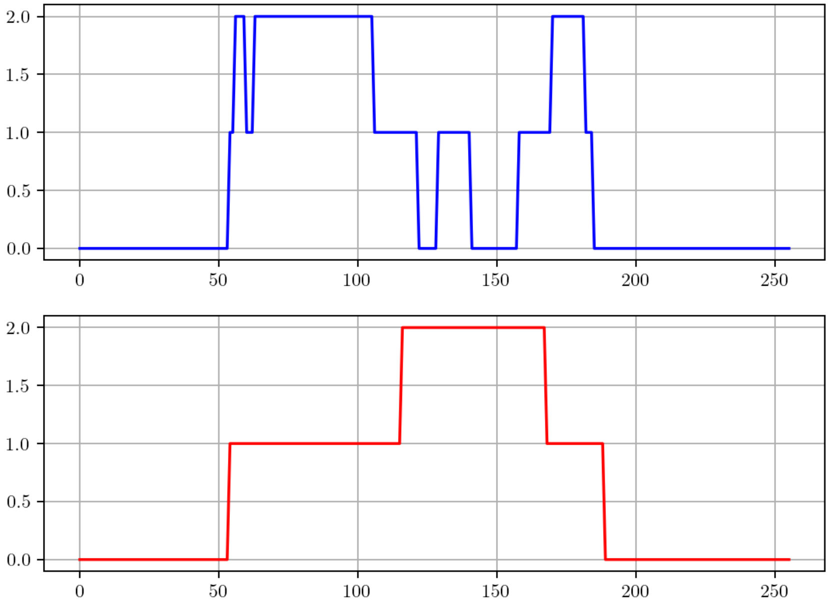
Inter-modal registration

Images have different intensity characteristics

$\mathcal{F}(\mathbf{x})$

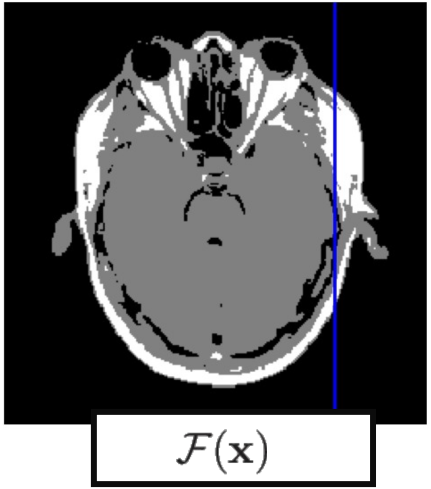
$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

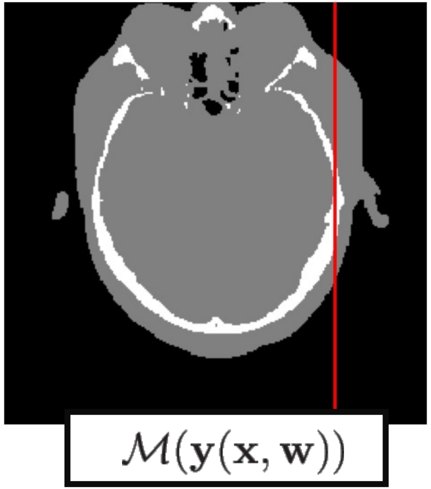


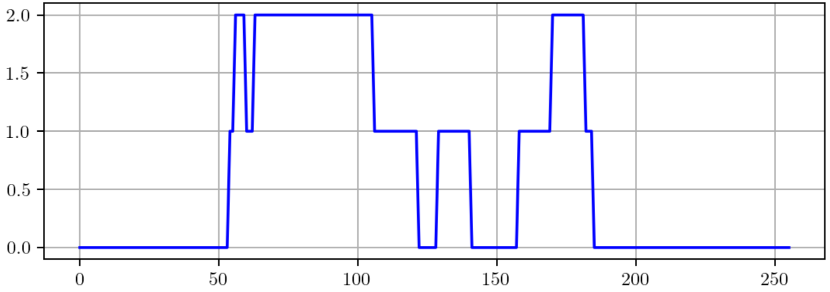
Easier task: what's a good energy function $E(\mathbf{w})$ now?

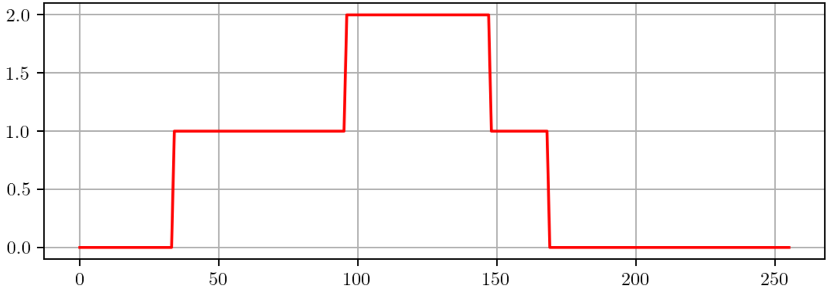
Inter-modal registration

Images have different intensity characteristics









Easier task: what's a good energy function $E(\mathbf{w})$ now?

Inter-modal registration

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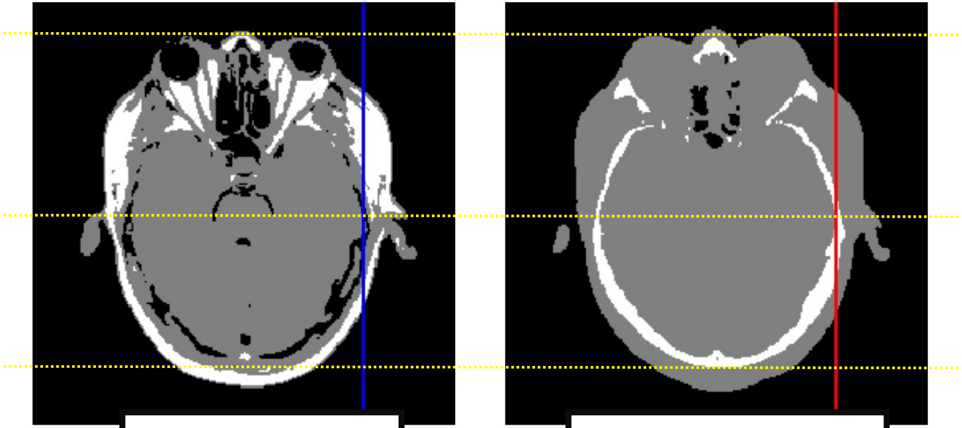
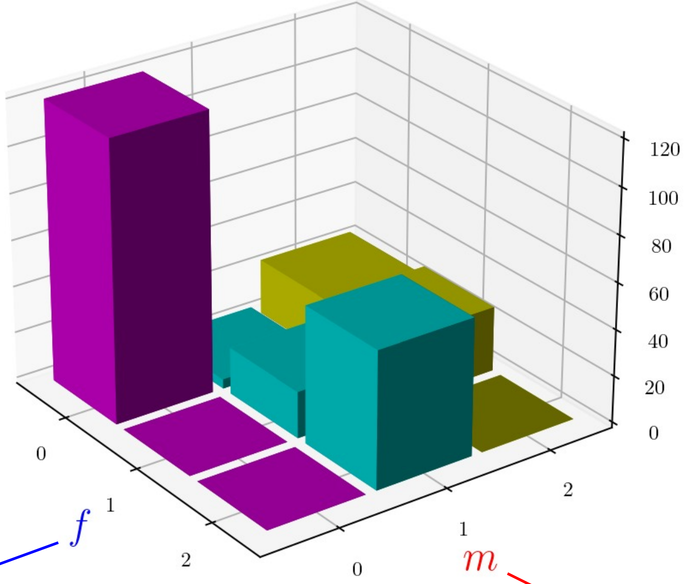


$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x},\mathbf{w}))$

joint histogram

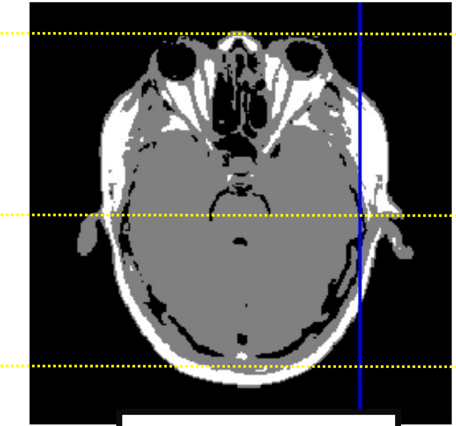


f intensity in fixed image

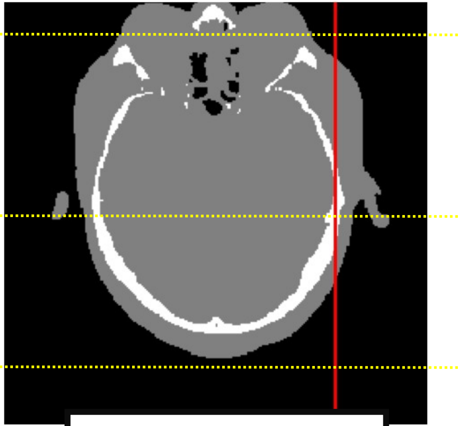
m intensity in moving image

Inter-modal registration

Images have different intensity characteristics

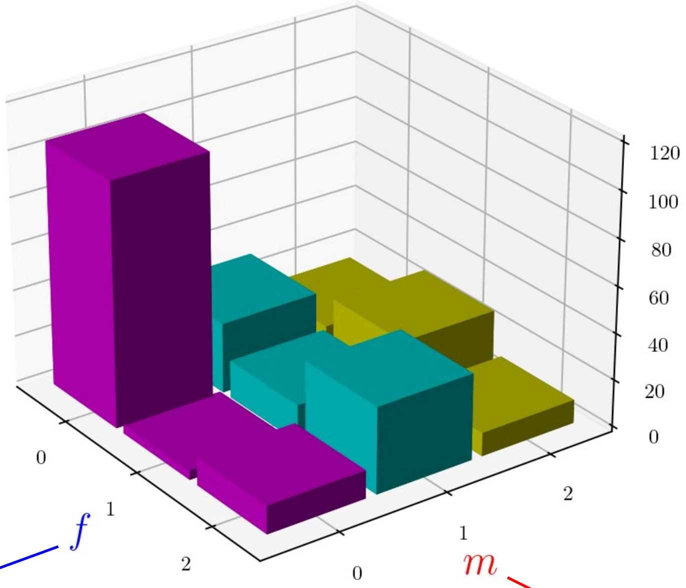


$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram



intensity in fixed image

intensity in moving image

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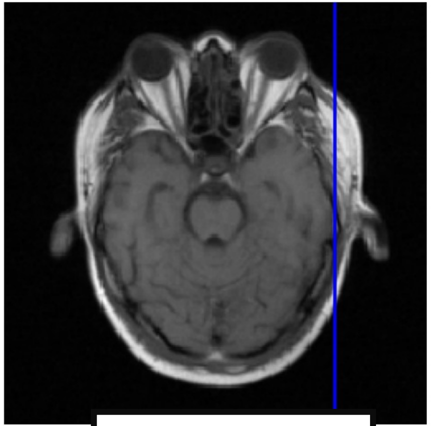
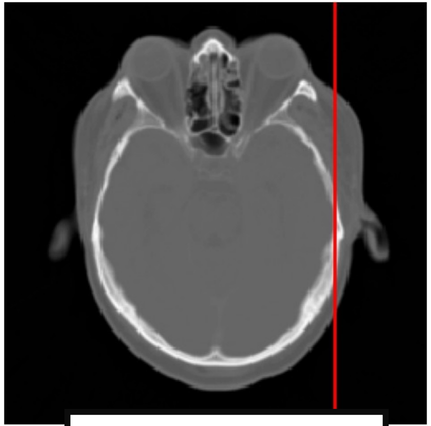
Inter-modal registration

Images have different intensity characteristics

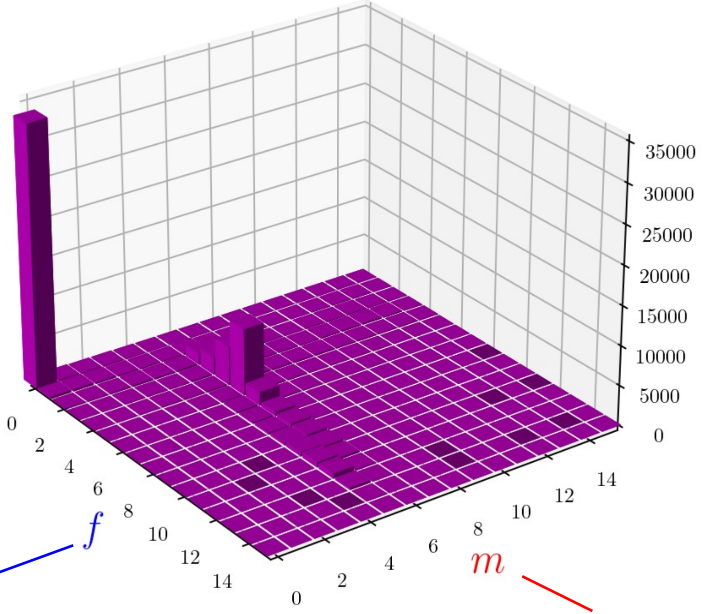


$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram



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Inter-modal registration

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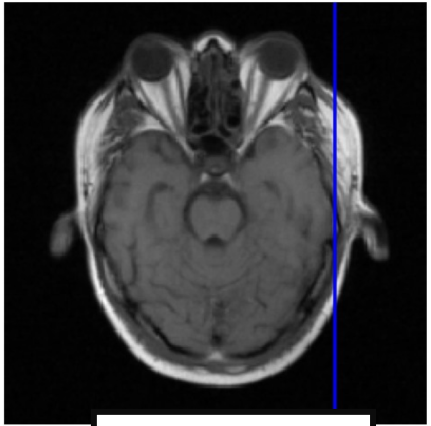
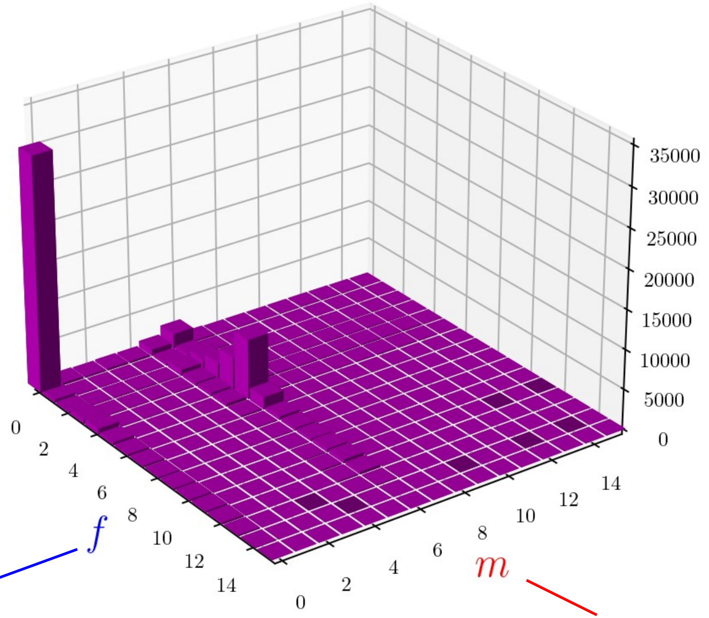


$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram



intensity in fixed image

intensity in moving image

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A bit of information theory...

Imagine that a coin is "rigged":

- ✔ lands on heads with probability $0 \leq \alpha \leq 1$
- ✔ I toss it *many* times, and the result is 1101000101111101110100010111110110100010 1...1

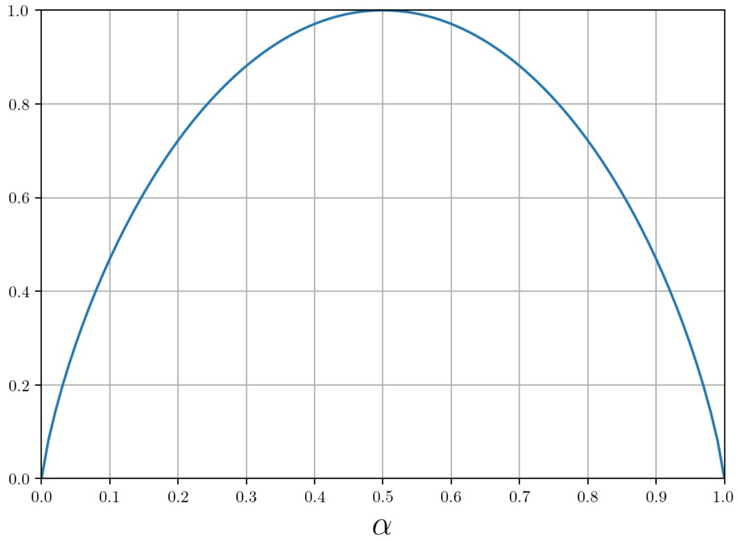
"heads"

"tails"

- ✔ The minimum number of bits required to store/communicate this result is (per toss):

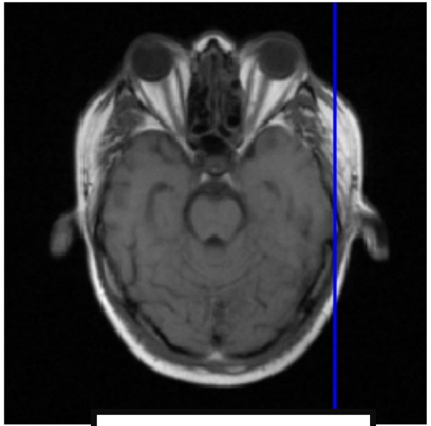
$$-\alpha \log_2(\alpha) - (1-\alpha)\log_2(1-\alpha)$$

"entropy"

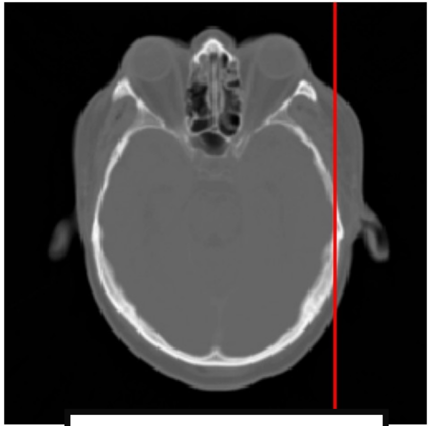


Inter-modal registration

Images have different intensity characteristics

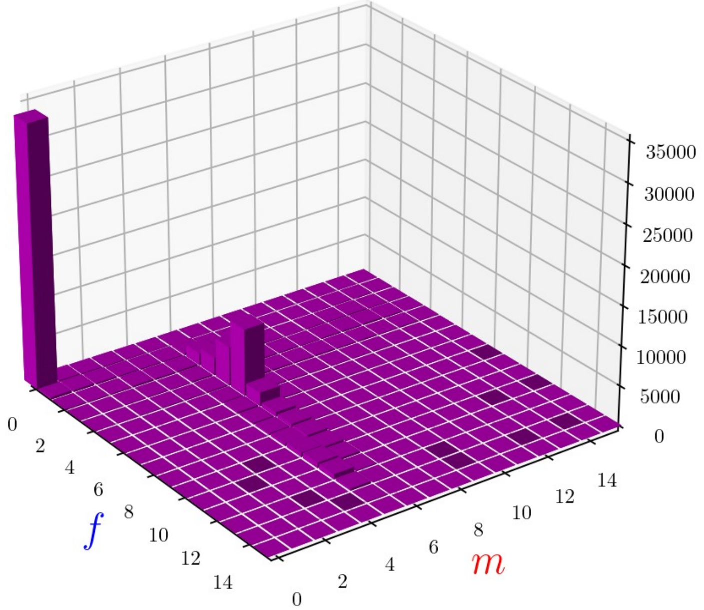


$\mathcal{F}(\mathbf{x})$



$\mathcal{M}(\mathbf{y}(\mathbf{x},\mathbf{w}))$

joint histogram

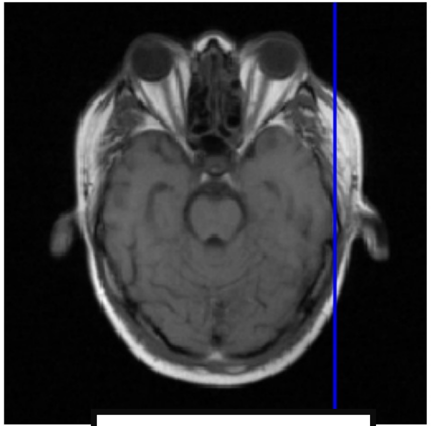


$$E(\mathbf{w}) = H_{F,M} \quad \text{where} \quad H_{F,M} = -\sum_{f=1}^{B}\sum_{m=1}^{B} p_{f,m} \log(p_{f,m})$$

normalized histogram counts

Inter-modal registration

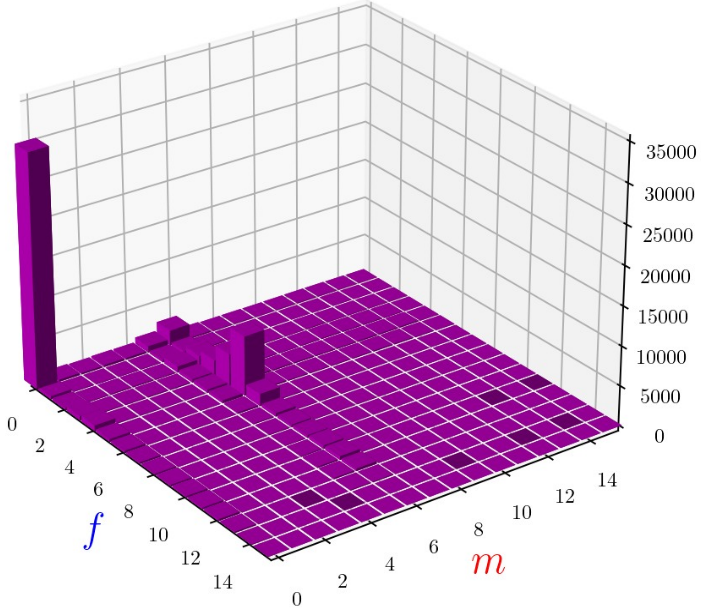
Images have different intensity characteristics



$\mathcal{F}(\mathbf{x})$

$\mathcal{M}(\mathbf{y}(\mathbf{x}, \mathbf{w}))$

joint histogram



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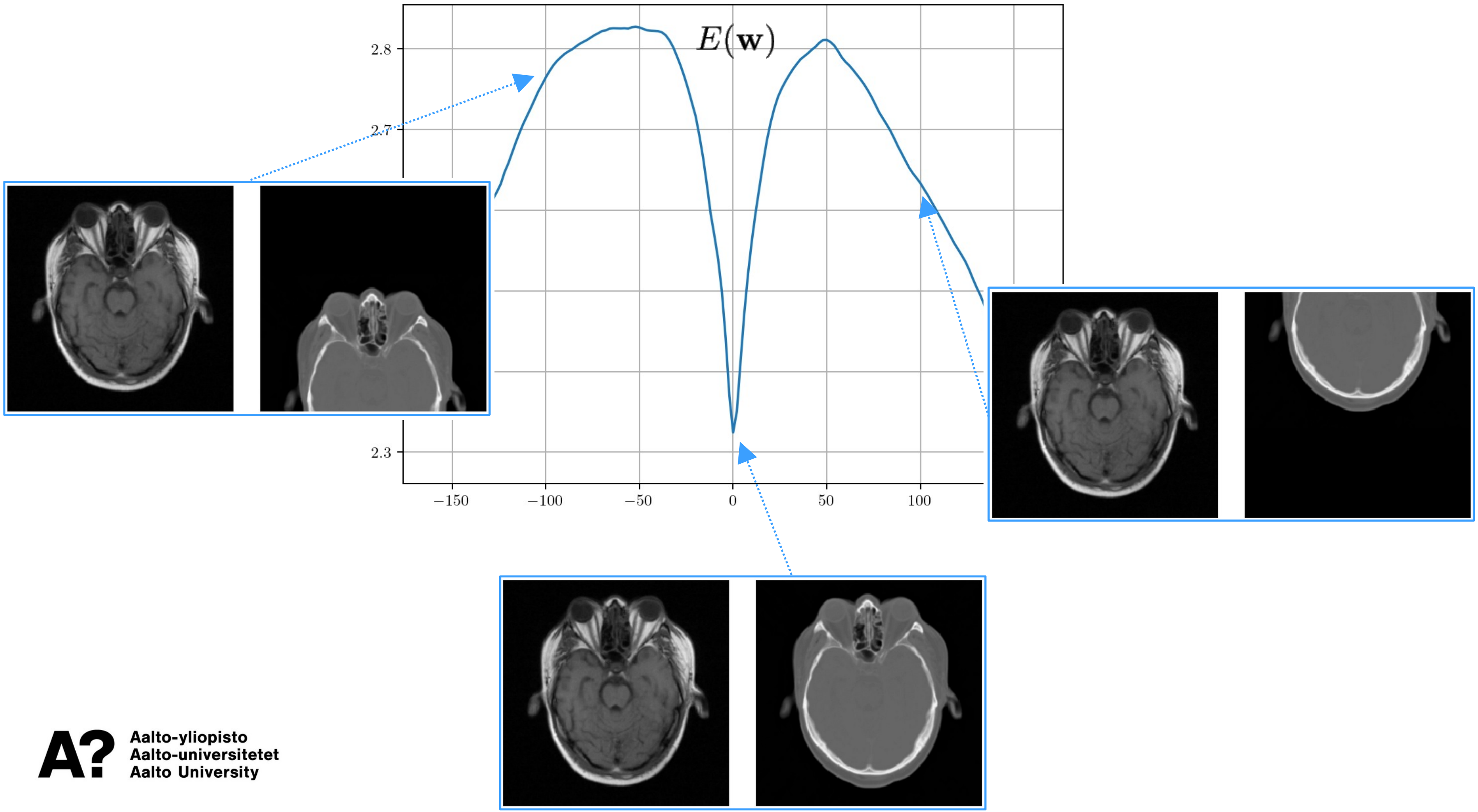
normalized histogram counts



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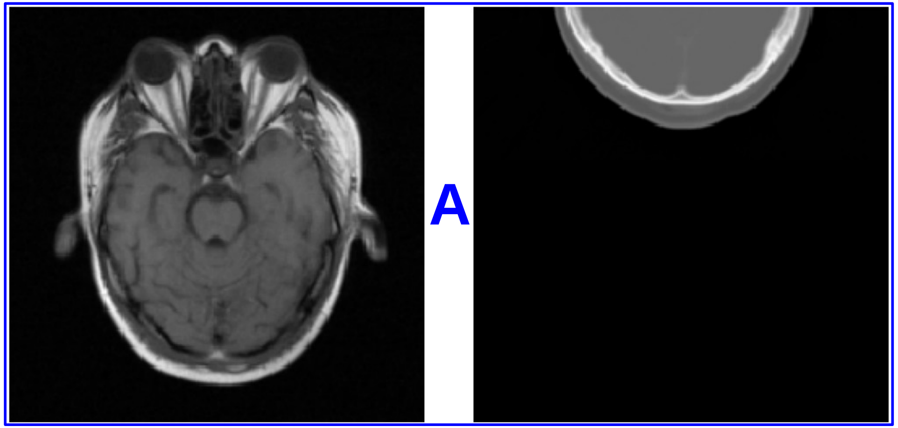
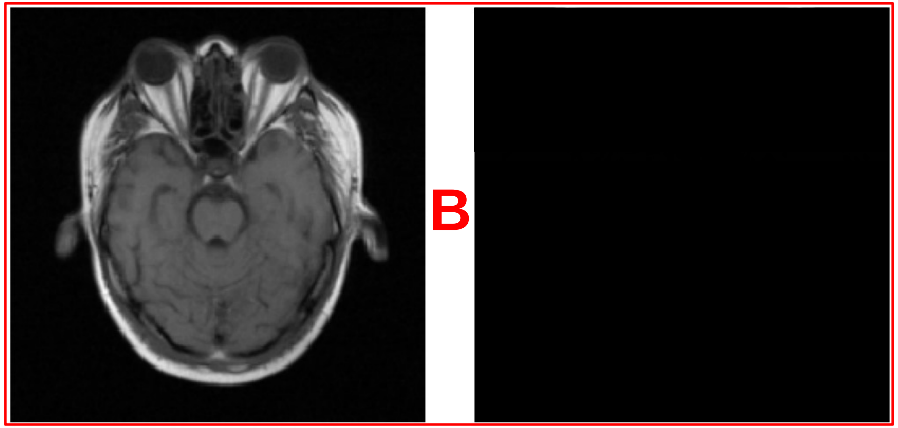
Diagnosing the problem

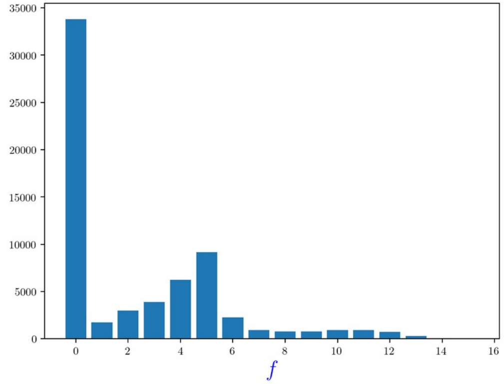
Question: which image pair takes more bits to encode?





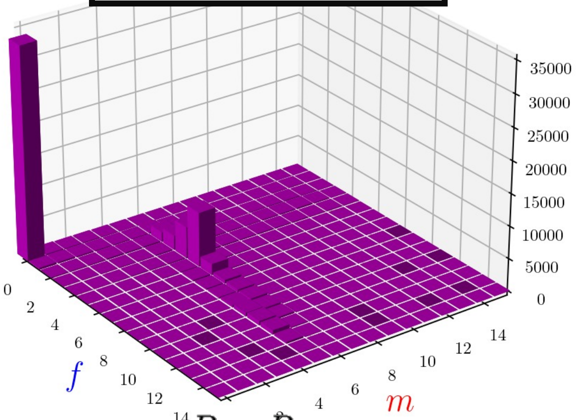
Solution

histogram fixed image



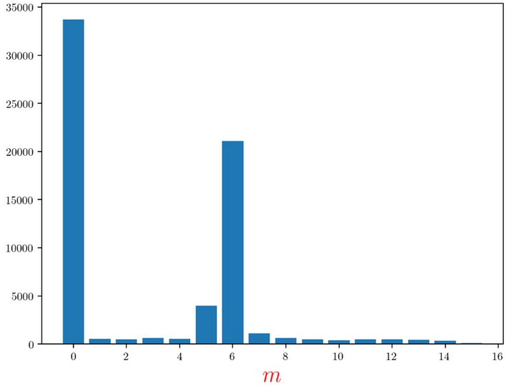
$$H_F = -\sum_{f=1}^{B} p_f \log(p_f)$$

joint histogram



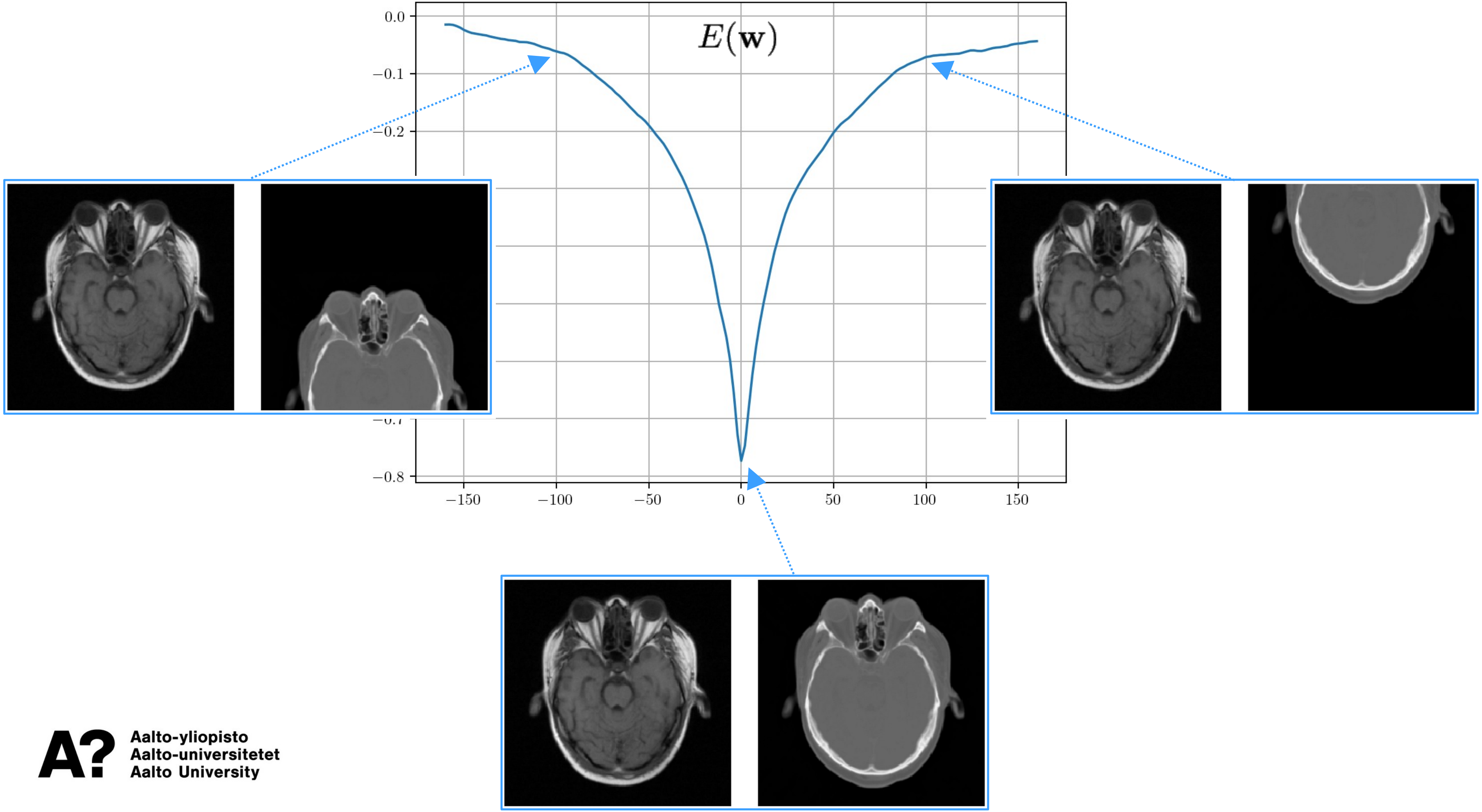
$$H_{F,M} = -\sum_{f=1}^{B} \sum_{m=1}^{B} p_{f,m} \log(p_{f,m})$$

histogram moving image



$$H_M = -\sum_{m=1}^{B} p_m \log(p_m)$$

$$E(\mathbf{w}) = H_{F,M} - H_F - H_M$$ (negative “mutual information”)



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Numerical optimization

Find transformation parameters $\mathbf{w}$ that minimize $E(\mathbf{w})$