

Parameter Sensitivity

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Abstract

1 Parameter Sensitivity

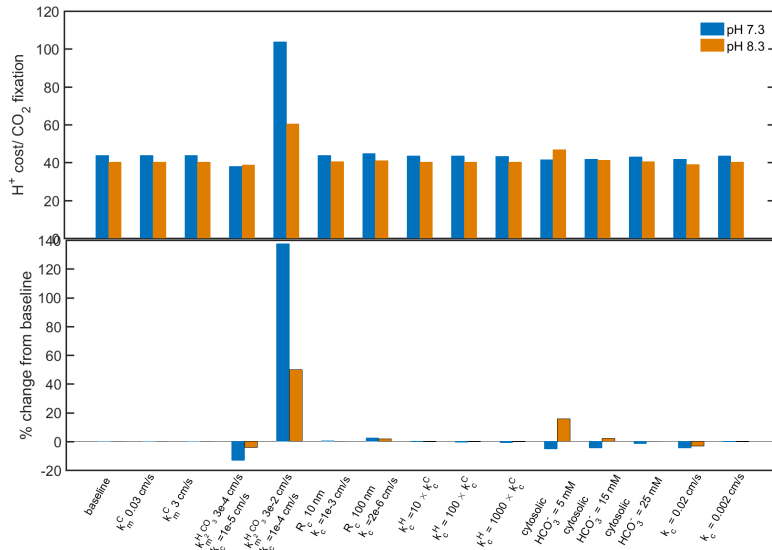


Figure 1: Cost and relative change from baseline for varying different parameters. For some parameters the carboxysome permeability is also changed to remain at optimal carboxysome permeability. Otherwise the main effect would be the shift away from optimum permeability.

2 Equation when RuBisCO is saturated

The analytic solution for the CO_2 and HCO_3^- concentration in the carboxysome when RuBisCO is saturated is:

$$C_{carboxysome} = \frac{N}{M} - \frac{R_c^3 V_{max} P}{3MD} \quad (1)$$

$$H_{carboxysome} = K_{eq}(pH)C_{carboxysome} \quad (2)$$

where,

$$N = (j_c + k_m^{eff}(pH_{out}))H_{out}((k_m^C + \alpha)G^C + \frac{D}{R_b^2}) + k_m^C C_{out}(k_m^{eff}G^H + \alpha G^C + \frac{D}{R_b^2}) \quad (3)$$

$$M = K_{eq} * k_m^{eff} \left((\alpha + k_m^C)G^C + \frac{D}{R_b^2} \right) + k_m^C \left(k_m^{eff}G^H + \frac{D}{R_b^2} \right) + \alpha k_m^{eff}G^H \quad (4)$$

$$P = ((\alpha + k_m^C)G^C + \frac{D}{R_b^2})(k_m^{eff}G^H + \frac{D}{R_b^2}) \quad (5)$$

$$G^C = \frac{D}{R_c^2 k_c^C} + \frac{1}{R_c} - \frac{1}{R_b} \quad (6)$$

$$G^H = \frac{D}{R_c^2 k_c^H} + \frac{1}{R_c} - \frac{1}{R_b} \quad (7)$$

The derivation of this equation can be found in the supplementary material of (Mangan 2014). Here we have made a few modifications: (1) kept track of the carboxysome permeability to CO_2 , k_c^C , and HCO_3^- , k_c^H , independently, (2) substituted the pH dependent equilibrium constant for the carbonic anhydrase reaction, $K_{eq}(ph) = \frac{V_{ca}K_{ba}}{V_{ba}K_{ca}}$, (3) written the $\text{CO}_2 \rightarrow \text{HCO}_3^-$ reaction with α as the linear reaction rate (in Mangan 2014 the linear rate was α/K_α), (4) we have replaced the membrane permeability to HCO_3^- with the effective membrane permeability to the bicarbonate pool, and designated when is dependent on the external pH. For all other k_m^{eff} values it is dependent on the pH inside the cell.

3 Reasonable reductions for physically relevant parameter values

3.1 Cell membrane permeability compared to diffusive velocities

Examining equations (6) we note that for large carboxysome permeability $1/R_c$ will be the dominant term, and for smaller carboxysome permeability values the first term will be larger and dominate. Therefore $G^C \geq 1/R_c$. Studying the equations we note that the terms $((\alpha + k_m^C)G^C + \frac{D}{R_b^2})$ appears repeatedly. We use the following argument:

$$(\alpha + k_m^C)G^C \geq (\alpha + k_m^C)/R_c \gg D/R_b^2, \quad (8)$$

$$\text{if } (\alpha + k_m^C) \gg DR_c/R_b^2 \quad (9)$$

For even a small 20 nm diameter ($R_c = 10^{-6}$ cm) carboxysome this will hold as $k_m^C \approx 0.3$ cm/s and $DR_c/R_b^2 = 4 \times 10^{-3}$ cm/s from the values in Table S1. So the membrane permeability to CO_2 could be an order of magnitude too high in our model and this would still be a reasonable assumption. Therefore we will substitute

$$(\alpha + k_m^C)G^C + D/R_b^2 \approx (\alpha + k_m^C)G^C. \quad (10)$$

Inserting this into equations (1-5) we get

$$C_{\text{carboxysome}} = \frac{(j_c + k_m^{\text{eff}}(pH_{\text{out}}))H_{\text{out}}(k_m^C + \alpha)G^C + k_m^C C_{\text{out}}(k_m^{\text{eff}}G^H + \alpha G^C + \frac{D}{R_b^2})}{K_{\text{eq}}k_m^{\text{eff}}(\alpha + k_m^C)G^C + k_m^C \left(k_m^{\text{eff}}G^H + \frac{D}{R_b^2}\right) + \alpha k_m^{\text{eff}}G^H} - \frac{R_c^3 V_{\text{max}}(\alpha + k_m^C)G^C (k_m^{\text{eff}}G^H + \frac{D}{R_b^2})/(3D)}{K_{\text{eq}}k_m^{\text{eff}}(\alpha + k_m^C)G^C + k_m^C \left(k_m^{\text{eff}}G^H + \frac{D}{R_b^2}\right) + \alpha k_m^{\text{eff}}G^H}. \quad (11)$$

We can divide through by $(k_m^C + \alpha)$ to obtain:

$$C_{\text{carboxysome}} = \frac{(j_c + k_m^{\text{eff}}(pH_{\text{out}}))H_{\text{out}}G^C + \frac{k_m^C}{(k_m^C + \alpha)}C_{\text{out}}(k_m^{\text{eff}}G^H + \alpha G^C + \frac{D}{R_b^2})}{K_{\text{eq}}k_m^{\text{eff}}G^C + \frac{k_m^C}{(k_m^C + \alpha)} \left(k_m^{\text{eff}}G^H + \frac{D}{R_b^2}\right) + \frac{\alpha}{(k_m^C + \alpha)}k_m^{\text{eff}}G^H} - \frac{R_c^3 V_{\text{max}}G^C (k_m^{\text{eff}}G^H + \frac{D}{R_b^2})/(3D)}{K_{\text{eq}}k_m^{\text{eff}}G^C + \frac{k_m^C}{(k_m^C + \alpha)} \left(k_m^{\text{eff}}G^H + \frac{D}{R_b^2}\right) + \frac{\alpha}{(k_m^C + \alpha)}k_m^{\text{eff}}G^H}. \quad (12)$$

We now want to examine the remaining terms in the membrane permeability to CO_2 , k_m^C .

3.2 Membrane permeability to CO_2 has little effect.

There are two parameter groupings in equation (12) containing k_m^C :

$$\frac{k_m^C}{k_m^C + \alpha} \quad (13)$$

$$\frac{\alpha}{k_m^C + \alpha} \quad (14)$$

Therefore if $k_m^C > \alpha$ or $\text{CO}_2 \rightarrow \text{HCO}_3^-$ conversion is negligible the first term (13) reduces to 1, and the second reduces to $1/k_m^C$. We will return to the case where this conversion is not negligible later.

With these two simplifications we obtain:

$$C_{\text{carboxysome}} = \frac{(j_c + k_m^{\text{eff}}(pH_{\text{out}}))H_{\text{out}}G^C + C_{\text{out}}(k_m^{\text{eff}}G^H + \alpha G^C + \frac{D}{R_b^2})}{K_{\text{eq}}k_m^{\text{eff}}G^C + \left(k_m^{\text{eff}}G^H + \frac{D}{R_b^2}\right) + \frac{1}{k_m^C}k_m^{\text{eff}}G^H} - \frac{R_c^3 V_{\text{max}}G^C (k_m^{\text{eff}}G^H + \frac{D}{R_b^2})/(3D)}{K_{\text{eq}}k_m^{\text{eff}}G^C + \left(k_m^{\text{eff}}G^H + \frac{D}{R_b^2}\right) + \frac{1}{k_m^C}k_m^{\text{eff}}G^H}. \quad (15)$$

Examining equation (15), note that the only appearance of the membrane permeability to CO_2 is now in the denominator which we can rewrite as $k_m^{eff}(G^C K_{eq} + \frac{G^H}{k_m^C}) + (k_m^{eff} G^H + \frac{D}{R_b^2})$. Using this equation, we can write a strong bound on when the membrane permeability will effect the function of the CCM.

We find k_m^C has no significant effect when $K_{eq} G^C > \frac{G^H}{k_m^C}$ or $k_m^C > \frac{G^H}{G^C K_{eq}}$. If we assume that the carboxysome permeability to CO_2 will always be smaller than or equal to the permeability to HCO_3^- ($k_c^C \geq k_c^H$) then $G^H \geq G^C$ and $\frac{G^H}{G^C} \leq 1$, so k_m^C will be negligible as long as $k_m^C > 1/K_{eq}$. For $\text{pH} > 6.6$, $1/K_{eq} > 0.3$ and therefore the assumed value of $k_m^C = 0.3$ will be negligible. However, if the cell operated in a lower pH regime and the membrane permeability was substantially lower to CO_2 it would begin to effect the CO_2 concentration.

Thus far we have made a series of observations about the size of terms compared to the membrane permeability to CO_2 and found that when $(\alpha + k_m^C) \gg DR_c/R_b^2$, $k_m^C > \alpha$ and $k_m^C > \frac{G^H}{G^C K_{eq}} \approx 1/K_{eq}$ the CO_2 concentration in the carboxysome reduces to

$$C_{carboxysome} = \frac{(j_c + k_m^{eff}(pH_{out}))H_{out}G^C + C_{out}(k_m^{eff}G^H + \alpha G^C + \frac{D}{R_b^2})}{k_m^{eff}(G^C K_{eq} + G^H) + \frac{D}{R_b^2}} - \frac{R_c^3 V_{max} G^C (k_m^{eff} G^H + \frac{D}{R_b^2}) / (3D)}{k_m^{eff}(G^C K_{eq} + G^H) + \frac{D}{R_b^2}}. \quad (16)$$

We can make a similar argument taking the equation for the CO_2 concentration at the cell membrane:

$$C_{cytosol}(r = R_b) = \frac{k_m^C C_{out} - (\alpha + k_m^C) C_{carboxysome}}{(\alpha + k_m^C) G^C + D/R_b^2} G^C + C_{carboxysome} \approx C_{out} \quad (17)$$

This means that the CO_2 leakage term will be negligible since the cytosolic CO_2 concentration will be approximately equal to the external CO_2 concentration. The HCO_3^- transport required to sustain a given internal inorganic carbon pool will then be:

$$j_c H_{out} = \left(\frac{R_c^3}{3R_b^2} V_{max} - k_m^C (C_{out} - C_{cytosol}) - k_m^{eff} H_{out} + k_m^{eff} H_{cytosol} \right) = \left(\frac{R_c^3}{3R_b^2} V_{max} - k_m^{eff} H_{out} + k_m^{eff} H_{cytosol} \right) \quad (18)$$

This equation is independent of the membrane permeability to CO_2 , as $H_{cytosol} = K_{eq} C_{cytosol}$. This observation is consistent with the low flux of CO_2 leakage in Figure 2.

3.3 Without facilitated CO₂ uptake external CO₂ has little effect

Unless conversion from CO₂ to HCO₃⁻ is large we note that the second C_{out} term in equation(15) is negligible for the regimes we study. We will revisit CO₂ uptake and recycling later. Comparing this term against the first term in the numerator, again allows us to put a quantitative description on when this regime holds. Additionally we find that when the transport of HCO₃⁻ is significant ($j_c > k_m^{eff}(pH_{out})$) we arrive at

$$C_{carboxysome} = \frac{j_c H_{out} G^C - R_c^3 V_{max} G^C (k_m^{eff} G^H + \frac{D}{R_b^2}) / (3D)}{k_m^{eff} (G^C K_{eq} + G^H) + \frac{D}{R_b^2}} \quad (19)$$

$$H_{carboxysome} = K_{eq} C_{carboxysome} \quad (20)$$

4 Effect of Carboxysome permeability

Recalling the equation for $G^C = \frac{D}{R_c^2 k_c^C} + \frac{1}{R_c} - \frac{1}{R_b}$, we can see that the carboxysome permeability to CO₂ will only matter if $\frac{D}{R_c^2 k_c^C} > \frac{1}{R_c}$. In other words the carboxysome permeability to CO₂, k_c^C , begins to effectively trap CO₂ in the carboxysome when $k_c^C < \frac{D}{R_c} \approx 2$ cm/s for our base case of a 100 nm carboxysome ($R_c = 50$ nm). Similarly $G^H \approx \frac{D}{R_c^2 k_c^H}$ as $k_c^H \leq k_c^C < \frac{D}{R_c}$. To begin let's examine the case when $k_c = k_c^C = k_c^H$.

In figure XX observe how the carboxysome radius shifts the onset of carboxysome permeability effecting the CCM function. Both the highest carboxysome permeability where RuBisCO is saturated (blue lines) and carbonic remains unsaturated (maroon lines) and the optimal carboxysome permeability (that requiring the lowest HCO₃⁻ transport to achieve the same CO₂ concentration) shift to lower permeability values with increasing carboxysome radii.

It is apparent that varying R_c is simply shifting these lines vertically, if we plot $k_c \times R_c^2$ on the y-axis, as motivated by the term $\frac{D}{R_c^2 k_c^C}$. All the curves collapse! Therefore, the optimal carboxysome permeability or maximum carboxysome permeability must be considered in combination with R_c , if R_c is varying. Until measurements of the carboxysome permeability are performed, if we want to consider smaller or larger carboxysomes we must also think about where we sit in the permeability phase space. Put another way, if you examine the cost of the CCM for varying R_c , it will be the same at the optimal k_c value for each R_c (and also the same for each maximal k_c value supporting CCM function – where maroon and blue lines meet).

4.0.1 Different carboxysome permeability for HCO₃⁻

An existing hypothesis in the CCM literature is that the carboxysome has differential permeability and is more permeable to HCO₃⁻ and less permeable to

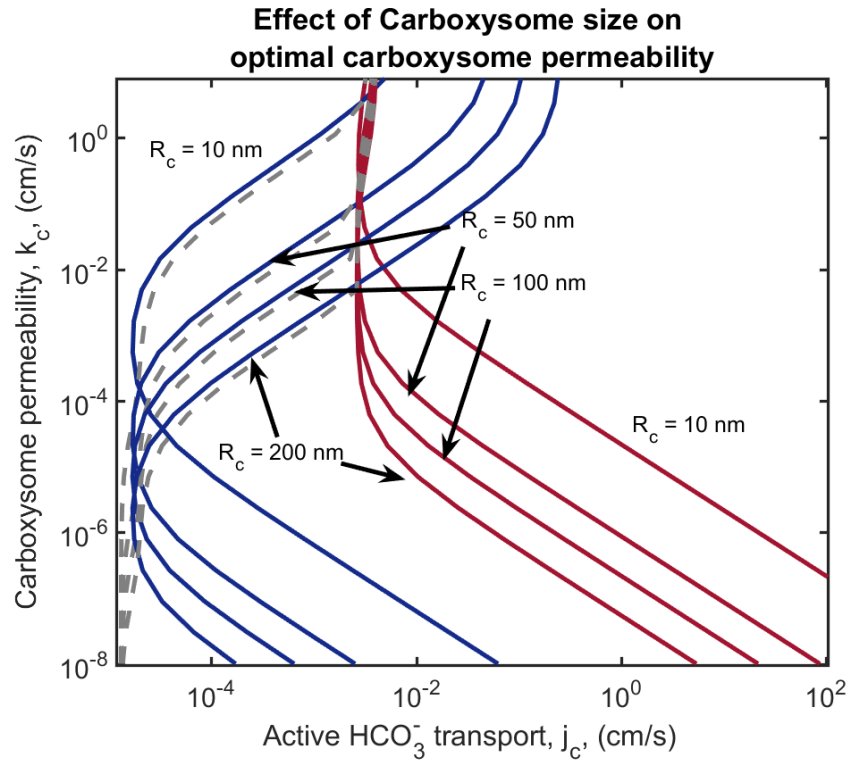


Figure 2: Changing the carboxysome radius shifts the onset of the carboxysome permeability effects and optimal carboxysome permeability. Blue lines show lines of constant concentration where RuBisCO is saturated (saturated to the right), and maroon shows where carbonic anhydrase is saturated (saturated to the right). Grey dashed lines shows where cytosolic HCO_3^- concentration is 30mM.

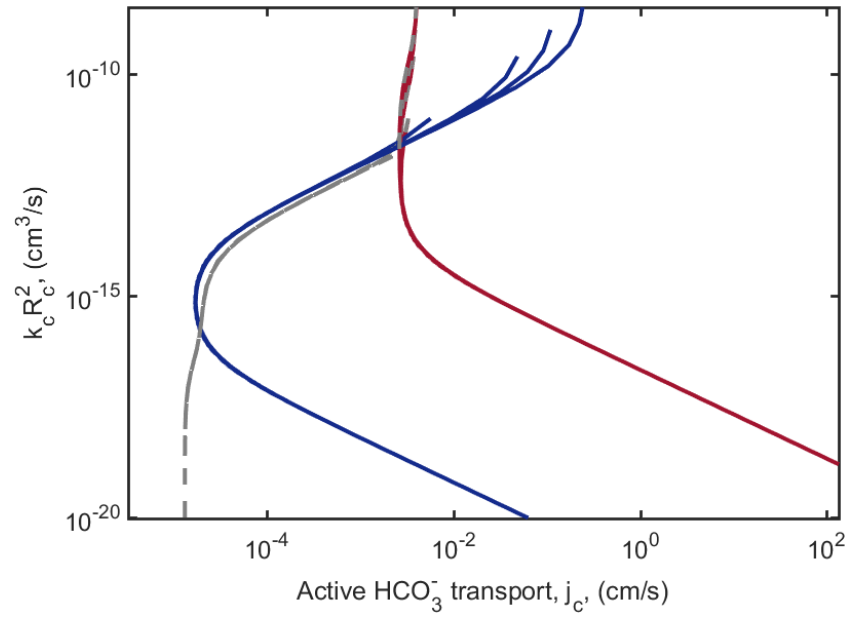


Figure 3: Same as previous figure, with y-axis scaled by R_c^2 . All blue, maroon and grey dashed lines have collapsed.

Effect of increasing differential carboxysome permeability to HCO_3^-

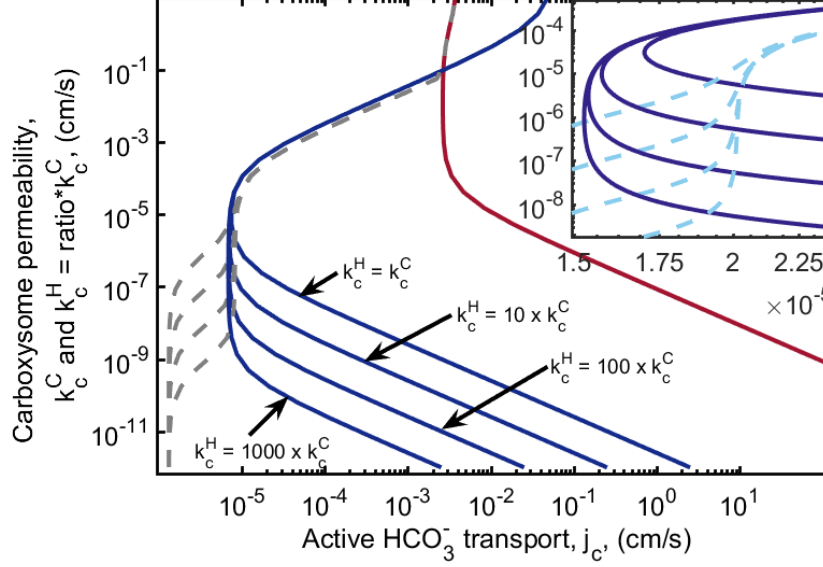


Figure 4: Increasing the carboxysome permeability to HCO_3^- relative to CO_2 mostly effects the required transport rates for CO_2 carboxysome permeabilities below the optimum. Inset shows that there is a small decrease in necessary flux near the optimum.

CO_2 . Intuitively this would allow more HCO_3^- into the carboxysome and trap more CO_2 , thereby accumulating more inorganic carbon in the form of CO_2 . We use our model to test whether differential carboxysome permeability enables higher carboxysomal CO_2 concentration for the same level of HCO_3^- transport. In the following figure we show the k_c vs j_c phase space where we have plotted the carboxysome permeability to CO_2 , k_c^C , on the y-axis. We plot different ratios (1, 10, 100, 1000) between k_c^C and the carboxysome permeability to HCO_3^- , $k_c^H = \text{ratio} \times k_c^C$.

Examining Figure XX, we see that making the carboxysome more permeable to HCO_3^- does not improve the function of the CCM as drastically as one might assume. The "turn on" of CO_2 accumulation with decreasing permeability is unaffected by changes to k_c^H , and depends only on the permeability to CO_2 , k_c^C . The "turn off" of accumulation for lower carboxysome permeabilities is greatly effected by the permeability of the carboxysome to HCO_3^- , k_c^H . These two effects are exactly what we previously discussed as defining the carboxysome permeability optimum.

As we start at the top of the y-axis and decrease the carboxysome permeability the following occurs: At high permeability not enough CO_2 is trapped, but HCO_3^- enters readily. As we moved to lower permeabilities CO_2 begins to

be trapped, but there is a window where HCO_3^- still enters enough to supply the system. Eventually the carboxysome begins to restrict HCO_3^- entry. If the carboxysome is more permeable to HCO_3^- than to CO_2 then the window where CO_2 trapping is effective without restricting HCO_3^- entry broadens. The width of this window (on the y-axis) will also depend strongly on how much of the CO_2 is being fixed.

The "turn off" of the optimum, caused by not allowing enough HCO_3^- into the carboxysome, does slightly increase the amount of transport required to saturate RuBisCO at the carboxysome optimum. The reduction in transport required, and therefore CCM cost is around 5% when going from a k_c^C to k_c^H ratio of 1 to 1000.

5 Effect of membrane permeability to H_2CO_3

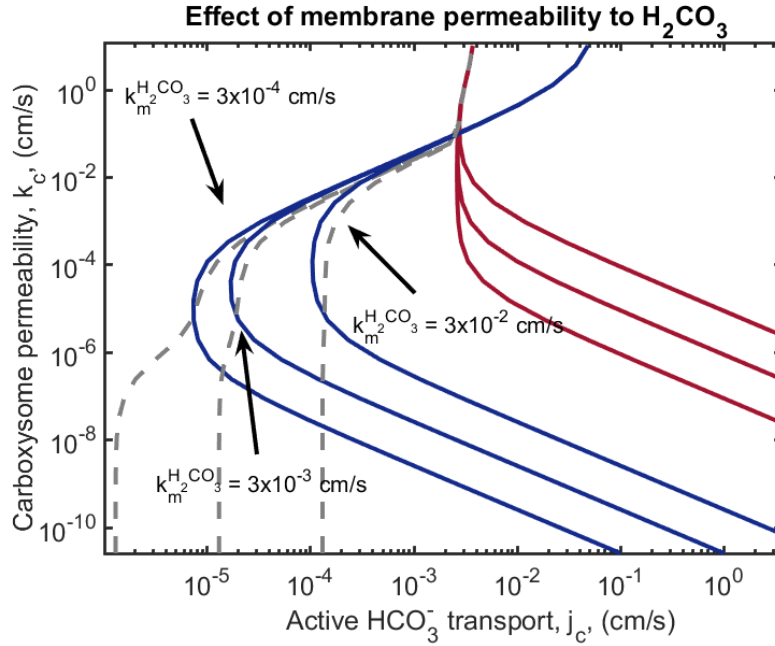


Figure 5: CCM functionality space for varying membrane permeability to H_2CO_3 , $k_m^{\text{H}_2\text{CO}_3}$. Blue lines indicate where RuBisCO is saturated. Red lines indicate where carbonic anhydrase is saturated. Grey dashed lines indicate where the HCO_3^- pool in the cytosol is 30mM. Each set of lines correspond to values $k_m^{\text{H}_2\text{CO}_3} = 3 \times 10^{-4}, 3 \times 10^{-3}, 3 \times 10^{-2}$ from right to left.

The sensitivity of the cost to our assumption for the value of the membrane permeability to H_2CO_3 can be determined from the equation derived previously.

If we are in a regime where CO_2 leakage is negligible, as is the regime presented in the main paper, the second line holds.

$$\begin{aligned} j_c H_{out} &= \left(\frac{R_c^3}{3R_b^2} V_{max} - k_m^C (C_{out} - C_{cytosol}) - k_m^{eff}(pH_{out})H_{out} + k_m^{eff}(pH_{in})H_{cytosol} \right) \\ &= \left(\frac{R_c^3}{3R_b^2} V_{max} - k_m^{eff}(pH_{out})H_{out} + k_m^{eff}(pH_{in})H_{cytosol} \right) \quad (21) \end{aligned}$$

In this equation $k_m^{eff} = k_m^{H_2CO_3} \times 10^{(pK_1 - pH)}$. Therefore, the leakage of H_{total} out of the cell will depend linearly on what we assume for $k_m^{H_2CO_3}$. This linear dependence is passed on to the active HCO_3^- transport required to replenish the leaked inorganic carbon, and therefore onto the CCM cost. In Figure XX you can see this effect, where going from $k_m^{H_2CO_3} = 3 \times 10^{-2}$ to $k_m^{H_2CO_3} = 3 \times 10^{-3}$ (an order of magnitude change), decreases the active HCO_3^- transport needed by an order of magnitude. Decreasing to $k_m^{H_2CO_3} = 3 \times 10^{-4}$ is a little less than an order of magnitude, indicating that the linear dependence breaks down and CO_2 leakage would become important for that value. There is also an order of magnitude change in the optimal carboxysome permeability from 10^{-4} to 10^{-5} across the 2 order of magnitude change in $k_m^{H_2CO_3}$ we are checking.

6 Selection and effect of cytosolic HCO_3^- pool size

The HCO_3^- cytosolic pool we assume in our cost calculation has a large effect on the absolute values for the cost calculation. The dependence of HCO_3^- transport required to support a given internal cytosolic pool can also be seen in the equation in the previous section.

In Figure XX the active HCO_3^- transport and carboxysome permeability values required to achieve a particular cytosolic pool are shown. For the RuBisCO half-max values assumed in the main text $K_m = 276 \mu\text{M}$ at internal pH 8, internal HCO_3^- cytosolic pools between 20 and 50 mM are required to saturate RuBisCO. For lower K_m values, lower cytosolic pools would be required. Additionally, as the K_m values are pH dependent, the internal pH of the cell will effect when saturation takes place.

It has recently been suggested that cytosolic HCO_3^- pools of around 5-10 mM can saturate RuBisCO. As was discussed in Whitehead et al., this is only possible with our given understanding of the CCM mechanism if the pH in the carboxysome is lower than the rest of the cell, or if the carbonic anhydrase does not act to bring CO_2 and HCO_3^- into equilibrium. Either of these possibilities seems physically questionable given our current understanding of the diffusion rate of protons and the mechanism of carbonic anhydrase – its speed is considered linked to its lack of directionality.

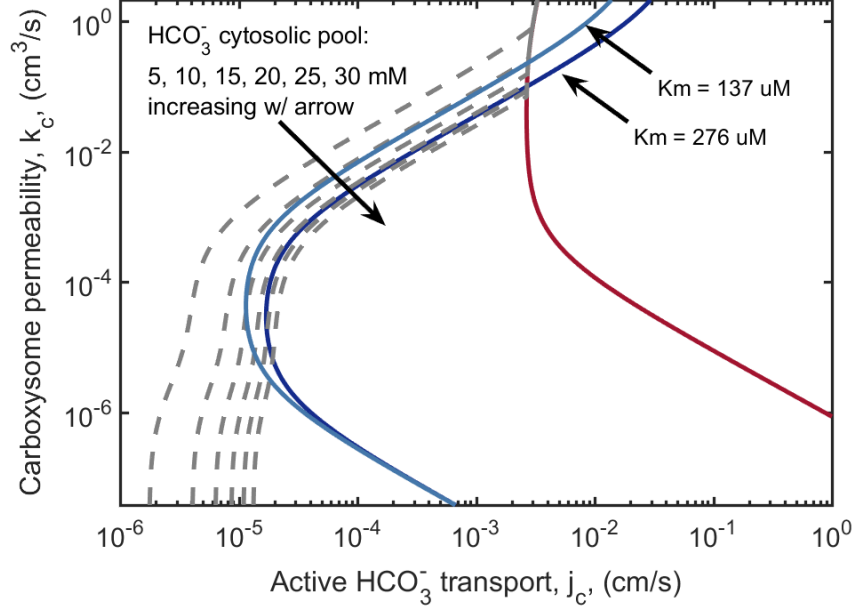


Figure 6: CCM functionality space showing carboxysome permeability and HCO_3^- transport rates resulting in varying cytosolic HCO_3^- pool. Grey dashed lines indicate where the HCO_3^- pool in the cytosol is 5, 10, 15, 20, 25, and 30 mM. Blue lines indicate where RuBisCO is saturated. Red lines indicate where carbonic anhydrase is saturated.

7 When can CO_2 scavenging have an effect

Next we examine for which parameter regimes $\text{CO}_2 \rightarrow \text{HCO}_3^-$ conversion activity acts as facilitated uptake, resulting in a flux of CO_2 into the cell, or scavenging, reducing the leakage of CO_2 out of the cell. As written, the conversion activity does not discriminate between CO_2 that has recently diffused into the cell and CO_2 which has already been in the carboxysome, but leaked back out. In other words, we do not "trace" the history of each CO_2 molecule, or even model each CO_2 molecule explicitly as we treat concentrations. Therefore, we must think about the net CO_2 flux at the cell membrane.

When there is a net flux of CO_2 out of the cell, there is by definition, no facilitated uptake. If there is zero net flux of CO_2 at the cell membrane, then the cell is scavenging 100% of the CO_2 leaking out of the carboxysome. If there is net flux of CO_2 into the cell, then not only is the conversion mechanism scavenging all CO_2 leaking out of the cell, but also reducing the cytosolic CO_2 concentration below the external concentration and facilitating uptake.

From Figure XX it is evident that increasing $\text{CO}_2 \rightarrow \text{HCO}_3^-$ decreases the

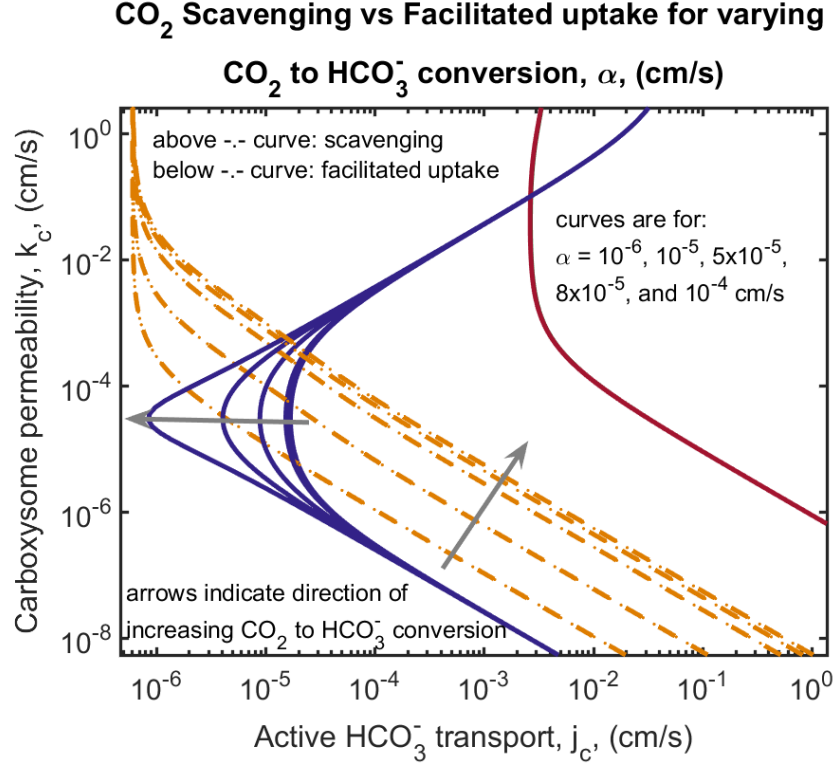


Figure 7: Effect of CO₂ → HCO₃⁻ conversion on the active transport vs carboxysome permeability plot. RuBisCO is unsaturated to the left of the blue line and saturated to the right. Carbonic anhydrase is unsaturated to the left of the red line and saturated to the right. Orange dashed lines show where the CO₂ flux across the membrane is zero. Above/to the right of an orange dashed line there is net CO₂ leaking out of the cell, and the main function of the conversion is scavenging. Below and to the left of an orange line the net CO₂ flux at the cell membrane is into the cell, and conversion is both scavenging and facilitating CO₂ uptake. Multiple lines are for increasing CO₂ → HCO₃⁻ conversion strength in the direction of the grey arrows.

need for active HCO₃⁻ uptake at optimal carboxysome permeability. We can also determine when the conversion mechanism switches from being primarily a scavenging mechanism to also acting as facilitated uptake. The orange dashed lines show the active HCO₃⁻ transport rate for each carboxysome permeability k_c value where the net CO₂ flux at the cell membrane is zero. Above this line the system is scavenging and below it the system is scavenging and causing facilitated uptake. We have plotted these curves for increasing CO₂ → HCO₃⁻ conversion strengths.

For $\alpha = 10^{-6}$ cm/s, the system is not facilitating uptake of CO_2 for the optimal permeability. At this point the minimum active HCO_3^- transport rate required to saturate RuBiSCO is approximately the same as without the CO_2 conversion mechanism. For higher α the conversion system is scavenging at the optimal carboxysome permeability. In this case the system requires less HCO_3^- transport because CO_2 conversion is acting as facilitated uptake, and contributing to the internal inorganic carbon pool. Note that the absolute flux from scavenging and facilitated uptake will vary drastically over this space.

In the main text we showed that the loss of CO_2 from leakage out of the cell is about an order of magnitude smaller than HCO_3^- leakage without CO_2 conversion. When we are at optimal carboxysome permeability, scavenging will have little effect, because there is just not very much CO_2 in the cytosol to scavenge. At higher carboxysome permeabilities though, the scavenging mechanism could compensate and matter much more. Thus the relative importance of the CO_2 conversion mechanism for scavenging vs facilitated uptake depends highly on the yet experimentally undetermined carboxysome permeability.

To identify when facilitated uptake starts, we can find for what CO_2 concentration in the carboxysome the flux at the cell membrane is zero, or when $C_{\text{cytosol}} = C_{\text{out}}$.

$$\text{CO}_2 \text{ flux} = k_m^C (C_{\text{cytosol}}(R_b) - C_{\text{out}}). \quad (22)$$

We do not know enough about the $\text{CO}_2 \rightarrow \text{HCO}_3^-$ mechanism to estimate a cost in the way we have done for HCO_3^- uptake. Therefore we cannot assess the effect of conversion on the cost.