

## Full length article

## Fast convergence optimization model for single and multi-purposes reservoirs using hybrid algorithm



Mohammad Ehteram<sup>a,\*</sup>, Sayed-Farhad Mousavi<sup>a</sup>, Hojat Karami<sup>a</sup>, Saeed Farzin<sup>a</sup>, Mohammad Emami<sup>a</sup>, Faridah Binti Othman<sup>b</sup>, Zahra Amini<sup>c</sup>, Ozgur Kisi<sup>d</sup>, Ahmed El-Shafie<sup>b</sup>

<sup>a</sup> Department of Water Engineering and Hydraulic Structures, Faculty of Civil Engineering, Semnan University, Semnan, Iran

<sup>b</sup> Department of Civil Engineering, Faculty of Engineering, University of Malaya, Malaysia

<sup>c</sup> Department of Civil Engineering, Faculty of Engineering, University Technology Malaysia, Kuala Lumpur, Malaysia

<sup>d</sup> Center for Interdisciplinary Research, International Black Sea University, Tbilisi, Georgia

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## ABSTRACT

Developing optimal operation policy for single or multi-purposes dams and reservoirs is a complex engineering application. The main reasons for such complexity are the stochastic nature of the system input and slow convergence of the optimization method. Furthermore, searching optimal operation for multi-purposes or chain reservoir systems, becomes even more complex because of interfering operations between successive dams. In this study, a new hybrid algorithm has been introduced by merging the genetic algorithm (GA) with the krill algorithm. In fact, the proposed hybrid algorithm amalgamates the advantages of both algorithms, first, the ability to converge fast for global optimum and, second, considering the effect of stochastic nature of the system. Three benchmark functions were used to evaluate the performance of this proposed optimization model. In addition, the proposed hybrid algorithm was examined for Karun-4 reservoir in Iran as an example for a hydro-power generation dam. Two benchmark problems of hydropower operations for multi-purposes reservoir systems, namely four-reservoir and ten-reservoir systems were considered in the study. Results showed that the proposed hybrid algorithm outperformed the well-developed traditional nonlinear programming solvers, such as Lingo 8 software.

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## 1. Introduction

Several dams and reservoirs have been constructed around the world for better operation of available water for irrigation, domestic and industrial uses, hydropower generation and flood mitigation. Dams and reservoirs are the hydraulic structures designed for storing and regularly releasing water to meet the downstream water needs based on the operator's decisions [1,2,20]. Despite the need for dams and reservoirs to match the increasing of the water demands, their operation is a highly challenging task to achieve. In fact, optimizing the operation of the existing dams and reservoirs is essential to maximize their benefit and to cope with present and future water demands [1]. In most cases, the decision-makers for dams and reservoirs depend on their experience to decide the appropriate timing and amount of water release. The main challenge related to dam and reservoir operations is that the release decisions should be made in light of the system's physical

constraints, including the stochastic nature of system parameters [3,21].

Over the last few decades, water resources managers have given serious attention to optimizing the operation policies of dams and reservoirs. Several optimization methods of this complex engineering application have been introduced due to improvements in analytical and computer technology [4]. Recently, evolutionary algorithms and other metaheuristics have been employed to achieve optimal operation and sustainable water resources management solutions for dam and reservoir operation [5]. In fact, evolutionary algorithms are iterative search strategies enclosing the following phases: consideration and explanation of decision variables and constraints; selection of the decision variables and corresponding values; computation of objectives and constraints for the selected decision variable values [6]. Furthermore, a simulation process is repeated and the set of decision values is updated until the values satisfy the selected stopping criteria; and the optimal solutions are obtained by a decision-making process [6].

Various metaheuristic methods have also been used to solve the optimization problems. Oliveira and Loucks [7] used a genetic

\* Corresponding author.

E-mail address: [eh.mohammad@yahoo.com](mailto:eh.mohammad@yahoo.com) (M. Ehteram).

algorithm (GA) to develop optimal water release policies for the complex dam and reservoir systems. Furthermore, the honey-bee mating algorithm (HBMA) and bat algorithm (BA) have been investigated as alternative methods for searching optimal water release decisions [8,9]. A further step towards long-term optimization has been taken in the multi-tier interactive GA [10]. GA has been modified by integrating it with a stepwise simulation model to solve large-scale dam and reservoir systems that include up to 16 dams [11]. Bozorg Haddad et al. [12] used water cycle algorithm to determine reservoir policies for the optimal operation of Karun-4 reservoir in Iran. They also compared the results of the water cycle algorithm with those obtained with GA and nonlinear programming method.

## 2. Problem statement and objectives

The above-mentioned optimization algorithms have several advantages and disadvantages. The major advantage in these algorithms is their ability to be adjusted to include several nonlinear systems in parallel or series with different constraints and objective functions. The major disadvantage is the difficulty of addressing the stochastic pattern of the system parameters, slow convergence and lack of ability to distinguish the optimal global solutions. In this context, this study introduces an improved krill algorithm in combination with GA to address the stochasticity pattern and improve the ability of the search procedure to return global optima with relatively faster convergence. The krill algorithm was first introduced in 2012 by Gandomi and Alavi [13]. It is a novel biological-based algorithm that considers time-independent parameters while searching for the optimal solution. In the krill algorithm, time interval can be fine-tuned to reflect the stochastic behavior of the system parameters, which is a unique advantage over other algorithms [14]. However, the krill algorithm is slow to converge, especially when applied to a complex stochastic system such as dam and reservoir system.

In this study, a proposal for improvement of krill algorithm has been made. It is necessary to integrate differential evaluation for the global numerical optimization to accelerate the convergence procedure. The proposed hybrid algorithm in the present research is different from standard krill algorithm in a few aspects. The major function of GA is to assure the uniformity of krill population by finding a starting candidate solution without prior knowledge of the solution. Such integration for both algorithms guarantees fast convergence and avoids trapping in local optima.

In this study, the proposed hybrid algorithm was introduced to investigate its ability to optimize single and multi-purposes dams and reservoirs operation. The efficiency and reliability of the proposed hybrid algorithm were first verified using three mathematical benchmark functions. Then, its performance was evaluated by using real case studies of dam and reservoir applications. These case studies are well-known which were selected primarily by other researchers to introduce a comparative analysis on the performance of the proposed hybrid algorithm and previously-developed optimization models.

## 3. Materials and methods

### 3.1. Krill algorithm

The krill algorithm is based on krill's food search behavior. The shortest distance of each krill from both the food and the center of krill community is taken as the target function for krill's movement.

In the krill optimization algorithm, krill movement is categorized by three factors:

1. The motion created by other organisms,
2. Food-finding behavior, and
3. Random distribution.

Krill swarm is aiming at increasing density and finding more food. Krill attraction to high-density locations is considered as the target function. In natural systems, the fitness of every creature is a combination of distance from food and the concentration in the krill swarm. In multidimensional spaces, the algorithm should be able to search multiple dimensions. Therefore, the following Lagrangian model is used for decision making in multidimensional space:

$$\frac{dX_i}{dt} = N_i + F_i + D_i \quad (1)$$

where  $N_i$  is the motion made by other creatures,  $F_i$  is the food-finding movement, and  $D_i$  is the physical distribution. Krill movements are explained as follows:

- Movements of other creatures: According to theory, krill tries to move towards the density center. The  $\alpha_i$  movement direction is approximated through the local density swarm, the swarm movement destination, and the factors avoided by the swarm.

This movement is shown as:  $\omega_n$

$$N_i^{new} = N_{max} \alpha_i + \omega_n N_i^{old} \quad (2)$$

$$\alpha_i = \alpha_i^{local} + \alpha_i^{target} \quad (3)$$

where  $N_{max}$  is maximum speed, and is usually taken 0.01 m/s,  $\omega_n$  is the inertia weight, in the range of zero and one,  $N_i^{old}$  is the last movement,  $\alpha_i^{target}$  is the target direction effect, which is showcased by the best krill. Neighborhood effects are the ratio of how much creatures are attracted to or repelled by certain areas for the local search. Neighbors' effects can be modeled as:

$$\alpha_i^{local} = \sum_{j=1}^{NN} \hat{K}_{ij} \hat{X}_{ij} \quad (4)$$

$$\hat{X}_{ij} = \frac{X_j - X_i}{\|X_j - X_i\| + \varepsilon} \quad (5)$$

$$\hat{K}_{ij} = \frac{K_j - K_i}{k^{worst} - k^{best}} \quad (6)$$

where  $k^{best}$  and  $k^{worst}$  are the best and worst values for krill fitness and  $K_i$  represents the fitness value of the current target function,  $K_j$  is the current neighbor's fitness value,  $X$  represents the corresponding position of the fitness value, and  $NN$  is the number of neighbors. There are several strategies for neighbor selection, one of which is related to the feel distance. Feel distance can be determined by the following equation:

$$d_{s,i} = \frac{1}{5N} \sum_{j=1}^N \|X_i - X_j\| \quad (7)$$

where  $d_{s,i}$  is the feel distance for getting the  $i^{th}$  krill and  $N$  is the krill population. Fig. 1 shows this distance. The five factors in the denominator are derived empirically. According to the above relation, if the distance between two krills is less than the one as yielded by Eq. (7), these two krills are neighbors. The following relation indicates the effect of the best-fitting function:

$$\alpha_i^{target} = C^{best} \hat{K}_{i,best}, \hat{X}_{i,best} \quad (8)$$

where  $C^{best}$  is the most fitting krill impact index. This index is defined on the basis that the solution converges to the global optimum.  $C^{best}$  is calculated as follows:

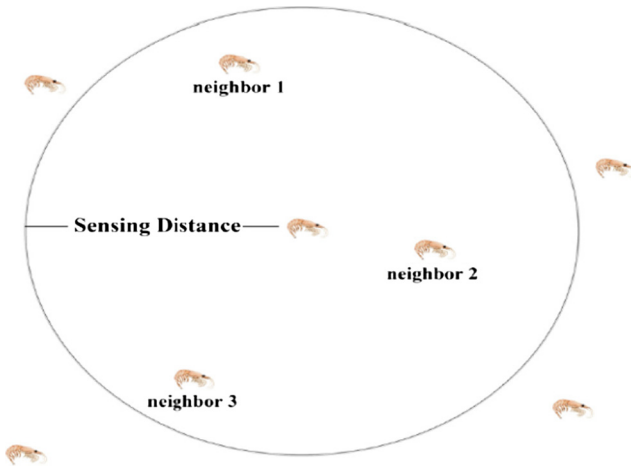


Fig. 1. Sensing distance.

$$C^{best} = 2 \left( rnd + \frac{I}{I_{max}} \right) \quad (9)$$

where  $rnd$  is a random value between zero and one,  $I$  is the number of repetitions and  $I_{max}$  is the maximum number of repetitions.

- Food-finding movement: Food-finding movement is formulated with two main influencing parameters. The first is food position and the second is the previous experience of food position:

$$F_i = V_f \beta_i + \omega_f F_i^{old} \quad (10)$$

$$\beta_i = \beta_i^{food} + \beta_i^{best} \quad (11)$$

where  $V_f$  is the food-finding speed,  $\omega_f$  is the food-finding inertia weight in the range of zero and one,  $F_i^{old}$  is the last food-finding move,  $\beta_i^{food}$  is the appeal of food, and  $\beta_i^{best}$  is the maximum krill fitness. The food effect is defined according to the position of food. First, the food center should be defined and then formulated. This value cannot be determined, but it can be approximated. The food center is approximated according to the following equation:

$$X^{food} = \frac{\sum_{i=1}^N \frac{1}{K_i} X_i}{\sum_{i=1}^N \frac{1}{K_i}} \quad (12)$$

Therefore, attraction to the  $i^{th}$  food is defined as follows:

$$\beta_i^{food} = C^{food} \hat{K}_{i,food} \hat{X}_{i,food} \quad (13)$$

where  $C^{food}$  is the food attraction factor. The food factor can be determined as follows:

$$C^{food} = 2 \left( 1 - \frac{I}{I_{max}} \right) \quad (14)$$

And it is defined as the possibility of krill swarm attraction to the global optimum. The impact of the best fit is defined using the following equation:

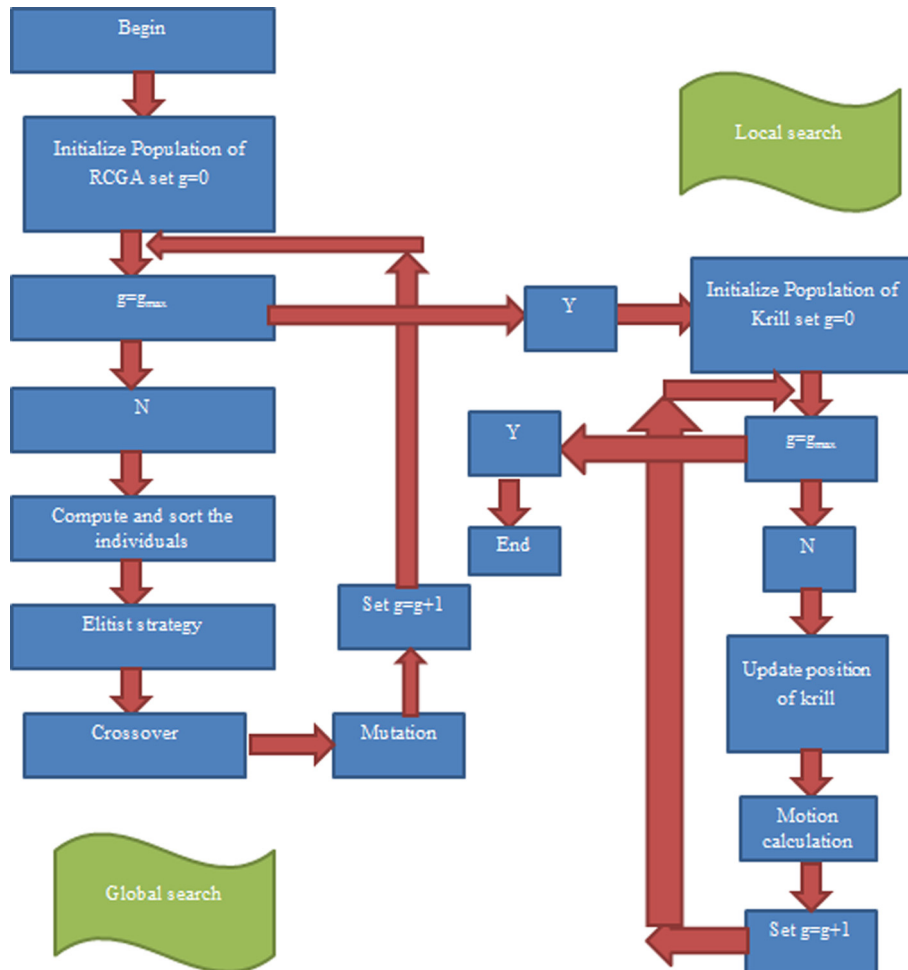


Fig. 2. Hybrid algorithm.

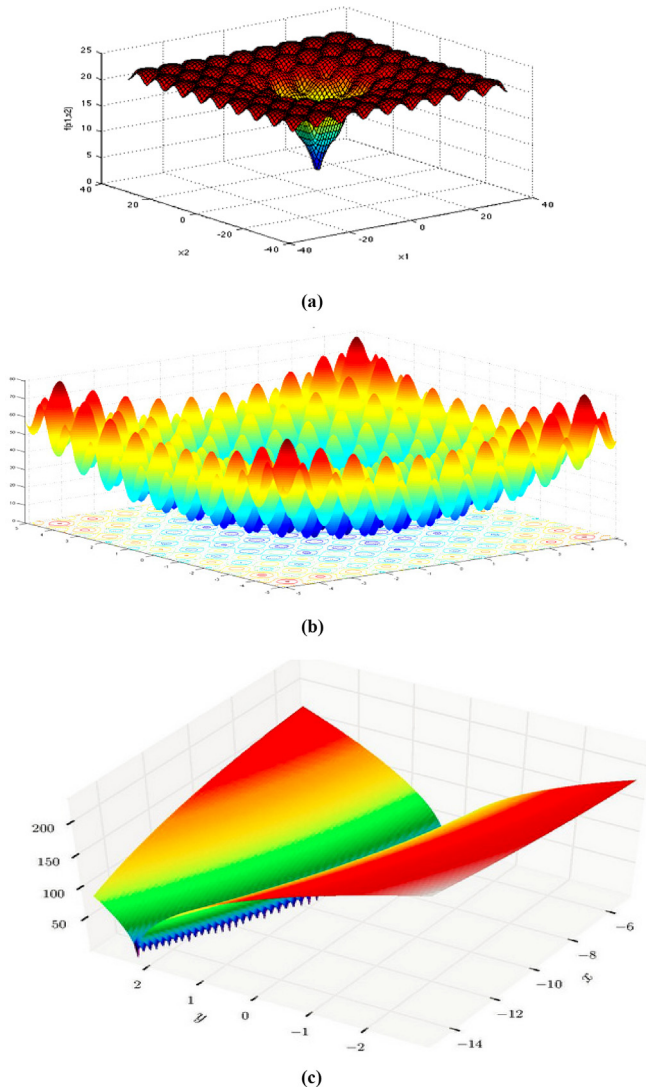


Fig. 3. Mathematical functions for (a) Ackley, (b) Rastrigin and (c) Bulkin-6.

**Table 1**  
Parameters for mathematical functions.

Characteristic	Ackley	Rastrigin	Bulkin-6
Population	50	50	50
Maximum foraging speed ( $V_f$ ) (m/s)	0.02	0.02	0.02
Maximum induced speed ( $N_{\max}$ ) (m/s)	0.01	0.01	0.01
Maximum diffusion speed ( $D_{\max}$ ) (m/s)	0.005	0.005	0.005
Random number (rnd)	0.9	0.9	0.9
Small positive number ( $\epsilon$ )	0.00001	0.00001	0.00001
Number of dimensions	30	2	30

$$\beta_i^{\text{best}} = \hat{K}_{i,\text{best}} \hat{X}_{i,\text{best}} \quad (15)$$

$\hat{K}_{i,\text{best}}$  is the best condition previously seen

- Physical distribution: The physical distribution of krill is a random process. This move can be stated as the highest dispersal speed and random direction vector:

$$D_i = D^{\max} \delta \quad (16)$$

where  $D^{\max}$  is the highest scatter speed and  $d$  is the random direction vector whose array values are random values between  $-1$  and  $1$ . The physical distribution of the vector is random and does not decrease over time. So, another phrase is added to it:

$$D_i = D^{\max} \left( 1 - \frac{I}{I_{\max}} \right) \delta \quad (17)$$

Using different parameters, a krill's position vector can be obtained as:

$$X_i(t + \Delta t) = X_i(t) + \Delta t \frac{dX_i}{dt} \quad (18)$$

$\Delta t$  is one of the most important constants and must be accurately set for optimization problems. The value of this constant is calculated as:

$$\Delta t = C_t \sum_{i=1}^{NV} (UB_j - LB_j) \quad (19)$$

### 3.2. Genetic algorithm

Genetic algorithms (GAs) are one of the well-known methods for solving optimization problems. The chromosomes are considered as the initial population and as a candidate solution. In this method, three operators including a mutation operator, a crossover operator, and a selection operator are considered for solving a problem. The meta-heuristic algorithms are random in nature; so, it is relatively difficult to solve for the best solution in the search space. Thus, if krill method is combined with GA (here is called the hybrid method, as shown in Fig. 2), it can upgrade the GA for local search. The GA explores the search space as a global search, and the krill algorithm searches the space as a local search. Also, the population in GA is regulated based on the krill algorithm. Such procedure causes the krill algorithm to converge faster on the optimization solution. The steps of the algorithm are as follows:

1. Select the initial population. Set the counter number equal to zero ( $g = 0$ ).
2. Control the counter number and if the condition is satisfied, go to step 8.
3. Compute the objective functions for each solution candidate based on its fitness value.
4. The best population in the current generation is determined, and they are designated as elite solutions ( $P_e$  is the population in this section). These elite solutions are copied to be used in the next generation.
5. In this level, several solution candidates from the remaining population,  $P_c$ , are selected. Then, the crossover operator is considered for a combination of each pair.
6. The remaining population is considered, and the mutation operator is applied for solution candidates.
7. Add one unit to counter number and go back to step 2.
8. Obtain the best parameters for the krill algorithm and select the number of solution candidates in this method.
9. Control the counter number. If the condition is satisfied, the algorithm is finished.
10. Motion computation: Compute the motion of the population induced by other individuals; foraging motion; physical diffusion.
11. Update the individual krill positions in the search space.
12. Set  $g = g + 1$ ; and go back to step 9.

### 4. Case studies

#### 4.1. Testing the hybrid algorithm on mathematical functions

To demonstrate and verify the search performance of the hybrid algorithm, selected benchmark test functions (Rastrigin, Ackley and Bulkin-6 functions) were used. In applied mathematics, test





Fig. 4. Karun-4 Dam.

functions, known as artificial landscapes, are useful to evaluate characteristics of optimization algorithms, such as:

- Convergence rate.
- Precision.
- Robustness.
- General performance.

Here some test functions are presented with the aim of giving an idea about the different situations that optimization algorithms have to face when coping with these kinds of problems. The functions listed below are some of the standard functions and datasets used for testing optimization algorithms. Central European Committee introduces the benchmark functions.

The Rastrigin and Ackley functions have one global minimum, whereas the Bulkin-6 function has several local minima, which complicates finding its global optimal solution. For comparison, the krill algorithm and GA were also applied to find the optimal solution of the previously mentioned benchmark function (Fig. 3). Because evolutionary algorithms generally start from a set of random solutions, assessing their ability requires multiple runs (Table 1). The objective is to minimize three objective functions in the following equations. These functions are used to verify the hybrid algorithm. The bulkin-6 function is:

$$f(x_1, x_2) = 100\sqrt{|x_2 - 0.01x_1^2|} + |0.01x_1 + 10| - 15 \leq x_1, \\ \leq -5, -3 \leq x_2 \leq 3 \quad (20)$$

Rastrigin function is:

$$f(x) = \sum_{i=1}^D (X_i^2 - 10 \cos 2\pi x_i + 10) f(x) \\ = \sum_{i=1}^D (X_i^2 - 10 \cos 2\pi x_i + 10) \quad (21)$$

Ackley function is:

$$f(x) = 20 + e - 20e^{0.2\sqrt{\frac{1}{n}\sum_{i=1}^n x_i^2}} - e^{\frac{1}{n}\sum_{i=1}^n \cos(2\pi x_i)} \quad (22)$$

#### 4.2. Karun-4 reservoir

To assess the hybrid method, optimization of power generation capacity of Karun-4 Dam Reservoir (located in the southwest of Iran) was studied (Fig. 4). Maximum and minimum volumes of this reservoir are 2191 and 1410 million cubic meters (MCM), respectively. In addition, the powerhouse capacity is  $1 \times 109$  W. The reservoir was simulated over a 5-year period (1975–1980). The target function is producing the highest amount of energy and electricity while simultaneously reducing water shortages. Eq. (23) was associated with the target function of this case study:

$$\text{Minimize } F = \sum_{t=1}^T \left(1 - \frac{P(t)}{PPC}\right) \quad (23)$$

$$P(t) = \min \left[ \frac{g \times \eta \times r(t)}{PF} \times \left( \frac{h(t)}{1000} \right), PPC \right] \quad (24)$$

$$h(t) = \frac{(H(t) + H(t+1))}{2} - TWL \quad (25)$$

$$H(t) = a + b(t) \times S(t) + c \times (S(t))^2 + d \times S(t)^3 \quad (26)$$

$$S_{\min} \leq S(t) \leq S_{\max} \quad (27)$$

$$R_{\min} \leq R(t) \leq R_{\max} \quad (28)$$

$$S(t+1) = S(t) + Q(t) + R(t) + spill(t) - loss(t) \quad (29)$$

where  $P(t)$  is the electricity produced by the power plant,  $PPC$  is total power plant capacity,  $g$  is gravitational acceleration,  $T$  is a total number of power plant performance periods. In addition, power  $\eta$  is the plant factor,  $h(t)$  is the water head (m),  $H(t)$  is water level in the tank and  $TWL$  is the depth of water. The coefficients are determined via fitting equations, where  $S(t)$  is the reservoir storage,  $I(t)$  is the reservoir inflow (MCM),  $loss(t)$  is the evaporation loss (MCM),  $S_{\min}$  is the minimum storage (MCM),  $S_{\max}$  is the maximum storage capacity (MCM). Furthermore,  $R_{\min}$  is the minimum amount of water released (MCM), and  $R_{\max}$  is the maximum amount of water released from the reservoir (MCM).

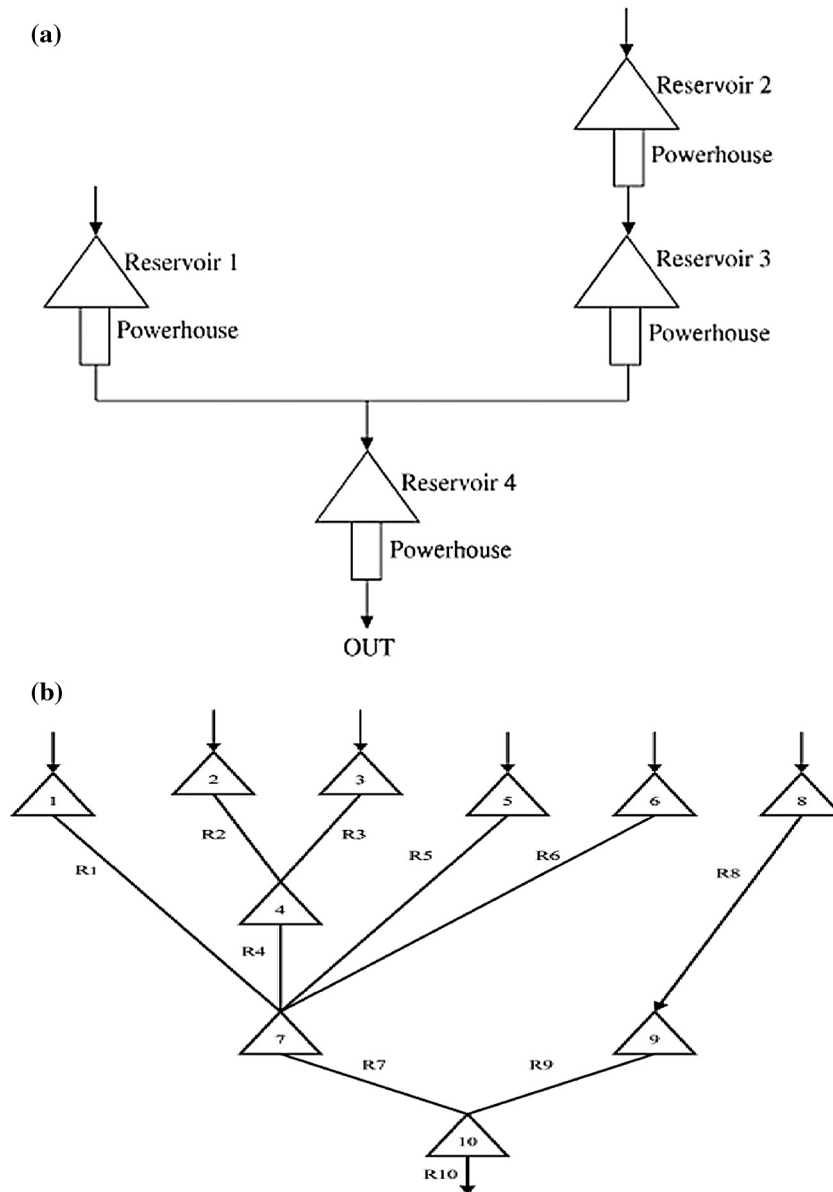


Fig. 5. System reservoir for (a) four-reservoir and (b) ten-reservoir.

#### 4.3. Four-reservoir and ten-reservoir systems

Schematics of four-reservoir and ten-reservoir systems are shown in Fig. 5. The released water from the system is used to generate hydropower. Hydropower generation is possible from each reservoir, and all water discharges pass through turbines. Hydropower benefits are quantified as functions of discharge. The objective is to maximize benefits from the system. There are inflows to the first and second reservoirs only, which include two and three units of hydropower, respectively.

The objective function for the above systems is based on the following equation:

$$\text{Maximize} \sum_{k=1}^K \sum_{t=1}^T (b_t^k \cdot R_t^k) \quad (30)$$

where  $K$  is the number of the reservoirs,  $T$  is the total number of courses,  $b_t^k$  is the reservoir inflow for each period, and  $R_t^k$  is the water released for each period.

The continuity equation is based on the following equation:

$$S_{i+1}^k = S_i^k + Q_i^k - R_i^k \quad (31)$$

where  $S_i^k$  is water storage in different periods,  $Q_i^k$  is inflow to the reservoir, and  $R_i^k$  is the water released.

Constraints related to water release and water supply are as follows:

$$\begin{aligned} S_{\min}^{(t)k} &\leq S_{i,t}^k \leq S_{\max}^{(t)k} \\ R_{\min}^{(t)k} &\leq R_{i,t}^k < R_{\max}^{(t)k} \end{aligned} \quad (32)$$

where  $S_{\min}^k$  is a minimum quantity of water storage,  $S_{\max}^k$  is maximum water storage,  $R_{\min(t)}^k$  is minimum water release,  $R_{\max(t)}^k$  is maximum water release. Here, the initial water storage values are 8, 6, 6, and 6 MCM and in the final stage are 8, 6, 6, and 6 MCM.

For the ultimate target function, a penalty function is used as follows:

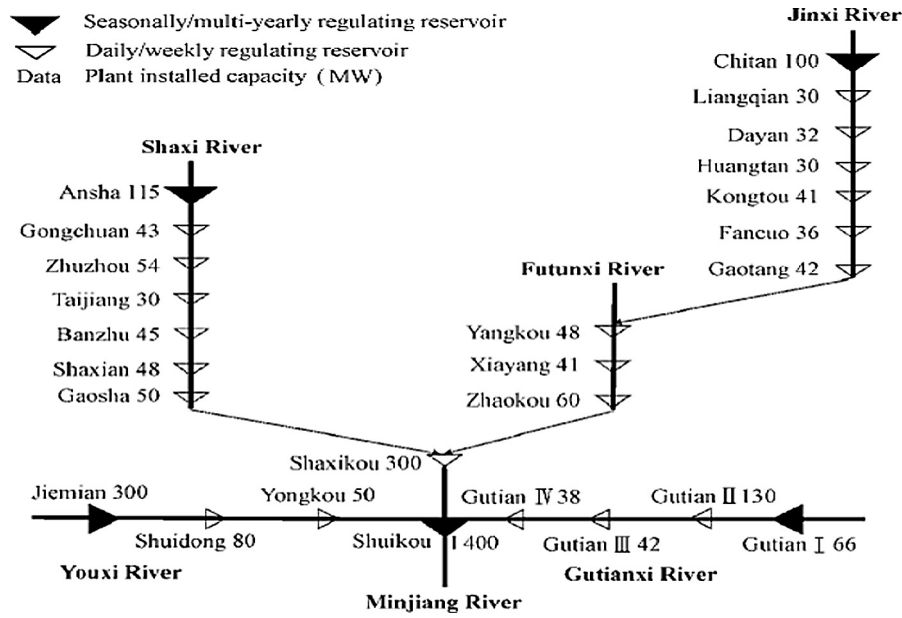


Fig. 6. Cascade reservoirs.

**Table 2**  
Results for mathematical functions.

Run number	Ackley			Rastrigin			Bulkin-6		
	GA (10 <sup>-5</sup> )	Krill (10 <sup>-6</sup> )	Hybrid (10 <sup>-9</sup> )	GA (10 <sup>-3</sup> )	Krill (10 <sup>-4</sup> )	Hybrid (10 <sup>-8</sup> )	GA (10 <sup>-3</sup> )	Krill (10 <sup>-4</sup> )	Hybrid (10 <sup>-5</sup> )
1	2.23	1.12	1.02	3.26	3.11	2.23	1.44	1.12	1.06
2	1.24	2.34	1.21	4.12	4.12	2.24	1.78	2.23	1.11
3	2.34	2.25	1.54	4.01	3.11	2.24	3.87	2.78	1.12
4	4.12	2.78	1.78	5.34	3.14	2.24	4.54	3.87	1.08
5	5.25	3.84	1.65	6.21	4.23	2.23	3.78	2.96	1.12
6	6.12	4.12	1.21	5.34	3.12	2.23	2.25	3.01	1.06
7	5.23	3.76	1.78	4.45	3.11	2.23	2.12	3.12	1.06
8	2.12	3.35	1.78	3.75	3.12	2.24	5.64	3.25	1.06
9	4.11	3.87	1.21	3.35	4.01	2.24	7.87	3.87	1.06
10	3.12	2.87	1.65	3.12	3.12	2.24	7.34	3.94	1.06
Worst	6.12	4.12	1.78	6.21	4.23	2.24	7.87	3.94	1.12
Best	1.24	1.12	1.02	3.12	3.11	2.23	1.44	1.12	1.06
Average	3.588	3.55	1.48	4.29	3.41	2.23	4.05	3.01	1.07
Standard deviation	1.49	1.00	0.25	0.84	0.53	0.008	2.01	0.56	0.014
Coefficient of variation	0.42	0.28	0.16	0.19	0.15	0.003	0.49	0.43	0.012

$$\begin{aligned}
 p_1 &= k_1 [S_{T+1} - S_{\text{target}}]^2 \leftarrow \text{if } (S_{T+1}) < S_{\text{target}} \\
 p_2 &= k_2 [S_{\min} - S_t] \leftarrow \text{if } (S_t) < S_{\min} \\
 p_3 &= k_3 [S_t - S_{\max}] \leftarrow \text{if } (S_t) > S_{\max} \\
 p &= p_1, p_2, p_3
 \end{aligned} \quad (33)$$

where  $p$  is penalty functions,  $k_1, k_2, k_3$  are penalty constants. In the end, the objective function is based on the following equation:

$$\text{Maximize } \sum_{t=1}^k \sum_{t=1}^T (b_t R_t^k) + \sum_{t=1}^k \sum_{t=1}^T p \quad (34)$$

#### 4.4. Chain reservoirs system

The chosen system for this study is located in Fujian, near Beijing, China. The study area is about 60,922 km<sup>2</sup> and contains five rivers: Shaxi, Jinxi, Futunxi, Youxi, and Gutianxi. The reservoir includes 26 power stations with a total capacity of 3250 MW. Fig. 6 depicts the details of the cascade reservoirs. About 60% of the inflows was used in this study.

The objective function is as follows:

$$E = \max \sum_{j=1}^n \sum_{i=1}^n P_{ij} \Delta \tau \quad (35)$$

In the above equation,  $\Delta \tau$  is time step for each level,  $E$  is total electricity production, and  $P_{ij}$  is the average output of the 26 power stations (Fig. 6).

The constraints are:

$$S_{ij+1} = S_{ij} + (I_{ij} - O_{ij} - O'_{ij}) \quad (36)$$

$$\sum_{i=1}^N P_{ij} \geq P_j^{\min} \quad (37)$$

$$S_{ij}^{\min} \leq S_{ij} \leq S_{ij}^{\max} \quad (38)$$

$$O_{ij}^{\min} \leq O_{ij} \leq O_{ij}^{\max} \quad (39)$$

$P_j^{\min}$  is the minimum output of the system,  $S_{ij}$  is the initial volume of the reservoir at any period,  $I_{ij}$  is inflow to the reservoirs,  $O_{ij}$  is outflow from the reservoirs, and  $O'_{ij}$  is overflow from the reservoirs.

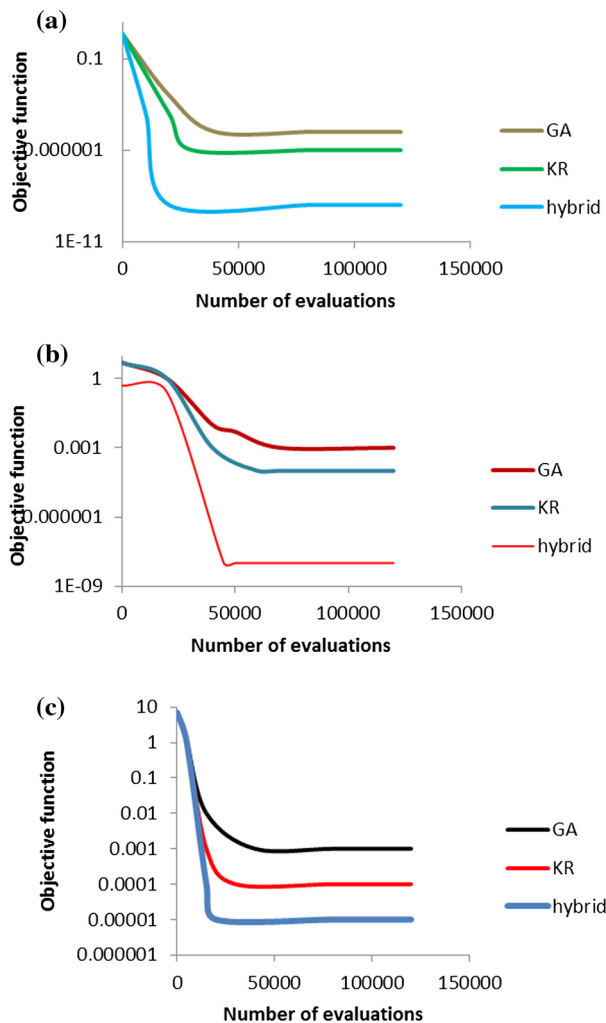


Fig. 7. Optimization of the objective functions for (a) Ackley, (b) Rastrigin and (c) Bulkin-6.

## 5. Results and discussions

The proposed hybrid algorithm was adjusted and applied to the benchmark functions and the real case studies. The following subsections present the results obtained from the hybrid algorithm

and compare them with the results of the nonlinear model, the genetic algorithm, and the krill algorithm.

### 5.1. Benchmark functions

Table 1 shows the parameters used in the calculation of the proposed benchmark mathematical functions shown in Fig. 3. The functions were evaluated 120,000 times. Table 2 shows the results for ten different runs in terms of the best, the worst, and average values for each function, the coefficient of variation, and standard deviation for GA, krill algorithm, and hybrid method. Results showed that the hybrid method had smaller standard deviation and coefficient of variation compared to the krill method and GA. In addition, a significant reduction in the best, worst, and average values could be reached for all functions using the hybrid algorithm. Fig. 7 presents a comparison of the three algorithms for the whole evaluation domain of “120,000”. It can be observed that the hybrid method showed faster convergence and more accurate results compared to the krill and GA.

### 5.2. Karun-4 reservoir

This case study was considered as an example of a single dam and reservoir problem. In fact, the major purpose of this case study was to maximize the hydropower generation of such systems, or in other words, to minimize the deficit according to the initial installation capacity. The krill algorithm and GA were executed in order to search for the optimal operation, and the results were compared with those of the hybrid algorithm. The optimal operation of this dam system has been explored in the last couple of years [6,9]. Several optimization methods have been examined to optimize the operation based on the objective function, which minimizes the deficit between the required release of water and the installation capacity of the hydropower unit. According to the previous achievements, a dimensionless value of 1.213 for the objective function was the global optimum value, which was achieved utilizing the nonlinear programming (NLP) method with Lingo 8 [14–16]. This solution was considered the reference value for examining the ability of other proposed algorithms to achieve this global optimal solution. Table 3 illustrates all the previously attained results in the literature and the proposed algorithms as well. According to Table 3, the best solution for this case study is 1.223, which was computed by Bozorg Haddad et al. [6] using the biogeography-based optimization (BBO) algorithm, with

Table 3  
Different results obtained for the Karun-4 Reservoir.

Study	Method	BO <sup>a</sup>	Percentage <sup>b</sup>	WO <sup>c</sup>	NFE <sup>d</sup>
Bozorg Haddad et al. [9]	GA	1.5124	79	1.96	70,070
	NLP	1.2131	100	–	70,070
	BA	1.239	97	1.254	70,070
Bozorg-Haddad et al. [12]	GA	1.531	79	1.980	70,000
	NLP	1.2130	100	–	70,000
	BBO	1.2230	98	1.239	–
Hosseini-Moghari et al. [17]	GA	6.196	84	7.338	100,000
	NLP	5.243	100	–	100,000
	ICA	5.586	93	6.999	100,000
	COA	5.246	99	5.7081	100,000
Present study	GA	1.610	84	1.780	50,000
	NLP	1.212	100	–	50,000
	Krill	1.300	93	1.400	50,000
	Hybrid	1.212	100	1.213	50,000

<sup>a</sup> Best optimal value of the objective function.

<sup>b</sup>  $\left(\frac{BO}{CO}\right) \times 100$ .

<sup>c</sup> Worst optimal value of the objective function.

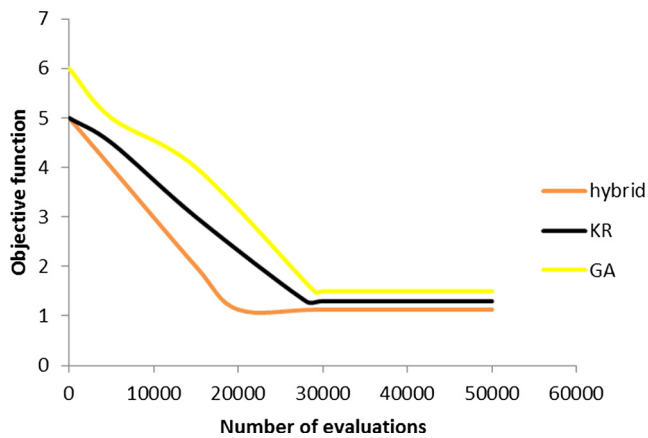
<sup>d</sup> Number of function evaluations.



**Table 4**

Results for Karun-4 Reservoir using Genetic, Krill and Hybrid algorithms.

Run number	Reservoir system		
	GA	Krill	Hybrid
1	1.78	1.30	1.213
2	1.77	1.34	1.212
3	1.85	1.32	1.213
4	1.61	1.40	1.213
5	1.61	1.30	1.212
6	1.61	1.32	1.213
7	1.62	1.33	1.213
8	1.61	1.31	1.213
9	1.61	1.30	1.213
10	1.61	1.30	1.213
Worst	1.78	1.40	1.213
Best	1.61	1.30	1.212
Average	1.66	1.32	1.213
Standard deviation	0.08	0.029	0.0006
Coefficient of variation	0.048	0.028	0.0005
Global solution	1.213		

**Fig. 8.** Convergence curve for Karun-4 Reservoir.

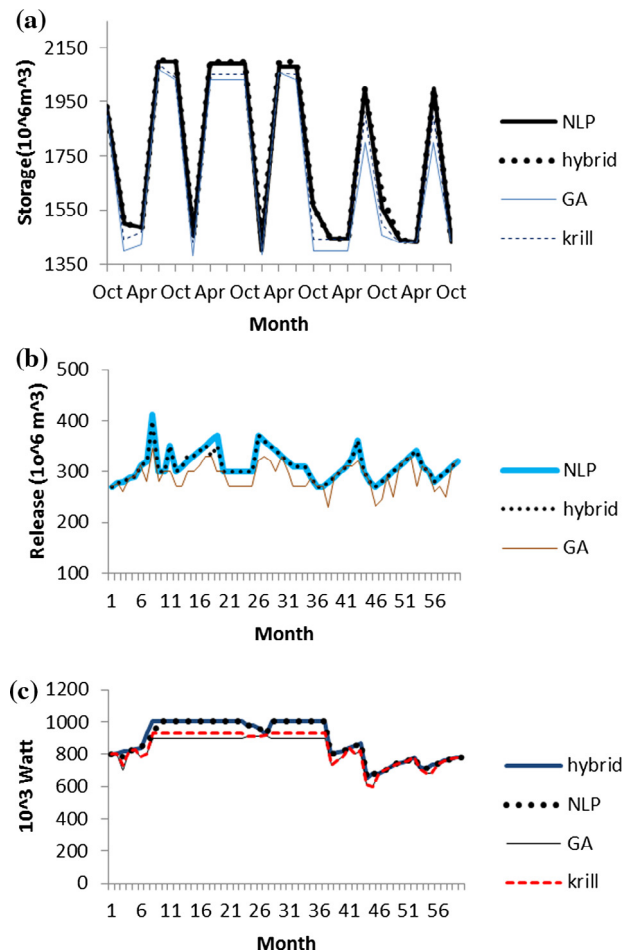
70,000 iterations. The best objective function could be minimized up to the optimal global value while using the proposed hybrid model after 50,000 iterations; in other words, the convergence time was reduced by almost 30%. In addition, Table 3 shows that the values attained for the optimal values of the objective function were 1.610 and 1.30, respectively, for GA and krill algorithm. For further analysis of this case study, Table 4 shows that the best results of the ten different runs computed by the hybrid algorithm could achieve an optimum solution value close to 100%. The amount of variation of the objective function generated from the hybrid algorithm is insignificant. In fact, the smaller the value of the coefficient of variation generated from the model, the higher the ability of the model to reach global optima. It can be observed in Table 4 that the hybrid method shows superior ability to achieve the smallest value for the coefficient of variation in comparison with other algorithms. It is also able to obtain the closest solution to the global optimum. In addition, the optimal objective function values were 93, 99, 93, and 97% for krill algorithm, cuckoo optimization algorithm (COA), the imperialist competitive algorithm (ICA), and bat algorithm (BA), respectively, while it reached 100% using the proposed hybrid algorithm. For comparison of the proposed methods applied in this study, the coefficient of variation achieved by the hybrid algorithm was 56 and 96 times smaller than those obtained by the krill algorithm and GA, respectively.

In order to examine the real performance of the proposed method on the operation of Karun Reservoir, Fig. 8 is presented.

Fig. 8 shows that the hybrid method converged to the global optimal value after 20,000 iterations, while the krill algorithm and GA converged after 30,000 iterations, which means a 33% reduction in convergence time. In fact, the faster the convergence to the global optimal, the more suitable the model is to be used for a real-time operation which is highly needed for operation of the dams and reservoirs. Fig. 8a, b and c shows release, storage, and power-generated patterns, respectively, at each time step of operation of Karun-4 reservoir using all the proposed algorithms of this study. It can be seen from Fig. 9a and b that the physical constraints of the reservoir are met for all algorithms. However, the storage pattern is more stable for the hybrid algorithm and retained a higher storage level during most of the operation time. At the same time, the hydropower generation (as shown in Fig. 9c) is significantly greater than the hydropower generation pattern from other algorithms.

### 5.3. Four-reservoir system

The four-reservoir system was first reported by Chow and Cortes-Rivera [18] as a case study for examining a particular optimization method for multi-reservoir operation. It has been reported that this case study resulted in a solid finding of a reduction in the local optimal solution for the objective function equal to 308.266 [18]. It should be noted here that the major objective of the study of the four-reservoir system was to maximize the benefit of the objective function value, which represents the amount of

**Fig. 9.** Simulation results for: (a) storage (MCM), (b) release (MCM) and (c) generated power (MW).

**Table 5**

Different results obtained for the four-reservoir system.

Study	Method	BO <sup>a</sup>	Percentage	WO <sup>c</sup>	NFE <sup>d</sup> × 10 <sup>6</sup>
Chow and Cortes-Rivera [18]	LP	308.26	99.99	–	–
Murray and Yakowitz [19]	DDP	308.23	99.98	–	–
Bozorg-Hadad et al. [8]	HBMO	308.07	99.92	306.71	1.1
	LP	308.29	100	–	–
Hosseini-Moghari et al. [17]	GA	302.42	98	299.89	0.1
	ICA	306.76	99.5	301.72	0.1
	COA	307.92	99.87	305.62	0.1
Asgari et al. [21]	GA	300.47	97	298.36	1.6
	WOA	308.15	99	306.99	1.6
Present study	NLP	308.29	100	307.12	0.05
	GA	307.12	98	305.45	0.05
	Krill	307.54	99	306.12	0.05
	Hybrid	308.29	100	308.11	0.05

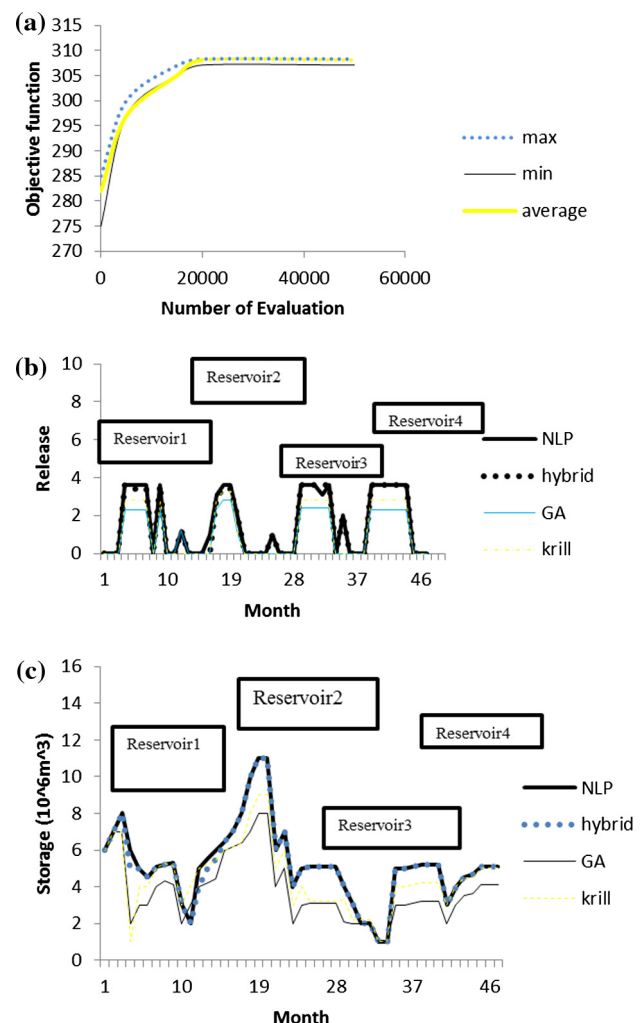
<sup>b</sup>  $\left(\frac{80}{30}\right) \times 100$ .<sup>a</sup> Best optimal value of objective function.<sup>c</sup> Worst optimal value of objective function.<sup>d</sup> Number of function evaluations.**Table 6**

Simulation results for the four-reservoir system using Genetic, Krill and Hybrid algorithms.

Run number	GA	Krill	Hybrid
1	307.12	306.87	308.29
2	305.45	307.54	308.11
3	307.43	307.52	308.29
4	307.04	307.01	308.29
5	306.12	307.23	308.28
6	306.78	307.14	308.27
7	306.74	307.51	308.29
8	306.93	307.43	308.29
9	307.01	307.11	308.29
10	307.00	307.23	308.29
Worst	305.45	306.12	307.11
Best	307.12	307.54	308.29
Average	306.72	307.26	308.17
Standard deviation	0.58	0.22	0.05
Coefficient of variation	0.001	0.0007	0.0001
Global solution	308.29		

hydropower generated. The global optimum of four-reservoir system operation was obtained using Lingo 8, computed by the linear programming method. Table 5 shows the previous research results of searching for the optimal operation in the four-reservoir case study. The optimal objective function values were 99, 99.87, 99.5, 99.92, and 99% for krill algorithm, COA, ICA, honey bee mating optimization (HBMO), and weed optimization algorithm (WOA), respectively. According to Table 5, the best objective function value is 308.26, attained by Chow and Cortes Rivera [17], while the optimal global value is achieved using the proposed hybrid algorithm. Table 6 shows the summary of computed results with GA, krill algorithm, and hybrid algorithm. According to Table 6, the best results of the ten different runs computed by the hybrid algorithm generated an optimum solution close to 100% of the global optimum, while the other two algorithms could only reach relatively small values for global optima, 307.12 and 307.54 for the genetic and krill algorithms, respectively. In addition, a very small coefficient of variation could be achieved utilizing the hybrid algorithm in ten different runs, while the value of the coefficient of variation was 7 and 10 times higher using the krill algorithm and GA, respectively. The convergence of the hybrid algorithm is presented in Fig. 10a. It shows that the hybrid algorithm could reach the global optima with fast convergence (after 20,000 iterations). For further

assessment of the proposed hybrid algorithm, four years of reservoir operation were examined for the stability of the release and storage patterns. Fig. 10b and c shows a stable release and storage pattern during four years of operations with approximate match-

**Fig. 10.** Simulation results for (a) convergence curve, (b) release (MCM) and (c) storage.

**Table 7**

Different results obtained for the ten-reservoir system.

Study	Method	BO <sup>a</sup>	Percentage <sup>b</sup>	WO <sup>c</sup>	NFE <sup>d</sup> × 10 <sup>6</sup>
Murray and Yakowitz [19]	DDP	1190.652 <sup>a</sup>	99.68%	–	–
Wardlow and Sharif [22]	DDP	1190.25	99.64%	–	1.25
Jalali et al. [23]	ACO	1192.30	99.82%	1181.12	0.5
Present study	NLP	1194.44	100%	1193.12	0.5
	GA	1190.01	99.62%	1187.23	0.5
	Krill	1191.23	99.73%	1189.14	0.5
	Hybrid	1194.44	100%	1190.12	0.5

<sup>a</sup> Best optimal value of objective function.<sup>b</sup>  $(\frac{BO}{GO}) \times 100$ .<sup>c</sup> Worst optimal value of objective function.<sup>d</sup> Number of function evaluations.**Table 8**

Simulation results for the ten-reservoir system using Genetic, Krill and Hybrid algorithms.

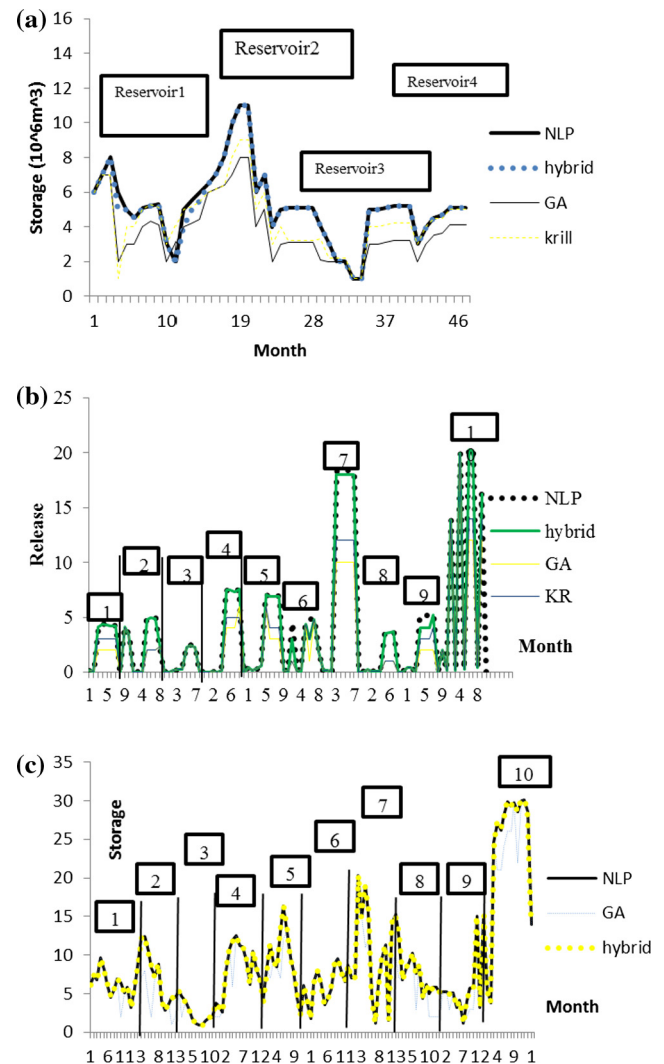
Run number	GA	Krill	Hybrid
1	1187.24	1189.24	1194.01
2	1190.01	1189.34	1194.10
3	1189.24	1189.14	1194.11
4	1188.12	1190.12	1193.12
5	1187.34	1189.32	1194.44
6	1188.11	1189.33	1194.44
7	1189.32	1191.23	1194.32
8	1189.23	1189.45	1194.11
9	1189.11	1189.67	1194.01
10	1189.12	1189.78	1194.01
Worst	1187.34	1189.24	1193.12
Best	1190.01	1190.12	1194.44
Average	1188.68	1189.66	1193.91
Standard deviation	1.13	0.707	0.31
Coefficient of variation	0.009	0.0005	0.0002
Global solution	1194.44		

ing with the pattern generated using NLP, which is the reference optimization operation for this case.

#### 5.4. Ten-reservoir system

Ten reservoirs case study represents a relatively complex optimization problem for dam and reservoir application. This multi-reservoir system problem was investigated by Murray and Yakowitz [19]. The objective function for this case study is to maximize the total hydropower generated from all ten connected reservoirs. It is difficult to reach the optimal value of the objective function because this case study has a very high local optimal value equal to 1190.652. However, a global optimum value of 1194.44 for the objective function in this case study was achieved using Lingo version 8. Table 7 shows the results of the previous studies on the ten-reservoir system. It can be seen from Table 7 that the highest percentage of the objective function relative to the absolute global optimal solution is 99.82%, obtained by Jalali et al. [23] using the ACO method. Alternatively, the hybrid method could reach the absolute global optima after 500,000 iterations, which indicates the ability of the hybrid algorithm to reach the full desired objective function.

In order to substantiate the results obtained by the proposed hybrid algorithm, Table 8 shows the results based on ten different performances. The hybrid method has the lowest standard deviation and coefficient of variation compared to other methods. Fig. 11a, b and c shows the convergence, water release, and storage patterns for the operation of the ten reservoirs using the NLP, GA, krill algorithm, and proposed hybrid algorithm, respectively. It can be seen that the nonlinear programming is almost entirely compat-

**Fig. 11.** Simulation results for (a) convergence curve, (b) release (MCM) and (c) storage (MCM).

ible with the hybrid algorithm, ensuring the outstanding performance of the hybrid algorithm.

#### 5.5. Cascade reservoirs

For the multi-reservoir system operation, the models based on the GA, krill algorithm, and hybrid algorithms were designed and

**Table 9**

Simulation results for the cascade reservoirs.

Hydroelectric power plant	Annual electricity generation (10 <sup>5</sup> MWh)		
	Krill	GA	Hybrid
Ansha	5.26	2.12	8.94
Chitan	6.12	4.56	7.12
Jiemian	1.24	1.12	11.01
Gutian I	2.33	2.12	4.45
Shuikou	70.01	60.25	90.23
Others	52.22	51.23	54.12
Total	137.18	121.4	175.87

evaluated using a case study where it has cascade reservoirs located in the Mingiang Basin, China and there is a power system containing 26 hydroelectric plants. The annual electricity generation obtained for each model is shown in Table 9. In this problem, annual electricity generation of the hybrid algorithm is 30% and 12% more than the GA and krill algorithm. The average times for optimization computations based on the hybrid algorithm, GA, and krill algorithm were 20, 35 and 30 s, respectively.

## 6. Conclusions

This study proposed a hybrid algorithm as an optimization tool for single and multi-dam and reservoir systems. The proposed hybrid algorithm is an integration of the GA and krill algorithm. Such integration overcomes the most common drawbacks of the existing optimization algorithms, which are slow convergence and subjection to local optima. It is vitally important for the optimization models to be able to reach the global optima rapidly. This is due to the fact that dam and reservoir application is a real-time operation with a sensitive objective function because of the highly complex nonlinear behavior of physical constraints of the reservoir system. To examine the proposed hybrid algorithm, several case studies including single, multiple, and cascade dam and reservoirs system were considered. In addition, three different benchmark mathematical functions were investigated. The proposed hybrid algorithm successfully obtained the global optima for all the presented case studies with more rapid convergence time. In addition, the results from the comparative analysis with previous researches applied to the same studies showed that the proposed hybrid algorithm outperformed the other optimization methods regarding its ability to obtain the global optimal and its higher convergence speed. The best solutions achieved by the hybrid algorithm for single Karun-4 Reservoir, the four-reservoir system, and the ten-reservoir system are 1.212, 308.29, and 1194.44, respectively. In brief, the proposed hybrid algorithm could apply to a wider range of real-world optimization applications, taking advantage of the proposed algorithm's fast convergence and high ability to obtain the global optimal solution.

However, and more specifically for dam and reservoir optimization, completing the ultimate optimization of the reservoir application requires an accurate simulation for the reservoir; in other words, an accurate elevation-storage-area formula is needed. This formula is highly nonlinear in most reservoirs. In this context, it is proposed to integrate the hybrid optimization algorithm with a highly nonlinear algorithm such as neural networks (NN) or the adaptive neuro-fuzzy inference system (ANFIS) as a simulation model for reservoirs to mimic their behavior accurately for any release decision.

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