Skewness

Computer Oriented Numerical and Statistical Methods

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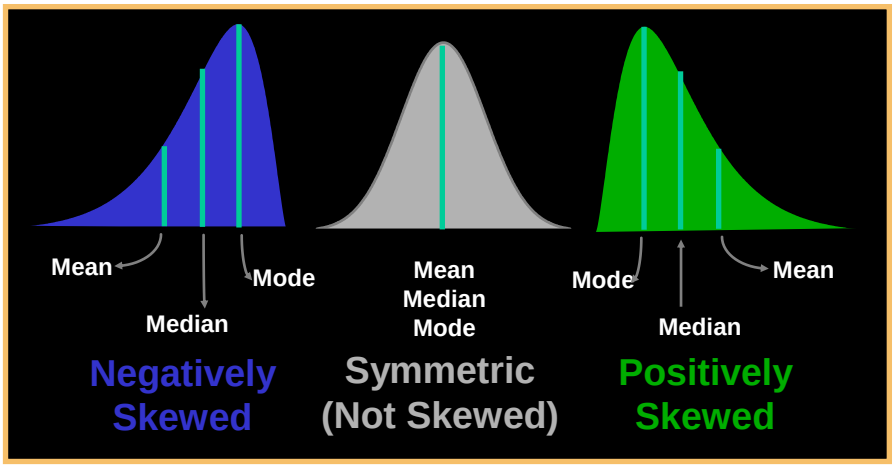
Outline

- Introduction
- Types of Skewness
- Measure of skewness.
- Karl Pearson's Measure
- Bowley's Measure

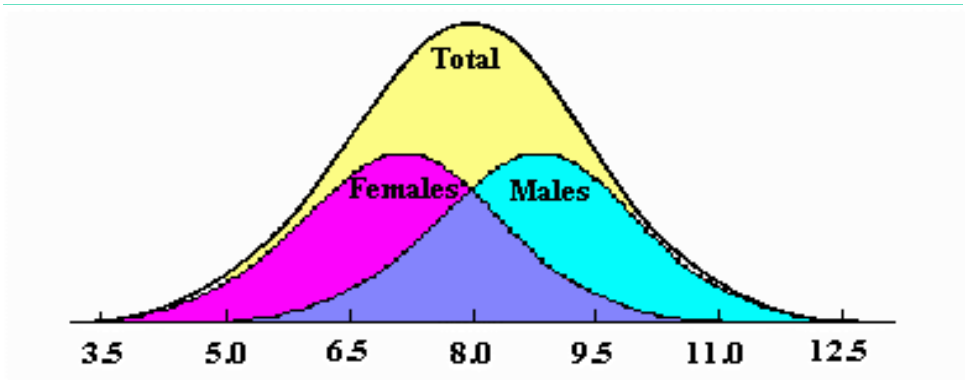
Introduction

- Measure of central tendency gives us an estimate of the representative value of a series, the measure of dispersion gives an indication of the extent to which the items cluster around or scatter away from the central value and the skewness is a measure that refers to the extent of symmetry or asymmetry in a distribution.
- It describes the shape of a distribution,

Skewness



- Teacher expects most of the students get good marks. If it happens, then the cure looks like the normal curve bellow:



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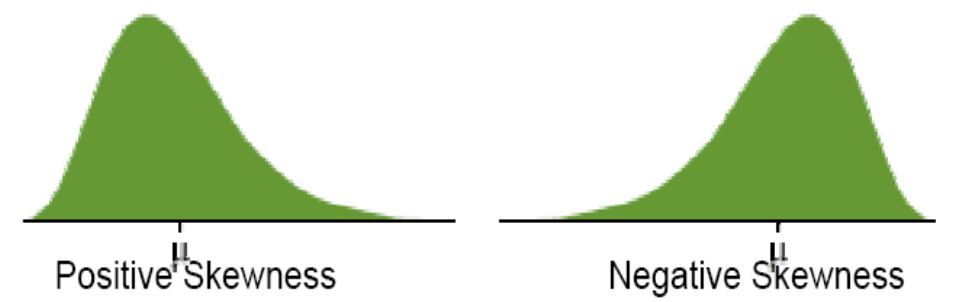
Example

- But for some reasons (e. g., lazy students, not understanding the lectures, not attentive etc.) it is not happening. So we get another two curves.



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Tests of Skewness

- Skewness is present in a distribution if:
  - The value of mean, median and mode do not coincide.
  - When the value are plotted on a graph paper, they do not yield a normal bell-shaped curve, or when divided vertically through the centre of the curve, the two halves are unequal.
  - Quartiles are not equidistance from the median i.e. $Q_3 - M_d$ is not equal to $M_d - Q_1$.
  - The sum of the positive deviations from the median is not equal to the sum of the negative deviation.
  - Frequencies on the either side of the mode are not equal.

Objectives of Skewness

- It helps in finding out the nature and the degree of concentration whether it is in higher or the lower values.
- The empirical relations of mean and median and mode are based on a moderately skewed distribution. The measure of skewness will reveal to what extent such empirical relationship holds good.
- It helps in knowing if the distribution is normal. Many statistical measures, such as the error of the mean are based on the assumption of a normal distribution.

Measures of Skewness

- To find out the direction and the extent of asymmetry in a series statistical measures of skewness are employed.
- This measure can be absolute or relative.
- Absolute measure of skewness tell us the extent of asymmetry and whether it is positive or negative.
- The absolute skewness is based on the difference between mean and mode. Symbolically,

  absolute S_k = Mean – Mode.
- If the value of mean is greater than the mode, skewness will be positive.

Measures of Skewness

- If the value of mean is less than the mode, skewness will be negative.
- Absolute measure of skewness is not adequate because it cannot be used for comparison of skewness in two distributions if they are in different units, since difference between mean and mode will be in terms of units of distribution.
- For comparison purpose we use the relative measure of skewness known as coefficient of skewness.

Measures of Skewness

- There are four types of relative measure of skewness :
  1. **The Karl Pearson's Coefficient of Skewness.**
  2. **The Bowley's Coefficient of Skewness.**
  3. The Kelly's Coefficient of Skewness.
  4. Measure of Skewness based on Moments and Kurtosis.

Karl Pearson's Coefficient of Skewness

- Karl Pearson's Coefficient of Skewness or Pearson Coefficient of skewness is given by the formula:

$$S_k = \frac{\text{Mean} - \text{Mode}}{\text{StandardDeviation}}$$

- If in a particular frequency distribution, it is difficult to determine precisely the mode, or the mode is ill-defined, the coefficient of skewness can be determined by the following formula:

$$S_k = \frac{3(\text{Mean} - \text{Median})}{\text{StandardDeviation}}$$

Bowley's Coefficient of Skewness

- Bowley's coefficient of skewness also known as Quartile coefficient of skewness and is especially useful :
  - When the mode is ill-defined and extreme observations are present in the data.
  - When the distribution has open-end classes or unequal class-interval.
- The quartile measure depends upon the fact that normally Q_3 and Q_1 are equidistance from the median, i.e. for symmetrical distribution $Q_3 - M_d = M_d - Q_1$.
- If a distribution is asymmetrical, then one quartile will be farther from the median than the other.

Bowley's Coefficient of Skewness

- In such a case skewness can be measured by the following formula given by Bowley :

  $$\text{Skewness} = (Q_3 - M_d) - (M_d - Q_1)$$

  $$\text{Skewness} = Q_3 + Q_1 - 2M_d$$
- If the first part is more than the second part, the skewness is positive and in the reverse situation it is negative.

Bowley's Coefficient of Skewness

- To make the measure a readily comparable, the coefficient of skewness is obtained by dividing it by quartile range viz $Q_3 - Q_1$

$$S_k = \frac{(Q_3 - M_d) - (M_d - Q_1)}{(Q_3 - M_d) + (M_d - Q_1)}$$

$$S_k = \frac{Q_3 + Q_1 - 2M_d}{Q_3 - Q_1}$$