COMP0009 Logic
Exercises 9: Relation Algebra.

Robin Hirsch

December 6, 2022

1. Let $\mathcal{A}_2$ be relation algebra with two atoms $\{1', 0'\}$ (so four elements $0, 1', 0'$ and $1 = 1' + 0'$, the identity is $1'$, all elements are self-converse, composition defined by $0; x = x; 0 = 0$ (all $x \in \mathcal{A}_2$), $1'; x = x; 1' = x$, $1; x = x; 1 = 1$ for $x \in \mathcal{A}_2 \setminus \{0\}$, and $0'; 0' = 1'$. Is $\mathcal{A}_2$ representable as a proper relation algebra? If so, what possible sizes could the base be?

2. Let $\mathcal{A}_2'$ be exactly the same as $\mathcal{A}_2$, except $0'; 0' = 1' + 0'$ $(= 1)$. Is $\mathcal{A}_2'$ representable as a proper relation algebra? If so, what possible sizes could the base be?

3. RCC_5 (Region Connection Calculus). A *region* is a subset of $\mathbb{R}^2$ whose boundary is topologically equivalent to a circle. You can draw regions on paper. R is the set of all regions. The *atomic relation* that holds between region r and region s is either

   - identity $(1')$
   - disjoint (d)
   - overlapping (o)
   - properly contained in $(\subset)$, or
   - properly contains $(\supset)$.

   RCC_5 is the proper relation algebra whose base is R, with five atoms $At = \{1', d, o, \subset, \supset\}$, where e.g. $d = \{(r, s) : r, s \in R,\ r \textit{ is disjoint from } s\}$.

   (a) For each atom, find the converse atom.

   (b) How many elements does the relation algebra RCC_5 have?

   (c) Define composition, by writing a composition table for the atoms.

   (d) Consider $RCC5$ as an abstract relation algebra. Take any representation of RCC_5. What can be said about the size of the base of the representation?