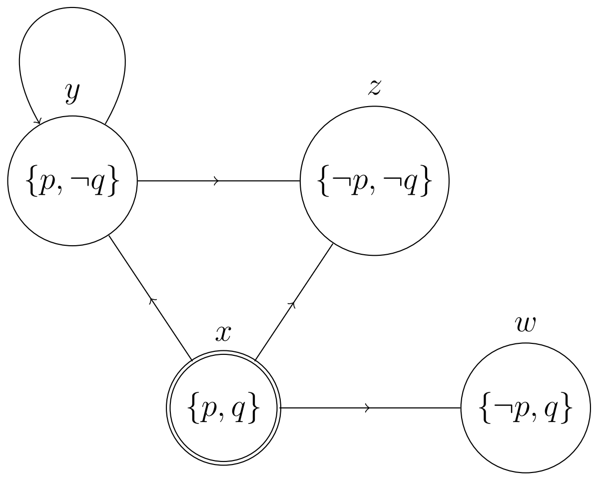
Marks for each part of each question are indicated in square brackets.

Calculators are NOT permitted.

1. a. Consider a Kripke frame with worlds $V = \{x, y, z, w\}$ and edges $E = \{(x, y), (x, z), (x, w), (y, y), (y, z)\}$. Let v be the propositional valuation $v(p) = \{x, y\},\ v(q) = \{x, w\}$.



Which of the following are true?

1. $(V, E), v, x \models \Box p$
2. $(V, E), v, x \models \Diamond p$
3. $(V, E), v, x \models \Diamond(p \wedge q)$
4. $(V, E), v, x \models \Diamond\Box\bot$
5. $(V, E), v, x \models \Diamond(p) \wedge \Box(p \to \Box\neg q)$.

b. Let (V, E) be the Kripke frame above. Which of the following hold?

1. $(V, E) \models (\Box p \to p)$
2. $(V, E) \models (\Box p \to \Box\Box p)$
3. $(V, E) \models (\Box(p \wedge q) \leftrightarrow (\Box p \wedge \Box q))$.

[Question 1 cont. over page]

c. For each formula below use a tableau to find a Kripke model of the formula.

1. $\Diamond p \wedge \Box(p \to \Diamond p)$
2. $\Diamond p \wedge \Box(p \to \Diamond\neg p) \wedge \Box(p \vee \Diamond p)$.

Also, use tableaus to find *transitive* Kripke models for both formulas.

d. A frame (V, E) is *dense* if $(v, w) \in E$ implies there is $u \in V$ such that $(v, u) \in E$ and $(u, w) \in E$. Write down a modal formula ϕ such that for all frames (V, E) we have $(V, E) \models \phi$ if and only if (V, E) is dense. Prove that your formula defines the class of dense frames.