# PCA for dim. reduction of geochemical data

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This notebook imports .csv geochemical data, converts to a data array, and performs dimensional reduction by principal component analysis (PCA) using sklearn singular value decomposition (SVD) based method. The output is then visualised to evaluate any emerging patterns.

To run the notebook on new data seamlessly, the .csv should be structured as follows:

- A row for each datapoint.
- The first column for location (or some other descriptor).
- A column per measured molecule (for geochemical data), or other dimensionality.

If your data is in another format (including file format), the code will need to be amended to suit.

As a prerequisite for running this notebook, we recommend setting up an **anaconda** environment for Python 3 using the following commands:

```
conda create -n geochem python=3.8
conda activate geochem

conda install -c conda-forge scikit-learn
conda install -c conda-forge pandas
conda install -c anaconda matplotlib
conda install -c anaconda seaborn
conda install -c anaconda jupyter

cd "your/file/path/here"

jupyter notebook
```

## Import packages

```
In []: # set matplotlib backend
%matplotlib inline

import matplotlib # visualisation, plotting etc [1]
import matplotlib.pyplot as plt # visualisation, plotting etc [1]
import seaborn as sns # more plotting, better linkage with pandas [2]

import pandas as pd # load .csv and dataframe management [3]
import numpy as np # matrix mathematics [4]

import sklearn as skl # machine learning, i.e. PCA etc... [5]
from sklearn.decomposition import PCA # PCA [5]
```

#### Load data

```
In [ ]: df = pd.read_csv('Plant_Chemistry_Major_and_Trace_Elements.csv') # read data as par
display(df) # display the table
```

Above, we can see all the data as a table. Each row represents a sample, and each column represents a dimension to the data. We want only the numerical data, so we reformat the data as a NumPy array and drop the first column representing location:

```
In [ ]: # create np array of data excluding location
    data = np.array(df.iloc[:, 2 : ]) # df.iloc[EVERY ROW, 3rd COLUMN : to LAST COLUMN
    print(data.shape) # print the shape of the data array
    print(data.dtype) # print the data type
```

All looks well, data has 41 samples and 13 dimensions (i.e., the measured molecules). The data is also in floating point format, which is what is required as an input for scikit-learn PCA.

```
In [ ]: # let's preserve a variable for number of samples (n_row) and number of dimensions n_row, n_col = data.shape # as above, we see data.shape outputs a tuple. So we asso
```

```
In [ ]: # plot data as heatmap
        plt.figure(figsize = (8,6), dpi=600) # create figure
        plt.title('Raw data as heatmap') # set title
        # plot data as image
        im = plt.pcolormesh(data, cmap='viridis', vmin=0, vmax=100) # use vmin and vmax to
        plt.colorbar(mappable=im, ax=plt.gca(), label='weight %') # add colorbar
        # get current axis
        ax = plt.gca()
        ax.set_box_aspect(1) # alter aspect ratio
        # x ticks settings
        x_tcks = np.arange(0, n_col, 1) # create an array of 1 to n dimensions
        x_{tck_labels} = []
        for comp in df.iloc[:,2:].columns: # use compositions as labels from data frame col
            x_tck_labels.append(comp)
        ax.set xticks(x tcks) # create x tick for each dimension
        ax.set_xticklabels(x_tck_labels, rotation = 90, ha = 'left') # add Label for each
        # y tick settings
        y_tcks = np.arange(0, n_row, 1) # create an array of 1 to n samples
        y_tck_labels = df.iloc[:, 0]
        ax.set_yticks(y_tcks) # create x tick for each dimension
        ax.set_yticklabels(y_tck_labels, ha = 'right', va = 'bottom', fontsize = 8) # add
        plt.grid(True, color='black')
        fig = plt.gcf()
        fig.savefig(f'figs/RawDataHeatmap.tif', format='tif')
        plt.close(fig=fig)
```

From above, we see can start to see which molecules show the most significant variation. Now let's try to statistically simplify the complexity of the problem, and try to identify correlation in the dataspace.

#### Sklearn PCA

Initial run, we do not define a smaller number of components than those in the data. This will allow us to determine the optimum heuristically with a scree plot.

```
In [ ]: # since scikit-learn PCA is an SVD-based implementation, we must first center the d
data_pca = skl.preprocessing.scale(data, axis=1) # set axis equal to the dimension

In [ ]: pca = PCA(random_state = 0) # create our linear decomposition model, random state j
pca.fit(data_pca) # fit model to data

exp_var = pca.explained_variance_ratio_*100 # get explained variance (as a %)
csum_var = np.cumsum(exp_var) # cumulative explained variance
print(f'Total explained variance = {exp_var.sum()}') # check explained variance sum
```

We want to determine an optimal number of components in the data. This is done heuristically with a scree plot below:

```
In [ ]: plt.figure(dpi=600)
    plt.title('Cumulative explained variance plot')

plt.plot(exp_var, marker = 'o', label = 'successive') # plot non cumulative explained plt.plot(csum_var, marker = 'x', label = 'cumulative') # plot cumulative explained plt.xticks(np.arange(0,n_col,2), np.arange(1,n_col+1,2))

plt.xlabel('Principal components')
    plt.ylabel('Explained variance (%)')

plt.legend() # add Legend

plt.savefig(f'figs/PCA_ScreePlot.tif', format='tif')

plt.close(fig=plt.gcf())

In [ ]: print(f'2 component explained variance = {csum_var[1]} %') # cumulative sum of 2 component format explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %') # cumulative sum of 3 component explained variance = {csum_var[2]} %')
```

We are looking for the "elbow" in the scree plot to determine the optimal number of components. This may exist at a few points (2, 3, or 4 components). We see that component 3 explains ~5% of variance, making it significant enough. 4th component and beyond only describe <1% of the total variance. Therefore, **4 components are optimal**.

print(f'4 component explained variance = {csum\_var[3]} %') # cumulative sum of 4 component(f'Maximum variance beyond 4 components = {exp\_var[4:].max()} %') # if PCA is

## Re-run PCA with optimal n\_comps

```
In [ ]: n_comps = 4 # choose optimal number of components
```

```
In [ ]: pca = PCA(n_components = n_comps, random_state = 0) # create our dim reduction mode
    pca.fit(data_pca) # fit model to data

    exp_var = pca.explained_variance_ratio_*100 # get explained variance (as a %)
    csum_var = np.cumsum(exp_var) # cumulative explained variance

    print(f'Total explained variance = {exp_var.sum()}') # check explained variance sum

In [ ]: scores = pca.transform(data_pca) # get score matrix
    print(scores.shape)

loads = pca.components_# get loadings matrix
    print(loads.shape)
```

Now our reduced dimensional model is divided into a scores matrix, and loadings matrix. Scores matrix is of shape [number of samples, number of principal components], whilst loadings matrix is of shape [number of components, number of input dimensions]. In this case, the scores matrix describes positive/negative correlation each sample has with each principal component, and the loadings matrix describes the positive/negative correlation each principal component has with each dimension.

We can consider the **loadings matrix** a **reduction in the number of samples** (i.e., if we only had  $n\_comps$  number of "samples", which "samples" would explain most of the variation), whilst the **scores matrix** represents a **dimensionality reduction** - each sample can now be desceribed in only  $n\_comps$  number of dimensions, rather than the original input number of dimensions.

#### Visualisation

First, let's plot the loadings matrix as though it were  $n\_comps$  number of new samples. We still have the original number of dimensions, but now we have  $n\_comps$  number of "samples" which explain almost the same amount of variation.

```
In [ ]: #### visualise the loadings matrix
        # plot data as heatmap
        plt.figure(figsize = (8,6), dpi=600) # create figure
        plt.title('PCA loadings matrix') # set title
        plt.pcolormesh(loads, cmap = 'viridis', vmin=loads.min(), vmax=loads.max()) # plot
        plt.colorbar(location='bottom', label='loading value') # add colorbar
        # get current axis
        ax = plt.gca()
        ax.set_box_aspect(n_comps/n_col) # alter aspect ratio
        ax.invert_yaxis() # flip y axis
        # x ticks settings
        x_tcks = np.arange(0, n_col, 1) # create an array of 1 to 13 (n dimensions)
        x_{tck_labels} = []
        for comp in df.iloc[:,2:].columns: # use compositions as labels from data frame col
            x_tck_labels.append(comp)
        ax.set_xticks(x_tcks) # create x tick for each dimension
        ax.set_xticklabels(x_tck_labels, rotation = 90, ha = 'left') # add label for each n
        # y tick settings
```

```
y_tcks = np.arange(0, n_comps, 1) # create an array of 1 to 3 (n components)
y_tck_labels = []
for i in range(1,n_comps+1):
    y_tck_labels.append(f'PC {i}')

ax.set_yticks(y_tcks) # create x tick for each dimension
ax.set_yticklabels(y_tck_labels, ha = 'right', va = 'top', fontsize = 14) # add Lat
plt.grid(True, color='black')

fig = plt.gcf()
fig.savefig(f'figs/PCA4_LoadingsHeatmap.tif', format='tif')
plt.close(fig=fig)
```

This visualisation helps us understand the assumptions of PCA. The columns "MgO" to "BaO" exist in trace abundance only, and as such have very little weight when it comes to supplying variance to the dataset. The first 3 components instead are explaining:

- 1. High SiO2 but low CaO, everthing else very moderate.
- 1. High SiO2 and high CaO, slightly lower Al2O3, everything else very moderate.
- 1. Moderate amounts of everything except high Fe2O3.
   Note: "high"/"low" is relative to the rest of the dataset, not as weight %.

Now let's see how that affects the distribution of our components in each of the original samples:

```
In [ ]: # plot data as heatmap
        plt.figure(figsize = (4,8), dpi=600) # create figure
        plt.title('PCA scores matrix') # set title
        # plot data as image
        im = plt.pcolormesh(scores, cmap='viridis', vmin=scores.min(), vmax=scores.max())
        plt.colorbar(mappable=im, ax=plt.gca(), label='PCA scores') # add colorbar
        # get current axis
        ax = plt.gca()
        ax.set_box_aspect(n_row/(n_comps*2)) # alter aspect ratio
        # x tick settings
        x_tcks = np.arange(0, n_comps, 1) # create an array of 1 to 3 (n components)
        x_{tck_labels} = []
        for i in range(1, n_comps+1):
            x_tck_labels.append(f'PC {i}')
        ax.set_xticks(x_tcks) # create x tick for each dimension
        ax.set_xticklabels(x_tck_labels, rotation = 45, ha = 'left') # add Label for each
        # y tick settings
        y_{t} = np.arange(0, n_{r}) + create an array of 1 to n samples
        y_tck_labels = df.iloc[:, 0]
        ax.set_yticks(y_tcks) # create x tick for each dimension
        ax.set_yticklabels(y_tck_labels, ha = 'right', va = 'bottom', fontsize = 8) # add
        plt.grid(True, color='black')
        fig = plt.gcf()
        fig.savefig(f'figs/PCA4_ScoresHeatmap.tif')
```

```
plt.close(fig=fig)
```

Finally let's try to better represent the dimensionally reduced dataspace with scatter plots and histograms:

```
In [ ]: | #### creating a colorblind friendly color map
        # create a float RGBA (red, green, blue, alpha) color for each category
        colors_cbf = np.array((np.array([35/255, 187/255, 20/255, 1]), np.array([216/255,
                           np.array([255/255, 193/255, 7/255, 1]), np.array([0/255, 77/255]
                           np.array([102/255, 179/255, 225/255, 1]), np.array([77/255, 11/2
In [ ]:
        # we create a key system for the location name only, by using .split()
        location = [] # create an empty list to append to
        for comm in df.loc[:, 'Community']: # for each entry in the 'Community' column of t
            location.append(comm.split(sep='_')[0]) # the separator is '_'
        # get list of unique locations
        loc_unique = np.unique(location)
        # create list of matplotlib markers for each category
        marker_lst = ['o', '^', 's', '*', 'D', 'X']
        for i in range(0, len(loc_unique)): # for each number of categories
            print(f'{loc_unique[i]} : {marker_lst[i]}') # print unique place name : marker
In [ ]: |
        #### Visualise the scores
        # histogram of scores
        fig, (ax1,ax2,ax3,ax4) = plt.subplots(1,4, figsize = (8,2))
        ax1.hist(scores[:,0], bins = 25, color = 'red', edgecolor = 'black') # create hist
        ax1.set_title('PCA score 1')
        ax2.hist(scores[:,1], bins = 25, color = 'green', edgecolor = 'black') # create his
        ax2.set_title('PCA score 2')
        ax3.hist(scores[:,2], bins = 25, color = 'blue', edgecolor = 'black') # create hist
        ax3.set title('PCA score 3')
        ax4.hist(scores[:,3], bins = 25, color = 'gray', edgecolor = 'black') # create his
        ax4.set_title('PCA score 4')
        plt.tight_layout()
        plt.show()
        # scatter plots
        fig, ((ax1,ax2,ax3),(ax4,ax5,ax6)) = plt.subplots(2,3, figsize = (9,5))
        fig.suptitle('PCA scores')
        sns.scatterplot(x=scores[:,0], y=scores[:,1], alpha = 0.75, hue = location, ax = ax
                        palette=colors_cbf, edgecolor = 'black', legend=False, style=locati
        ax1.set_xlabel('PC 1')
        ax1.set_ylabel('PC 2')
        sns.scatterplot(x=scores[:,0], y=scores[:,2], alpha = 0.75, hue = location, ax = ax
                         palette=colors_cbf, edgecolor = 'black', legend=False, style=locat:
        ax2.set_xlabel('PC 1')
        ax2.set_ylabel('PC 3')
        sns.scatterplot(x=scores[:,0], y=scores[:,3], alpha = 0.75, hue = location, ax = ax
```

```
palette=colors_cbf, edgecolor = 'black', legend=False, style=locat;
        ax3.set_xlabel('PC 1')
        ax3.set_ylabel('PC 4')
        sns.scatterplot(x=scores[:,1], y=scores[:,2], alpha = 0.75, hue = location, ax = ax
                         palette=colors cbf, edgecolor = 'black', legend=False, style=locati
        ax4.set xlabel('PC 2')
        ax4.set_ylabel('PC 3')
        sns.scatterplot(x=scores[:,1], y=scores[:,3], alpha = 0.75, hue = location, ax = ax
                         palette=colors_cbf, edgecolor = 'black', legend=False, style=locat:
        ax5.set_xlabel('PC 2')
        ax5.set_ylabel('PC 4')
        sns.scatterplot(x=scores[:,2], y=scores[:,3], alpha = 0.75, hue = location, ax = ax
                         palette=colors_cbf, edgecolor = 'black', legend=False, style=locat'
        ax6.set_xlabel('PC 3')
        ax6.set_ylabel('PC 4')
        plt.tight_layout()
        fig = plt.gcf()
        fig.savefig('figs/PCA4_ScoresScatter_subplots.tif', format='tif')
        plt.close(fig=fig)
In [ ]: # generate a custom legend for plt.legend()
        leg = [] # create empty list
        for i in range(0, len(loc_unique)): # for each unique location
            leg.append(matplotlib.lines.Line2D([], [], marker = marker_lst[i], color = colo
In [ ]: # create plot for each combination of components
        for a in range(0, n_comps): # from 0 to n_comps
            for b in range(0, n_comps): # from 0 to n_comps again
                if a<=b: # remove repeats and straight lines</pre>
                else: # otherwise create scatter plot
                    plt.figure(dpi=600)
                     ax = plt.gca()
                     sns.scatterplot(x=scores[:,a], y=scores[:,b], alpha = 0.75, hue = locat
                                     palette=colors_cbf, edgecolor = 'black', legend=False,
                                     markers=marker lst)
                     # add custom Legend
                     plt.legend(leg, loc_unique, loc = (1.04, 0.18), markerscale = 1.5, font
                                labelspacing = 1)
                     # set x and y labels
                     plt.xlabel(f'PC {a+1}', fontsize=12, fontweight='bold')
                     plt.ylabel(f'PC {b+1}', fontsize=12, fontweight='bold')
                    ax.set_box_aspect(1)
                     plt.savefig(f'figs/scores_scatter_PC{a+1}vsPC{b+1}.tif', format='tif')
                     plt.close(fig=plt.gcf())
In [ ]: #### now with "correct" aspect ratio
        # create plot for each combination of components
        for a in range(0, n_comps): # from 0 to n_comps
            for b in range(0, n_comps): # from 0 to n_comps again
                if a<=b: # remove repeats and straight lines</pre>
                else: # otherwise create scatter plot
                     plt.figure(dpi=600)
```

```
ax = plt.gca()
                                          ax.set_box_aspect((scores[:,b].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,b].min())/(scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores[:,a].max()-scores
                                          sns.scatterplot(x=scores[:,a], y=scores[:,b], alpha = 0.75, hue = locat
                                                                            palette=colors_cbf, edgecolor = 'black', legend=False,
                                                                            markers=marker lst)
                                          # add custom Legend
                                          plt.legend(leg, loc_unique, loc = (1.04, 0.18), markerscale = 1.5, font
                                                                 labelspacing = 1)
                                          # set x and y labels
                                          plt.xlabel(f'PC {a+1}', fontsize=12, fontweight='bold')
                                          plt.ylabel(f'PC {b+1}', fontsize=12, fontweight='bold')
                                          plt.savefig(f'figs/scores_scatter_CorrectedAspectRatio_PC{a+1}vsPC{b+1}
                                          plt.close(fig=plt.gcf())
                 # change backend for interactive plotting
In [ ]:
                 %matplotlib notebook
                 fig = plt.figure(figsize=(8,8)) # create figure
                 ax = fig.add_subplot(projection='3d', ) # add subplot for 3d plotting
                 scatter = ax.scatter(scores[:,0], scores[:,1], scores[:,2], c=scores[:,3], cmap='v
                 cb = plt.colorbar(mappable=scatter, ax=ax, location='bottom')
                 cb.set_label(label='PC 4', weight='bold')
                 # set axis limits
                 ax.set_xlim(scores.min(), scores.max())
                 ax.set_ylim(scores.min(), scores.max())
                 ax.set_zlim(scores.min(), scores.max())
                 # add axis labels
                 ax.set_xlabel('PC 1', fontweight='bold')
                 ax.set_ylabel('PC 2', fontweight='bold')
                 ax.set_zlabel('PC 3', fontweight='bold')
                 plt.show()
In [ ]: | %matplotlib notebook
                 fig = plt.figure(figsize=(8,8)) # create figure
                 ax = fig.add_subplot(projection='3d', ) # add subplot for 3d plotting
                 # set axis limits
                 ax.set_xlim(scores.min(), scores.max())
                 ax.set_ylim(scores.min(), scores.max())
                 ax.set_zlim(scores.min(), scores.max())
                 # add axis labels
                 ax.set_xlabel('PC 1', fontweight='bold')
                 ax.set_ylabel('PC 2', fontweight='bold')
                 ax.set_zlabel('PC 3', fontweight='bold')
                 idx = 0
                 for loc in location:
                          loc_idx = 0
                          for locU in loc_unique:
                                  if loc == locU:
                                           ax.scatter(scores[idx,0], scores[idx,1], scores[idx,2], color=colors_ct
                                                                 marker=marker_lst[loc_idx], edgecolor='black', depthshade=Ti
                                          idx = 1
                                  else:
                                          loc idx += 1
```

```
#scatter = ax.scatter(scores[:,0], scores[:,1], scores[:,2], c=scores[:,3], cmap='v
#cb = plt.colorbar(mappable=scatter, ax=ax, location='bottom')
#cb.set_label(label='PC 4', weight='bold')

plt.legend(leg, loc_unique, markerscale = 2, fontsize = 10, labelspacing = 1.25)

plt.show()
```

The scatter plots and histograms above highlight the fact that there is not enough samples for a robust cluster analysis. There are several areas where points are deviating from one another; however, it is hard to know if these points are noise in the data, or if with a larger dataset these would form groups of their own.

#### **Export data**

Convert loadings and score matrices back into a dataframe with the appropriate headers, and save as .csv.

```
In [ ]: # x ticks settings
        x_tcks = np.arange(0, n_col, 1) # create an array of 1 to 13 (n dimensions)
        x tck labels = []
        for comp in df.iloc[:,2:].columns: # use compositions as Labels from data frame col
            x_tck_labels.append(comp)
        # y tick settings
        y_tcks = np.arange(0, n_comps, 1) # create an array of 1 to 3 (n components)
        y_tck_labels = []
        for i in range(1, n_comps+1):
            y_tck_labels.append(f'PC {i}')
        # create pandas dataframe for exporting
        df_loads = pd.DataFrame(data = loads, index = y_tck_labels, columns = x_tck_labels)
        df loads
In []: df_loads.to_csv('PlantChemistry_MajorTraceElements_PCA4_loadings.csv') # save as cs
In []: df_scores = pd.DataFrame(data = scores, columns = y_tck_labels) # create df with de
        df_scores['Community'] = df.loc[:,'Community'] # add Locations column
        df_scores
In [ ]: df_scores.to_csv('PlantChemistry_MajorTraceElements_PCA4_scores.csv') # save as csv
```

# References

#### **Plotting**

```
[1] J.D. Hunter. 2007. Matplotlib: A 2D graphics environment. Computing in Science & Engineering, 9(3), pp.90-95, doi:10.1109/MCSE.2007.55
[2] M.L. Waskom. 2021. seaborn: statistical data vizualisation. The Open Journal, 6(60), pp. 3021, doi:10.21105/joss.03021
```

#### Data management

[3] The pandas development team. 2020. pandas-dev/pandas: Pandas. *Zenodo*, doi:10.5281/zenodo.3509134

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### Principal component analysis

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