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Unit-2

Predicate Calculus

> A predicate is a verb phrase template that describes a property of objects, or a relationship among objects represented by the variables.

> The logic based upon the analysis of predicates in any statement is called predicate logic.

Example

John is a bachelor.
Smith is a bachelor.
→ the part "is a bachelor" is called a predicate

> Symbolize a predicate by a capital letter and names of individuals or objects in general by small letters.

> Every predicate describes about one or more objects.

> Therefore a statement could be written symbollically in terms of the predicate letter followed by the name or names of the objects to which the predicate is applied.

Example
1. John is a bachelor.
2. Smith is a bachelor.

> Here, "is a bachelor" symbolically denoted by the predicate letter B, "John" by 'j', "Smith" by 's'.

> Statements (1) and (2) can be written as B(j) and B(s) respectively.

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Ex: Jack is taller than Jill

J_1: Jack, J_2: Jill

$T(J_1, J_2)$

> A statement function of one variable is defined to be an expression consisting of a predicate symbol and an individual variable.

$B(j)$

> Such a statement function becomes a statement when the variable is replaced by the name of any object.

$B(j)$ where 'j' means Jack

> It is possible to form student function of two variables by using statement functions of one variable.

Example: $M(x)$: x is a man

$H(y)$: y is a mortal.

> $M(x) \wedge M(y)$: x is a man and y is a mortal.

Quantifiers: allow us to quantify (count) how many objects in the universe of discourse satisfy a given predicate.

Universe of discourse - the particular domain of the variable in a propositional function.

Two types of quantifiers:

- Universal
- Existential

Universal & Existential Quantifiers

The quantifier 'all' is called as the Universal quantifier denoted as $\forall x$.

Represents each of the following phrases:

For all x, All x are such that

For every x, Every x is such that

For each x, Each x is such that

The quantifier 'some' is the Existential quantifier, denoted as $\exists x$,

Represents each of the following phrases:

There exists an x such that...

There is an x such that...

For some x...

There is atleast one x such that...

Some x is such that...

☆ The symbol "$\exists! x$" is read 'there is a unique x such that...' or 'There is one and only one x such that...'.

Ex: There is one and only one even prime.

$\exists! x$, [x is an even prime]

$\exists! x$, $P(x)$ where $P(x) = x$ is an even prime integer

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Negation:

To form the negation of a statement involving one quantifier, change the quantifier form universal to existential, or from existential to universal, and negate the statement, which it quantifies.

Statement	Negation
$\forall x, F(x)$	$\exists x, [\sim F(x)]$
$\forall x, [\sim F(x)]$	$\exists x, F(x)$

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① ~~Let S~~ All squares are rectangles.

Let 'S' denote the set of all squares

For all $x \in S$, x is rectangle.

Symbollically $\forall x \in S, P(x)$

② For every integer x, x^2 is a non-negative integer.

$\forall x \in Z, Q(x)$

'Z' is set of integers

$Q(x)$ is open statement "x^2 is a non-negative integer"

③ Some determinants are equal to zero.

$\exists x \in D, P(x)$

D → set of determinants

P() → is equal to zero

8) For the universe of all integers, $P(x): x>0$ and $Q(x): x$ is even, $R(x): x$ is perfect square, $S(x): x$ is divisible by 3, $T(x): x$ is divisible by 7.

Write down the following quantifier statement in symbollic form.

i) Atleast one integer is even. $\exists x, Q(x)$

ii) There exists a positive integer which is even. $\exists x, P(x) \wedge Q(x)$

iii) Some even integers are divisible by 3. $\exists x, Q(x) \wedge S(x)$.

iv) Every integer either even or odd. ~~$\forall x, Q(x) \vee \neg Q(x)$~~

$\forall x, Q(x) \underline{\vee} \neg Q(x)$

v) If x is even and a perfect square, then x is not divisible by 3.

$$\forall x, (Q(x) \wedge R(x)) \longrightarrow \neg S(x)$$

vi) If x is odd or is not divisible by 7 then x is divisible by 3.

$$\forall x, (\neg Q(x) \vee \neg T(x)) \longrightarrow S(x)$$

Negate and simplify each of the following:

(1) $\exists x\ [P(x) \vee Q(x)]$

$$\forall x \sim[P(x) \vee Q(x)] = \forall x\ [\sim P(x) \wedge \sim Q(x)]$$

(2) $\forall x\ [P(x) \wedge \sim Q(x)]$

$$\exists x\ [\sim P(x) \vee Q(x)]$$

(3) $\forall x\ [P(x) \rightarrow Q(x)] = \forall x, [\sim P(x) \vee Q(x)]$

$$\exists x, [P(x) \wedge \sim Q(x)]$$

9) Let P(x) denotes the statement

$P(x)$: x is a professional Athlet.

$Q(x)$: x plays soccer.

The universe is a set of people. Write each of the propositions in english.

i) $\forall x, P(x) \rightarrow Q(x)$

Every professional athlet plays soccer.

ii) $\exists x, (P(x) \wedge Q(x))$

Some professional athlets play soccer.

iii) $\forall x, (P(x) \vee Q(x))$

Every person ~~is~~ either a professional athlet or plays soccer.

9) Consider a statement 'given any +ve integer, there is a greater +ve integer.' Symbolize with and without using the set of +ve integers as universe of discourse

i) with using set of +ve integers.

$\cancel{\forall x, \exists y,}$

$\forall x \in Z^+, \exists y \in Z^+, G(y, x)$

$G()$: is greater than

ii) without using set of the integers.

$\cancel{\forall x \in Z}$ $P()$: is a +ve integer

$G()$: is greater than

$\cancel{\forall}$

$\forall x, \exists y, P(x, y) \rightarrow G(y, x)$

Equivalent Formulas :

All true $\{\forall x, F(x)\} \equiv \{\sim[\exists x, \sim F(x)]\}$ None false

All false $\{\forall x, \sim F(x)\} \equiv \{\sim[\exists x, F(x)]\}$ None true

Not all true $\{\sim[\forall x, F(x)]\} \equiv \{\exists x, [\sim F(x)]\}$ Atleast one false

Not all false $\{\sim[\forall x, \{\sim F(x)\}]\} \equiv \{\exists x, F(x)\}$ Atleast one true

Sentence	Abbreviated Meaning
$\forall x, F(x)$	All true
$\forall x, [\sim F(x)]$	All false
$\sim\{\forall x, [F(x)]\}$	Not all true / Atleast one false
$\sim[\forall x, [\sim F(x)]]$	Not all false / Atleast one true
$\exists x, F(x)$	Atleast one true
$\exists x, [\sim F(x)]$	Atleast one false
$\sim\{\exists x, [F(x)]\}$	None true
$\sim\{\exists x, [\sim F(x)]\}$	None false

Free and Bound Variables

A formula containing a part of the form $(x)P(x)$ or $(\exists x)P(x)$, such an part is called an x-bound part of the formula.

Any occurrence of x in an x-bound part of a formula is called a bound occurrence of x, while any occurrence of x or of any variable that is not a bound occurrence is called a free occurrence.

The formula $P(x)$ either in $(x)P(x)$ or in $(\exists x)P(x)$ is described as the scope of the quantifier.

Quantified Propositions:

Fundamental rule US: (Universal Specification)

If a statement of a form $\forall x, P(x)$ is assumed to be true, then the universal quantifier can be ~~representat~~ dropped to obtain $P(c)$ is true for an arbitary object c in the universe. This may be represented as

$$\forall x, P(x)$$
$$P(c) \text{ for all } c.$$

Fundamental Rule UG: (Universal Generalisation)

If $\exists x, P(x)$ is assumed to be true, then there is an element c in the universe such that $P(c)$ is true. This may be represented as $\exists x, P(x) \therefore P(c)$ for some c.

Fundamental Rule ES: (Existential Specification)

If a statement $P(c)$ is true of each element c of the universe, then the universal quantifier may be prefixed to obtain $\forall x, P(x)$. It is represented as $P(c)$ for all c
$\therefore \forall x, P(x)$

Fundamental Rule EG: (Existential Generalisation)

If $P(c)$ is true for some element c in the universe then $\exists x, P(x)$ is true. This may be represented as

$$P(c) \text{ for some } c$$
$$\therefore \exists x, P(x)$$

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Q) Consider the argument.

All men are fallible.

All kings are men.

∴ All kings are fallible. Symbolize the argument & check its validity.

A) Let $M(x)$ denote the assertion "x is a man".

$K(x)$ denote the assertion "x is a king".

$F(x)$ denote the assertion "x is fallible".

The above argument is symbolized as

$$\forall x, [M(x) \rightarrow F(x)]$$
$$\forall x, [K(x) \rightarrow M(x)]$$
$$\overline{\therefore \forall x, [K(x) \rightarrow F(x)]}$$

Proof:

1) $\forall x, [K(x) \rightarrow M(x)]$	Premise 2
2) $K(c) \rightarrow M(c)$	by US
3) $\forall x, [M(x) \rightarrow F(x)]$	Premise 1
4) $M(c) \rightarrow F(c)$	by US
5) $K(c) \rightarrow F(c)$	by 2) & 4) and transitive Rule
6) $\forall x, [K(x) \rightarrow F(x)]$	by 5) and Rule UG

9) Symbolize the following argument and check for its validity.

Lions are dangerous animals.

There are lions.

$\therefore$ There are dangerous animals.

A)

Let $L(x)$ denotes 'x is a lion'.

$D(x)$ denotes 'x is dangerous'.

Symbolically

$$\forall x, [L(x) \rightarrow D(x)]$$

$$\frac{\exists x, L(x)}{\therefore \exists x, D(x)}$$

Proof:

1) $\forall x, [L(x) \rightarrow D(x)]$ Premise 1

2) $L(c) \rightarrow D(c)$ by 1) and US

3) $\exists x, L(x)$ Premise 2

4) $L(c)$ by 3) and Rule ES

5) $D(c)$ by 2) & 4) and transitive rule

6) $\exists x, D(x)$ by 5) and Rule EG

Universe of Discourse

> Variables which are qualified stand for only those objects which are members of a particular set or class.

> Such a restricted class is called the universe of discourse or the domain of individuals or simply universe.

> If the discussion refers to human beings only, then the universe of discourse is the class of human beings.

> In elementary algebra or number theory, the universe of discourse could be numbers (real, complex, rational, etc.)