4/7/19

# Unit-1

## Propositinal Logic

Concepts:

> Statements and Notations
> Connectives
> Well Formed Formulas
> Tautology and Contradiction
> Equivalence, implication
> Rules of inference and arguments
> Proof by Contradiction, conditional proof
> Normal forms
> Automatic theorem proving

Statements / Propositions and notations:

Statement: A declarative sentence which is either true or false but not both.

Example:

Delhi is capital of the India – True

$2+3=5$ – True

SNIST is located at Miyapur – False

Scanned with CamScanner

Not a Statement - Neither True nor False

> Is Canada a Country: it is a question.

> Please close the door: it is a request.

> $X+3=7$: truth value depends upon the value of X

> $1+101=110$: truth value depends upon the context.

> Get out of the class: it is a command.

> Jungle book is greatest movie of all the time: it is an opinion

Types of Statements:

1) Primitive / Primary / Atomic / Simple statement

A declarative statements which cannot be further broken down.

Have one and only of two possible truth values.

true (T)
false (F)

Denoted by distinct symbols A, B, C, ..., P, Q, ...

Ex:

P: The weather is cloudy

Q: It is raining today.

R: It is snowing.

Scanned with CamScanner

2) Compound / molecule statement:

Statements that involve one or more of the connectives are called Compound statements.

Example:

- If you finish your homework then you can watch T.V.
- I have read this and I understand the concepts.

Truth Table:

Truth table displays the relationship among the ~~truth~~ truth values of propositions (statements).

The truth value of a proposition 'P' is made up off individual propositions $P_1, P_2, \ldots P_n$

Connectives:

Negation ($\sim$)
Conjunction ($\wedge$)
Disjunction ($\vee$)
Conditional / Implication ($\rightarrow$)
Bi-conditional / bi-implication ($\leftrightarrow$, $\Leftrightarrow$)

Scanned with CamScanner

Negation ( ~, not)

> Connective that modifies a statement.

> Unary operation operates on a single statement.

> Formed by introducing the word "not" at a proper place in the statement (OR) by prefixing the statement with the phrase "It is not the case that".

> If P is a statement its negation is ; ~P or not P or negation of P or the denial of P.

Truth Table:

| P | ~P |
|---|---|
| T | F |
| F | T |

1. P : London is a city

~P: London is not a city

~P: It is not the case that London is a city

2. P: I went to my class yesterday.

~P: I did not go to my class yesterday.

~P: It is not the case that I went to my class yesterday

3. Einstein is a genius is a statement

It is not the case that Einstein is a genius.

or

Einstein is not a genius.

or

It is false that Einstein is a genius.

Scanned with CamScanner

5/7/19

## Conjunction ($\wedge$)

$P \wedge Q$ (P and Q)

Truth value is T whenever both P and Q have the truth-values T; otherwise truth-value is F.

| P | Q | $P \wedge Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Ex:

P: The weather is cloudy.

Q: It is raining today.

$P \wedge Q$: The weather is cloudy and it is raining today

Jack and Jill went up the hill.

→ Jack went up the hill and Jill went up the hill

P: Jack went up the hill.

Q: Jill went up the hill.

## Disjunction ($\vee$)

$P \vee Q$ (P or Q)

Truth value F only when both P and Q have the truth value F; otherwise truth-value T.

| P | Q | $P \vee Q$ |
|---|---|---|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Ex:

P: The weather is cloudy

Q: It is raining today.

$P \vee Q$: The weather is cloudy or it is raining today.

Scanned with CamScanner

## Exclusive or

Either P is true or Q is true, but not both,

| P | Q | $P \oplus Q$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

## Implication or Conditional

$P \rightarrow Q$

P – premise, hypothesis, or antecedent of the implication.

Q – conclusion or consequent of the implication.

| P | Q | $P \rightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

$$\boxed{P \rightarrow Q = (\sim P) \cup Q}$$

Ex:

P: the sun is shining today

Q: $2+7>4$

$P \rightarrow Q$ : If the sun is shining today, then $2+7>4$.

## Biconditional

$(P \leftrightarrow Q)$

Conjunction of the conditionals $\{ (P \rightarrow Q) \wedge (Q \rightarrow P) \}$

True: when P and Q have the same truth-values

False: when P and Q have different truth-values

($\leftrightarrow$) P if and only if Q

- P iff Q
- P is necessary and sufficient for Q

| P | Q | $P \leftrightarrow Q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

$$P \leftrightarrow Q = (P \rightarrow Q) \wedge (Q \rightarrow P)$$
$$= (\sim P \vee Q) \wedge (\sim Q \vee P)$$

$$P \leftrightarrow Q = \sim (P \oplus Q)$$

| P | Q | $P \rightarrow Q$ | $Q \rightarrow P$ | $P \leftrightarrow Q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

Scanned with CamScanner

Precedence of logical connectives is :

$\sim$ - highest

$\wedge$ - second highest

$\vee$ - third highest

$\rightarrow$ - fourth highest

$\leftrightarrow$ - fifth highest

Q) If Jerry takes Calculus or Ken takes Sociology, then Larry will take English

J: Jerry takes Calculus

K: Ken takes Sociology

L: Larry takes English.

$(J \vee K) \rightarrow L$

Q) The crop will be destroyed if there is a flood

C: The crop will be destroyed

F: There is a flood.

$F \rightarrow C$

| $p \leftrightarrow q$ | $q \leftrightarrow p$ | $p \rightarrow q$ | $p$ | $q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | T | T | T | T |
| T | T | T | T | T |
| T | T | T | T | T |

Scanned with CamScanner

8/7/19

9) Consider the propositions:

p: David is playing pool.

q: David is inside.

r: David is doing his homework.

s: David is listening to music

Translate the following statements into symbolic notation using p, q, r, s, ~, ∨, ∧, and parenthesis only.

ⓐ Either David is playing inside or he is inside.

$p \vee q$

ⓑ Neither is David playing pool, nor is he doing his homework.

~~$(\sim p) \vee (\sim r)$~~ $(\sim p) \wedge (\sim r) = \sim(p \vee r)$

ⓒ David is playing pool and not doing his homework.

$p \wedge (\sim r)$

ⓓ David is inside doing his homework, not playing pool.

$q \wedge r \wedge (\sim p)$

ⓔ David is inside doing his homework, while listening to music, and he is not playing pool.

$q \wedge r \wedge s \wedge (\sim p)$

ⓕ David is not listening to music, nor is he playing pool, neither is he doing home work.

$(\sim s) \wedge (\sim p) \wedge (\sim r)$

Scanned with CamScanner

Using the specifications of p, q, r, s translate the following propositions into acceptable English.

ⓐ $(\sim p) \wedge (\sim q)$

Neither is David playing pool nor he is inside.

ⓑ $p \vee (q \wedge r)$

David is either player pool or he is inside and doing his homework.

ⓒ $\sim ((\sim p) \wedge r)$

It is not the case that David is not playing pool and he is doing his homework.

ⓓ $[(\sim p) \vee q] \wedge [\sim p \vee s]$. X

David is not playing pool and he is either inside or listening to music.

ⓔ $[(\sim p) \wedge q] \vee [(\sim p) \wedge s]$ X

David is either not playing pool or he is inside and listening to music.

ⓓ $[(\sim p) \vee q] \wedge [\sim r \vee s]$

David is not playing pool or he is inside and he is not doing his homework or listening to music

ⓔ $[\sim p \wedge q] \vee [\sim r \wedge s]$

David is either not doing his homework and is inside or he is not doing his homework and listening to music

Scanned with CamScanner

Q) Construct the truth table for the formula

$(p \vee q) \rightarrow (p \wedge \sim q)$

| $p$ | $q$ | $\sim q$ | $p \vee q$ | $p \wedge \sim q$ | $(p \vee q) \rightarrow (p \wedge \sim q)$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |

Q) Construct the truth table for...

$[(p \wedge q) \vee \sim r] \longleftrightarrow p$

| $p$ | $q$ | $r$ | $p \wedge q$ | $\sim r$ | $[(p \wedge q) \vee (\sim r)]$ | $[(p \wedge q) \vee (\sim r)] \leftrightarrow p$ |
|---|---|---|---|---|---|---|
| F | F | F | F | T | T | F |
| F | F | T | F | F | F | T |
| F | T | F | F | T | T | F |
| F | T | T | F | F | F | T |
| T | F | F | F | T | T | T |
| T | F | T | F | F | F | F |
| T | T | F | T | T | T | T |
| T | T | T | T | F | T | T |

Q) Construct truth tables for the following:

ⓐ ~~$[(p \vee q) \wedge (\sim p)$~~ $[(p \vee q) \wedge (\sim r)] \leftrightarrow q$

ⓑ $(p \vee q) \wedge ((\sim p) \vee (\sim r))$

ⓒ $\{(p \wedge q) \vee (\sim p \wedge r)\} \vee (q \wedge r)$.

ⓓ $[(p \vee q) \wedge (\sim r)] \longleftrightarrow (q \rightarrow r)$.

Scanned with CamScanner

a)

| $p$ | $q$ | $\sim r$ | $(p \vee q)$ | $(p \vee q) \wedge (\sim r)$ | $[(p \vee q) \wedge (\sim r)] \leftrightarrow q$ |
|---|---|---|---|---|---|
| F | F | T | F | F | T |
| F | F | F | F | F | T |
| F | T | T | T | T | T |
| F | T | F | T | F | F |
| T | F | T | T | T | F |
| T | F | F | T | F | T |
| T | T | T | T | T | T |
| T | T | F | T | F | F |

b)

| $p$ | $q$ | $\sim r$ | $p \vee q$ | $\sim p$ | $\sim p \vee \sim r$ | $(p \vee q) \wedge (\sim p \vee \sim r)$ |
|---|---|---|---|---|---|---|
| F | F | T | F | T | T | F |
| F | F | F | F | T | T | F |
| F | T | T | T | T | T | T |
| F | T | F | T | T | T | T |
| T | F | T | T | F | T | T |
| T | F | F | T | F | F | F |
| T | T | T | T | F | T | T |
| T | T | F | T | F | F | F |

c)

| $p$ | $q$ | $r$ | $\sim p$ | $p \wedge q$ | $\sim p \wedge r$ | A $(p \wedge q) \vee (\sim p \wedge r)$ | B $q \wedge r$ | $A \vee B$ |
|---|---|---|---|---|---|---|---|---|
| F | F | F | T | F | F | F | F | F |
| F | F | T | T | F | T | T | F | T |
| F | T | F | T | F | F | F | F | F |
| F | T | T | T | F | T | T | T | T |
| T | F | F | F | F | F | F | F | F |
| T | F | T | F | F | F | F | F | F |
| T | T | F | F | T | F | T | F | T |
| T | T | T | F | T | F | T | T | T |

Scanned with CamScanner

11/7/19

## Well-formed formulas (WFF)

Expression consisting of

> statements (propositions/variables)
> parentheses
> connecting symbols.

→ A statement variable standing alone is a well-formed formula.

→ If A is a well-formed formula, then ¬A is a well-formed formula.

→ If A and B are well-formed formulas, then (A∧B), (A∨B), (A→B), and (A↔B) are well-formed formulas.

→ A string of symbols containing the statement variables, connectives, and parentheses is a well-formed formula, iff it can be obtained by finitely many applications of 1, 2 and 3 above.

## Tautologies:

A statement formula which is true regardless of the truth values of the statements is called a tautology (or) universally valid formula (or) a logical truth.

## Contradiction:

A statement formula which is false regardless of the truth values of the statements is called a contradiction.

## Contingency:

Propositional function that is neither a tautology nor a contradiction

Scanned with CamScanner

Tautology Example: $\boxed{\sim(P \wedge Q) \leftrightarrow (\sim P \vee \sim Q)}$

| $P$ | $Q$ | $P \wedge Q$ | $\sim(P \wedge Q)$ | $\sim P$ | $\sim Q$ | $\sim P \vee \sim Q$ | $\sim(P \wedge Q) \leftrightarrow (\sim P \wedge \sim Q)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | T |
| T | F | F | T | F | T | T | T |
| F | T | F | T | T | F | T | T |
| F | F | F | T | T | T | T | T |

Q) Show that $\sim(P \to Q) \to (\sim Q)$ is a tautology

| $P$ | $Q$ | $P \to Q$ | $\sim(P \to Q)$ | $\sim Q$ | $\sim(P \to Q) \to (\sim Q)$ |
|---|---|---|---|---|---|
| T | T | T | F | F | T |
| T | F | F | T | T | T |
| F | T | T | F | F | T |
| F | F | T | F | T | T |

Q) Tautology or not. $((P \to Q) \wedge (Q \to R)) \to (P \to R)$

| $P$ | $Q$ | $R$ | $P \to Q$ | $Q \to R$ | $(P \to Q) \wedge (Q \to R)$ | $P \to R$ | $((P \to Q) \wedge (Q \to R)) \to (P \to R)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

Scanned with CamScanner

Q) $(\sim P \wedge (P \vee Q)) \rightarrow Q$

| $P$ | $Q$ | $\sim P$ | $P \vee Q$ | $\sim P \wedge (P \vee Q)$ | $(\sim P \wedge (P \vee Q)) \rightarrow Q$ |
|---|---|---|---|---|---|
| T | T | F | T | F | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | F | T |

Q) $\sim (P \rightarrow Q) \rightarrow P$

| $P$ | $Q$ | $P \rightarrow Q$ | $\sim (P \rightarrow Q)$ | $\sim (P \rightarrow Q) \rightarrow P$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | F | T |

Q) $\sim P \rightarrow (P \rightarrow Q)$

| $P$ | $Q$ | $\sim P$ | $P \rightarrow Q$ | $\sim P \rightarrow (P \rightarrow Q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | F | F | T |
| F | T | T | T | T |
| F | F | T | T | T |

Scanned with CamScanner

12/7/19

Check whether the following tautology or contradiction.

$Q \wedge \sim(P \to Q)$

| P | Q | $P \to Q$ | $\sim(P \to Q)$ | $Q \wedge \sim(P \to Q)$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | F | F |

→ $(P \vee Q) \wedge (\sim P \wedge \sim Q)$

| P | Q | $\sim P$ | $\sim Q$ | $P \vee Q$ | $\sim P \wedge \sim Q$ | $(P \vee Q) \wedge (\sim P \wedge \sim Q)$ |
|---|---|---|---|---|---|---|
| T | T | F | F | T | F | F |
| T | F | F | T | T | F | F |
| F | T | T | F | T | F | F |
| F | F | T | T | F | T | F |

→ $(P \wedge Q) \wedge \sim P$

| P | Q | $P \wedge Q$ | $\sim P$ | $(P \wedge Q) \wedge \sim P$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | F |
| F | T | F | T | F |
| F | F | F | T | F |

→ $(P \to Q) \wedge (Q \to R) \wedge \sim(P \to R)$

| P | Q | R | $P \to Q$ | $Q \to R$ | $(P \to Q) \wedge (Q \to R)$ | $P \to R$ | $\sim(P \to R)$ | $(P \to Q) \wedge (Q \to R) \wedge \sim(P \to R)$ |
|---|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | F | F |
| T | T | F | T | F | F | F | T | F |
| T | F | T | F | T | F | T | F | F |
| T | F | F | F | T | F | F | T | F |
| F | T | T | T | T | T | T | F | F |
| F | T | F | T | F | F | T | F | F |
| F | F | T | T | T | T | T | F | F |
| F | F | F | T | T | T | T | F | F |

Scanned with CamScanner

If $P \to Q$ is a statement then.....

> Converse

Converse of $P \to Q$ is $Q \to P$

> Inverse/ Opposite

Opposite/ Inverse of $P \to Q$ is $(\sim P) \to (\sim Q)$

> Contrapositive

Contrapositive of $P \to Q$ is $(\sim Q) \to (\sim P)$

→ Converse and Inverse are Same.

Example:

P: Triangle 1 and Triangle 2 are similar.

Q: Corresponding sides of Triangle 1 and 2 are proportional.

Converse $(Q \to P)$

If the corresponding sides of triangles 1 and 2 are proportional, then triangle 1 and triange 2 are similar.

Opposite $(\sim P \to \sim Q)$

If triangle 1 and triangle 2 are not similar, then the corresponding sides of triangles 1 and 2 are not proportional.

Contrapositive $(\sim Q \to \sim P)$

If the corresponding sides of triangles 1 and 2 are not proportional, then triangle 1 and 2 are not similar.

Scanned with CamScanner

## Duality Law:

> Two formulas A and $A^*$ are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

A: $P \vee (Q \wedge R)$
B: $P \wedge (Q \vee R)$

> If the formula A contains special variables T or F then its dual $A^*$ is obtained by replacing T by F and F by T.

> The connectives $\wedge$ and $\vee$ are called duals of each other.

## Equivalences

Commutative Laws

> $P \wedge Q \Leftrightarrow Q \wedge P$

> $Q \wedge P \Leftrightarrow P \wedge Q$

Associative laws

> $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$

> $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

Distributive Laws

> $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$

> $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

Demorgan's laws

$\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$

Scanned with CamScanner

## More Equivalences

> $\sim(\sim P) = P$

> $P \vee P \equiv P$

> $P \wedge P \equiv P$

> $P \vee \sim P \Leftrightarrow T$

> $P \wedge \sim P \Leftrightarrow F$

> $R \vee (P \wedge \sim P) \Leftrightarrow R$

> $R \wedge (P \vee \sim P) \Leftrightarrow R$

> $R \vee (P \vee \sim P) \Leftrightarrow T$

> $R \wedge (P \wedge \sim P) \Leftrightarrow F$

> $P \rightarrow Q \Leftrightarrow (\sim P \vee Q)$

> $\sim (P \rightarrow Q) \Leftrightarrow (P \wedge \sim Q)$

> $P \rightarrow Q \Leftrightarrow (\sim Q \rightarrow \sim P)$

15/2/19

> $P \vee F \Leftrightarrow P$

> $P \vee T \Leftrightarrow T$

> $P \wedge F \Leftrightarrow F$

> $P \wedge T \Leftrightarrow P$

> $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$

> $\sim (P \leftrightarrow Q) \Leftrightarrow (P \leftrightarrow \sim Q)$

> $(P \leftrightarrow Q) \Leftrightarrow [(P \rightarrow Q) \wedge (Q \rightarrow P)]$

> $P \leftrightarrow Q \Leftrightarrow [(P \wedge Q) \vee (\sim P \wedge \sim Q)]$

Absorption laws:

$P \vee (P \wedge Q) \Leftrightarrow P$

$P \wedge (P \vee Q) \Leftrightarrow P$

Scanned with CamScanner

9) Without using truth tables, show that

$$P \to (Q \to R) \Leftrightarrow (P \wedge Q) \to R$$

$$\begin{aligned}
LHS &= P \to (Q \to R) \\
&= \sim P \vee (Q \to R) && \text{(Since } A \to B \Leftrightarrow (\sim A \vee B)) \\
&= \sim P \vee (\sim Q \vee R) \\
&= (\sim P \vee \sim Q) \vee R && \text{(By associative property)} \\
&= \sim (P \wedge Q) \vee R && \text{(By demorgan's law)} \\
&= (P \wedge Q) \to R
\end{aligned}$$

LHS = RHS

Q) Without using truth table, show that $(P \vee Q) \vee \sim P$ is a tautology.

$$\begin{aligned}
LHS &= (P \vee Q) \vee \sim P \\
&= (Q \vee P) \vee \sim P && \text{(Commutative law)} \\
&= Q \vee (P \vee \sim P) && \text{(Associative law)} \\
&= Q \vee T \\
&= T
\end{aligned}$$

Scanned with CamScanner

Q) Show that the statement formula $\sim(P \vee Q) \vee (\sim P \wedge Q) \vee P$ is a tautology

$$LHS = \sim(P \vee Q) \vee (\sim P \wedge Q) \vee P$$

$$\cancel{= \sim P \wedge \sim Q \vee Q \cdot (\sim P \vee P) \wedge (\sim Q \vee P)}$$

$$\cancel{= \sim(P \vee Q) \vee [(\sim P \vee P) \vee (\sim P \vee P}$$

$$= \sim(P \vee Q) \vee [(\sim P \vee P) \wedge (\sim Q \vee P)] \quad \text{(Distributive law)}$$

$$= \sim(P \vee Q) \vee [T \wedge (Q \vee P)]$$

$$= \sim(P \vee Q) \vee (P \vee Q)$$

$$= T$$

Q) Show that $[\{(P \to Q) \leftrightarrow (\sim P \vee Q)\} \wedge R] = R$

$$LHS = [\{(P \to Q) \leftrightarrow (\sim P \vee Q)\} \wedge R]$$

$$= \{(P \to Q) \leftrightarrow (P \to Q)\} \wedge R$$

$$= T \wedge R$$

$$= R$$

Q) Show that $\{(P \wedge Q) \wedge \sim(P \wedge Q)\}$ is a contradiction

Let $P \wedge Q = R$

$$LHS = R \wedge \sim R = F$$

Scanned with CamScanner

Q) Show that $((\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

$$LHS = (\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$$

$$= ((\sim P \wedge \sim Q) \wedge R) \vee ((Q \vee P) \wedge R)$$

$$= ((\sim P \wedge \sim Q) \vee (Q \vee P)) \wedge R$$

$$= (\sim (P \vee Q) \vee (P \vee Q)) \wedge R$$

$$= T \wedge R$$

$$= R$$

Q) S.T. $((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$ is a tautology

$$LHS = ((P \vee Q) \wedge \sim((\sim P \wedge \sim Q) \vee (\sim P \wedge R))) \vee [(\sim Q \vee \sim R) \wedge \sim P]$$

$$= [(P \vee Q) \wedge (P \vee Q) \wedge (P \vee R)] \vee [\sim(Q \wedge R) \wedge \sim P]$$

$$= [(P \vee Q) \wedge (P \vee R)] \vee \sim[(Q \wedge R) \vee P]$$

$$= [P \vee (Q \wedge R)] \vee \sim[P \vee (Q \wedge R)]$$

Let $P \vee (Q \wedge R) = S$

$$= S \vee \sim S$$

$$= T$$

Scanned with CamScanner

18/7/19

Q) Show that $((p \vee q) \wedge ((p \to r) \wedge (q \to r))) \to r$ is a tautology

$$((p \vee q) \wedge ((p \to r) \wedge (q \to r))) \to r$$

$$\cancel{= (p \vee q) \to (((p \to r) \wedge (q \to r)) \to r)}$$

$$= ((p \vee q) \wedge ((\sim p \vee r) \wedge (\sim q \vee r))) \to r$$

$$= ((p \vee q) \wedge ((\sim p \wedge \sim q) \vee r)) \to r$$

$$= ((p \vee q) \wedge (\sim (p \vee q) \vee r)) \to r$$

$$= (((p \vee q) \wedge \sim (p \vee q)) \vee ((p \vee q) \wedge r)) \to r$$

$$= [F \vee ((p \vee q) \wedge r)] \to r$$

$$= ((p \vee q) \wedge r) \to r$$

$$= \sim ((p \vee q) \wedge r) \vee r$$

$$= \sim (p \vee q) \vee \sim r \vee r$$

$$= \sim (p \vee q) \vee T$$

$$= T$$

Scanned with CamScanner

Q) Prove that $(p \wedge q) \rightarrow p$ is a tautology

$$(p \wedge q) \rightarrow p = \sim(p \wedge q) \vee p$$
$$= (\sim p \vee \sim q) \vee p$$
$$= (\sim p \vee p) \vee \sim q$$
$$= T \vee \sim q$$
$$= T$$

Q) Prove $\sim p \leftrightarrow q \Leftrightarrow p \leftrightarrow \sim q$

$$LHS = \sim p \leftrightarrow q = (\sim p \rightarrow q) \wedge (q \rightarrow \sim p)$$
$$\cancel{= (q \rightarrow \sim p) \wedge (\sim p \rightarrow q)}$$
$$= (p \vee q) \wedge (\sim q \vee \sim p)$$
$$\cancel{= (p \vee q) \wedge \sim(q \wedge p)}$$
$$= (\sim\sim q \vee p) \wedge (\sim p \vee \sim q)$$
$$= (\sim q \rightarrow p) \wedge (p \rightarrow \sim q)$$
$$= (p \leftrightarrow \sim q)$$
$$= RHS$$

Q) Without using truth tables, P.T. $((p \vee q) \wedge \sim p) \rightarrow q$ is a tautology.

$$((p \vee q) \wedge \sim p) \rightarrow q = ((p \wedge \sim p) \vee (q \wedge \sim p)) \rightarrow q$$
$$= (F \vee (q \wedge \sim p)) \rightarrow q$$
$$= (q \wedge \sim p) \rightarrow q$$
$$= \sim(q \wedge \sim p) \vee q$$
$$= (\sim q \vee p) \vee q$$
$$= (\sim q \vee q) \vee p$$
$$= T \vee p$$
$$= T$$

Scanned with CamScanner

Q) P.T. $\sim(p\wedge q) \to (\sim p \vee (\sim p \vee q)) \Leftrightarrow (\sim p \vee q)$

$\sim(p\wedge q) \to (\sim p \vee (\sim p \vee q))$

$= \sim(p\wedge q) \to (\sim p \vee q)$

$= (p\wedge q) \vee (\sim p \vee q)$

$= \cancel{((p\wedge q)\vee \sim p)} \vee (p\wedge q) \vee \sim p \vee q$

$= ((p \vee \sim p) \wedge (\sim p \vee q)) \vee q$

$= \cancel{F \wedge (\sim p \vee q)}\ (T \wedge (\sim p \vee q)) \vee q$

$= \sim p \vee q \vee q$

$= \sim p \vee q$

~~Indicate~~ Write the duals of

① $(p \vee q) \wedge R$

② $(p \wedge q) \vee T$

③ $\sim(p \vee q) \wedge (p \vee \sim(q \wedge \sim s))$

19/7/19

## Normal forms

Elementary product: A product of the variables and their negations in a formula is called an Elementary product.

Ex: $\sim p$, $p \wedge q$, $p \wedge \sim q$, $p \wedge \sim q \wedge \sim R$

Elementary Sum: A sum of the variables and their negations in a formula is called an Elementary sum.

Ex: $p$, $p \vee q$, $p \vee \sim q$, $p \vee \sim q \vee \sim R$

Scanned with CamScanner

## Disjunctive Normal Forms

A formulla equivalent to a given formula and consists of a sum of elementary products of the given formula.

Examples:

1. Obtain Disjunctive Normal Form of $P \wedge (P \to Q)$

$$P \wedge (P \to Q) \Leftrightarrow P \wedge (\sim P \vee Q) \quad [\text{since } P \to Q \equiv (\sim P \vee Q)]$$
$$\Leftrightarrow (P \wedge \sim P) \vee (P \wedge Q) \quad [\text{Distributive Law}]$$

Q) Obtain Disjunctive Normal Form of $(P \to Q) \to (\sim R \wedge Q)$

$$(P \to Q) \to (\sim R \wedge Q)$$
$$= \sim (P \to Q) \vee (\sim R \wedge Q)$$
$$= \sim (\sim P \vee Q) \vee (\sim R \wedge Q)$$
$$= (P \wedge \sim Q) \vee (\sim R \wedge Q)$$

## Conjuctive Normal Forms

A formula equivalent to a given formula and consists of a product of elementary sums of the given formula

Examples:

1. Obtain Conjunctive Normal forms of $P \wedge (P \to Q)$.

$$P \wedge (P \to Q) \Leftrightarrow P \wedge (\sim P \vee Q)$$

Q) Obtain conjunctive normal form of $\sim (P \vee Q) \leftrightarrow (P \wedge Q)$.

$$\sim (P \vee Q) \leftrightarrow (P \wedge Q) = (\sim (P \vee Q) \to (P \wedge Q)) \wedge ((P \wedge Q) \to \sim (P \vee Q)) \quad [R \leftrightarrow S = (R \to S) \wedge (S \to R)]$$
$$= ((P \vee Q) \vee (P \wedge Q)) \wedge (\sim (P \wedge Q) \vee (\sim P \wedge \sim Q)) \quad [P \to Q = (\sim P \vee Q)]$$
$$= ((P \vee Q \vee P) \wedge (P \vee Q \vee Q)) \wedge ((\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q))$$
$$= (P \vee Q \vee P) \wedge (P \vee Q \vee Q) \wedge (\sim P \vee \sim Q \vee \sim P) \wedge (\sim P \vee \sim Q \vee \sim Q)$$

Scanned with CamScanner

Q) Obtain Conjunctive Normal form of $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$.

$$Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q) = Q \vee ((P \vee \sim P) \wedge \sim Q)$$

$$\cancel{= Q \vee ((P \vee Q) \wedge \sim Q)}$$

$$= (Q \vee P \vee \sim P) \wedge (Q \vee \sim Q)$$

$$\cancel{= (P \vee Q \vee Q) \wedge (Q \vee \sim Q)}$$

Q) Obtain Conjunctive Normal Form of $(P \to Q) \to (\sim r \wedge q)$

$$(P \to Q) \to (\sim r \wedge q) = \sim (P \to Q) \vee (\sim r \wedge q)$$

$$= \sim (\sim P \vee Q) \vee (\sim r \wedge q)$$

$$= (P \wedge \sim Q) \vee (\sim r \wedge q)$$

$$\cancel{= (P \vee \sim r) \wedge q}$$

$$= ((P \wedge \sim Q) \vee \sim r) \wedge ((P \wedge \sim Q) \vee q)$$

$$= ((P \vee \sim r) \wedge (\sim Q \vee \sim r)) \wedge ((P \vee Q) \wedge (\sim Q \vee q))$$

$$= (P \vee \sim r) \wedge (\sim Q \vee \sim r) \wedge (P \vee Q) \wedge (\sim Q \vee q)$$

<u>Examples:</u>

1. $P \to \{(P \to Q) \wedge \sim(\sim Q \vee \sim P)\}$
2. $\sim\{P \to (Q \wedge R)\}$
3. $(\sim P \vee \sim Q) \to (P \leftrightarrow \sim Q)$
4. $(P \to (Q \to R)) \wedge (\sim P \to (\sim Q \wedge \sim R))$
5. $(\sim P \vee \sim Q) \to (P \leftrightarrow \sim Q)$ $\boxed{\text{CNF}}$

Scanned with CamScanner

~~① $p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$~~

~~$= p \to [(\sim p \vee q) \wedge (p \wedge \sim q)]$~~

~~$= p \to [\sim p \vee (q \wedge \sim q)]$~~

~~$= \sim p \vee [\sim p \vee (q \wedge \sim q)]$~~

~~$= \sim p \vee \sim p \vee (q \wedge \sim q)$~~

~~$= p \to [(\sim p \vee q) \wedge (\sim p \vee \sim q)]$~~

~~$= p \to [(\sim p \vee q) \wedge$~~

①

$$p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$$

$$= p \to [(\sim p \vee q) \wedge (q \wedge p)]$$

$$= p \to \{[(\sim p \wedge q) \vee (q \wedge q)] \wedge p\}$$

$$= \sim p \vee \{[(\sim p \wedge q) \vee q] \wedge p\} \quad \text{①}$$

$$= \{\sim p \vee [(\sim p \wedge q) \vee q]\} \wedge (\sim p \vee p)$$

$$= (\sim p \vee (\sim p \wedge q) \vee q) \wedge (\sim p \vee p)$$

$$= \cancel{[(\sim p \vee \sim p) \wedge (\sim p \vee q)] \vee q} \quad \sim p \vee [(\sim p \vee q) \wedge (q \vee q)] \wedge (\sim p$$

$$= [\sim p \vee (\sim p \vee q)] \wedge [\sim p \vee (q \vee q)] \wedge (\sim p \vee p)$$

$$= (\sim p \vee \sim p \vee q) \wedge (\sim p \vee q \vee q) \wedge (\sim p \vee p)$$

CNF

$$\text{①} = \sim p \vee \{[(\sim p \wedge q) \vee q] \wedge p\}$$

$$= \sim p \vee \{(\sim p \wedge q \wedge p) \vee (q \wedge p)\}$$

$$= \sim p \vee (\sim p \wedge q \wedge p) \vee (q \wedge p)$$

DNF

Scanned with CamScanner

② $\sim\{P \to (Q \wedge R)\}$

$= \sim\{\sim P \vee (Q \wedge R)\} = P \wedge \sim(Q \wedge R)$

$= P \wedge (\sim Q \vee \sim R)$ CNF

$= (P \wedge \sim Q) \vee (P \wedge \sim R)$ DNF

③ $(\sim P \vee \sim Q) \to (P \leftrightarrow \sim Q)$

$= (\sim P \vee \sim Q) \to \{(P \to \sim Q) \wedge (\sim Q \to P)\}$

$= (\sim P \vee \sim Q) \to \{(\sim P \vee \sim Q) \wedge (Q \vee P)\}$

$= \sim(\sim P \vee \sim Q) \vee \{(\sim P \vee \sim Q) \wedge (Q \vee P)\}$

$= (P \wedge Q) \vee \{(\sim P \vee \sim Q) \wedge (Q \vee P)\}$

$= \{(P \wedge Q) \vee (\sim P \vee \sim Q)\} \vee \{(P \wedge Q) \vee (Q \vee P)\}$

$= \cancel{(P \wedge Q)}\,[(P \vee (\sim P \vee \sim Q)) \wedge (Q \vee (\sim P \vee \sim Q))] \vee [(P \vee Q \vee P) \wedge (Q \vee Q \vee P)]$

$= \{(P \vee \sim P \vee \sim Q) \wedge (Q \vee \sim P \vee \sim Q)\} \vee \{(P \vee Q \vee P) \wedge (Q \vee Q \vee P)\}$

Scanned with CamScanner

2014

## Principle Disjunctive Normal Forms

> Let P and Q be two statement variables.

> Let us construct all possible formulas which consists of conjunctions of P or its negation and conjunctions of Q or its negation.

> None of the formulas should contain both a variable and its negation.

Ex: Either $P \wedge Q$ or $Q \wedge P$ is included but not both.

> For two variables P and Q, there are $2^2$ such formulas given by:

Ex: $P \wedge Q$, $P \wedge \sim Q$, $\sim P \wedge Q$ and $\sim P \wedge \sim Q$

→ these formulas are called minterms.

For a given formula, an equivalent formula consisting of disjunction of minterms only is known as its principal disjunction normal form.

> Also called sum-of-products canonical form.

Q) Obtain the PDNF of $P \rightarrow Q$

$$
\begin{aligned}
P \rightarrow Q &= \sim P \vee Q \\
&= (\sim P \wedge T) \vee (Q \wedge T) \\
&= [\sim P \wedge (Q \vee \sim Q)] \vee [Q \wedge (P \vee \sim P)] \\
&= (\sim P \wedge Q) \vee (\sim P \wedge \sim Q) \vee (Q \wedge P) \vee (Q \wedge \sim P) \\
&= (\sim P \wedge Q) \vee (Q \wedge P) \vee (\sim P \wedge \sim Q)
\end{aligned}
$$

Scanned with CamScanner

Q) PDNF of $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

$$= (P \wedge Q \wedge T) \vee (\sim P \wedge R \wedge T) \vee (Q \wedge R \wedge T)$$

$$= (P \wedge Q \wedge (R \vee \sim R)) \vee (\sim P \wedge R \wedge (Q \vee \sim Q)) \vee (Q \wedge R \wedge (P \vee \sim P))$$

$$= \{(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R)\} \vee \{(\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)\} \vee \{(P \wedge Q \wedge R) \vee (\sim P \wedge Q \wedge R)\}$$

$$= (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$$

Q) Find the PDNF of $P \vee (P \wedge Q)$

$$P \vee (P \wedge Q) = (P \wedge T) \vee (P \wedge Q)$$

$$= (P \wedge (Q \vee \sim Q)) \vee (P \wedge Q)$$

$$= (P \wedge Q) \vee (P \wedge \sim Q) \vee (P \wedge Q)$$

$$= (P \wedge Q) \vee (P \wedge \sim Q)$$

Q) Find the PDNF of $P \vee (\sim P \wedge Q)$

$$P \vee (\sim P \wedge Q) = (P \wedge T) \vee (\sim P \wedge Q)$$

$$= (P \wedge (Q \vee \sim Q)) \vee (\sim P \wedge Q)$$

$$= (P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$$

Q) Obtain $P \rightarrow ((P \rightarrow Q) \wedge \sim(\sim Q \vee \sim P))$ is PDNF.

Scanned with CamScanner

22/7/19

## Principal Conjunctive Normal Forms

> Let us construct all possible formulas which consists of conjunctions of P or its negation and conjunctions of Q or its negation.

> None of the formulas should contain both a variable and its negation.

Ex: either $P \vee Q$ or $Q \vee P$ is included but not both.

> For two variable P and Q, there are $2^2$ such formulas given by $P \vee Q$, $P \vee \sim Q$, $\sim P \vee Q$ and $\sim P \vee \sim Q$.

→ these formulas are maxterms.

Q) Obtain the PCNF for $(P \to R) \wedge (Q \leftrightarrow P)$

$(P \to R) \wedge (Q \leftrightarrow P) = (P \to R) \wedge (Q \to P) \wedge (P \to Q)$

$= (\sim P \vee R) \wedge (\sim Q \vee P) \wedge (\sim P \vee Q)$

$= (\sim P \vee R \vee F) \wedge (\sim Q \vee P \vee T) \wedge (\sim P \vee Q \vee F)$

$= (\sim P \vee R \vee (Q \wedge \sim Q)) \wedge (\sim Q \vee P \vee (R \wedge \sim R)) \wedge (\sim P \vee Q \vee (R \wedge \sim R))$

$= (\sim P \vee Q \vee R) \wedge (\sim P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

$= (\sim P \vee Q \vee R) \wedge (\sim P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee \sim R)$

Q) Obtain the PCNF for $(Q \to P) \wedge (\sim P \wedge Q)$

$= (\sim Q \vee P) \wedge (\sim P \wedge Q) = \cancel{(\sim Q \wedge P \wedge Q) \vee (P \wedge \sim P \wedge Q)}$

$= (\sim Q \vee P) \wedge (\sim P \vee F) \wedge (Q \vee F)$

$= (P \vee \sim Q) \wedge (\sim P \vee (Q \wedge \sim Q)) \wedge (Q \vee (P \wedge \sim P))$

$= (P \vee \sim Q) \wedge (\sim P \vee Q) \wedge (\sim Q) \wedge (P \vee Q) \wedge (\sim P \vee Q)$

$= (P \vee Q) \wedge (\sim P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee \sim Q)$

Scanned with CamScanner

25/7/19

## Theory of Inference:

> Now we come to the questions of what we mean by the rules & theory of inference.

> The rules of inference are criteria for determining the validity of an argument.

> These rules are stated in terms of the forms of the statements involved rather that in terms of the actual statements or their truth value.

→ Consider a set of propositions $P_1, P_2 \ldots P_n$ & a proposition of the form $(P_1 \wedge P_2 \wedge \ldots P_n) \rightarrow Q$ is called an argument. Here $P_1, P_2 \ldots P_n$ are called premises & $Q$ is called conclusion.

## Consistency of Premises:

The premises $P_1, P_2 \ldots P_n$ of an argument is said to be inconsistent if their conjunction $(P_1 \wedge P_2 \wedge \ldots P_n)$ is false in every situation on the other hand $P_1, P_2 \ldots P_n$ is said to be consistent if their conjunction $(P_1 \wedge P_2 \ldots P_n)$ is true in atleast one possible situation.

## Valid Argument:

An argument with premises $P_1, P_2 \ldots P_n$ & a conclusion $Q$ is said to be valid if whenever each of the premises $P_1, P_2 \ldots P_n$ is true then the conclusion is true.

Scanned with CamScanner

## Rules of Inference :-

$I_1 : P \wedge Q \Rightarrow P$

$I_2 : P \wedge Q \Rightarrow Q$ } (simplification)

$I_3 : P \Rightarrow P \vee Q$

$I_4 : Q \Rightarrow P \vee Q$ } (addition)

$I_5 : \cancel{Q \Rightarrow P \vee Q}\ \sim P \Rightarrow P \rightarrow Q$

$I_6 : Q \Rightarrow P \rightarrow Q$

$I_7 : \sim(P \rightarrow Q) \Rightarrow P$

$I_8 : \sim(P \rightarrow Q) \Rightarrow \sim Q$

$I_9 : P, Q \Rightarrow P \wedge Q$

$I_{10} : \sim P, P \vee Q \Rightarrow Q$ (disjunctive syllogism)

$I_{11} : P, P \rightarrow Q \Rightarrow Q$ (modus ponens)

$I_{12} : \sim Q, P \rightarrow Q \Rightarrow \sim P$ (modus tollens)

$I_{13} : P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$ (hypothetical syllogism)

$I_{14} : P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$ (dilemma)

8) Determine whether the conclusion C follows logically from the premises $H_1$ & $H_2$.

i) $H_1 : P \rightarrow Q$ $\quad H_2 : P$ $\quad C : Q$

Sol: $H_1 \wedge H_2 \Rightarrow C$

| $H_2 : P$ | $C : Q$ | $H_1 : P \rightarrow Q$ | $H_1 \wedge H_2$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | F |
| F | F | T | F |

valid

ii) $H_1: P \to Q$ $H_2: \sim P$ $C: Q$

| $P$ | $Q$ | $P \to Q$ | $\sim P$ | $(P \to Q) \wedge \sim P$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

valid

iii) $H_1: P \to Q$ $H_2: \sim(P \wedge Q)$ $C: \sim P$

| $P$ | $Q$ | $P \to Q$ | $P \wedge Q$ | $\sim(P \wedge Q)$ | $(P \to Q) \wedge \sim(P \wedge Q)$ | $\sim P$ |
|---|---|---|---|---|---|---|
| T | T | T | T | F | F | F |
| T | F | F | F | T | F | F |
| F | T | T | F | T | T | T |
| F | F | T | F | T | T | T |

valid

## Rules of inference:

Rule P: A premise may be introduced at any point in the division

Rule T: A formula 'S' may be introduced in a derivation if 'S' is implied by any one or more of the preceeding formulas in a derivation.

Scanned with CamScanner

Q) Demonstrate that R is a valid inference from the premises.

$$\left.\begin{array}{l} P \to Q \\ Q \to R \\ P \end{array}\right\}$$

A)

| | | | |
|---|---|---|---|
| {1} | (1) | $P \to Q$ | rule P |
| {2} | (2) | $P$ | rule P |
| {1,2} | (3) | $Q$ | rule T (1) (2) modus ponens |
| {4} | (4) | $Q \to R$ | rule P |
| {1,2,4} | (5) | $R$ | rule T (3) (4) modus ponens rule |

Q) Show that $S \vee R$ is implied by

$(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$

$$\left.\begin{array}{l} P \vee Q \\ P \to R \\ Q \to S \end{array}\right\}$$

A)

| | | | |
|---|---|---|---|
| {1} | (1) | $P \vee Q$ | Rule P |
| {1} | (2) | $\sim P \to Q$ | Rule T $(P \to Q \Leftrightarrow \sim P \vee Q)$ |
| {3} | (3) | $Q \to S$ | Rule P |
| {1,3} | (4) | $\sim P \to S$ | Rule T (Hypothetical syllogism) |
| {5} | (5) | $P \to R$ | Rule P |
| {1,3} | (6) | $\sim S \to P$ | Rule T (4) $P \to Q \Leftrightarrow \sim Q \to \sim P$ |
| {1,3,5} | (7) | $\sim S \to R$ | Rule T (5)(6) (hypothetical syllogism) |
| {1,3,5} | (8) | $S \vee R$ | Rule T (7) $(P \to Q \Leftrightarrow \sim P \vee Q)$ |

Scanned with CamScanner

1/8/19

Q) S.T. $R \wedge (P \vee Q)$ is a valid conclusion from the premises

$P \vee Q$, $Q \to R$, $P \to M$, $\sim M$

A)

| | | | |
|---|---|---|---|
| {1} | (1) | $P \to M$ | Rule P |
| {2} | (2) | $\sim M$ | Rule P |
| {1,2} | (3) | $\sim P$ | Rule T (1) (2) (M.T.) |
| {4} | (4) | $P \vee Q$ | Rule P |
| {4} | (5) | $\sim P \to Q$ | Rule T ($P \to Q \Rightarrow \sim P \vee Q$) |
| {1,2,4} | (6) | $Q$ | Rule T (3) (5) (M.P.) |
| {7} | (7) | $Q \to R$ | Rule P |
| {1,2,4,7} | (8) | $R$ | Rule T (6)(7) |
| {1,2,4,7,4} | (9) | $R \wedge (P \vee Q)$ | Rule T (8)(4) |

Conditional Proof:

~~Q) S.T. $R \to S$ can be d~~

Rule CP: If we can derive S from R and a set of premises, then we can derive $R \to S$ from the same set of premises alone.

Rule CP is also called "deduction theorem" and is generally used if the "conclusion of the form $R \to S$."

In such cases "R is taken as an additional premise" and S is derived from the given premises and R.

Scanned with CamScanner

Q) S.T. $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\sim R \vee P$, and $Q$.

| | | | |
|---|---|---|---|
| {1} | (1) | $\sim R \vee P$ | Rule P |
| {2} | (2) | $R$ | Rule P (assumed premise) |
| {1,2} | (3) | $P$ | Rule T (1) (2) |
| {4} | (4) | $P \to (Q \to S)$ | Rule P |
| {1,2,4} | (5) | $Q \to S$ | Rule T (3) (4) |
| {6} | (6) | $Q$ | Rule P |
| {1,2,4,6} | (7) | $S$ | Rule T (5) (6) |
| {1,4,6} | (8) | $R \to S$ | ~~Rule~~ CP |

Q) "If there are two ball games then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrive on time. Therefore there was no ball game." Show that these statements constitute valid argument.

A) Let P: There was a ball game

Q: Travelling was difficult

R: They arrived on time.

$$\left.\begin{array}{l} P \to Q \\ R \to \sim Q \\ R \end{array}\right\} \text{premises} \qquad \text{Conclusion: } \sim P$$

Scanned with CamScanner

| | | | |
|---|---|---|---|
| {1} | (1) | $R \to \sim Q$ | Rule P |
| {2} | (2) | $R$ | Rule P |
| {1,2} | (3) | $\sim Q$ | Rule T (1) (2) (M.P.) |
| {4} | (4) | $P \to Q$ | Rule P |
| {1,2,4} | (5) | $\sim P$ | Rule T (3) (4) (M.T.) |

Q) "If A works hard then either B or C will enjoy themselves. If B enjoys himself, then A will not work hard. If D enjoys himself, then C will not. Therefore, if A works hard, D will not enjoy himself."

A)

A: A works hard
B: B enjoy himself
C: C enjoy himself
D: D enjoy himself

$$\left.\begin{array}{l} A \to B \vee C \\ B \to \sim A \\ D \to \sim C \end{array}\right\} \text{premises} \qquad \text{Conclusion: } A \to \sim D$$

| | | | |
|---|---|---|---|
| {1} | (1) | $A \to B \vee C$ | Rule P |
| {2} | (2) | $A$ | P: Assumed |
| {1,2} | (3) | $B \vee C$ | Rule T (1) (2) (M.P.) |
| {4} | (4) | $\sim B \to C$ | Rule T (3) |
| {5} | (5) | $\sim C \to B$ | Rule T (4) |
| {6} | (6) | $D \to \sim C$ | Rule P |
| {6,5} | (7) | $D \to B$ | Rule T (6) (5) (H.S.) |
| {8} | (8) | $B \to \sim A$ | Rule P |
| {6,5,8} | (9) | $D \to \sim A$ | Rule T (7) (8) (H.S.) |
| {2,6,5,8} | (10) | $\sim D$ | Rule T (9) (2) (M.T.) |
| {1} | (11) | $A \to \sim D$ | CP rule |

Scanned with CamScanner

2/8/19

## Consistency of premises and indirect method of proof:

A set of formulas $H_1, H_2, \ldots H_m$ is said to be consistent if their conjunction has truth value 'T'.

If atleast one of the formulas $H_1, H_2, \ldots H_m$ is false so that their conjunction is false. Then the formulas $H_1, H_2, \ldots H_m$ are called inconsistent.

Alternatively, a set of formulas $H_1, H_2, \ldots H_m$ is inconsistent if their conjunction implies a contradiction.

$$H_1 \wedge H_2 \ldots \wedge H_m \Rightarrow R \wedge \sim R$$

The notion of inconsistency used in a procedure called proof by contradiction or Reductio ad absurdum or indirect method of proof.

Q) S.T. $\neg(P \wedge Q)$ follows from $\neg P \wedge \neg Q$

Solution: We introduce $\neg\neg(P \wedge Q)$ as an additional premise and show that this additional premise leads to a contradiction.

| | | | |
|---|---|---|---|
| {1} | (1) | $\neg\neg(P \wedge Q)$ | P (assumed) |
| {1} | (2) | $P \wedge Q$ | T (1), $E_1$ |
| {1} | (3) | $P$ | T (2), $I_1$ |
| {4} | (4) | $\neg P \wedge \neg Q$ | P |
| {4} | (5) | $\neg P$ | T (4), $I_1$ |
| {1,4} | (6) | $P \wedge \neg P$ | T (3), (5), $I_9$ |

Scanned with CamScanner

9) S.T. the following premises are inconsistent.

1. If Jack misses many classes through illness, then he fails high school.
2. If Jack fails high school, then he is uneducated.
3. If Jack read a lot of books, then he is not educated.
4. Jack misses many classes through illness and reads a lot of books.

Solution: E: Jack misses many classes

S: Jack fails high school

A: Jack read a lot of books

H: Jack is uneducated.

$$\left.\begin{array}{l} E \to S \\ S \to H \\ A \to \sim H \end{array}\right\} \text{premises}$$

$E \wedge A$ } conclusion

| | | | |
|---|---|---|---|
| {1} | (1) | $E \to S$ | Rule P |
| {2} | (2) | $S \to H$ | Rule P |
| {1,2} | (3) | $E \to H$ | Rule T (1),(2), H.S. |
| {4} | (4) | $A \to \sim H$ | Rule P |
| {4} | (5) | $H \to \sim A$ | Rule T (4) |
| {1,2,4} | (6) | $E \to \sim A$ | Rule T (3),(5), H.S. |
| {1,2,4} | (7) | $\sim E \vee \sim A$ | Rule T (6), Logical implication ~~Demorgan's law~~ |
| {1,2,4} | (8) | $\sim(E \wedge A)$ | Rule T (7), Demorgan's law |
| {9} | (9) | $E \wedge A$ | Rule P |
| {1,2,4,9} | (10) | $(E \wedge A) \wedge \sim(E \wedge A)$ | Rule T (8),(9), $P \wedge \sim P \Rightarrow F$ |

Scanned with CamScanner

Q) Prove by indirect method of proof that R follows from $\sim Q$, $P \to Q$, $P \vee R$.

| | | | |
|---|---|---|---|
| {1} | (1) | $\sim Q$ | Rule P |
| {2} | (2) | $P \to Q$ | Rule P |
| {2} | (3) | $\sim Q \to \sim P$ | Rule T (2) |
| {1, 2} | (4) | $\sim P$ | Rule T (1) (3) |
| {5} | (5) | $P \vee R$ | Rule P |
| {5} | (6) | $\sim P \to R$ | Rule T (5) |
| {1, 2, 5} | (7) | $R$ | Rule T (4) (6) |
| {8} | (8) | $\sim R$ | Rule P (assumed) |
| {1, 2, 5, 8} | (9) | $R \wedge \sim R$ | Rule T (7) (8) |

Scanned with CamScanner

29/8/19

# Unit-1

## Automatic Theorem Proving:

Variables: The capital letters $A, B, C, \ldots, P, Q, R \ldots$ are used as statement variables. Also used as statement formulas.

Connectives: The connectives $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$ appear in the formulas with the precedence as $\sim$ with highest followed by $\wedge$. and so on.

String of Formulas: A string of formulas is defined as follows.

A) Any formula is a string of formulas

B) If $\alpha$ and $\beta$ are strings of formulas, then $\alpha, \beta$ and $\beta, \alpha$ are strings of formulas.

C) Only those strings which are obtained by steps (A) and (B) are strings of formulas, with the exception of empty string which is also a string of formulas.

Sequents: If $\alpha$ and $\beta$ are strings of formulas, then $\alpha \rightarrow \beta$ is called a sequents in which $\alpha$ is called antecedent and $\beta$ is called consequent.

## Rules for Automatic Theorem Proving

Antecedent rules:

- Rule $\sim \Rightarrow$: If $\alpha, \beta \overset{S}{\Rightarrow} X, \gamma$ then $\alpha, \sim X, \beta \overset{S}{\Rightarrow} \gamma$
- Rule $\wedge \Rightarrow$: If $\alpha, \beta, X, Y \overset{S}{\Rightarrow} \gamma$ then $\alpha, X \wedge Y, \beta \overset{S}{\Rightarrow} \gamma$
- Rule $\vee \Rightarrow$: If $X, \alpha, \beta \overset{S}{\Rightarrow} \gamma$ and $Y, \alpha, \beta \overset{S}{\Rightarrow} \gamma$.
  then $\alpha, X \vee Y, \beta \overset{S}{\Rightarrow} \gamma$
- Rule $\rightarrow \Rightarrow$ If $Y, \alpha, \beta \Rightarrow \gamma$ and $\alpha, \beta \Rightarrow X, \gamma$
  then $\alpha, X \rightarrow Y, \beta \Rightarrow \gamma$
- Rule $\leftrightarrow \Rightarrow$ If $X, Y, \alpha, \beta \Rightarrow \gamma$ and $\alpha, \beta \Rightarrow X, Y, \gamma$
  then $\alpha, X \leftrightarrow Y, \beta \Rightarrow \gamma$

Scanned with CamScanner

Rules for Automatic Theorem proving (contd.,)

*Consequent rules:

Rule $\Rightarrow\sim$ : If $X, \alpha \overset{S}{\Rightarrow} \beta, \gamma$ then $\alpha \overset{S}{\Rightarrow} \beta, \sim X, \gamma$

Rule $\Rightarrow\wedge$ : If $\alpha \overset{S}{\Rightarrow} X, \beta, \gamma$ and $\alpha \overset{S}{\Rightarrow} Y, \beta, \gamma$
then $\alpha \overset{S}{\Rightarrow} \beta, X \wedge Y, \gamma$

Rule $\Rightarrow\vee$ : If $\alpha \overset{S}{\Rightarrow} X, Y, \beta, \gamma$ then $\alpha \overset{S}{\Rightarrow} \beta, X \vee Y, \gamma$

Rule $\Rightarrow\rightarrow$ : If $X, \alpha \overset{S}{\Rightarrow} Y, \beta, \gamma$ then $\alpha \overset{S}{\Rightarrow} \beta, X \rightarrow Y, \gamma$

Rule $\Rightarrow\leftrightarrow$ : If $X, \alpha \overset{S}{\Rightarrow} Y, \beta, \gamma$ and $Y, \alpha \overset{S}{\Rightarrow} X, \beta, \gamma$
then $\alpha \overset{S}{\Rightarrow} \beta, X \leftrightarrow Y, \gamma$

Scanned with CamScanner