Assignment 5

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Download latex-tikz codes from

https://github.com/SavaranaDatta/AI1103/blob/main/Assignment5/Assignment5.tex

PROBLEM(UGC 2018(DEC MATH SET-A), Q.111)

Let $X_1, X_2, X_3, .., X_n$ be independent random variables follow a common continuous distribution **F**, which is symmetric about 0. For i=1,2,3,..n, define

$$S_i = \begin{cases} 1 & if \;\; X_i > 0 \\ -1 & if \;\; X_i < 0 \;\; and \\ 0 & if \;\; X_i = 0 \end{cases} \tag{1.1}$$

R_i=rank of $|X_i|$ in the set$\{|X_1|, |X_2|, .., |X_n|\}$.Which of the following statements are correct?

(A) $S_1, S_2, .., S_n$ are independent and identically distributed.
(B) $R_1, R_2, .., R_n$ are independent and identically distributed.
(C) $S = (S_1, S_2, .., S_n)$ and $R = (R_1, R_2, .., R_n)$ are independent.

SOLUTION(UGC 2018(DEC MATH SET-A), Q.111)

A sequence $\{X_i\}$ is an Independent and identical if and only if

$$F_{X_n}(x) = F_{X_k}(x)$$

$\forall$ n,k,x and any subset of terms of the sequence is a set of mutually independent random variables. Where F is the probability density function.

Option(A)

As the probability distribution function of $\{X_i\}$ is symmetric about origin we can say that

$$F_{S_n}(s) = F_{S_k}(s) \quad \forall s, k, n$$

Any subset of terms of sequence $\{S_i\}$ is a set of mutually independent random variables. So, the sequence $\{S_i\}$ is independent and identical.

Option (B)

Ranking of a sequence depend on every elements of the sequence.As $\{R_i\}$ is a ranking function of $\{X_i\}$, we can say that $\{R_i\}$ is not an independent function. Hence, it is not independent and identical.

Option (C)

As the i^{th} element of sequence R depends only on X_i, we can say that sequence S and R are independent.
Answer:A,C