Assignment 6

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Download the latex code from

https://github.com/SavaranaDatta/AI1103/tree/main/Assignment_6.tex

1 PROBLEM

Let X_1 and X_2 be a random sample of size two from a distribution with probability density function

$$f_\theta(x) = \theta\left(\frac{1}{\sqrt{2\pi}}\right)e^{-\frac{1}{2}x^2} + (1-\theta)\left(\frac{1}{2}\right)e^{-|x|},$$

$-\infty < x < \infty$,
where $\theta \in \left\{0, \frac{1}{2}, 1\right\}$. If the observed values of X_1 and X_2 are 0 and 2, respectively, then the maximum likelihood estimate of θ is

1) 0
2) $\frac{1}{2}$
3) 1
4) not unique

2 SOLUTION

Definition 2.1 (Maximum Likelihood Estimation (MLE))**.** Let $x_1, x_2, \ldots, x_n$ be observations from an independent and identically distributed random variables drawn from a Probability Distribution f_0 ,where f_0 is known to be from a family of distributions f that depend on some parameters θ.

The goal of MLE is to maximize the likelihood function:

$$L(\theta) = f(x_1, x_2, \ldots, x_n \mid \theta) \tag{2.0.1}$$

$$= f(x_1 \mid \theta) \times f(x_2 \mid \theta) \times .... \times f(x_n \mid \theta) \tag{2.0.2}$$

Lemma 1. *Let $f(x)$ be a differentiable function.If $f'(x) < 0$ in the interval $x \in [a, b]$ then $f(x)$ attains its maximum value at $x = a$ in the interval $x \in [a, b]$.*

The likelihood function for given data is given by

$$L(\theta \mid X_1 = 0, X_2 = 2) = f_\theta(X_1 = 0) \times f_\theta(X_2 = 2) \tag{2.0.3}$$

$$= \left(\theta\left(\frac{1}{\sqrt{2\pi}} - \frac{1}{2}\right) + \frac{1}{2}\right)^2 e^{-2} \tag{2.0.4}$$

$$\implies L'(\theta) = \left(\sqrt{\frac{2}{\pi}} - 1\right)e^{-2}\left(\theta\left(\frac{1}{\sqrt{2\pi}} - \frac{1}{2}\right) + \frac{1}{2}\right) \tag{2.0.5}$$

$L'(\theta) < 0$ in the interval [0,1]. So from Lemma 1 we can say $L(\theta)$ is maximum at $\theta = 0$.So the required option is (1).