Assignment 5

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Download latex-tikz codes from

https://github.com/SavaranaDatta/AI1103/blob/main/Assignment5/Assignment5.tex

Problem(UGC 2018(Dec math set-a), Q.111)

Let $X_1, X_2, X_3, .., X_n$ be independent random variables follow a common continuous distribution **F**, which is symmetric about 0. For i=1,2,3,..n, define

$$S_i = \begin{cases} 1 & if \ \ X_i > 0 \\ -1 & if \ \ X_i < 0 \ \ and \\ 0 & if \ \ X_i = 0 \end{cases} \tag{1.1}$$

R_i=rank of $|X_i|$ in the set$\{|X_1|, |X_2|, .., |X_n|\}$.Which of the following statements are correct?

(A) $S_1, S_2, .., S_n$ are independent and identically distributed.
(B) $R_1, R_2, .., R_n$ are independent and identically distributed.
(C) $S = (S_1, S_2, .., S_n)$ and $R = (R_1, R_2, .., R_n)$ are independent.

Solution(UGC 2018(Dec math set-a), Q.111)

A sequence $\{X_i\}$ is an Independent and identical if and only if

$$F_{X_n}(x) = F_{X_k}(x)$$

$\forall$ n,k,x and any subset of terms of the sequence is a set of mutually independent random variables. Where F is the probability density function.

(A) As the probability distribution function of $\{X_i\}$ is symmetric about origin we can say that

$$F_{X_i}(-x) = F_{X_i}(x) \forall x \in R \tag{2.1}$$

and the mean of the distribution(μ)

$$\mu = 0 \tag{2.2}$$

The sequence S_i depend on X_i as mention in 1.1, as each S_i depend only on X_i we can say that sequence S_i is independent.

$$\Pr(S_1 = 1, S_2 = 1, ..., S_n = 1) = \Pi_{i=1}^{n} \Pr(S_i = 1) \tag{2.3}$$

Any subset of terms of sequence $\{S_i\}$ is a set of mutually independent random variables and its distribution is identical.

$$F_{S_n}(s) = F_{S_k}(s) \quad \forall s, k, n \tag{2.4}$$

So, the sequence $\{S_i\}$ is independent and identical.

(B) **Ranking** refers to the data transformation in which the numerical or ordinary values are replaced by the rank of numerical value when compared to a list of other values.Usually we follow increasing order for ranking.

Ranking of a sequence depend on every elements of the sequence.Let $\{R_i\}$ be the output sequence of the ranking function of $\{|X_i|\}$.

$$R_k = \text{rank of } |X_k| \text{ in the set}\{|X_1|, |X_2|, .., |X_n|\} \tag{2.5}$$

As R_k depend not only on $|X_k|$ but on the rest of the elements of the set$\{|X_1|, |X_2|, .., |X_n|\}$. So the sequence R_i is not independent. Hence R_i is not an independent and identical distribution.

(C) As the i^{th} element of sequence R depends only on set$\{|X_1|, |X_2|, .., |X_n|\}$, we can say that sequence S and R are independent.

Answer:A,C