

Challenging problem 19

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and latex-tikz code from

https://github.com/SavaranaDatta/AI1103/tree/main/Challenging_problem_19.tex

1 PROBLEM(CSIR UGC NET EXAM (DEC 2012) Q 51)

Suppose X_1, X_2, X_3, X_4 are i.i.d random variables taking values 1 and -1 with probability $1/2$ each. Then $E(X_1 + X_2 + X_3 + X_4)^4$ equals

- 1) 4 2) 76 3) 16 4) 12

2 SOLUTION

Let PMF of random variable X is given by

$$P_X(n) = \begin{cases} 0.5 & n=1 \\ 0.5 & n=-1 \end{cases} \quad (2.0.1)$$

Z-transform of X is

$$P_X(z) = 0.5(z^{-1} + z) \quad (2.0.2)$$

Let Y be a random variable, defined as

$$Y = X_1 + X_2 + X_3 + X_4 \quad (2.0.3)$$

Z-transform of Y is

$$P_Y(z) = P_{X_1}(z)P_{X_2}(z)P_{X_3}(z)P_{X_4}(z) \quad (2.0.4)$$

$$= (0.5)^4(z^{-1} + z)^4. \quad (2.0.5)$$

$$= (0.5)^4 \left(\binom{4}{0}z^{-4} + \binom{4}{1}z^{-2} + \binom{4}{2}z^0 + \binom{4}{3}z^2 + \binom{4}{4}z^4 \right) \quad (2.0.6)$$

$$P_Y(n) = \begin{cases} \binom{4}{0}(0.5)^4 & n=-4 \\ \binom{4}{1}(0.5)^4 & n=-2 \\ \binom{4}{2}(0.5)^4 & n=0 \\ \binom{4}{3}(0.5)^4 & n=2 \\ \binom{4}{4}(0.5)^4 & n=4 \end{cases} \quad (2.0.7)$$

$$P_{Y^4}(n) = \begin{cases} 2\binom{4}{0}(0.5)^4 & n=256 \\ 2\binom{4}{1}(0.5)^4 & n=16 \\ \binom{4}{2}(0.5)^4 & n=0 \end{cases} \quad (2.0.8)$$

$$E(Y^4) = 256(0.5)^3 + 16(0.5) \quad (2.0.9)$$

$$= 40 \quad (2.0.10)$$