AI1103 : Assignment 3

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Download all python codes from

https://github.com/SavaranaDatta/AI1103/tree/main/Assignment3/codes

and latex codes from

https://github.com/SavaranaDatta/AI1103/blob/main/Assignment3/Assignment3.tex

Problem(GATE 1996(MA) 25)

Let $Y_1, Y_2, ..., Y_{15}$ be a random sample of size 15 from the probability density function
$f_y(y) = 3(1-y)^2, 0 < y < 1$
Use the central limit theorem to approximate $P\left(\frac{1}{8} < \bar{Y} < \frac{3}{8}\right)$

Solution(GATE 1996(MA) 25)

The **central limit theorem** states that whenever a random sample of size n is taken from any distribution with mean and variance, then the sample mean will be approximately normally distributed with mean and variance. The larger the value of the sample size, the better the approximation to the normal.

$$Z_n = \frac{\bar{Y} - \mu(y)}{\frac{\sigma}{\sqrt{n}}} \tag{1.1}$$

$\bar{Y}$: Mean of the randomly selected 15 variables.

$$\bar{Y} = \frac{Y_1 + Y_2 + .. + Y_{15}}{15} \tag{1.2}$$

$\mu(y)$: Mean of the probability density function.
σ^2: Variance of the probability density function.
Z_n: Standard Normal Variant.
Mean of probability density function is

$$\mu(y) = \int_{-\infty}^{\infty} y f(y) dy \tag{1.3}$$

$$= \int_0^1 y \times 3(1-y)^2 dy \tag{1.4}$$

$$= \frac{1}{4} \tag{1.5}$$

Variance of probability density function is

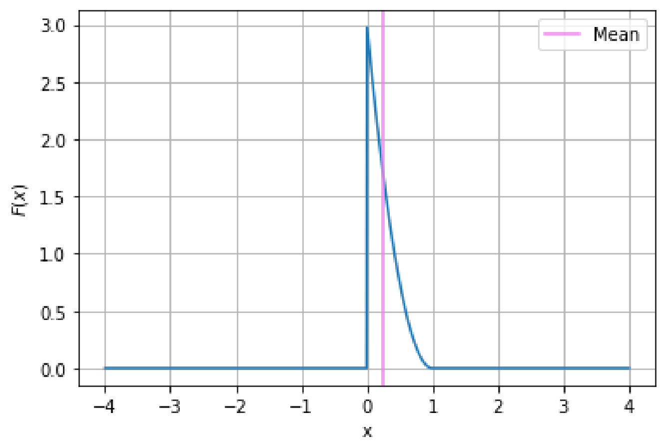

Fig. 0

$$\sigma^2(y) = E[y^2] - (E[y])^2 \tag{1.6}$$

$$= \left(\int_0^1 y^2 f(y) dy\right) - \left(\frac{1}{4}\right)^2 \tag{1.7}$$

$$\int_0^1 y^2 f(y) dy = \int_0^1 y^2 \times 3(1-y)^2 dy \tag{1.8}$$

$$= 3\int_0^1 (y - y^2)^2 dy \tag{1.9}$$

$$= \frac{1}{10} \tag{1.10}$$

Substituting equation 1.10 in equation 1.7

$$\sigma^2 = \frac{1}{10} - \frac{1}{16} \tag{1.11}$$

$$= \frac{3}{80} \tag{1.12}$$

As $\bar{Y}$ follows the standard gaussian distribution, we have

$$\bar{Y} \sim N\left(\mu, \sigma^2\right) \tag{1.13}$$

$$\bar{Y} = \left(\frac{1}{(\sqrt{2\pi})\sigma} e^{\frac{-(\bar{Y}-\mu)^2}{2\sigma^2}}\right) \tag{1.13.1}$$

From equation 1.1 $\bar{Y}$ can be expressed in Z as,

$$\bar{Y} = Z_n\left(\frac{\sigma}{\sqrt{n}}\right) + \mu(y) \tag{1.14}$$

Now,

$$\Pr\left(\frac{1}{8} < \bar{Y} < \frac{3}{8}\right) = \Pr\left(\frac{1}{8} < Z_n\left(\frac{\sigma}{\sqrt{n}}\right) + \mu(y) < \frac{3}{8}\right) \tag{1.15}$$

$$= \Pr\left(\frac{\frac{1}{8} - \mu(y)}{\frac{\sigma}{\sqrt{n}}} < Z_n < \frac{\frac{3}{8} - \mu(y)}{\frac{\sigma}{\sqrt{n}}}\right) \tag{1.16}$$

$$= Q\left(\frac{\frac{-1}{8}}{\sqrt{\frac{3}{80}}}\right) - Q\left(\frac{\frac{1}{8}}{\sqrt{\frac{3}{80}}}\right) \tag{1.17}$$

$$= 1 - 2Q\left(\frac{\frac{1}{8}}{\sqrt{\frac{3}{80}}}\right) \tag{1.18}$$

$$= 1 - 2Q\,(0.645) \tag{1.19}$$

$$= 0.9938 \tag{1.20}$$

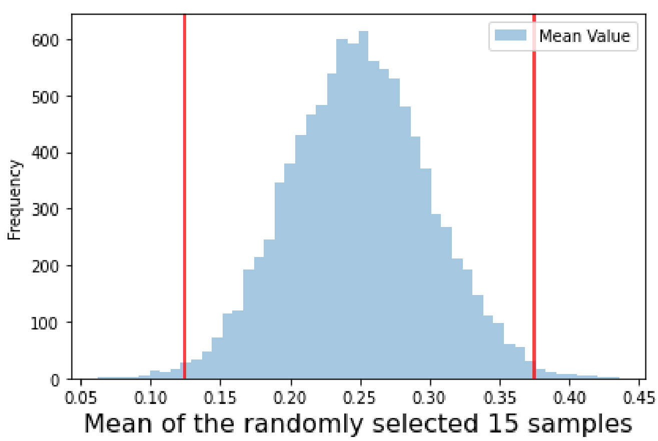

Fig. 0