AI1103 : Assignment 3

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Download all python codes from

https://github.com/SavaranaDatta/AI1103/tree/main/Assignment3/codes

and latex codes from

https://github.com/SavaranaDatta/AI1103/blob/main/Assignment3/Assignment3.tex

Problem(GATE 1996(MA) 25)

Let $Y_1, Y_2, ..., Y_{15}$ be a random sample of size 15 from the probability density function
$f_y(y) = 3(1-y)^2, 0 < y < 1$
Use the central limit theorem to approximate $P\left(\frac{1}{8} < \bar{Y} < \frac{3}{8}\right)$

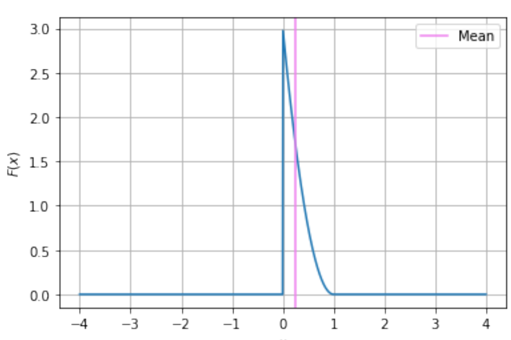
Solution(GATE 1996(MA) 25)

Mean of probability density function is

$$E[y] = \int_{-\infty}^{\infty} yf(y)dy \tag{1.1}$$

$$= \int_0^1 y \times 3(1-y)^2 dy \tag{1.2}$$

$$= \frac{1}{4} \tag{1.3}$$



Variance of probability density function is

$$var[y] = E[y^2] - (E[y])^2 \tag{1.4}$$

$$= \left(\int_0^1 y^2 f(y)dy\right) - \left(\frac{1}{4}\right)^2 \tag{1.5}$$

$$= \frac{3}{80} \tag{1.6}$$

From central limit theorem we can approximately say that

$$Z_n = \frac{y - E[y]}{\frac{\sqrt{var[y]}}{\sqrt{n}}} \tag{1.7}$$

$$Z_{15}\left(\frac{1}{8}\right) = \frac{\frac{1}{8} - \frac{1}{4}}{\frac{\sqrt{\frac{3}{80}}}{\sqrt{15}}} = -\frac{5}{2} \tag{1.8}$$

$$Z_{15}\left(\frac{3}{8}\right) = \frac{\frac{3}{8} - \frac{1}{4}}{\frac{\sqrt{\frac{3}{80}}}{\sqrt{15}}} = \frac{5}{2} \tag{1.9}$$

$$P\left(\frac{1}{8} < \bar{Y} < \frac{3}{8}\right) = P\left(-\frac{5}{2} < Z < \frac{5}{2}\right) \tag{1.10}$$

The probability for standard normal variant in above range is 0.9938.