Assignment 6

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Download the python code from

https://github.com/SavaranaDatta/AI1103/tree/main/Assignment_6.py

and latex-tikz code from

https://github.com/SavaranaDatta/AI1103/tree/main/Assignment_6.tex

1 PROBLEM

Let X_1 and X_2 be a random sample of size two from a distribution with probability density function

$$f_\theta(x) = \theta\left(\frac{1}{\sqrt{2\pi}}\right)e^{-\frac{1}{2}x^2} + (1-\theta)\left(\frac{1}{2}\right)e^{-|x|},$$

$-\infty < x < \infty$,

where $\theta \in \left\{0, \frac{1}{2}, 1\right\}$. If the observed values of X_1 and X_2 are 0 and 2, respectively, then the maximum likelihood estimate of θ is

1) 0
2) $\frac{1}{2}$
3) 1
4) not unique

2 SOLUTION

Definition 2.1 (Maximum Likelihood Estimation (MLE))**.** In statistics maximum likelihood estimation is a method of estimating the parameters of a probability distribution by maximising a likelihood function so that under the assumed statistical model the observed data is more probable .The point in the parameter space that maximises the likelihood function is called maximum likelihood estimate .

The likelihood function is given by

$$L(\theta \mid X_1 = 0, X_2 = 2) = f_\theta(X_1 = 0) \times f_\theta(X_2 = 2) \tag{2.0.1}$$

$$= \left(\theta\left(\frac{1}{\sqrt{2\pi}} - \frac{1}{2}\right) + \frac{1}{2}\right)^2 e^{-2} \tag{2.0.2}$$

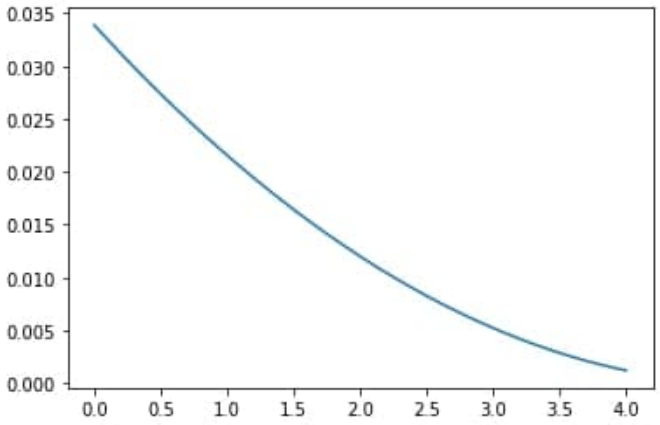

Fig. 1: Graph of $L(\theta)$

But $\theta \in \left\{0, \frac{1}{2}, 1\right\}$. From figure we can see that maximum value of $L(\theta)$ occurs at $\theta = 0$. Therefore required option is **1**.