Savarana Datta Reddy

IITH(AI)

September 13, 2021

Question

Find the angle between the following pair of lines

1. $$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \tag{1}$$
$$\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4} \tag{2}$$

2. $$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} \tag{3}$$
$$\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-3}{8} \tag{4}$$

Solution

Let a and b be the direction vectors of the two lines and θ be the angle between the lines. The angle is given by

$$\cos\theta = \frac{\mathrm{a}^\top \mathrm{b}}{\|\mathrm{a}\| \, \|\mathrm{b}\|} \tag{5}$$

Part 1

- Let the direction vectors of lines be a and b

$$\mathrm{a} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} \tag{6}$$

$$\mathrm{b} = \begin{pmatrix} -1 \\ 8 \\ 4 \end{pmatrix} \tag{7}$$

- A vector joining the 2 lines is given by

$$v_1 = \begin{pmatrix} 2-(-2) \\ 1-4 \\ -3-5 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ -8 \end{pmatrix} \tag{8}$$

$$a^\top b = \begin{pmatrix} 2 & 5 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ 8 \\ 4 \end{pmatrix} \tag{9}$$

$$= 26 \tag{10}$$

$$\|a\| = \sqrt{38} \tag{11}$$

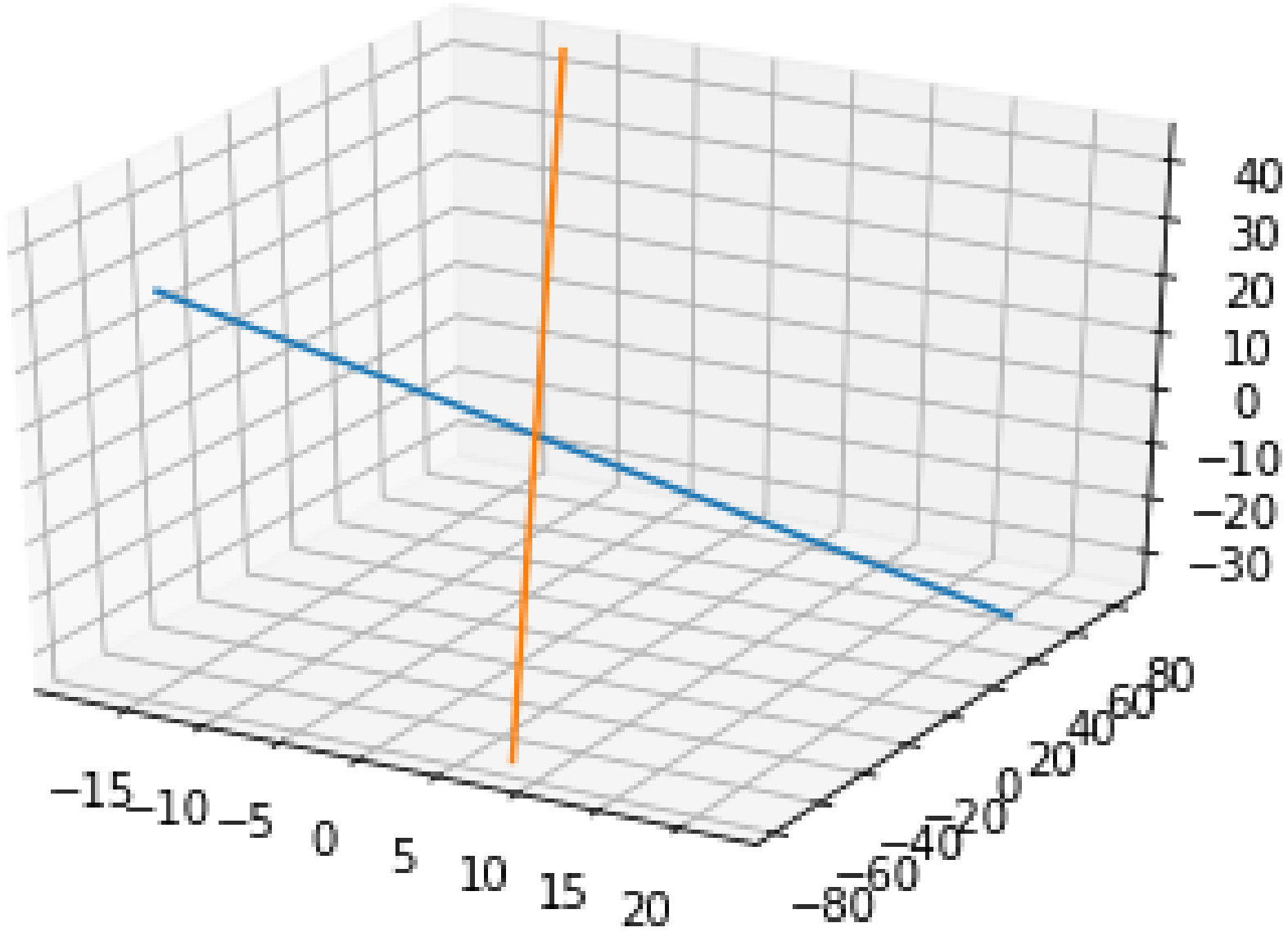
$$\|b\| = 9 \tag{12}$$

$$\implies \cos\theta = \frac{26}{9\sqrt{38}} \tag{13}$$

$$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right) \tag{14}$$

$$= 62.053 \tag{15}$$

Plot of the straight lines



Part 2

- Let the direction vectors of lines be c and d

$$\mathrm{c} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \tag{16}$$

$$\mathrm{d} = \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} \tag{17}$$

$$\mathbf{c}^\top \mathbf{d} = \begin{pmatrix} 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} = 18 \tag{18}$$

$$\|\mathbf{c}\| = 3 \tag{19}$$

$$\|\mathbf{d}\| = 9 \tag{20}$$

$$\implies \cos\theta = \frac{18}{9 \times 3} \tag{21}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right) \tag{22}$$

$$= 48.189 \tag{23}$$

Plot of the straight lines