Assignment 5

Savarana Datta - AI20BTECH11008

Download all python codes from

https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_GATE_Assignment2/codes

Download latex-tikz codes from

https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_GATE_Assignment2/GATE_Assignment2.tex

1 Problem(GATE EC Q.32)

The signal x(t) is described by

$$x(t) = \begin{cases} 1 & ,\text{for} -1 \le t \le +1 \\ 0 & ,otherwise \end{cases} \tag{1.0.1}$$

Two of the angular frequencies at which its fourier transform becomes zero are

1) π,2π
2) 0.5π,1.5π
3) 0,π
4) 2π,2.5π

2 Solution

Lemma 2.1. *The fourier transform of a rect function is sinc function*

$$rect\left(\frac{t}{\tau}\right) \overset{\mathcal{F}}{\rightleftharpoons} \tau sinc\left(f\tau\right) \tag{2.0.1}$$

Proof.

$$\int_{-\infty}^{\infty} rect\left(\frac{x}{\tau}\right) e^{-i2\pi xt} dx = \int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} e^{-i2\pi xt} dx \tag{2.0.2}$$

$$= \left[\frac{e^{-i2\pi xt}}{-i2\pi t}\right]_{\frac{-\tau}{2}}^{\frac{\tau}{2}} \tag{2.0.3}$$

$$= \frac{e^{-i\pi t\tau} - e^{i\pi t\tau}}{-i2\pi t} \tag{2.0.4}$$

$$= \tau \frac{\sin \pi t\tau}{\pi t\tau} \tag{2.0.5}$$

$$= \tau sinc(t\tau) \tag{2.0.6}$$

□

We can observe that

$$x(t) = rect(\frac{t}{2}) \tag{2.0.7}$$

From the lemma 2.1, fourier transform of x(t) is

$$x(f) = 2sinc(2f) \tag{2.0.8}$$

$$= 2\frac{sin\left(2\pi f\right)}{(2\pi f)} \tag{2.0.9}$$

$$= 2\frac{sin(\omega)}{\omega} \tag{2.0.10}$$

x(f) is zero when $\omega = n\pi$, where $n \in I - \{0\}$. Hence, option 1 is true.

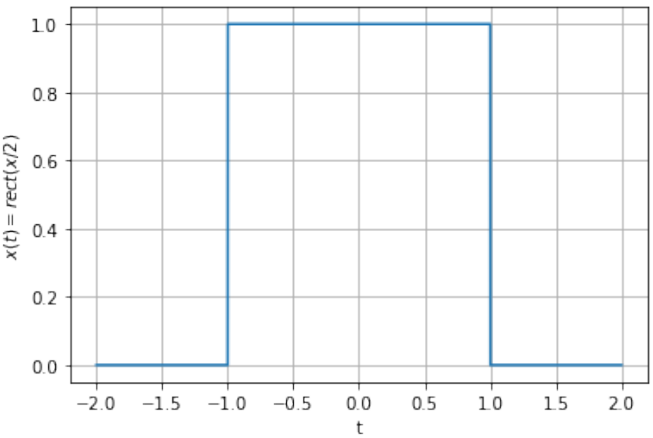

Fig. 4: Plot of x(t)

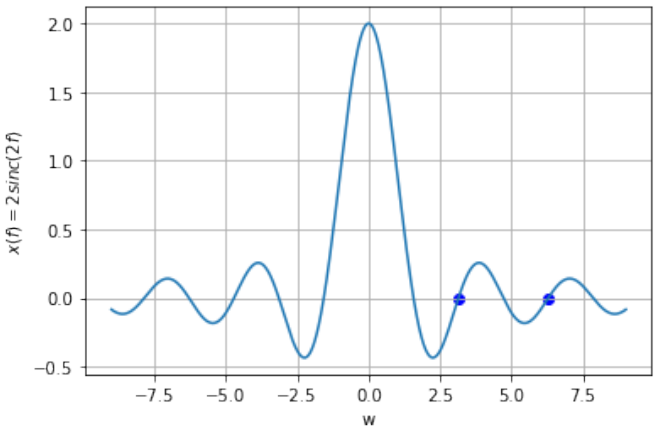

Fig. 4: Plot of the fourier transform