

# GATE EC 21

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## Question

Consider the signal

$$f(t) = 1 + 2 \cos(\pi t) + 3 \sin\left(\frac{2\pi}{3}t\right) + 4 \cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right) \quad (1)$$

, where  $t$  is in seconds. Its fundamental time period in seconds, is

## Solution

Individual natural frequencies of each term are

$$f_1 = \frac{1}{2} \quad (2)$$

$$f_2 = \frac{1}{3} \quad (3)$$

$$f_3 = \frac{1}{4} \quad (4)$$

Individual fundamental time periods of each term are

$$T_1 = \frac{1}{f_1} = 2 \quad (5)$$

$$T_2 = \frac{1}{f_2} = 3 \quad (6)$$

$$T_3 = \frac{1}{f_3} = 4 \quad (7)$$

Fundamental time period( $T$ ) of the signal

$$T = LCM(T_1, T_2, T_3) \quad (8)$$

$$= LCM(2, 3, 4) \quad (9)$$

$$= 12\text{sec} \quad (10)$$

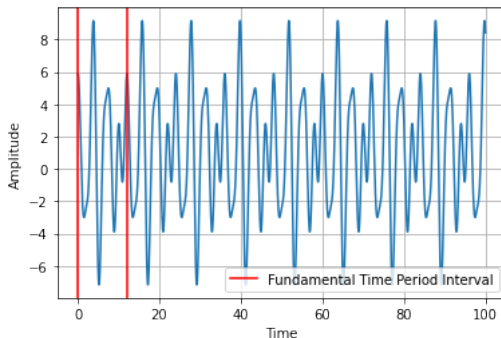


Figure: Plot of the signal

## Linearity of Fourier Transform

$$\mathcal{F}\{c_1g(t) + c_2h(t)\} = c_1\mathcal{F}\{g(t)\} + c_2\mathcal{F}\{h(t)\} \quad (11)$$

## Fourier Transform of Cosine function

Let  $x(t) = \cos(2\pi At)$ ,

$$\cos(2\pi At) = \frac{e^{i2\pi At} + e^{-i2\pi At}}{2} \quad (12)$$

The Fourier transform of  $x(t)$

$$G_x(f) = \int_{-\infty}^{\infty} \frac{e^{i2\pi At} + e^{-i2\pi At}}{2} e^{-i2\pi ft} dt \quad (13)$$

$$= \frac{1}{2} \left[ \int_{-\infty}^{\infty} e^{-i2\pi t(f-A)} dt + \int_{-\infty}^{\infty} e^{-i2\pi t(A+F)} dt \right] \quad (14)$$

$$= \frac{1}{2} [\delta(f-A) + \delta(f+A)] \quad (15)$$

## Fourier Transform of sine function

Let  $x(t) = \sin(2\pi At)$ ,

$$\cos(2\pi At) = \frac{e^{i2\pi At} - e^{-i2\pi At}}{2i} \quad (16)$$

The Fourier transform of  $x(t)$

$$G_x(f) = \int_{-\infty}^{\infty} \frac{e^{i2\pi At} - e^{-i2\pi At}}{2i} e^{-i2\pi ft} dt \quad (17)$$

$$= \frac{1}{2i} \left[ \int_{-\infty}^{\infty} e^{-i2\pi t(f-A)} dt - \int_{-\infty}^{\infty} e^{-i2\pi t(A+F)} dt \right] \quad (18)$$

$$= \frac{1}{2i} [\delta(f - A) - \delta(f + A)] \quad (19)$$

Using 11,15 and 19

$$\begin{aligned} G_x(f) = & \delta(f) + \delta(f - (1)2) + \delta\left(f + \frac{1}{2}\right) \\ & + \sqrt{2}\delta\left(f - \frac{1}{4}\right) + \sqrt{2}\delta\left(f + \frac{1}{4}\right) + \sqrt{2}i\delta\left(f - \frac{1}{4}\right) - \sqrt{2}i\delta\left(f + \frac{1}{4}\right) - \\ & \frac{3}{2}i\delta\left(f - \frac{1}{3}\right) + \frac{3}{2}i\delta\left(f + \frac{1}{3}\right) \quad (20) \end{aligned}$$

Minimum frequency is zero and the maximum frequency is  $\frac{1}{2}$ . So, the bandwidth is  $\frac{1}{2}$