Quiz 1

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Download latex-tikz codes from

https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_Quiz1

1 Question

For the following signal determine whether the system is (1) stable, (2) casual, (3) linear and (4) time invariant.

$$T(x[n]) = x[n^2] \tag{1.0.1}$$

2 Solution

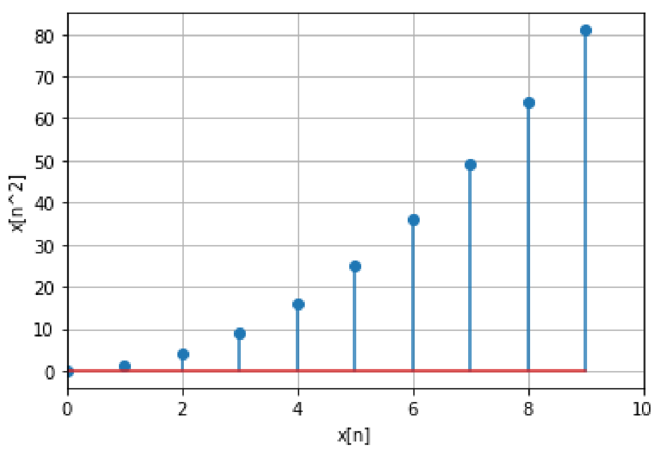

Fig. 0: plot of the system

Definition 2.1. ***Stable*** *A system is said to be BIBO stable if the response to a bounded input is always bounded.*

As the given signal input $x[n]$ is bounded,

$$|x[n]| < M \text{ for some real } M \tag{2.0.1}$$

$$\text{Hence } |x[n^2]| < M \tag{2.0.2}$$

So $x[n^2]$ is also bounded. Hence, the system is stable i.e, bounded input bounded output stable.

Definition 2.2. ***Casual*** *The output at any instant does not depend on the future inputs i.e, for at n_0 $y[n_0]$ does not depend on $x[n]$ for $n > n_0$.*

Here, for this signal the output depends on x$[n_0^2]$. As n_0 is an integer $n_0^2 > n_0$ for $n_0 > 1$.
For example consider n=2

$$x[2] \implies x[4] \tag{2.0.3}$$

Here the output for n=2 depends on n=4. So the output depends on the future input. Hence, the system is non casual.

Definition 2.3. ***Linear*** *The response to an arbitary linear combination of input signals is always the same linear combinations of the individual responses to these signals*

$$x_1[n] \implies x_1[n^2] \tag{2.0.4}$$

$$x_2[n] \implies x_2[n^2] \tag{2.0.5}$$

$$ax_1[n] + bx_2[n] \implies ax_1[n^2] + bx_2[n^2] \tag{2.0.6}$$

As this system obeys both law of addition and law of homogenity, the given system is linear.

Definition 2.4. ***Time Invariant*** *The response to an arbitrary translated set of inputs is always the response to the original set, but translated by the same amount.*
If

$$x[n] \implies y[n] \tag{2.0.7}$$

then

$$x[n - n_0] \implies y[n - n_0] \tag{2.0.8}$$

for all x and n_0.

Here

$$x[n] \implies x[n^2] \tag{2.0.9}$$

adding time delay(n_0) to the output signal

$$x[n^2] \implies x[(n - n_0)^2] \tag{2.0.10}$$

adding time delay(n_0) to the input signal

$$x[n] \implies x[n-n_0] \tag{2.0.11}$$

Now the ouput signal

$$x[n-n_0] \implies x[n^2-n_0] \tag{2.0.12}$$

As 2.0.10 and 2.0.12 are not same, the given signal is time variant.