Assignment 3

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Download all python codes from

https://github.com/SavaranaDatta/EE3900/tree/main/Assignment3/codes

and latex codes from

https://github.com/SavaranaDatta/EE3900/tree/main/Assignment3/Assignment3.tex

1 Construction 2.16

Let ABC be a right triangle in which a=8 , c=6 and $\angle B = 90°$. BD is the perpendicular from $\mathbf{B}$ on AC (altitude). The circle through $\mathbf{B}, \mathbf{C}, \mathbf{D}$(circumcircle of ΔBCD) is drawn. Construct the tangents from $\mathbf{A}$ to this circle.

2 Solution

Lemma 2.1. *If ΔABC is a right angled triangle at $\mathbf{B}$, then the points of contact of tangents from $\mathbf{A}$ to the circumcircle of ΔBCD (where $\mathbf{D}$ is the foot of perpendicular from $\mathbf{B}$ to side AC) satisfies the equation*

$$\left(\mathbf{x} - \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\right)\left(\mathbf{A} - \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\right)^{\top} = \frac{a^2}{4} \tag{2.0.1}$$

Proof. Given,
ΔABC is a right angled triangle at $\mathbf{B}$ and $\mathbf{D}$ is the foot of perpendicular from $\mathbf{B}$ to the side AC.
As ΔBCD is right angled at $\mathbf{D}$, the circumcentre($\mathbf{C_1}$) is the midpoint of side BC and radius(r) is half the length of BC

$$\mathbf{C_1} = \frac{\mathbf{B}+\mathbf{C}}{2} \tag{2.0.2}$$

$$r = \frac{a}{2} \tag{2.0.3}$$

As it is tangent to the circle

$$(\mathbf{x}-\mathbf{A})^{\top}\left(\mathbf{x} - \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\right) = 0 \tag{2.0.4}$$

$$\mathbf{x}^{\top}\mathbf{x} - \mathbf{x}^{\top}\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right) - \mathbf{A}^{\top}\mathbf{x} + \mathbf{A}^{\top}\left(\frac{\mathbf{B}+\mathbf{c}}{2}\right) = 0 \tag{2.0.5}$$

As the point of intersection lies on the circle

$$\left(\mathbf{x} - \frac{\mathbf{B}+\mathbf{C}}{2}\right)^{\top}\left(\mathbf{x} - \frac{\mathbf{B}+\mathbf{C}}{2}\right) = \frac{a^2}{4} \tag{2.0.6}$$

$$\mathbf{x}^{\top}\mathbf{x} - 2\mathbf{x}^{\top}\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right) + \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)^{\top} = \frac{a^2}{4} \tag{2.0.7}$$

Subtracting equations 2.0.5 and 2.0.7 we get

$$\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)^{\top} - \mathbf{A}^{\top}\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right) + \mathbf{A}^{\top}\mathbf{x} - \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)^{\top}\mathbf{x} = \frac{a^2}{4} \tag{2.0.8}$$

$$\left(\mathbf{x} - \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\right)\left(\mathbf{A} - \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\right)^{\top} = \frac{a^2}{4} \tag{2.0.9}$$

□

Let

$$\mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.10}$$

$$\implies \mathbf{C} = \begin{pmatrix}8\\0\end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{A} = \begin{pmatrix}0\\6\end{pmatrix} \tag{2.0.12}$$

The centre and the radius of circumcircle is given by

$$\mathbf{C_1} = \begin{pmatrix}4\\0\end{pmatrix} \tag{2.0.13}$$

$$r = \frac{a}{2} = 4 \tag{2.0.14}$$

The equation of circumcircle of ΔBCD is

$$\mathbf{x}^{\top}\mathbf{x} - 2\mathbf{C_1}\mathbf{x} + f = 0 \tag{2.0.15}$$

$$\implies \mathbf{x}^{\top}\mathbf{x} - \begin{pmatrix}8 & 0\end{pmatrix}\mathbf{x} = 0 \tag{2.0.16}$$

Let $\mathbf{T}$ be the point of intersection of tangent to the circle

$$\mathbf{T} = \begin{pmatrix}\alpha\\\beta\end{pmatrix} \tag{2.0.17}$$

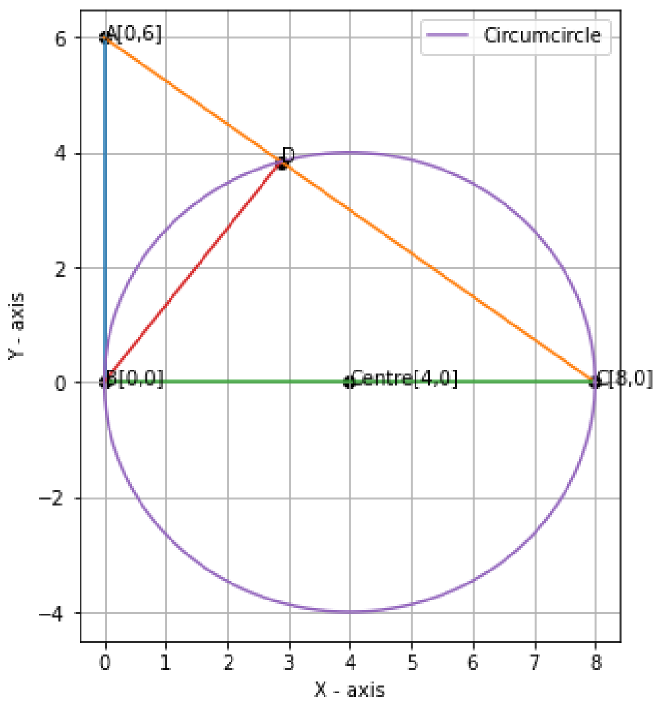

Fig. 0: The plot of circumcircle of ΔBCD

As $\mathbf{T}$ satisfies 2.0.1 we have

$$\left(\mathbf{T}-\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\right)\left(\mathbf{A}-\left(\frac{\mathbf{B}+\mathbf{C}}{2}\right)\right)^\top = \frac{a^2}{4} \tag{2.0.18}$$

$$\left(\begin{pmatrix}\alpha\\ \beta\end{pmatrix}-\begin{pmatrix}4\\0\end{pmatrix}\right)\left(\begin{pmatrix}0\\6\end{pmatrix}-\begin{pmatrix}4\\0\end{pmatrix}\right)^\top = 16 \tag{2.0.19}$$

$$\begin{pmatrix}\alpha-4\\ \beta\end{pmatrix}\begin{pmatrix}-4 & 6\end{pmatrix} = 16 \tag{2.0.20}$$

$$2\alpha = 3\beta \tag{2.0.21}$$

As point $\mathbf{T}$ lies on the circumcircle it satisfies the equation 2.0.16

$$\mathbf{T}^\top\mathbf{T}-\begin{pmatrix}8 & 0\end{pmatrix}\mathbf{T} = 0 \tag{2.0.22}$$

$$\implies \alpha^2+\beta^2-8\alpha = 0 \tag{2.0.23}$$

Solving equations 2.0.21 and 2.0.23 we have

$$\mathbf{T_1} = \begin{pmatrix}0\\0\end{pmatrix}(or) \tag{2.0.24}$$

$$\mathbf{T_2} = \begin{pmatrix}\frac{72}{13}\\ \frac{48}{13}\end{pmatrix} \tag{2.0.25}$$

The equation of tangent 1 is given by

$$x-0 = \frac{0-0}{6-0}(y-0) \tag{2.0.26}$$

$$\implies \begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 0 \tag{2.0.27}$$

The equation of tangent 2 is given by

$$y-6 = \frac{6-\frac{48}{13}}{0-\frac{72}{13}}(x-0) \tag{2.0.28}$$

$$y-6 = \frac{-30}{72}(x) \tag{2.0.29}$$

$$y+\frac{30}{72}x = 6 \tag{2.0.30}$$

$$\implies \begin{pmatrix}\frac{30}{72} & 1\end{pmatrix}\mathbf{x} = 6 \tag{2.0.31}$$

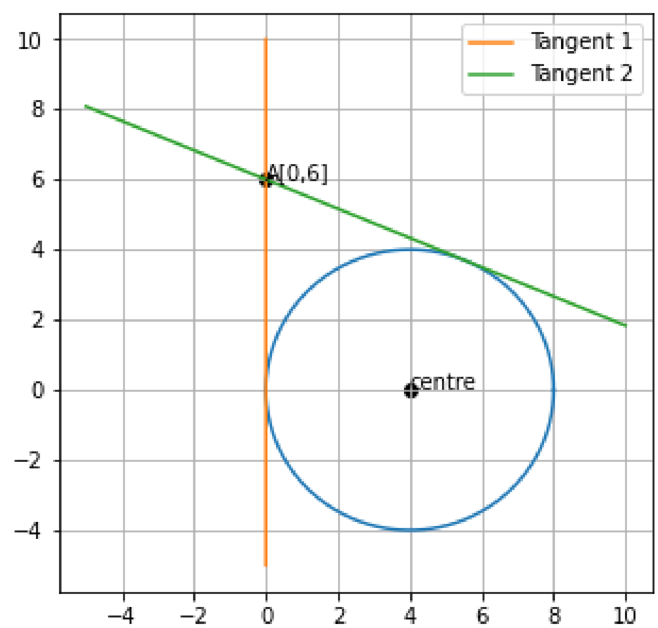

Fig. 0: The plot of tangents to the circle