Assignment 5

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Download all python codes from

https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_As5/codes/EE3900_As5.py

Download latex-tikz codes from

https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_As5/EE3900_As5.tex

1 Problem(Quadratic Forms Q.2.5)

Find the area of the region in the first quadrant enclosed by x-axis, line $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0$ and the circle $\mathbf{x}^\top\mathbf{x} = 4$.

2 Solution

Lemma 2.1. *The points of intersection of line L:$\mathbf{x} = \mathbf{q} + \mu\mathbf{m}$ with the conic*

$$\mathbf{x}^\top\mathbf{V}\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{2.0.1}$$

are given by

$$\mathbf{x_i} = \mathbf{q} + \mu_i\mathbf{m} \tag{2.0.2}$$

where

$$\mu_i = \frac{1}{\mathbf{m}^\top\mathbf{V}\mathbf{m}} - \mathbf{m}^\top\left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) \pm \sqrt{[\mathbf{m}^\top(\mathbf{V}\mathbf{q} + \mathbf{u})] - (\mathbf{q}^\top\mathbf{V}\mathbf{q} + 2\mathbf{u}^\top\mathbf{q} +)(\mathbf{m}^\top\mathbf{V}\mathbf{m})} \tag{2.0.3}$$

The equation of line $\begin{pmatrix}1 & -\sqrt{3}\end{pmatrix}\mathbf{x} = 0$ can also be expressed as $\mathbf{x} = \mu\begin{pmatrix}1\\ \frac{1}{\sqrt{3}}\end{pmatrix}$.

The matrix parameters of the circle $\mathbf{x}^\top\mathbf{x} = 4$ are

$$\mathbf{V} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \tag{2.0.4}$$

$$\mathbf{u} = \begin{pmatrix}0\\ 0\end{pmatrix} \tag{2.0.5}$$

$$f = -4 \tag{2.0.6}$$

The points of intersection of the line $\mathbf{x} = \mu\begin{pmatrix}1\\ \frac{1}{\sqrt{3}}\end{pmatrix}$ and the circle $\mathbf{x}^\top\mathbf{x} = 4$ are

$$\mathbf{x_i} = \mathbf{q} + \mu_i\mathbf{m} \tag{2.0.7}$$

$$= \mu_i\begin{pmatrix}1\\ \frac{1}{\sqrt{3}}\end{pmatrix} \tag{2.0.8}$$

where

$$\mu_i = \pm\sqrt{3} \tag{2.0.9}$$

As the required point of intersection lies in first quadrant we have $\mu_i > 0$

$$\implies \mathbf{A} = \sqrt{3}\begin{pmatrix}1\\ \frac{1}{\sqrt{3}}\end{pmatrix} = \begin{pmatrix}\sqrt{3}\\ 1\end{pmatrix} \tag{2.0.10}$$

The points of intersection of the line $\mathbf{x} = \mu\begin{pmatrix}1\\ 0\end{pmatrix}$ and the circle $\mathbf{x}^\top\mathbf{x} = 4$ are

$$\mathbf{X_j} = \mathbf{q} + \mu_j\mathbf{m} \tag{2.0.11}$$

$$= \mu_j\begin{pmatrix}1\\ 0\end{pmatrix} \tag{2.0.12}$$

where

$$\mu_j = \pm 2 \tag{2.0.13}$$

As the required point of intersection lies in first quadrant we have $\mu_j > 0$

$$\implies \mathbf{B} = 2\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}2\\ 0\end{pmatrix} \tag{2.0.14}$$

The angle(θ) of the sector AOB is

$$\cos\theta = \frac{\mathbf{A}^\top\mathbf{B}}{||\mathbf{A}||||\mathbf{B}||} \tag{2.0.15}$$

$$= \frac{2\sqrt{3}}{2\times 2} \tag{2.0.16}$$

$$= \frac{\sqrt{3}}{2} \tag{2.0.17}$$

$$\implies \theta = 30^\circ \tag{2.0.18}$$

$$\text{Area of the sector} = \left(\frac{\theta}{360^\circ}\right)\pi r^2 \tag{2.0.19}$$
$$= \frac{\pi}{3} \tag{2.0.20}$$

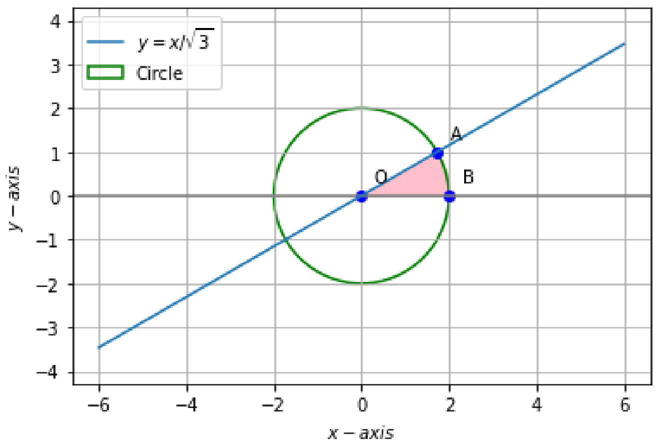

Fig. 0: Reference plot