Assignment 5

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1 PROBLEM(GATE EC Q.32)

The signal x(t) is described by

$$x(t) = \begin{cases} 1 & , \text{for} -1 \leq t \leq +1 \\ 0 & , otherwise \end{cases} \quad (1.0.1)$$

Two of the angular frequencies at which its fourier transform becomes zero are

1) $\pi, 2\pi$
2) $0.5\pi, 1.5\pi$
3) $0, \pi$
4) $2\pi, 2.5\pi$

2 SOLUTION

Lemma 2.1. *The fourier transform of a rect function is sinc function*

$$rect\left(\frac{t}{\tau}\right) \overset{\mathcal{F}}{\rightleftharpoons} \tau sinc\left(f\tau\right) \quad (2.0.1)$$

Proof.

$$\int_{-\infty}^{\infty} rect\left(\frac{x}{\tau}\right) e^{-i2\pi xt} dx = \int_{\frac{-\tau}{2}}^{\frac{\tau}{2}} e^{-i2\pi xt} dx \quad (2.0.2)$$

$$= \left[\frac{e^{-i2\pi xt}}{-i2\pi t}\right]_{\frac{-\tau}{2}}^{\frac{\tau}{2}} \quad (2.0.3)$$

$$= \frac{e^{-i\pi t\tau} - e^{i\pi t\tau}}{-i2\pi t} \quad (2.0.4)$$

$$= \tau \frac{\sin \pi t\tau}{\pi t\tau} \quad (2.0.5)$$

$$= \tau sinc(t\tau) \quad (2.0.6)$$

$\square$

We can observe that

$$x(t) = rect\left(\frac{t}{2}\right) \quad (2.0.7)$$

From the lemma 2.1, fourier transform of x(t) is

$$x(f) = 2sinc(2f) \quad (2.0.8)$$

$$= 2\frac{sin\,(2\pi f)}{(2\pi f)} \quad (2.0.9)$$

$$= 2\frac{sin(\omega)}{\omega} \quad (2.0.10)$$

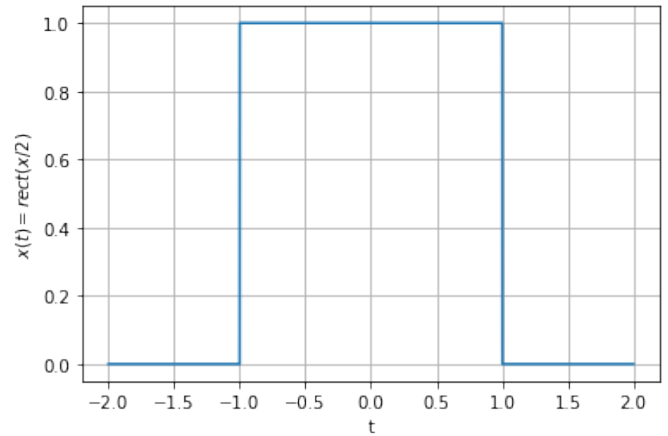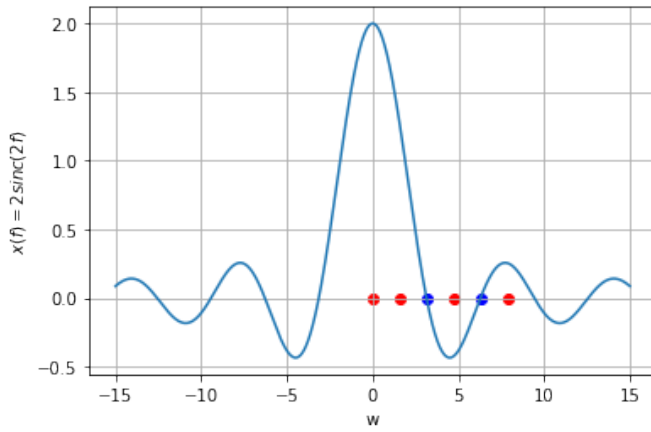x(f) is zero when $\omega = n\pi$, where $n \in I - \{0\}$. Hence, option 1 is true.

Fig. 4: Plot of x(t)

Fig. 4: Plot of the fourier transform