Assignment 5

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Download all python codes from

https://github.com/SavaranaDatta/EE3900/blob/main/EE3900 GATE Assignment2/codes

Download latex-tikz codes from

https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_GATE_Assignment2/GATE_Assignment2.tex

1 Problem(GATE EC Q.32)

The signal x(t) is described by

$$x(t) = \begin{cases} 1 & , \text{for } -1 \le t \le +1 \\ 0 & , otherwise \end{cases}$$
 (1.0.1)

Two of the angular frequencies at which its fourier transform becomes zero are

- 1) π , 2π
- 2) $0.5\pi, 1.5\pi$
- 3) $0.\pi$
- 4) $2\pi, 2.5\pi$

2 Solution

Lemma 2.1. The fourier transform of a rect function is sinc function

$$rect\left(\frac{t}{\tau}\right) \stackrel{\mathcal{F}}{\rightleftharpoons} \tau sinc\left(f\tau\right)$$
 (2.0.1)

Proof.

$$\int_{-\infty}^{\infty} rect\left(\frac{x}{\tau}\right) e^{-i2\pi xt} dx = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-i2\pi xt} dx \qquad (2.0.2)$$

$$= \left[\frac{e^{-i2\pi xt}}{-i2\pi t} \right]^{\frac{1}{2}}$$
 (2.0.3)

$$=\frac{e^{-i\pi t\tau}-e^{i\pi t\tau}}{-i2\pi t} \qquad (2.0.4)$$

$$= \tau \frac{\sin \pi t \tau}{\pi t \tau} \tag{2.0.5}$$

$$= \tau sinc(t\tau) \tag{2.0.6}$$

We can observe that

$$x(t) = rect\left(\frac{t}{2}\right) \tag{2.0.7}$$

From the lemma 2.1, fourier transform of x(t) is

$$x(f) = 2\operatorname{sinc}(2f) \tag{2.0.8}$$

$$=2\frac{\sin(2\pi f)}{(2\pi f)}$$
 (2.0.9)

$$=2\frac{\sin(\omega)}{\omega}\tag{2.0.10}$$

x(f) is zero when $\omega = n\pi$, where $n \in I - \{0\}$. Hence, option 1 is true.

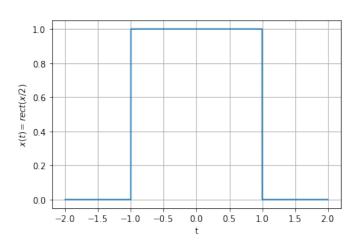


Fig. 4: Plot of x(t)

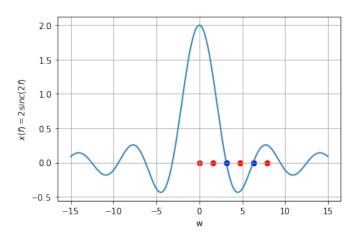


Fig. 4: Plot of the fourier transform