Assignment 3

Savarana Datta - AI20BTECH11008

Download all python codes from

https://github.com/SavaranaDatta/EE3900/tree/main/Assignment3/codes

and latex codes from

https://github.com/SavaranaDatta/EE3900/tree/main/Assignment3/Assignment3.tex

1 Construction 2.16

Let ABC be a right triangle in which a=8 , c=6 and $\angle B = 90°$. BD is the perpendicular from **B** on AC (altitude). The circle through $\mathbf{B}, \mathbf{C}, \mathbf{D}$(circumcircle of ΔBCD) is drawn. Construct the tangents from **A** to this circle.

2 Solution

Lemma 2.1. *If ΔABC is a right angled triangle at $\mathbf{B}$(Be $\mathbf{O}$), then the points of contact of tangents from $\mathbf{A}$ to the circumcircle of ΔBCD (where $\mathbf{D}$ is the foot of perpendicular from $\mathbf{B}$ to side AC) are $\mathbf{B}$ and $\mathbf{P}$*

$$\mathbf{P} = 2\mathbf{A} + 2\lambda\left(\frac{\mathbf{C}}{2} - \mathbf{A}\right) \tag{2.0.1}$$

$$\textit{where } \lambda = \frac{a^2}{a^2 + \frac{c^2}{4}} \tag{2.0.2}$$

Proof. Given,
ΔABC is a right angled triangle at $\mathbf{B}$(be $\mathbf{O}$) and $\mathbf{D}$ is the foot of perpendicular from $\mathbf{B}$ to the side AC. As ΔBCD is right angled at $\mathbf{D}$, the circumcentre($\mathbf{C_1}$) is the midpoint of side BC and radius(r) is half the length of BC

$$\mathbf{C_1} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{2.0.3}$$

$$\implies \mathbf{C_1} = \frac{\mathbf{C}}{2} \tag{2.0.4}$$

$$r = \frac{a}{2} \tag{2.0.5}$$

As ΔABC is right angled at $\mathbf{B}$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B}) = 0 \tag{2.0.6}$$

$$\implies \mathbf{A}^\top \mathbf{C} = 0 \tag{2.0.7}$$

$\mathbf{B}$ is one of the points of contact of tangent to the circle. Let $\mathbf{M}$ be the point of intersection of BP and AC_1

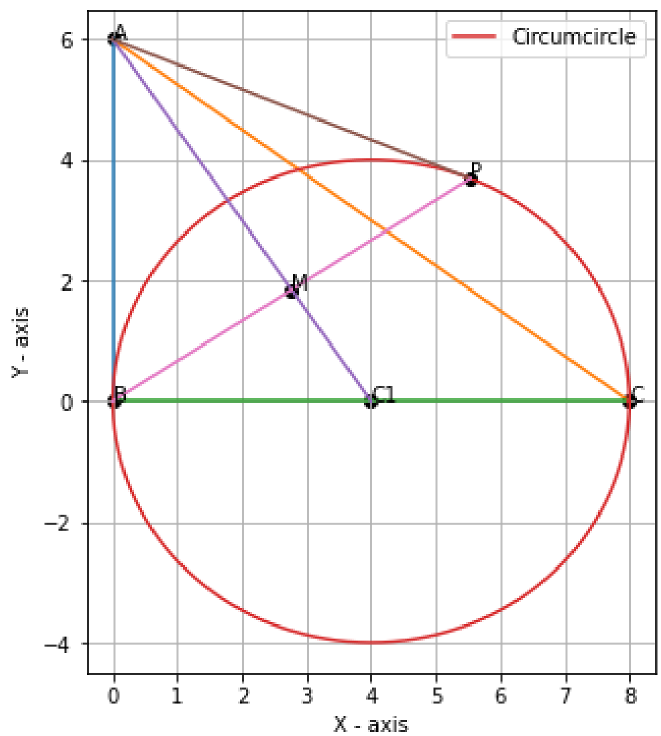

Fig. 0: Reference plot

As $\mathbf{M}$ lies on the line AC_1

$$\mathbf{M} = \mathbf{A} + \lambda\left(\frac{\mathbf{C}}{2} - \mathbf{A}\right) \tag{2.0.8}$$

As BM is perpendicular to AC_1

$$(\mathbf{M} - \mathbf{B})^\top \left(\frac{\mathbf{C}}{2} - \mathbf{A}\right) = 0 \tag{2.0.9}$$

$$\left(\mathbf{A} + \lambda\left(\frac{\mathbf{C}}{2} - \mathbf{A}\right)\right)^\top \left(\frac{\mathbf{C}}{2} - \mathbf{A}\right) = 0 \tag{2.0.10}$$

$$\mathbf{A}^\top \frac{\mathbf{C}}{2} - \mathbf{A}^\top\mathbf{A} + \lambda\left(\|\frac{\mathbf{C}}{2} - \mathbf{A}\|^2\right) = 0 \tag{2.0.11}$$

$$\lambda = \frac{\mathbf{A}^\top\mathbf{A} - \mathbf{A}^\top\frac{\mathbf{C}}{2}}{\|\frac{\mathbf{C}}{2} - \mathbf{A}\|^2} \tag{2.0.12}$$

From 2.0.7 we have

$$\lambda = \frac{a^2}{a^2 + \frac{c^2}{4}} \tag{2.0.13}$$

As $\mathbf{M}$ is midpoint of $\mathbf{B}$ and $\mathbf{P}$

$$\mathbf{M} = \frac{\mathbf{B} + \mathbf{P}}{2} \tag{2.0.14}$$

$$\mathbf{P} = 2\mathbf{M} \tag{2.0.15}$$

$$\implies \mathbf{P} = 2\left(\mathbf{A} + \lambda\left(\frac{\mathbf{C}}{2} - \mathbf{A}\right)\right) \tag{2.0.16}$$

□

Let

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.17}$$

$$\implies \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{2.0.18}$$

$$\implies \mathbf{A} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{2.0.19}$$

The centre and the radius of circumcircle is given by

$$\mathbf{C_1} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{2.0.20}$$

$$r = \frac{a}{2} = 4 \tag{2.0.21}$$

The equation of circumcircle of ΔBCD is

$$\mathbf{x}^\top \mathbf{x} - 2\mathbf{C_1}\mathbf{x} + f = 0 \tag{2.0.22}$$

$$\implies \mathbf{x}^\top \mathbf{x} - \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.23}$$

The points of contact of tangents from $\mathbf{A}$ are

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.24}$$

$$\mathbf{P} = 2\mathbf{A} + 2\lambda\left(\frac{\mathbf{C}}{2} - \mathbf{A}\right) \tag{2.0.25}$$

$$\mathbf{P} = \begin{pmatrix} \frac{72}{13} \\ \frac{48}{13} \end{pmatrix} \tag{2.0.26}$$

Equation of tangent 1 (AB)

$$\mathbf{x} - \mathbf{B} = \mu_1 (\mathbf{A} - \mathbf{B}) \tag{2.0.27}$$

$$\mathbf{x} = \mu_1 \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{2.0.28}$$

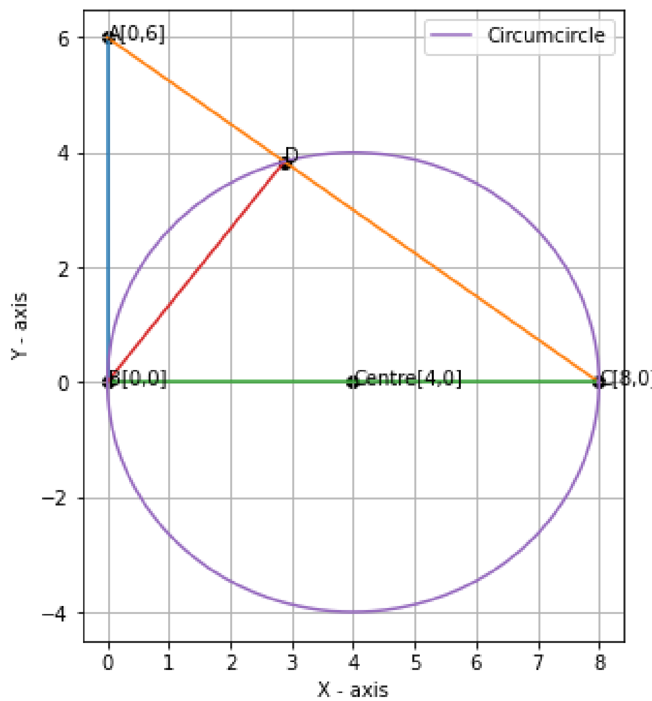

Fig. 0: The plot of circumcircle of ΔBCD

Equation of tangent 2 (AP)

$$\mathbf{x} - \mathbf{A} = \mu_2 (\mathbf{A} - \mathbf{P}) \tag{2.0.29}$$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \mu_2 \left(\begin{pmatrix} 0 \\ 6 \end{pmatrix} - \begin{pmatrix} \frac{72}{13} \\ \frac{48}{13} \end{pmatrix}\right) \tag{2.0.30}$$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \mu_2 \begin{pmatrix} \frac{-72}{13} \\ \frac{30}{13} \end{pmatrix} \tag{2.0.31}$$

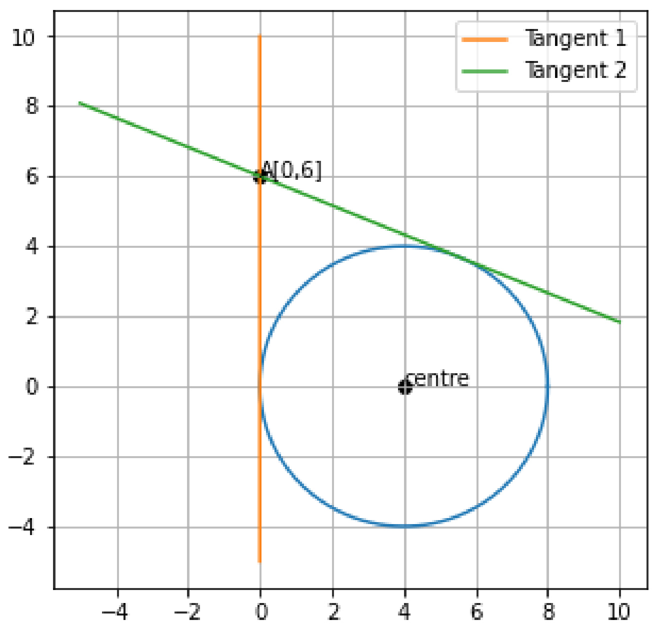

Fig. 0: The plot of tangents