

# GATE Assignment 3

Savarana Datta - AI20BTECH11008

Download all python codes from

[https://github.com/SavaranaDatta/EE3900/tree/main/GATE Assignment3/codes](https://github.com/SavaranaDatta/EE3900/tree/main/GATE%20Assignment3/codes)

and latex codes from

[https://github.com/SavaranaDatta/EE3900/tree/main/GATE Assignment3/main.tex](https://github.com/SavaranaDatta/EE3900/tree/main/GATE%20Assignment3/main.tex)

## 1 PROBLEM(GATE 1999(EC) 2.18)

The Nyquist sampling frequency (in Hz) of a signal is given by

$$6 \times 10^4 \text{Sinc}^2(400t) + 10^6 \text{Sinc}^3(100t) \quad (1.0.1)$$

is?

- 1) 200
- 2) 300
- 3) 600
- 4) 1000

## 2 SOLUTION

**Lemma 2.1.** Fourier transform of  $\text{sinc}^2$  function

$$\text{sinc}^2(at) \stackrel{\mathcal{F}}{\rightleftharpoons} \frac{1}{|a|} \text{tri}\left(\frac{f}{a}\right) \quad (2.0.1)$$

**Lemma 2.2.** Fourier transform of  $\text{sinc}^3$  function

$$\begin{aligned} \text{sinc}^3(at) \stackrel{\mathcal{F}}{\rightleftharpoons} & \frac{1}{|a|} (3(a-f)^2 \text{sgn}(f-a) - (f-3a)^2 \text{sgn}(f-3a) \\ & - 3(a+f)^2 \text{sgn}(a+f) + (3a+f)^2 \text{sgn}(3a+f)) \end{aligned} \quad (2.0.2)$$

Using lemma 2.1

$$X_1(f) = 6 \times 10^4 \text{sinc}^2(400t) \stackrel{\mathcal{F}}{\rightleftharpoons} 6 \times 10^4 \frac{1}{400} \text{tri}\left(\frac{f}{400}\right) \quad (2.0.3)$$

$X_1(f) = 0$  for  $f > 200\text{Hz}$  Using lemma 2.2

$$\begin{aligned} X_2(f) = 10^6 \times \text{sinc}^3(100t) \stackrel{\mathcal{F}}{\rightleftharpoons} & 10^6 \times \frac{1}{100} (3(100-f)^2 \\ & \text{sgn}(f-100) - (f-300)^2 \text{sgn}(f-300) - 3(100+f)^2 \\ & \text{sgn}(100+f) + (300+f)^2 \text{sgn}(300+f)) \end{aligned} \quad (2.0.4)$$

$X_2(f) = 0$  for  $f > 300\text{Hz}$  Using the above equations, we have

$$X(f) = \frac{6 \times 10^4}{400} \text{tri}\left(\frac{f}{400}\right) + \frac{10^6}{100} (3(100-f)^2$$

$$\text{sgn}(f-100) - (f-300)^2 \text{sgn}(f-300) - 3(100+f)^2 \text{sgn}(100+f) + (300+f)^2 \text{sgn}(300+f)) \quad (2.0.5)$$

$X(f)$  is zero for  $f > 300\text{Hz}$

$$\text{Nyquist sampling frequency} = 2 \times \text{max frequency} \quad (2.0.6)$$

$$= 600\text{Hz} \quad (2.0.7)$$

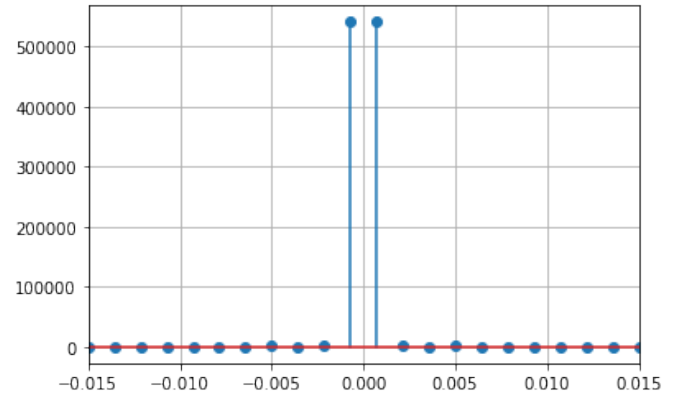


Fig. 4: Plot of  $x(t)$  sampled at 1kHz

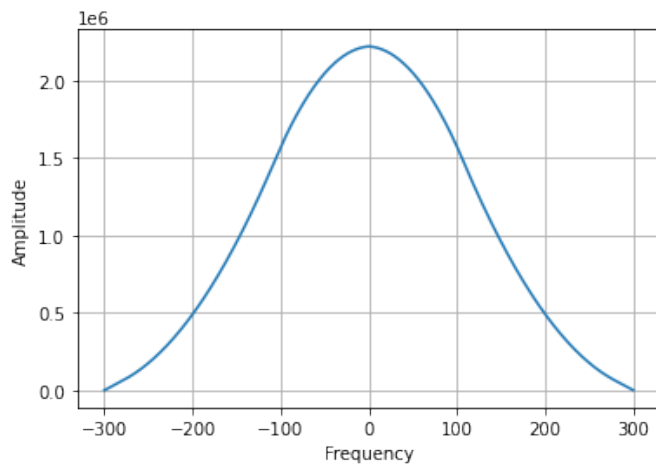


Fig. 4: Plot of  $x(t)$  sampled at 1kHz