GATE Assignment 3

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Download all python codes from

https://github.com/SavaranaDatta/EE3900/tree/main/GATE Assignment3/codes

and latex codes from

https://github.com/SavaranaDatta/EE3900/tree/main/GATE Assignment3/main.tex

1 Problem(GATE 1999(EC) 2.18)

The Nyquist sampling frequency (in Hz) of a signal is given by

$$6 \times 10^4 Sinc^2(400t) + 10^6 Sinc^3(100t) \tag{1.0.1}$$

is?

1) 200
2) 300
3) 600
4) 1000

2 Solution

Lemma 2.1. *Fourier transform of $sinc^2$ function*

$$sinc^2(at) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{|a|} tri\left(\frac{f}{a}\right) \tag{2.0.1}$$

Lemma 2.2. *Fourier transform of $sinc^3$ function*

$$\begin{aligned} sinc^3(at) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{|a|}(3(a-f)^2 sgn(f-a) - (f-3a)^2 sgn(f-3a) \\ - 3(a+f)^2 sgn(a+f) + (3a+f)^2 sgn(3a+f)) \end{aligned} \tag{2.0.2}$$

Using lemma 2.1

$$X_1(f) = 6 \times 10^4 sinc^2(400t) \overset{\mathcal{F}}{\rightleftharpoons} 6 \times 10^4 \frac{1}{400} tri\left(\frac{f}{400}\right) \tag{2.0.3}$$

$X_1(f) = 0$ for $f > 200Hz$ Using lemma 2.2

$$\begin{aligned} X_2(f) = 10^6 \times sinc^3(100t) \overset{\mathcal{F}}{\rightleftharpoons} 10^6 \times \frac{1}{100}(3(100-f)^2 \\ sgn(f-100) - (f-300)^2 sgn(f-300) - 3(100+f)^2 \\ sgn(100+f) + (300+f)^2 sgn(300+f)) \end{aligned} \tag{2.0.4}$$

$X_2(f) = 0$ for $f > 300Hz$ Using the above equations, we have

$$\begin{aligned} X(f) = \frac{6 \times 10^4}{400} tri\left(\frac{f}{400}\right) + \frac{10^6}{100}(3(100-f)^2 \\ sgn(f-100) - (f-300)^2 sgn(f-300) - 3(100+f)^2 \\ sgn(100+f) + (3a+f)^2 sgn(300+f)) \end{aligned} \tag{2.0.5}$$

$X(f)$ is zero for $f > 300Hz$

$$\text{Nyquist sampling frequency} = 2 \times \text{ max frequency} \tag{2.0.6}$$
$$= 600Hz \tag{2.0.7}$$

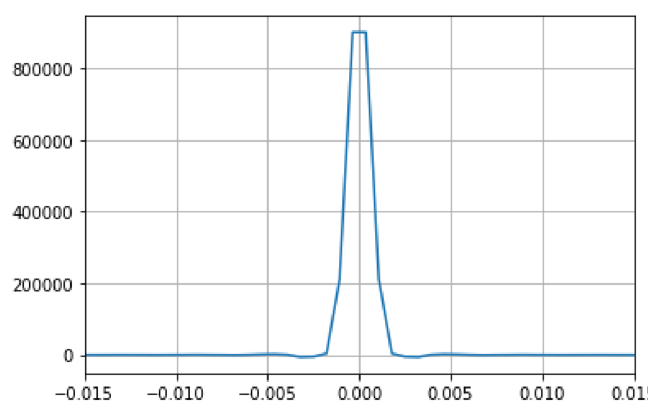

Fig. 4: Plot of x(t) sampled at 1kHz

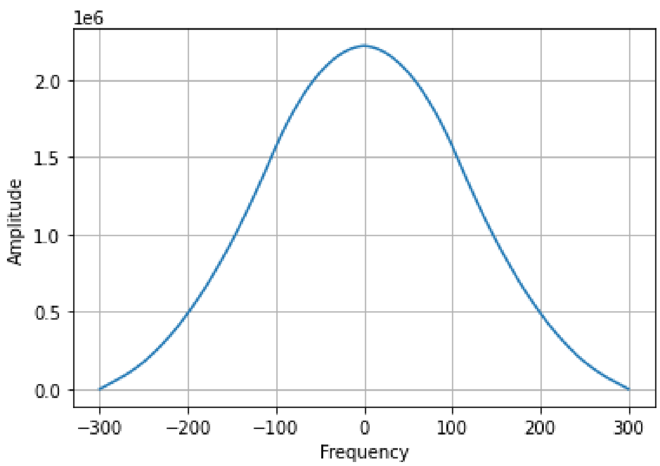

Fig. 4: Plot of x(t) sampled at 1kHz