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Question

Find the angle between the following pair of lines

1. $$\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} \tag{1}$$
$$\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4} \tag{2}$$

2. $$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} \tag{3}$$
$$\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-3}{8} \tag{4}$$

Solution

Let a and b be the direction vectors of the two lines and θ be the angle between the lines. The angle is given by

$$\cos\theta = \frac{\mathsf{a}^\top \mathsf{b}}{\|\mathsf{a}\| \, \|\mathsf{b}\|} \tag{5}$$

Part 1

- Let the direction vectors of lines be a and b

$$\mathrm{a} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} \tag{6}$$

$$\mathrm{b} = \begin{pmatrix} -1 \\ 8 \\ 4 \end{pmatrix} \tag{7}$$

$$a^\top b = \begin{pmatrix} 2 & 5 & -3 \end{pmatrix} \begin{pmatrix} -1 \\ 8 \\ 4 \end{pmatrix} \tag{8}$$

$$= 26 \tag{9}$$

$$\|a\| = \sqrt{38} \tag{10}$$

$$\|b\| = 9 \tag{11}$$

$$\implies \cos\theta = \frac{26}{9\sqrt{38}} \tag{12}$$

$$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right) \tag{13}$$

$$= 62.053 \tag{14}$$

- A vector joining the 2 lines V_1

$$V_1 = \begin{pmatrix} 2-(-2) \\ 1-4 \\ 3-5 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix} \tag{15}$$

- Matrix M be

$$\mathrm{M} = \begin{pmatrix} \mathrm{b} \\ \mathrm{a} \\ \mathrm{V_1} \end{pmatrix} \tag{16}$$

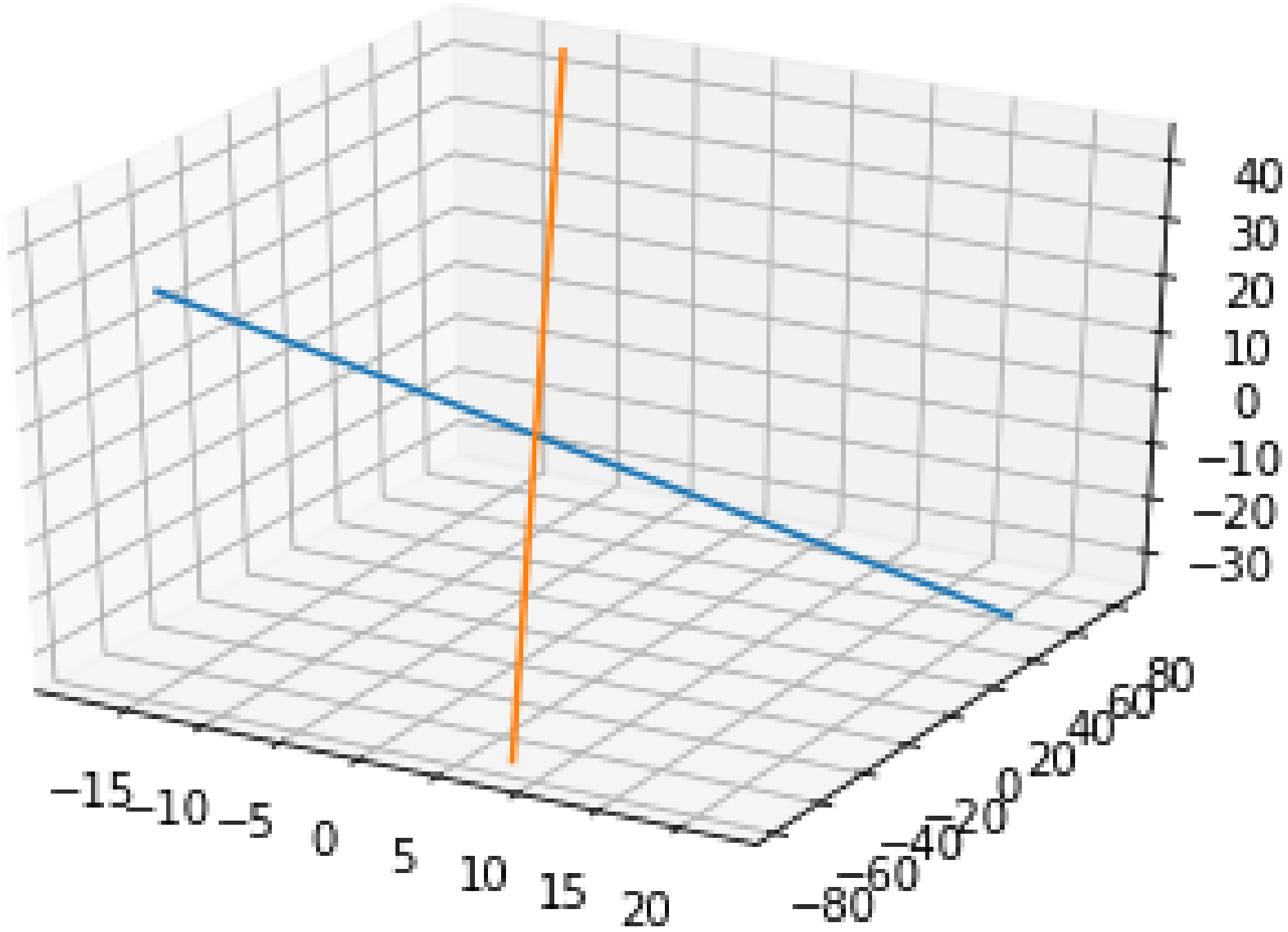
$$= \begin{pmatrix} -1 & 8 & 4 \\ 2 & 5 & -3 \\ 4 & -3 & -2 \end{pmatrix} \tag{17}$$

- The row echelon form of matrix M

$$\mathrm{M} = \begin{pmatrix} -1 & 8 & 4 \\ 0 & 21 & 5 \\ 0 & 0 & \frac{149}{21} \end{pmatrix} \tag{18}$$

- As the rank of the matrix M is 3, the three vectors a,b and $\mathrm{V_1}$ are non co-planar. Hence the lines do not intersect.

Plot of the straight lines



Part 2

- Let the direction vectors of lines be c and d

$$\mathrm{c} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \tag{19}$$

$$\mathrm{d} = \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} \tag{20}$$

$$\mathbf{c}^\top \mathbf{d} = \begin{pmatrix} 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} = 18 \tag{21}$$

$$\|\mathbf{c}\| = 3 \tag{22}$$

$$\|\mathbf{d}\| = 9 \tag{23}$$

$$\implies \cos\theta = \frac{18}{9 \times 3} \tag{24}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right) \tag{25}$$

$$= 48.189 \tag{26}$$

- A vector joining the 2 lines V_2

$$V_2 = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} \tag{27}$$

- Matrix M be

$$\mathrm{M} = \begin{pmatrix} \mathrm{c} \\ \mathrm{d} \\ \mathrm{V_2} \end{pmatrix} \tag{28}$$

$$= \begin{pmatrix} 2 & 2 & 1 \\ 4 & 1 & 8 \\ 5 & 4 & 3 \end{pmatrix} \tag{29}$$

- The row echelon form of matrix M

$$\mathrm{M} = \begin{pmatrix} 2 & 2 & 1 \\ 0 & -3 & 6 \\ 0 & 0 & 3 \end{pmatrix} \tag{30}$$

- As the rank of the matrix M is 3, the three vectors c,d and $\mathrm{V_2}$ are non co-planar. Hence the lines do not intersect.

Plot of the straight lines