GATE EC 21

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Question

Consider the signal

$$f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right) \quad (1)$$

, where t is in seconds. Its fundamental time period in seconds, is

Solution

Individual natural frequencies of each term are

$$f_1 = \frac{1}{2} \tag{2}$$

$$f_2 = \frac{1}{3} \tag{3}$$

$$f_3 = \frac{1}{4} \tag{4}$$

Individual fundamental time periods of each term are

$$T_1 = \frac{1}{f_1} = 2 \tag{5}$$

$$T_2 = \frac{1}{f_2} = 3 \tag{6}$$

$$T_3 = \frac{1}{f_3} = 4 \tag{7}$$

Fundamental time period(T) of the signal

$$
\begin{aligned}
T &= LCM\left(T_1, T_2, T_3\right) && (8)\\
&= LCM\left(2, 3, 4\right) && (9)\\
&= 12sec && (10)
\end{aligned}
$$

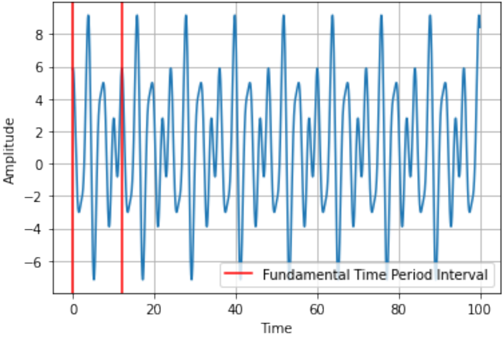

Figure: Plot of the signal

Linearity of Fourier Transform

$$\mathcal{F}\left\{c_1 g\left(t\right) + c_2 h\left(t\right)\right\} = c_1 \mathcal{F}\left\{g\left(t\right)\right\} + c_2 \mathcal{F}\left\{h\left(t\right)\right\} \tag{11}$$

Fourier Transform of Cosine function

Let $x\left(t\right) = \cos\left(2\pi A t\right)$,

$$\cos\left(2\pi A t\right) = \frac{e^{i2\pi A t} + e^{-i2\pi A t}}{2} \tag{12}$$

The Fourier transform of $x\left(t\right)$

$$G_x\left(f\right) = \int_{-\infty}^{\infty} \frac{e^{i2\pi A t} + e^{-i2\pi A t}}{2} e^{-i2\pi f t} dt \tag{13}$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-i2\pi t(f-A)} dt + \int_{-\infty}^{\infty} e^{-i2\pi t(A+F)}\right] \tag{14}$$

$$= \frac{1}{2}\left[\delta\left(f - A\right) + \delta\left(f + A\right)\right] \tag{15}$$

Fourier Transform of sine function

Let $x(t) = \sin(2\pi At)$,

$$\cos(2\pi At) = \frac{e^{i2\pi At} - e^{-i2\pi At}}{2i} \tag{16}$$

The Fourier transform of $x(t)$

$$G_x(f) = \int_{-\infty}^{\infty} \frac{e^{i2\pi At} - e^{-i2\pi At}}{2i} e^{-i2\pi ft} dt \tag{17}$$

$$= \frac{1}{2i} \left[\int_{-\infty}^{\infty} e^{-i2\pi t(f-A)} dt - \int_{-\infty}^{\infty} e^{-i2\pi t(A+F)} \right] \tag{18}$$

$$= \frac{1}{2i} \left[\delta(f-A) - \delta(f+A) \right] \tag{19}$$

Using 11,15 and 19

$$G_x(f) = \delta(f) + \delta\left(f - \frac{1}{2}\right) + \delta\left(f + \frac{1}{2}\right) + \sqrt{2}\delta\left(f - \frac{1}{4}\right) + \sqrt{2}\delta\left(f + \frac{1}{4}\right) + \sqrt{2}i\delta\left(f - \frac{1}{4}\right) - \sqrt{2}i\delta\left(f + \frac{1}{4}\right) - \frac{3}{2}i\delta\left(f - \frac{1}{3}\right) + \frac{3}{2}i\delta\left(f + \frac{1}{3}\right) \quad (20)$$

Minimun frequency is zero and the maximum frequency is $\frac{1}{2}$. So, the bandwidth is $\frac{1}{2}$

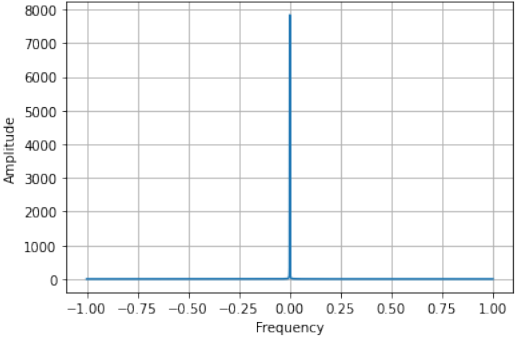

Figure: Fourier transform of y(t)