

# Assignment 5

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Download all python codes from

[https://github.com/SavaranaDatta/EE3900/blob/main/EE3900\\_GATE\\_Assignment2/codes](https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_GATE_Assignment2/codes)

Download latex-tikz codes from

[https://github.com/SavaranaDatta/EE3900/blob/main/EE3900\\_GATE\\_Assignment2/GATE\\_Assignment2.tex](https://github.com/SavaranaDatta/EE3900/blob/main/EE3900_GATE_Assignment2/GATE_Assignment2.tex)

## 1 PROBLEM(GATE EC Q.32)

The signal  $x(t)$  is described by

$$x(t) = \begin{cases} 1 & , \text{for } -1 \leq t \leq +1 \\ 0 & , \text{otherwise} \end{cases} \quad (1.0.1)$$

Two of the angular frequencies at which its fourier transform becomes zero are

- 1)  $\pi, 2\pi$
- 2)  $0.5\pi, 1.5\pi$
- 3)  $0, \pi$
- 4)  $2\pi, 2.5\pi$

## 2 SOLUTION

**Lemma 2.1.** The fourier transform of a rect function is sinc function

$$\text{rect}\left(\frac{t}{\tau}\right) \xrightarrow{\mathcal{F}} \tau \text{sinc}(f\tau) \quad (2.0.1)$$

*Proof.*

$$\int_{-\infty}^{\infty} \text{rect}\left(\frac{x}{\tau}\right) e^{-i2\pi xt} dx = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-i2\pi xt} dx \quad (2.0.2)$$

$$= \left[ \frac{e^{-i2\pi xt}}{-i2\pi t} \right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \quad (2.0.3)$$

$$= \frac{e^{-i\pi t\tau} - e^{i\pi t\tau}}{-i2\pi t} \quad (2.0.4)$$

$$= \tau \frac{\sin \pi t\tau}{\pi t\tau} \quad (2.0.5)$$

$$= \tau \text{sinc}(t\tau) \quad (2.0.6)$$

□

We can observe that

$$x(t) = \text{rect}\left(\frac{t}{2}\right) \quad (2.0.7)$$

From the lemma 2.1, fourier transform of  $x(t)$  is

$$x(f) = 2 \text{sinc}(2f) \quad (2.0.8)$$

$$= 2 \frac{\sin(2\pi f)}{(2\pi f)} \quad (2.0.9)$$

$$= 2 \frac{\sin(\omega)}{\omega} \quad (2.0.10)$$

$x(f)$  is zero when  $\omega = n\pi$ , where  $n \in \mathbb{I} - \{0\}$ . Hence, option 1 is true.

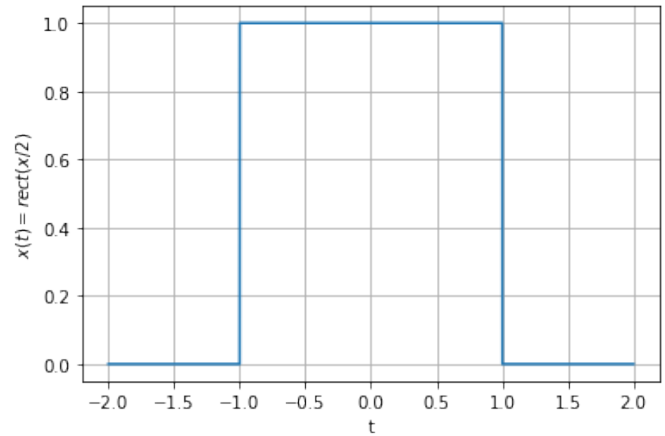


Fig. 4: Plot of  $x(t)$

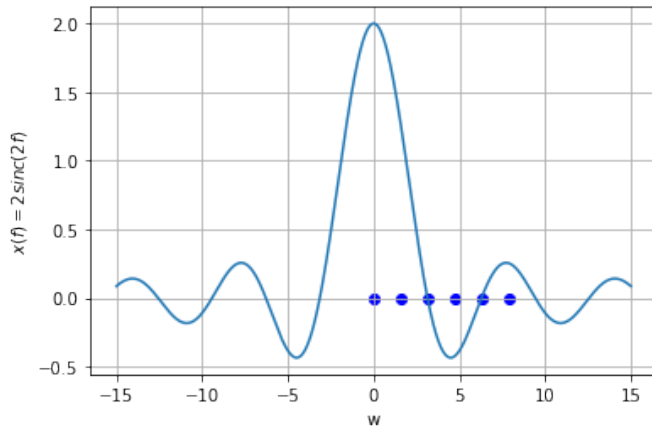


Fig. 4: Plot of the fourier transform