Quiz-2

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1 QUESTION

For the following pair of input z-transform $X(z)$ and system function $H(z)$, determine the region of convergence for the output z-transform $Y(z)$:

$$X(z) = \frac{1}{\left(1 - \frac{1}{5}z^{-1}\right)(1 + 3z^{-1})}, \quad \frac{1}{5} < |z| < 3 \quad (1.0.1)$$

$$H(z) = \frac{1 + 3z^{-1}}{1 + \frac{1}{3}z^{-1}}, \qquad |z| > \frac{1}{3} \quad (1.0.2)$$

2 SOLUTION

Lemma 2.1. *Properties of ROC: The ROC doesnot contain any poles.*
For

$$X(s) = \frac{N(s)}{D(s)} \quad (2.0.1)$$

The poles of $X(s) \implies D(s) = 0$

Lemma 2.2. *The poles of $X(s)$ consists of a strip parallel to $j\omega$ axis in the s-plane.*

We know that the z-transform of output signal $(Y(z))$

$$Y(z) = H(z)X(z) \quad (2.0.2)$$

Here

$$Y(z) = \left(\frac{1}{\left(1 - \frac{1}{5}z^{-1}\right)(1 + 3z^{-1})}\right)\left(\frac{1 + 3z^{-1}}{1 + \frac{1}{3}z^{-1}}\right) \quad (2.0.3)$$

$$= \frac{1}{\left(1 - \frac{1}{5}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)} \quad (2.0.4)$$

$$= 1 - \frac{5}{24\left(z + \frac{1}{3}\right)} + \frac{3}{40\left(z - \frac{1}{5}\right)}, \quad \frac{1}{3} < |z| < 3 \quad (2.0.5)$$

The poles of $Y(z)$ are

$$z = \frac{-1}{3}, \frac{1}{5} \quad (2.0.6)$$

The zeros of $Y(z)$ are

$$z = 0 \quad (2.0.7)$$

From the above 2 lemmas we can say that the possible ROCs of

$$Y(z) = \frac{1}{\left(1 - \frac{1}{5}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)} \quad (2.0.8)$$

are

$$z < \frac{-1}{3} \quad (2.0.9)$$

$$\frac{-1}{3} < z < \frac{1}{5} \quad (2.0.10)$$

$$z > \frac{1}{5} \quad (2.0.11)$$

But this expression of $Y(z)$ is valid for the interval $\frac{1}{3} < |z| < 3$. Hence, the ROC of $Y(z)$ is $z > \frac{1}{3}$ and $z < \frac{-1}{3}$ i.e, $|z| > \frac{1}{3}$.
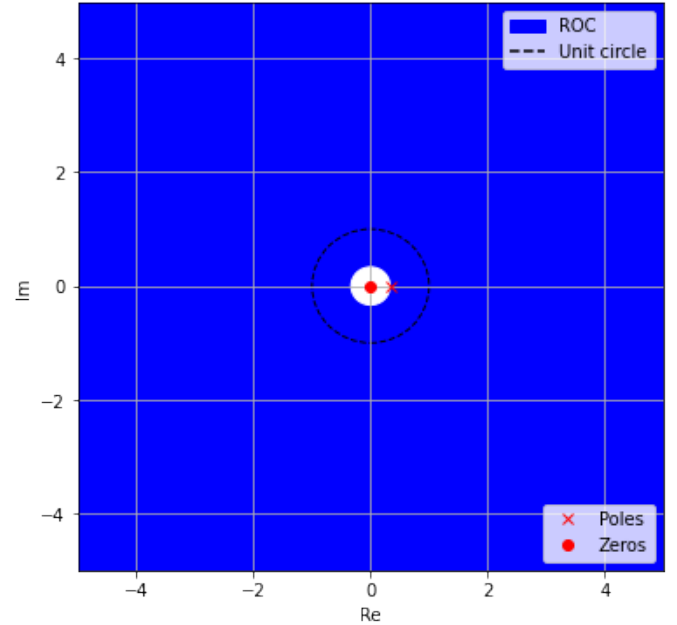

Fig. 1: ploe-zero plot of the system