

Assignment 3

Savarana Datta - AI20BTECH11008

Download all python codes from

<https://github.com/SavaranaDatta/EE3900/tree/main/Assignment3/codes>

and latex codes from

<https://github.com/SavaranaDatta/EE3900/tree/main/Assignment3/Assignment3.tex>

1 CONSTRUCTION 2.16

Let ABC be a right triangle in which $a=8$, $c=6$ and $\angle B = 90^\circ$. BD is the perpendicular from B on AC (altitude). The circle through B, C, D (circumcircle of $\triangle BCD$) is drawn. Construct the tangents from A to this circle.

2 SOLUTION

Lemma 2.1. *If $\triangle ABC$ is a right angled triangle at B (be O), then the points of contact of tangents from A to the circumcircle of $\triangle BCD$ (where D is the foot of perpendicular from B to side AC) are B and P*

$$\mathbf{P} = 2\mathbf{A} + 2\lambda \left(\frac{\mathbf{C}}{2} - \mathbf{A} \right) \quad (2.0.1)$$

$$\text{where } \lambda = \frac{a^2}{a^2 + \frac{c^2}{4}} \quad (2.0.2)$$

Proof. Given,

$\triangle ABC$ is a right angled triangle at B (be O) and D is the foot of perpendicular from B to the side AC . As $\triangle BCD$ is right angled at D , the circumcentre (C_1) is the midpoint of side BC and radius (r) is half the length of BC

$$\mathbf{C}_1 = \frac{\mathbf{B} + \mathbf{C}}{2} \quad (2.0.3)$$

$$\Rightarrow \mathbf{C}_1 = \frac{\mathbf{C}}{2} \quad (2.0.4)$$

$$r = \frac{a}{2} \quad (2.0.5)$$

As $\triangle ABC$ is right angled at B

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B}) = 0 \quad (2.0.6)$$

$$\Rightarrow \mathbf{A}^\top \mathbf{C} = 0 \quad (2.0.7)$$

B is one of the points of contact of tangent to the circle. Let M be the point of intersection of BP and

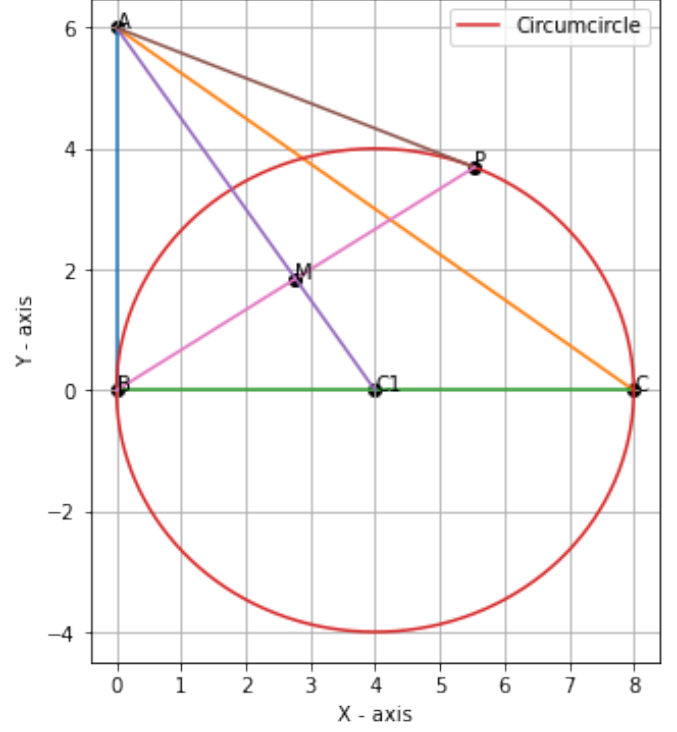


Fig. 0: Reference plot

AC_1

As M lies on the line AC_1

$$\mathbf{M} = \mathbf{A} + \lambda \left(\frac{\mathbf{C}}{2} - \mathbf{A} \right) \quad (2.0.8)$$

As BM is perpendicular to AC_1

$$(\mathbf{M} - \mathbf{B})^\top \left(\frac{\mathbf{C}}{2} - \mathbf{A} \right) = 0 \quad (2.0.9)$$

$$\left(\mathbf{A} + \lambda \left(\frac{\mathbf{C}}{2} - \mathbf{A} \right) \right)^\top \left(\frac{\mathbf{C}}{2} - \mathbf{A} \right) = 0 \quad (2.0.10)$$

$$\mathbf{A}^\top \frac{\mathbf{C}}{2} - \mathbf{A}^\top \mathbf{A} + \lambda \left(\left\| \frac{\mathbf{C}}{2} - \mathbf{A} \right\|^2 \right) = 0 \quad (2.0.11)$$

$$\lambda = \frac{\mathbf{A}^\top \mathbf{A} - \mathbf{A}^\top \frac{\mathbf{C}}{2}}{\left\| \frac{\mathbf{C}}{2} - \mathbf{A} \right\|^2} \quad (2.0.12)$$

From 2.0.7 we have

$$\lambda = \frac{a^2}{a^2 + \frac{c^2}{4}} \quad (2.0.13)$$

As \mathbf{M} is midpoint of \mathbf{B} and \mathbf{P}

$$\mathbf{M} = \frac{\mathbf{B} + \mathbf{P}}{2} \quad (2.0.14)$$

$$\Rightarrow \mathbf{P} = 2\mathbf{M} \quad (2.0.15)$$

$$\Rightarrow \mathbf{P} = 2\left(\mathbf{A} + \lambda\left(\frac{\mathbf{C}}{2} - \mathbf{A}\right)\right) \quad (2.0.16)$$

□

Let

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.17)$$

$$\Rightarrow \mathbf{C} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \quad (2.0.18)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (2.0.19)$$

The centre and the radius of circumcircle is given by

$$\mathbf{C}_1 = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (2.0.20)$$

$$r = \frac{a}{2} = 4 \quad (2.0.21)$$

The equation of circumcircle of $\triangle BCD$ is

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{C}_1 \mathbf{x} + f = 0 \quad (2.0.22)$$

$$\Rightarrow \mathbf{x}^T \mathbf{x} - \begin{pmatrix} 8 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.23)$$

from the above lemma, points of contact of tangents from \mathbf{A} are

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and } \quad (2.0.24)$$

$$\mathbf{P} = 2\mathbf{A} + 2\lambda\left(\frac{\mathbf{C}}{2} - \mathbf{A}\right) \quad (2.0.25)$$

$$\Rightarrow \mathbf{P} = \begin{pmatrix} \frac{72}{13} \\ \frac{48}{13} \end{pmatrix} \quad (2.0.26)$$

Equation of tangent 1 (AB)

$$\mathbf{x} = \mathbf{B} + \lambda \mathbf{m} \quad (2.0.27)$$

$$\mathbf{x} = \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.28)$$

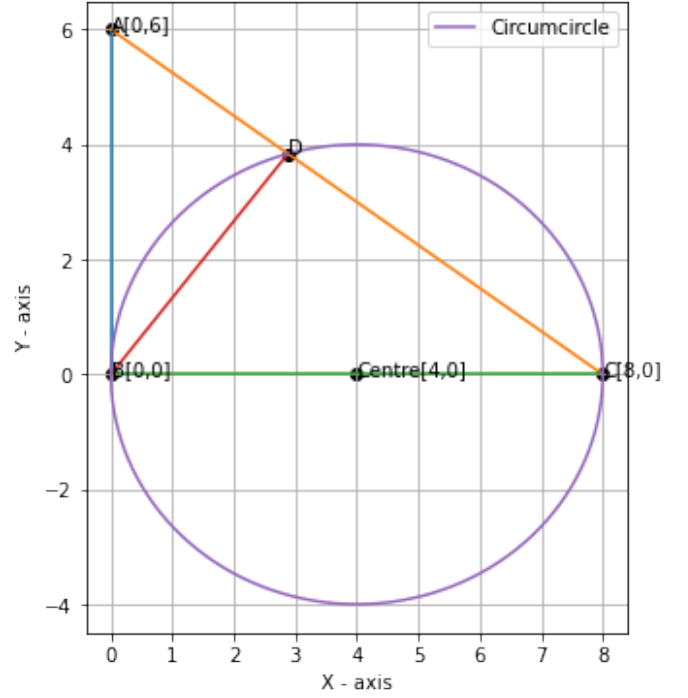


Fig. 0: The plot of circumcircle of $\triangle BCD$

Equation of tangent 2 (AP)

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (2.0.29)$$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \lambda \left(\begin{pmatrix} 0 \\ 6 \end{pmatrix} - \begin{pmatrix} \frac{72}{13} \\ \frac{48}{13} \end{pmatrix} \right) \quad (2.0.30)$$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -\frac{5}{12} \end{pmatrix} \quad (2.0.31)$$

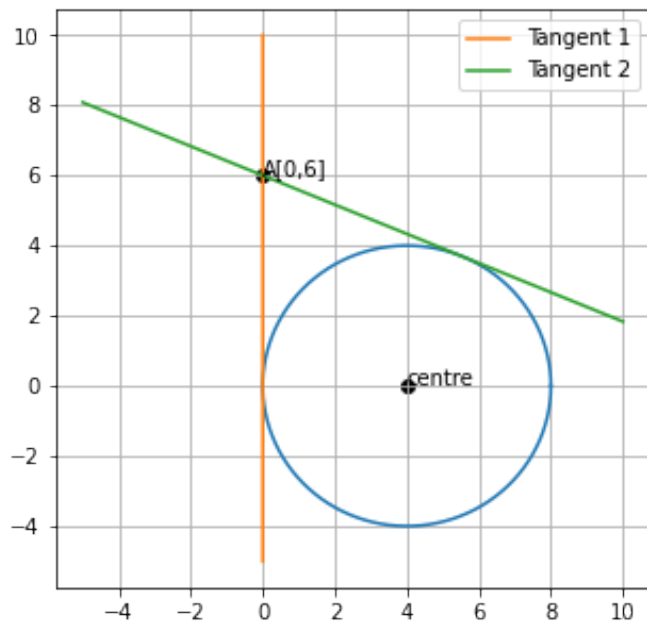


Fig. 0: The plot of tangents