Laboratory Session 08 : May 25, 2020

Exercises due : June 10, 2020

Exercise 1

- Students from the Bachelor degree in Physics performed an experiment to study the Zeeman effect. The apparatus contains a Ne source lamp whose position can be changed. During the setting up of the apparatus, the source position has to be adjusted in order to maximize the intensity of the detected light signal.
- The following table gives the position of the source (in mm) and the corresponding height of the peak (arbitrary units) for the wavelength under study:

x_i	2.44	3.49	3.78	3.31	3.18	3.15	3.1	3.0	3.6	3.4
y_i	129	464	189	562	589	598	606	562	360	494

- Assume a quadratic dependence of the peak height, y_i as a function of the source position x_i,

$$f(x) = c_\circ + c_1 x + c_2 x^2$$

- All the measured values are affected by a Gaussian noise with zero mean, such that

$$y_i = f(x_i) + \epsilon$$

- where ϵ follows a normal distribution with mean $\mu = 0$ and unknown standard deviation, σ

A) Build a Markov Chain Monte Carlo to estimate the best parameters of the quadratic dependence of the data and the noise that affects the measured data.

- as can be seen from our data, the students forgot to take measurements in the region $x \in (2.44, 3.0)$

B) run a Markov Chain Monte Carlo to predict peak height measurements at $x_1 = 2.8$ mm and $x_2 = 2.6$ mm

Exercise 2

- the number of British coal mine disasters has been recorded from 1851 to 1962. By looking at the data it seems that the number of incidents decreased towards the end of the sampling period. We model the data as follows:

- before some year, we call τ, the data follow a Poisson distribution, where the logarithm of the mean value, $\log \mu_t = b_0$, while for later years, we can model it as $\log \mu_t = b_0 + b_1$

- the dependence can be modeled as follows $y_t \sim \text{Pois}(\mu_t)$, where $\log \mu_t = b_0 + b_1 \text{Step}(t - \tau)$

- implement the model in `jags`, trying to infer the parameters b_0, b_1 and τ

- the step function is implemented, in BUGS, as `step(x)` and return 1 if $x \geq 0$ and 0 otherwise

- assign a uniform prior to b_0, b_1 and a uniform prior in the interval $(1, N)$, where $N = 112$ is the number of years our data span on

- finally, here is our data:

```
data <- NULL
data$D <- c(4,5,4,1,0,4,3,4,0,6,
   3,3,4,0,2,6,3,3,5,4,5,3,1,4,4,1,5,5,3,4,2,5,2,2,3,4,2,1,3,2,
   1,1,1,1,1,3,0,0,1,0,1,1,0,0,3,1,0,3,2,2,
   0,1,1,1,0,1,0,1,0,0,0,2,1,0,0,0,1,1,0,2,
   2,3,1,1,2,1,1,1,1,2,4,2,0,0,0,1,4,0,0,0,
   1,0,0,0,0,0,1,0,0,1,0,0)
data$N <- 112
```

- before running `jags`, assign an initial value to the parameters as follows: $b_0 = 0$, $b_1 = 0$ and $\tau = 50$

- explore the features of the chains and try to understand the effects of the burn-in, and thinning

- plot the posterior distributions of the parameters and extract their mean values, and 95