Ising Model

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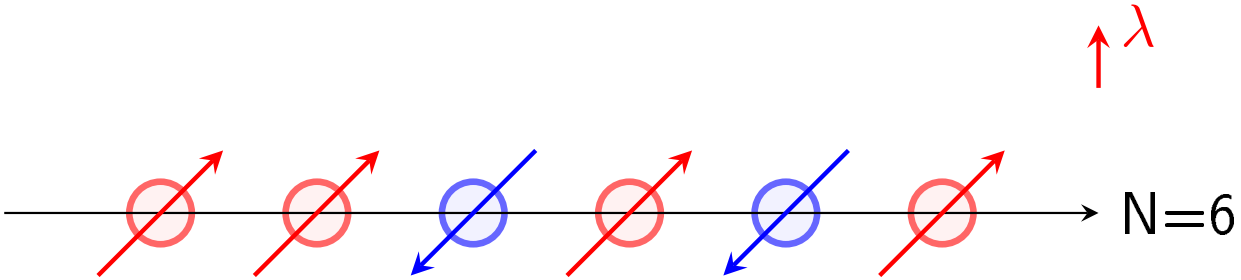
1-D Ising Model:



$$H = \lambda \sum_{i}^{N} \sigma_z^i - \sum_{i}^{N-1} \sigma_x^{i+1} \sigma_x^i$$

where

$$\sigma_z^i = \underbrace{\mathbb{I} \otimes \mathbb{I} \otimes \ldots \otimes \mathbb{I}}_{i-1} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes \underbrace{\mathbb{I} \otimes \ldots \otimes \mathbb{I}}_{N-i}$$

$$\sigma_x^i = \underbrace{\mathbb{I} \otimes \mathbb{I} \otimes \ldots \otimes \mathbb{I}}_{i-1} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \underbrace{\mathbb{I} \otimes \ldots \otimes \mathbb{I}}_{N-i}$$

Code development

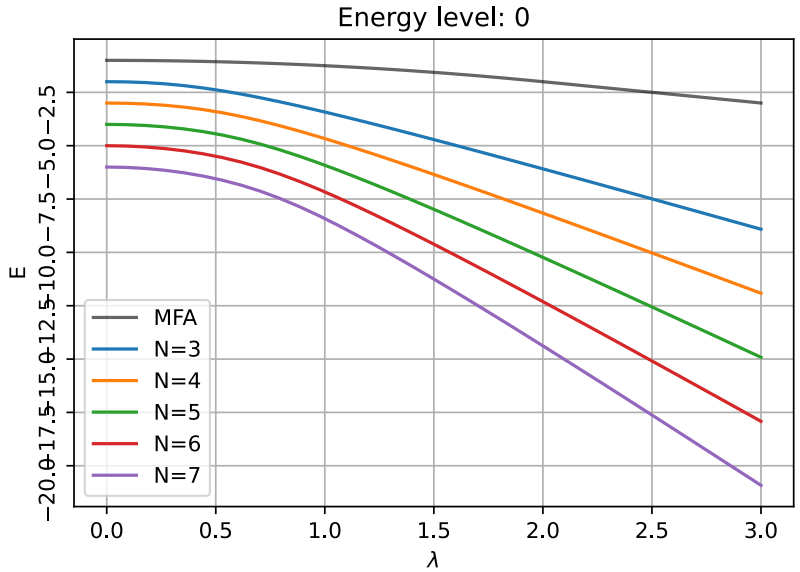
Building the Hamiltonian

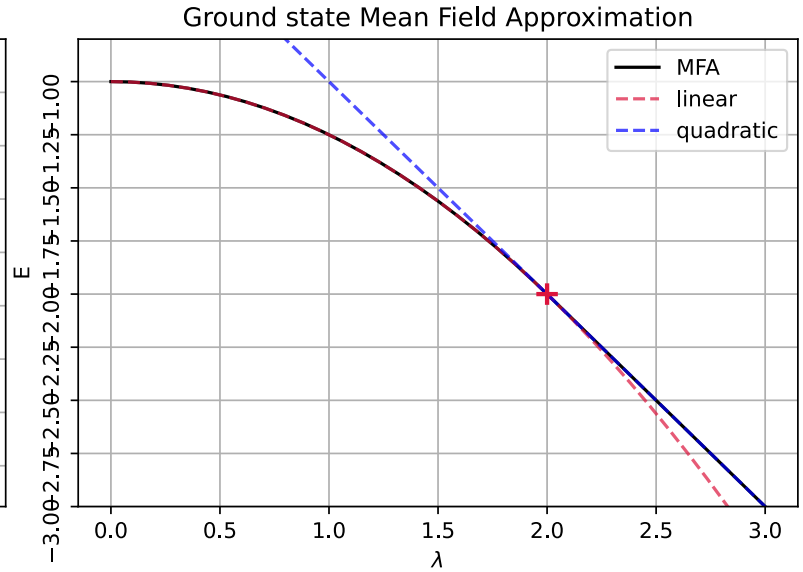
$$H = \underbrace{\lambda \sum_{i}^{N} \sigma_z^i}_{A} - \underbrace{\sum_{i}^{N-1} \sigma_x^{i+1} \sigma_x^i}_{B}$$

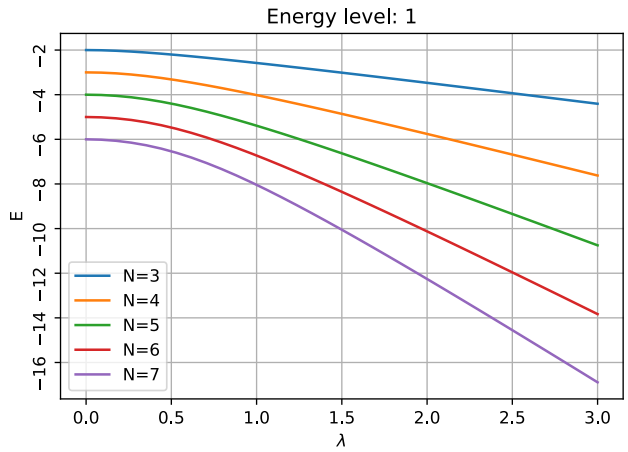
```
function ising_init_H(N,lambda) result(H)
  integer   :: N
  double precision :: lambda
  double precision, dimension(:,:), allocatable :: H, int_A, int_B
  integer*16 :: ii,jj,kk,ll
  allocate(H(2**N,2**N))
  do ii = 1, N, 1                                                    (A)
    do jj = 1, 2**N, 1
      H(jj,jj) = H(jj,jj) + -2*(modulo( (jj-1)/int(2**(N-ii)),2) ) +1
    end do
  end do
  do ii = 1, N-1, 1                                                  (B)
    do kk = 0,2**(ii-1)-1,1
      do jj=1,2**(N-ii),1
        int_A(kk*(2**(N-ii+1))  + 2**(N-ii)+jj, kk*(2**(N-ii+1))  + jj) = 1
        int_A(kk*(2**(N-ii+1))  + jj, kk*(2**(N-ii+1))  + 2**(N-ii)+jj) = 1
      end do
    end do
    do kk = 0,2**(ii)-1,1
      do jj=1,2**(N-ii-1),1
        int_B(kk*(2**(N-ii)) + 2**(N-ii-1)+jj, kk*(2**(N-ii)) + jj) = 1
        int_B(kk*(2**(N-ii)) + jj, kk*(2**(N-ii)) + 2**(N-ii-1)+jj) = 1
      end do
    end do
    H = H - matmul(int_B,int_A)
  end do
```

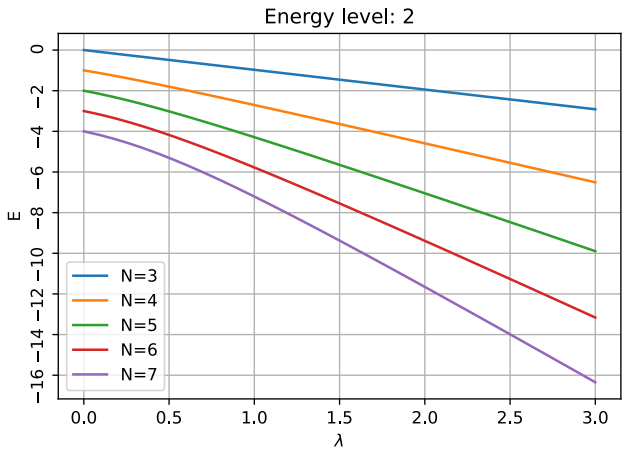
Results

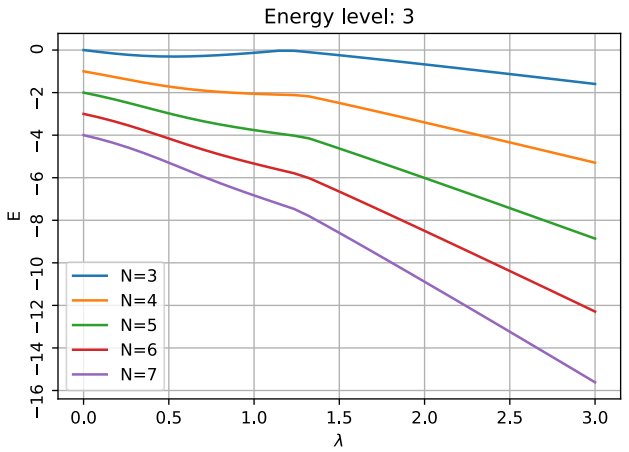
Energy spectra







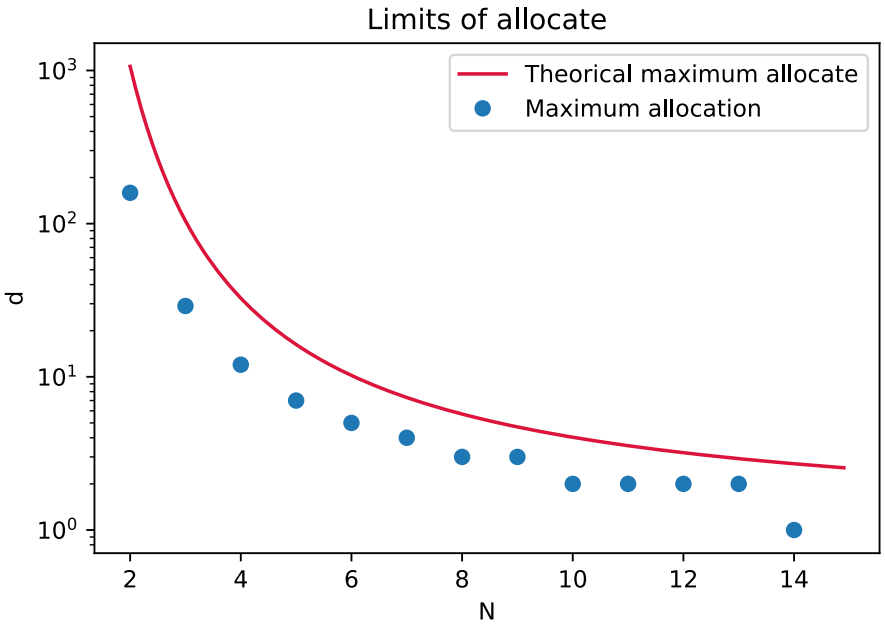




Given an hamiltonian

```
double precision, dimension(:,:), allocatable :: H
```

Maximum number of spins: $N_{max} = 13$



Data was generated using a Ubuntu machine with 8GB of RAM