E & M cheat sheet

Coulomb's law:

$$E_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\underline{r}}_{12}$$

Permittivity:

$$\varepsilon = \varepsilon_r \varepsilon_0$$

Relative permittivity: $\varepsilon_r = 1 + \chi$ ↳ susceptible to polarization

Electric field

$$\underline{E} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\hat{\underline{r}}$$

$$\Sigma \underline{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{\underline{r}}_i$$

if in an arbitrary volume V

$$dq = \rho dV$$

$$E(R) = \int_V \frac{1}{4\pi\varepsilon_0}\frac{R-r}{|R-r|^3}\rho(r)\, dV$$

Electric flux:

$$\Phi = \frac{\Sigma q_{enclosed}}{\varepsilon_0}$$

$$\Phi_E = \int_S E\cdot\hat{n}\, da = \int \underline{E}\cdot d\underline{a}$$

Gauss's law:

$$\Phi_{net} = \int_S \underline{E}\cdot d\underline{a} = \frac{Q_{internal}}{\varepsilon_0} \qquad \int_S D\cdot d\underline{a} = Q_{internal}$$

$$\nabla\cdot\underline{E} = \frac{\rho}{\varepsilon_0}$$

$$\int_S \underline{E}\cdot d\underline{a} = \int_V \frac{\rho}{\varepsilon_0}\, dV$$

Solid angle Ω

$$d\Omega = \frac{da}{r^2}$$

Work done in an electric field

$$W = -q\int_a^b \underline{E}\cdot d\underline{s}$$

$$W_{unit} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = V(b) - V(a)$$

Potential @ point P

$$V(P) = -\int_{P_0=0}^{P} \underline{E}\cdot d\underline{s}$$

Electric potential energy:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Electric field as grad of potential

$$\underline{F} = -\underline{\nabla} W$$

$$q\underline{E} = -\underline{\nabla}(qV)$$

$$\underline{E} = -\underline{\nabla} V$$

Cartesian coordinate:

$$\underline{E} = -\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} V \Rightarrow \begin{pmatrix}\hat{e}_x\\ \hat{e}_y\\ \hat{e}_z\end{pmatrix}$$

Cylindrical coordinates:

$$\underline{E} = -\begin{pmatrix}\frac{\partial}{\partial\rho}\\ \frac{1}{\rho}\frac{\partial}{\partial\phi}\\ \frac{\partial}{\partial z}\end{pmatrix} V \Rightarrow \begin{pmatrix}\hat{e}_\rho\\ \hat{e}_\phi\\ \hat{e}_z\end{pmatrix}$$

Spherical coordinates:

$$E = -\begin{pmatrix}\frac{\partial}{\partial r}\\ \frac{1}{r}\frac{\partial}{\partial\theta}\\ \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\end{pmatrix} V \Rightarrow \begin{pmatrix}\hat{e}_r\\ \hat{e}_\theta\\ \hat{e}_\phi\end{pmatrix}$$

Discrete

potential for a discrete charge distribution

$$V(x_i, y_i, z_i) = \frac{1}{4\pi\varepsilon_0}\sum_j \frac{q_i}{r_{ij}}$$

$r_{ij} = |\mathbf{r}_{ij}| = |\mathbf{r}_i - \mathbf{r}_j|$

Electric dipole:

$$V_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{2d\cos\theta}{r^2}$$

$(r >> d)$

Electrostatic potential energy for discrete charges

$$U = \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\sum_{i,j\neq 1}^{n}\frac{q_i q_j}{r_{ij}}$$

Continuous

Potential for a continuous charge distribution

$$V(x_i, y_i, z_i) = \frac{1}{4\pi\varepsilon}\int_\tau \frac{\rho(x_i, y_i, z_i)}{r_{ij}}\,d\tau$$

Electrostatic potential energy for continuous charges.

see geometry

$dU = dQ V_r \qquad dQ = \rho A\, dr$

$dU = \frac{Q_r}{4\pi\varepsilon_0 r}\, dQ \qquad Q_r = V\rho$

Electric displacement

$\mathbf{D} = \epsilon\,\mathbf{E}$

Poisson equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$$

Lorentz Force

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$$

Current:

$I = n q v_d A \qquad \mathbf{J} = \sigma\mathbf{E}$

$I = \int_A \mathbf{J}\cdot d\mathbf{a} \qquad \rho = \frac{1}{\sigma}$

Magnetic Flux

$$\Phi_B = \int_S \mathbf{B}\cdot d\mathbf{S}$$

Hall voltage

$$\Delta V_H = \frac{1}{nq}\frac{IB}{t}$$

$q v_d B = q E_H$

Magnetic force on a wire:

$d\mathbf{F}_B = I\, d\mathbf{s}\times\mathbf{B}$

$\oint d\mathbf{s} = 0$

Force on a wire:

$$\mathbf{F} = I\int_L d\mathbf{l}\times\mathbf{B}$$

velocity selector

$$v = \frac{E}{B}$$

Torque:

$\boldsymbol{\tau} = I\mathbf{A}\times\mathbf{B} = IAB\sin\theta$

mass spectrometer

$$r = \frac{mv}{qB_0}$$

magnetic dipole moment:

$\boldsymbol{\mu} = I\mathbf{A}$

Force acting on a charged particle:

$\mathbf{F}_B = q\mathbf{v}\times\mathbf{B} = qvB\,\hat{\ell}$

Potential energy for magnetic dipole:

$U = -\boldsymbol{\mu}\cdot\mathbf{B}$

$= -I\mathbf{A}\cdot\mathbf{B}$

$= -IAB\cos\theta$

Biot-Savart Law

$$dB = \frac{\mu_0 I}{4\pi} \frac{ds \times \hat{r}}{r^2}$$

Ampere's Law:

$$\oint B \cdot ds = \mu_0 I$$

Gauss's Law for magnetism:

$$\nabla \cdot B = 0$$

$$* \ \nabla \cdot (P \times Q) = Q \cdot (\nabla \times P) - P \cdot (\nabla \times Q)$$

$$\Phi_B = \int_S B \cdot ds = \int_V \nabla \cdot B \, dV = 0$$

Ampère-Maxwell Law

$$\oint B \cdot ds = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} \text{ (displacement current)}$$

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{dE}{dt}$$

Magnetic Susceptibility

$$B_m = \chi_m B_0$$

Magnetic Field Strength

$$H = \frac{1}{\mu_0} B - M$$

$$H = (\mu_0 \mu_r)^{-1} B$$

Faraday's Law

$$\mathcal{E}_{emf} = -\frac{d\phi_B}{dt}$$

$$\phi_B = \int_S B \cdot da$$

Time varying B-field → EMF

$$\nabla \times E = -\frac{dB}{dt}$$

Self inductance

$$\mathcal{E}_L = -\frac{d\phi_B}{dt}$$

$$\mathcal{E}_L = -L \frac{dI}{dt}$$

$$L = \frac{\mu_0 N^2 A}{l} \text{ (for solenoid)}$$

Energy stored in the magnetic field

$$U = \frac{1}{2} I^2 L$$

Energy density ~~of~~ of the field:

$$u = \frac{B^2}{2\mu_0}$$

3D wave equation

$$\nabla^2 A = \frac{1}{v^2} \frac{\delta^2 A}{\delta t^2}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Circuits

Capacitance

$$C = \frac{Q}{\Delta V} \simeq \frac{\varepsilon_0 A}{d}$$

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{\parallel} = C_1 + C_2$$

Energy stored in a charged capacitor

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$U = \frac{1}{2}C(\Delta V)^2$$

Electric polarization

$$\underline{P} = \frac{\Delta P}{\Delta V}$$

$$\underline{P} = \epsilon_0 \chi \underline{E}$$

Standard Stuff

$$\Delta V = IR$$

$$R = \frac{L}{\sigma A} \qquad \rho = \frac{1}{\sigma}$$

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

$$\varepsilon = \frac{dW}{dq}$$

$$\varepsilon = IR$$

AC currents:

$$I(t) = \frac{dq}{dt} = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

$$I = I_{max}\sin(\omega t - \phi)$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

RL circuits

$$I(t) = \frac{\varepsilon}{R}(1 - e^{-\frac{t}{\tau}}) = \frac{\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$$

$$\tau = \frac{L}{R}$$

LC circuits

$$U = U_E + U_B = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

$$I_{max} = -\omega Q_{max} \qquad U(t) = \frac{Q_{max}^2}{2C}$$

$$Q = A\cos(\omega t + \phi)$$

$$\omega = \frac{1}{\sqrt{LC}}$$

RLC circuits:

$$Q(t) = Q_{max}\, e^{-\frac{Rt}{2L}} \cos(\omega_d t)$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

AC w/ capacitor

$$\varepsilon = \Delta V_{max}\sin\omega t = \Delta V_C \qquad I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{X_C}$$

$$Q = C\Delta V_{max}\sin\omega t \qquad |X_C| = \frac{1}{\omega C}$$

$$I = I_{max}\sin(\omega t + \frac{\pi}{2})$$

AC w/ resistor

$$V = I_{max}\sin\omega t$$

AC w/ Inductor

$$I_L = I_{max}\sin(\omega t - \frac{\pi}{2})$$

$$I_{max} = \frac{\Delta V_{max}}{X_L}$$

$$|X_L| = \omega L$$

RLC circuit

$$Z = R + i(X_L - X_C) = Ze^{i\phi}$$

$$\phi = \arctan\frac{X_L - X_C}{R}$$

$$I = \frac{\varepsilon_{max}}{Z} e^{i(\omega t - \phi)}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\langle P \rangle_t = \frac{1}{2} I_{max}\Delta V_{max}\cos\phi$$

$$\langle P \rangle_t = I_{rms}\Delta V_{rms}\cos\phi$$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$\Delta V_{rms} = \frac{V_{max}}{\sqrt{2}}$$