Financial Signal Processing

Advanced topics

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- In real world we might hold other assets, apart from non defaultable bonds, e.g. stocks.
- They do not guarantee the return over one year, rather the return value follows a distribution.
- Assume historical daily returns for a stock $r_1, r_2, \cdots, r_{256}$ (working days in a year). Assume that daily returns identically independently distributed.
- Then yearly stock return is $E(r) = \mathrm{mean}(r_1, r_2, \cdots, r_{256}) \times 256$
- Yearly volatility: $Vol(r) = std(r_1, r_2, \cdots, r_{256}) \times 16 = \sqrt{var(r_1, r_2, \cdots, r_{256}) \times 256}$
- Quite often it is more convenient to work with *var* since it is additive
- The above logic is distribution free, quite often we assume normal distribution (strong caveats, but much easier calculations)
- Assume prices at $P_0 = 100, P_1 = 102$, two consecutive days, Daily return can be defined either:
  - arithmetic returns: $ret = (101 - 100)/100 = 0.01$
  - log returns: $ret = ln(101/100) = .0099$, so that $P_1 = P_0 \times exp(ret)$
  - Unless you want to be very precise (pricing) the two methods yield similar results

- The *Vol* of the return distribution is extremely important
- Would you prefer to hold a stock $(E, Vol) = (5\%, 10\%)$ or $(3\%, 5\%)$
- The ratio E/Vol - Sharpe ratio: a key criterion (Signal to Noise ratio)
- *Vol* generally not constant through time, returns can vary dramatically
- So given historic returns up to t, how do we calculate an estimate for *Vol* at $t + 1$?

- In general we impose some structure $Vol(t+1) = F(Vol(t), \cdots)$. A simpler model is autoregressive conditional heteroscedasticity model ARCH:
  - Estimate the best fitting autoregressive model AR(q):
$$y_t = a_0 + a_1 y_{t-1} + \cdots + a_q y_{t-q} + \epsilon_t = a_0 + \sum_{i=1}^{q} a_i y_{t-i} + \epsilon_t$$
  - Obtain the squares of the error $\hat{\epsilon}^2$ and regress them on a constant and q lagged values:
$$\hat{\epsilon}^2 = \hat{\alpha}_0 + \sum_{i=1}^{q} \hat{\alpha}_i \hat{\epsilon}^2_{t-i}$$
where q is the length of ARCH lags.

- Typical models are the generalised ARCH - GARCH(p,q):
$$h_t = w + \sum_{i=1}^{p} \alpha_i (R_{t-i} - \mu)^2 + \sum_{j=1}^{q} \beta_j h_{t-j}$$
- Here w, α and β are parameters to be estimated (MLE), R is the return and h is the historic vol.
- One could say that p and q are the hyper-parameters.

http://www.stern.nyu.edu/rengle/EnglePattonQF.pdf

https://en.wikipedia.org/wiki/Autoregressive_conditional_heteroskedasticity

- Volatility of vol is an important metric, shows how volatile the vol is
- Typically vol is mean reverting
- Volatility clustering
- Exogenous (non asset) factors can affect volatility
- Assuming returns is the signal vol can be considered its power
- Typically one can estimate vol of a period by adding var of the subperiods as a first approximation

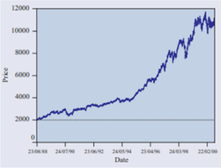

Figure 1. The Dow Jones Industrial Index, 23 August 1988 to 22 August 2000.

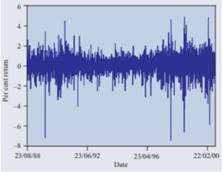

Figure 2. Returns on the Dow Jones Industrial Index.

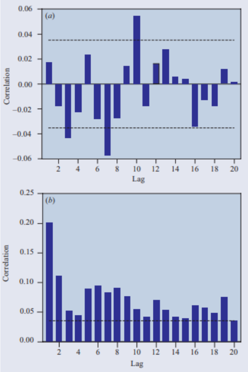

Figure 3. Correlograms of returns and squared returns.

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- We assume that the return of an asset at t, $R(t)$ is random.
- However there is intuition that there are exogenous factors affecting it:
  - If the market (all stocks) rally a given stock will probably also rally
  - if the oil price rallies, an oil's producer stock price will probably increase
  - if a country is in economic expansion probably its stocks will perform well
- We model the returns using a linear model, $R_{factor}(t)$ is the return of a factor
  - So $R(t) = a + b \times R_{factor}(t) + e$, a and b to be estimated, e the error term
  - b is the exposure of the asset to the factor
  - e is the specific return (after accounting for systematic factor return)
  - a small in magnitude otherwise arbitrage (could be set to 0)
- How do you estimate those?
- Why a linear model?

CAPM

$$\mathbb{E}(\boldsymbol{r}_i) = r_f + \beta_i(\mathbb{E}(\boldsymbol{r}_m) - r_f)$$

- $\mathbb{E}(\boldsymbol{r}_i)$ is the expected return on the capital asset
- r_f is the risk-free rate of interest such as interest arising from government bonds
- β_i (the beta) is the sensitivity of the expected excess asset returns to the expected excess market returns, or also $\beta_i = \frac{Cov(\boldsymbol{r}_i,\boldsymbol{r}_m)}{Var(\boldsymbol{r}_m)} = \rho_{i,m}[\frac{\sigma_i}{\sigma_m}]$
- $\mathbb{E}(\boldsymbol{r}_m)$ is the expected return of the market
- $\mathbb{E}(\boldsymbol{r}_m) - r_f$ is sometimes known as the market premium (the difference between the expected market rate of return and the risk-free rate of return).
- $\mathbb{E}(\boldsymbol{r}_i) - r_f$ is also known as risk premium
- $\rho_{i,m}$ denotes the correlation coefficient between the investment i and the market m
- σ_i is the standard deviation for the investment i
- σ_m is the standard deviation for the market m.

Fama–French model I

$$r = R_f + \beta(R_m - R_f) + b_s \cdot SMB + b_v \cdot HML + \alpha$$

- r is the portfolio's expected rate of return, R_f is the risk-free return rate, and R_m is the return of the portfolio.
- The "three factor" β is analogous to the classical β but not equal to it, since threre are now two additional factors to do some of the work.
  - SMB stands for "***S***mall [market capitalization] ***M***inus ***B***ig" - size premium → return fo. small mkt caps - big mkt caps
  - HML for "***H***igh [book-to-market ratio] ***M***inus ***L***ow" - value premium → return high book to mkt ratio - return low book to mkt

  They measure the historic excess returns of small caps over big caps and of value stocks over growth stocks.

Fama–French model II

Book to mkt ratio = $\frac{\text{mkt cap}}{\text{book val}}$ = assets - liabilities

mkt cap calc example:

$$V_A = \sum_t \frac{D}{(1+r)^t}$$ = discount future dividends

- What are the factors?
- What is the market?
- Factor Time invariance?

APT I

- Risky asset returns are said to follow a factor structure if they can be expressed as:
  - $r_j = a_j + b_{j1}F_1 + b_{j2}F_2 + ... + b_{jn}F_n + \epsilon_j$

    where
    - a_j is a constant for asset j
    - F_n is a systematic factor
    - b_{jn} is the sensitivity of the jth asset to factor n, also called factor loading,
    - and ϵ_j is the risky asset's idiosyncratic random shock with mean zero.
- Idiosyncratic shocks are assumed to be uncorrelated across assets and uncorrelated with the factors.

APT II

- The APT states that if asset returns follow a factor structure then the following relation exists between expected returns and the factor sensitivities:
  - $\mathbb{E}(r_j) = r_f + b_{j1}RP_1 + b_{j2}RP_2 + ... + b_{jn}RP_n$
  - where
    - RP_n is the risk premium of the factor,
    - r_f is the risk-free rate,
- That is, the expected return of an asset j is a linear function of the asset's sensitivities to the n factors.

APT Example: French – Fama 5 Factor I

The model says the market value of a share of stock is the discounted value of expected dividends per share,

$$m_t = \sum_{\tau}^{\infty} \mathbb{E}(d_{t+\tau})/(1+r)^{\tau}$$

discount expected vals

In this equation, m_t is the share price at time t, $\mathbb{E}(d_{t+\tau})$ is the expected dividend per share for period $t+\tau$, and τ is (approximately) the long-term average expected stock return or, more precisely, the internal rate of return on expected dividends.

APT Example: French – Fama 5 Factor II

$$m_t = \sum_{\tau}^{\infty} \mathbb{E}(d_{t+\tau})/(1+r)^{\tau}$$

With a bit of manipulation, we can extract the implications of the above equation for the relations between expected return and expected profitability, expected investment, and B/M. Miller and Modigliani(1961) show that that at the time t total market value of the firm's stock implied by the above equation is,

$E[d_{t+\tau}] \to E[Y - dB]$

$$M_t = \sum_{\tau}^{\infty} \mathbb{E}(\overset{\text{earnings}}{Y_{t+\tau}} - d\overset{\text{book val}}{B_{t+\tau}})/(1+r)^{\tau}$$

liabilities: 10m : equity

assets: 10M

assume no debt -> assets : liabilities = eq.

$\frac{\text{mkt}}{\text{book}}$ ratio tells you about state of company - higher ratio: more profitable, healthier

APT Example: French – Fama 5 Factor III

eg: mkt val = \$100m
book val = \$10m

Book

mkt val   earnings - book val

$$\frac{M_t}{B_t} = \frac{\sum_{\tau}^{\infty} \mathbb{E}(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t}$$

book val

This equation makes three statements about the expect stock returns:

- Fix everything except the current value of the stock, M_t, and the expected stock return, r. Then a lower value of M_t, or equivalently a higher book-to-market equity ratio, B_t/M_t, implies a higher expected return. $M_t \downarrow$, $r \uparrow$ or $\frac{M_t}{B_t} \uparrow$, $r \uparrow$
- Fix M_t and the value of everything except expected future earnings and the expected stock return. The equation then tells us that higher expected earnings imply a higher expected return. $Y \uparrow$, $r \uparrow$
- For fixed values of B_t, M_t, and the expected earnings, higher expected growth in book equity - investment - implies a lower expected return. Stated in perhaps more familiar terms, the equations says that B_t/M_t is a noisy proxy for expected return because market cap M_t also responds to forecasts of earnings and investment.

APT Example: French – Fama 5 Factor IV

SMB = premium for small company
HML: premium for companies w/ small book val vs big book val

Tests of the three-factor model center on the time-series regression,

$$R_{it} - R_{Ft} = a_i + b_i R_{mt} - R_{Ft} + s_i SMB_t + h_i HML_t + e_{it}$$

In this equation, R_{it} is the return on security or portfolio i for period t, R_{Ft} is the risk free return, R_{Mt} is the return on value-weight (VW) market portfolio, SMB_t is the return on a diversified portfolio of small stocks minus the return on a diversified portfolio of big stocks, HML_t is the difference between the returns on diversified portfolios of high and low B/M stocks, and e_{it} is a zero mean residual. Treating the parameters in this equation as true values rather than estimates, if the factor exposures b_i, s_i and h_i capture all variation in expected returns, the intercept a_t is zero for all securities and portfolios i.

APT Example: French – Fama 5 Factor V

$$R_{it} - R_{Ft} = a_i + b_i R_{mt} - R_{Ft} + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

In this equation, RMW_t is the difference between the returns on diversified portfolios of stocks with robust and weak profitability, and CMA_t is the difference between the returns on diversified portfolios of the stocks of low and high investment firms, which we call conservative and aggressive. If the exposures to the five factors, b_i, s_i, h_i, r_i and c_i captures all variation in the expected returns, the intercept a_t is zero for all securities and portfolios i.

APT Example: French – Fama 5 Factor VI

Average percent returns, standard deviations (Std Dev) and *t*-statistics for the average return for the portfolios used to construct *SMB*, *HML*, *RMW*, and *CMA*; July 1963 - December 2013, 606 months

We use independent sorts to form two *Size* groups, and two or three *B/M*, operating profitability (*OP*), and investment (*Inv*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label the portfolios with two or four letters. The first is small (*S*) or big (*B*). In the 2x3 and 2x2 sorts, the second is the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (C), neutral (*N*), or aggressive (A). In the 2x2x2x2 sorts, the second character is the *B/M* group, the third is the *OP* group, and the fourth is the *Inv* group.

	2x3 Sorts						2x2 Sorts			
Size-B/M	*SL*	*SN*	*SH*	*BL*	*BN*	*BH*	*SL*	*SH*	*BL*	*BH*
Mean	0. 93	1.31	1.46	0.89	0.94	1.10	1.03	1.43	0.88	1.04
Std Dev	6.87	5.44	5.59	4.65	4.34	4.68	6.41	5.42	4.50	4.38
t-statistic	3.32	5.93	6.44	4.69	5.36	5.78	3.95	6.51	4.82	5.86
Size-OP	*SW*	*SN*	*SR*	*BW*	*BN*	*BR*	*SW*	*SR*	*BW*	*BR*
Mean	1.02	1.27	1.35	0.81	0.87	0.98	1.10	1.32	0.82	0.95
Std Dev	6.66	5.32	5.96	4.98	4.38	4.39	6.16	5.69	4.53	4.39
t-statistic	3.77	5.87	5.60	4.00	4.91	5.50	4.41	5.71	4.47	5.33
Size-Inv	*SC*	*SN*	*SA*	*BC*	*BN*	*BA*	*SC*	*SA*	*BC*	*BA*
Mean	1.41	1.34	0.96	1.07	0.94	0.85	1.40	1.06	0.99	0.88
Std Dev	6.12	5.22	6.59	4.38	4.08	5.18	5.73	6.17	4.09	4.69
t-statistic	5.66	6.35	3.59	5.99	5.69	4.03	6.01	4.25	5.98	4.62

	2x2x2x2 *Size-B/M-OP-Inv* Sorts							
	SLWC	*SLWA*	*SLRC*	*SLRA*	*SHWC*	*SHWA*	*SHRC*	*SHRA*
Mean	1.13	0.70	1.36	1.16	1.43	1.24	1.64	1.54
Std Dev	7.18	7.36	5.38	6.15	5.55	5.62	5.23	5.52
t-statistic	3.89	2.34	6.24	4.64	6.34	5.42	7.72	6.88
	BLWC	*BLWA*	*BLRC*	*BLRA*	*BHWC*	*BHWA*	*BHRC*	*BHRA*
Mean	0.77	0.78	1.02	0.91	1.02	0.93	1.24	1.17
Std Dev	5.16	5.47	4.16	4.74	4.36	4.69	4.79	5.51
t-statistic	3.69	3.51	6.04	4.75	5.78	4.87	6.38	5.23

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Assume for 1 asset CAPM holds:

model for CAPM

- $r = b * f + e$ return of asset: return of factor error
- $var(r) = var(b * f + e) = var(b * f) + var(e) = b^2 * var(f) + var(e)$
  uncorrelated
- where: $r \sim$ asset return, $b \sim$ market exposure, $f \sim$ market return, $e \sim$ specific return
- f and e independent by construction (time series regression)
- So the variance of an asset is a linear combination of a the variance of the market factor and its specific variance

Assume APT holds for 100 assets (portfolio) and 5 factors (no constant term)

- $R = B * F' + E$: R(100x1) returns, B(100x5) betas, F(1x5) factor returns, E(100x1) specific returns

  $\overset{}{100}\begin{bmatrix} R \end{bmatrix}^{1} = 100\begin{bmatrix} B \end{bmatrix}^{5} \begin{bmatrix} F' \end{bmatrix} + \begin{bmatrix} E \end{bmatrix}$

  want ~ 20 assets per factor

- The equation holds in vector form for one period (cross sectional regression)
- The portfolio variance follows:

$$var(R) = var(B * F' + E) = var(B * F') + var(E) = B * VCV * B' + D$$

VCV(5x5) factor covariance matrix, D(100x100) diagonal e^2

assume B is const so comes out of var

- Advantages
  - Mathematical stability (VCV low dimensionality)
  - Intuition
  - Easier to estimate with factor portfolios
  - Less data needed

VCV : variance - covariance matrix of factors.

$$
\underset{100\times 5}{\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{100} \end{bmatrix}} = \underset{100\times 5}{\begin{bmatrix} \underbrace{b_1^1 \; b_1^2 \; b_1^3 \; b_1^4 \; b_1^5}_{\text{betas for stock 1 for all 5 factors}} \\ \vdots \\ b_{100}^1 \; b_{100}^2 \; b_{100}^3 \; b_{100}^4 \; b_{100}^5 \end{bmatrix}} \underset{\substack{5\times 1 \\ \text{(transposed for mult)}}}{\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}} + \underset{100\times 1}{\begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{100} \end{bmatrix}}
$$

$$
VCV = \underset{5\times 5}{\begin{bmatrix} var_1 & cov_{12} & \cdots & cov_{15} \\ cov_{21} & var_2 & & \\ \vdots & & \ddots & \\ cov_{15} & & & var_5 \end{bmatrix}}
$$

$$
D = \begin{bmatrix} var_{e_1} & cov_{e_1 e_2} & \cdots & \vdots \\ cov_{e_2 e_1} & var_{e_2} & & \vdots \\ \vdots & & \ddots & \\ \cdots & & & var_{e_{100}} \end{bmatrix}
$$

cov_{e_i, e_j} for $i \neq j$ $= 0$

Choosing factors – estimating exposures (betas)

Economic val:
ln (mkt cap debt)

- Fundamental models
  - Calculate beta, regress for factor exposure (100 equations 5 unknowns)
  - French Fama factors
  - Macro factors (oil price), exposures?
  - Industry
  - Geography ...
- Statistical models
  - PCA on asset by asset VCV (100x100) - components don't mean anything, diff no. of components needed for diff time periods
  - Get e.g. 5 factors, 5 first components, or explaining 50% of variance
  - Factor VCV diagonal
  - Timeseries regression for exposures

Choosing factors – estimating exposures (betas)

- Issues
  - Intuition
  - Interpretability - In PCA fctor definition changes, PCA on Corr or VCV?
  - In fundamental collinearity
  - Thin industries

An example with two assets

- Main objective function: Trying to optimise this
  - argmax{expected_return − portfolio_variance}
  - We want to maximise that by changing portfolio weights, vector $W(w_1, w_2)$
- Expected return:

$$\mathbb{E}(R_p) = \sum_i w_i \, \mathbb{E}(R_i)$$

= weight × exp. of asset

where R_p is the return on the portfolio, R_i is the return on asset i and w_i is the weighting of component asset i (that is, the proportion of asset "i" in the portfolio).

- Portfolio return variance:

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

$\sigma_p^2 : \sum_{ij} w_i \sigma_{ij} w_j$

correlation: $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$

↳ $\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$

weights sum to 1

For a two asset portfolio:

- Portfolio return: $\mathbb{E}(R_p) = w_A \, \mathbb{E}(R_A) + w_B \, \mathbb{E}(R_B) = w_A \, \mathbb{E}(R_A) + (1 - w_A) \, \mathbb{E}(R_B)$
- Portfolio variance: $\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB}$

https://en.wikipedia.org/wiki/Modern_portfolio_theory

In practical applications we would use

- factor model for risk
- between 5-20 signals for return forecasting

Assume APT holds for 100 assets (portfolio) and 5 factors (no constant term)

- Main objective function:
  - argmax{return – lf*factor_risk – ls*specific_risk – tc*transaction_cost}

    lf, ls, tc = weights for each. also Lagrange mults.
  - We want to maximise that by changing portfolio weights, vector W (100 x 1)
  - Subject e.g. to total risk < K

    risk can be var
  - Alternative specifications might include return > L, return/total_risk > S
  - Additional constraints might include:

$$sum(W) = 1, W > 0,\ (Long\ only),$$

$$sum(W) < L * fund\ size(leverage),\ if\ \ W > 0\ long\ only$$

$$sum(W_i) < I\ \ for\ \ i\ \ in\ [...],\ \ factor\ \ exposure$$

equities, leverage ~3
credit ~8
govt bonds ~10

- Return:expected portfolio return
  - $W' \times R$ 1x100 x 100x1
  - R : expected asset return vector (100x1)
- Factor_risk: the variance attributed to factors
  - $\overset{1\times 100}{W'} \times X \times \overset{100\times 100}{VCV} \times X' \times \overset{100\times 1}{W}$
  - X exposures to factors (100x5)
  - VCV factor covariance (5x5)
- Specific_risk: the variance attributed to specific asset return
  - $\overset{1\times 100}{W'} \times \overset{100\times 100}{D} \times \overset{100\times 1}{W}$
  - D specific covariance matrix (100x100), can be assumed diagonal
- Transaction_cost:
  - $(W' - W'_{t-1}) * T$ (diff between what want - what curr. have) x cost per asset
  - T, tcost vector per asset (100x1) strictly $|w' - w'_{t-1}| \times T$
  - W_{t-1}, holdings in previous period (100x1)

- Universe selection and implications
  - Generally the more uncorrelated assets the better
  - Use all the universe? Liquid assets? Semi liquid?
  - Can we trade them? esp in violent market moves?
  - Tcosts?
- Model speed
  - Trade-off between performance, t-costs, slippage?
  - Market impact costs

- Forecasting returns I:
  - Where financial signal processing shines
  - Where major research is happening
  - We have a time series (signal, filtration at t-1), forecasting returns at t
  - Quite often signal at t-2 (cannot trade on opening of day t)
  - Signal processing techniques
  - Characteristics
    - Turnover
    - Biases, cohort behaviour
    - Stability
    - Dependence on macro factors eg. car of company, assets

- Forecasting returns II:
  - Inter Signal correlation crucial   ideally no correlation but in reality ideally ~20-30 %
  - Lead lag effects, cross market information
  - Different signal classifications*
    - Momentum
    - Value - Mean reversion
    - Fundamental
    - Sentiment

Example (Momentum): Equity Momentum

- $s = sum(w_t \times ret_t), t \ in \ [1..6]$
- Trade s
- Asset agnostic

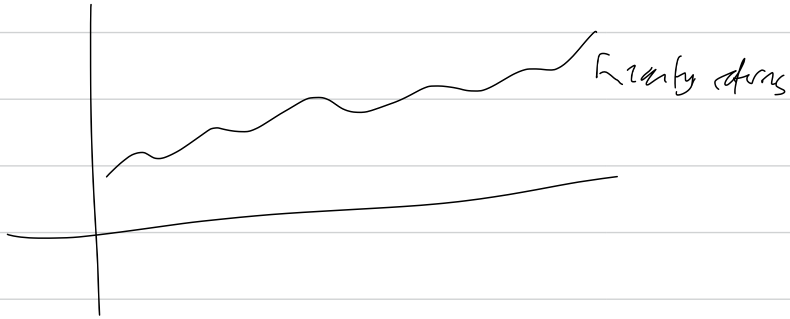
Example (Value) : Spread vs Probability of default

exp. prob of default

- $Spr = a + b \times edf + e$
- Trade e

$*http : //pages.stern.nyu.edu/ \sim lpederse/papers/ValMomEverywhere.pdf$

Momentum:



momentum is same as using filter in sig. processing terms

- We have a signal that returns r per year (time-series of daily returns)
- Information ratio and its meaning
  - Define $IR = \mathbb{E}(r)/std(r)$, the ratio of the first and the second moment Sharpe
  - Return per unit of risk
  - How many deviations we are from having 0 or negative returns
  - Assuming normality completely describes the return distribution
  - In a well diversified portfolio returns are approximately normal
- Caveats
  - Nomality? (depending on the asset class) not always
  - Returns serially uncorrelated (Markov processes)? returns iid
  - Ignoring higher moments – non continuous return process (jumps)
  - Point in time metric, e.g. trailing twelve months
  - Stability of IR also important

- The fundamental law of Active portfolio management*
  - IR = IC * sqrt(breadth) how many assets
  - IC = information coefficient, correlation between signal values and returns
  - Breadth = N: number of names in the universe (assuming uncorrelated)
  - $IR = N * \mathbb{E}(r)/sqrt(N * var(r)) \sim sqrt(N) * qssetIR \sim$
    - Nominator : number of bets * expected return on bet
    - Denominator : std of N uncorellated bets
    - Asset IR : assuming risk of 1, the sqrt(b) of a time series regression s = b*r + e (signal on asset returns)
- IR does depend on market efficiency
  - IR decay, esp with high tcosts very important
  - Market impact costs

*Active Portfolio Management : A quantative approach for producing superior returns and selecting superior money managers, Richard Grinold, Ronald Kahn

Risk Forecasting

- Assume we have an APT like risk model, holds for 100 assets (portfolio) and 5 factors (no constant term)*
- $R = B * F' + E$: R(100x1) returns, B(100x5) betas, F(1x5) factor returns, E(100x1) specific returns
- $var(R) = var(B * F' + E) = var(B * F') + var(E) = B * VCV * B' + D$

*Portfolio Risk Analysis, Gregory Connor, Lisa R. Goldberg, Robert A. Korajczyk

Risk Forecasting I

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*Portfolio Risk Analysis, Gregory Connor, Lisa R. Goldberg, Robert A. Korajczyk

Risk Forecasting II

- We nee to forecast the factor and specific risk at t, with information at t-1
- VCV is easier
  - Factor is a portfolio, close to normal return distribution, as a first approximation an

    exp. weighted moving average

    EWMA weighted std of return OK
  - That said conditioning on macro factors (regimes) can improve, forecasting the forecasting factors dangerous though (martingale tower property)
- Specific risk is more important
  - Factor risk can be diversified, specific cannot
  - Where non normality emerges
  - As first approximation timeseries std (specific returns)
  - Better approximation, linear cross sectional model : e^2 or $|e| \sim$ regressed on factors

*Portfolio Risk Analysis, Gregory Connor, Lisa R. Goldberg, Robert A. Korajczyk

Topics in optimisation I

- Objective function:
  - argmax{return – lf*factor_risk – ls*specific_risk – tc*transaction_cost}
  - We want to maximise that by changing portfolio weights, vector W (100 x 1)
- Return based on signals, risk based on risk model, t-costs as a first approximation known (assume t-cost at t same as t-1)
- Quadratic optimisation:
  - Why break the variance in two? Constraints, Same units!
- Assumes no path dependency (only in tcost)
- VCV should be well behaved

Topics in optimisation II

- Robust optimisation
  - Different VCV
  - Account for uncertainty in $\mathbb{E}(r)$ estimation
- Adding terms (e.g. skew – kurtosis)
- Multiperiod optimisation
  - Assume an alpha decay (empirical) and run optimisation for t-1, t-2, . . ., t-k
  - Average out k optimised portfolios