Quantum Walk

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1 Introduction

Quantum random walks quantum analogues of classical random walks. Unlike the continuous time quantum walk, the discrete time quantum walk algorithm requires the use of one or more coin qubits representing the number of movement choices from each graph vertex.In Continuous time quantum walk uses a transition matrix commonly expressed as Hamilton whose evolution over time is simulated or taken into account. Intuitively, the quantum walk is very similar to its classical counter part. In a classical walk, the walker observes some random process, say a coin toss, and decides on his next step conditioned on the output of this random process. So for a classical random walk, the walker is given a probability to make a transition. In a quantum walk, on the other hand, the random process is replaced by a quantum process,say by a Quantum Coin,which is a unitary matrix.
So the next step of the walker is controlled by a complex amplitude rather than a probability. This generalization, from positive numbers to complex numbers, makes quantum walks more powerful than classical random walks. For a graph G=(V,E) we can have it adjacency matrix as $A_{j,k} = \begin{cases} 1, & \text{if} \quad (j,k) \in E, \\ 0, & \text{if} \quad otherwise \end{cases}$

Now the transition matrix used in analogous to our coin toss in classical counterpart can be obtained from our adjacency matrix as the Hamiltonian H $= \frac{A}{d}$ where d is the differentiation operator $\frac{d}{dt}$.Amplitude wave function of the particle is given by Schrodinger's Equation $ih\frac{d}{dt}|\psi_t\rangle = H|\psi_t\rangle$.If hd happens to be 1 then our Hamiltonian reduce to our adjacency matrix and $|\psi_t\rangle = e^{iHt}|\psi_0\rangle$.The unitary evolutionary operator for various matrix of 2,4,8 vertex can be obatined by using e^{iHt}.As a example for graph with 2 matrix

$$U_{C_2} = \begin{bmatrix} \cos t & -i\sin t \\ -i\sin t & \cos t \end{bmatrix}$$

Which can be easily implemented using the U gate in Qiskit.