CONTINUOUS TIME QUANTUM WALKS

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Quantum Walk :Intuition

A random walk(Classical Version) is the process by which randomly-moving objects wander away from where they started.
Quantum random walks quantum analogues of classical random walks.
Quantum Walk can be of two form

1. Discrete Time Quantum Walk :One or more coin qubits representing the number of movement choices from each graph vertex.
2. Continuous Time Quantum Walk :Transition matrix commonly expressed as Hamilton whose evolution over time is simulated or taken into account.

Mathematical model

For a graph G=(V,E) we can have it adjacency matrix as $A_{j,k} = \begin{cases} 1, & \text{if} \quad (j,k) \in E, \\ 0, & \text{if} \quad \textit{otherwise} \end{cases}$
Transition matrix can be obtained from our adjacency matrix A as the Hamiltonian H $= \frac{A}{d}$ where d is the differentiation operator $\frac{d}{dt}$.
Amplitude wave function of the particle is given by Schrodinger's Equation

$$ih\frac{d}{dt}|\psi_t\rangle = H|\psi_t\rangle$$

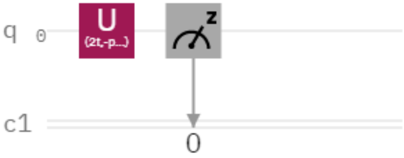
Mathematical Model(Contd.)

If hd happens to be 1 then our Hamiltonian reduce to our adjacency matrix and $|\psi_t\rangle = e^{iHt}|\psi_0\rangle$.The unitary evolutionary operator for various matrix of 2,4,8 vertex can be obatined by using e^{iHt}.As a example for graph with 2 matrix

$$U_{C_2} = \begin{bmatrix} \cos t & -i \sin t \\ -i \sin t & \cos t \end{bmatrix}$$

Implementation

The quantum state at time t, $|\psi(t)\rangle$ is calculated by multiplying e^{-iHt} matrix of corresponding graph with qubit image of $|i\rangle$ state, in other words it will be i^{th} column of e^{-iHt} matrix. Probability of $|i\rangle^{th}$ state at any instant of time is given by ; $P_i = |\langle i|\psi(t)\rangle|^2$.
The above circuit corresponds to the unitary for C_2(2 vertex graph). Circuit for two vertex graph is



Reference

1.J. Kempe, "Quantum random walks - an introductory overview" ,arXiv :quant-ph/0303081v1 13 Mar 2003.