

Chapter 8: Time, Speed and Distance

Basic Concepts

1. Speed

The rate at which the body covers the distance is known as its speed. Mathematically, this can be represented with the help of following formula:

$$\text{Speed} = \text{Distance}/\text{Time}$$

The basic units in which speed is represented are kmph, m/s etc.

If the distance to be covered is constant then the speed of any body will be inversely proportional to the time required to travel the same distance.

Thus, if s_1 , s_2 and t_1 , t_2 are the speeds and times for a given body to cover a given distance then it can be inferred that: $s_1:s_2 :: t_2, t_1$

Note: The average speed is given by the formula $(2xy)/(x + y)$, where x and y are the constant speeds at which a body has travelled any given distance to and fro.

Note: If two persons P and Q start at the same time from two points A and B respectively towards each other and after crossing each other they take X and Y hours in reaching B and A respectively. Then, $V_Q/V_P = (X/Y)^{1/2}$.

2. Trains

These are problems concerning the motions of trains or any other bodies with length that is not negligible and hence has to be considered for the purpose of calculation. The following points will help to visualize problems on trains.

1. The time taken by a train L meter long in crossing a stationery man or a pole or any other object with negligible length is given by L_1/V_1 , where V_1 is the speed at which train is moving.

Extending the above point, the time taken by the same train to cross another stationary object L_2 meter long is $(L_1 + L_2)/V_1$.

2. Suppose both the objects are moving then the time is given by $T = (L_1 + L_2)/(V_1 \pm V_2)$. +ve sign is used when objects are moving in opposite direction. -ve sign is used when objects are moving in the same direction.

3. Relative Speed

Relative speed is the most important fundamental concept used on solving variety of questions on time, speed, and distance. Whenever we consider the two bodies in motion simultaneously we must consider the relative speed of the two bodies in order to compute different problems.

1. If two bodies travel in the same direction then the relative speed is given by the difference of the speeds of individual bodies. Similarly, if the two bodies move in opposite directions the relative speed is given by the sum of the individual speeds of the given bodies.
2. If the speed of A be V_a and that of B be V_b and if A and B are moving in the same direction and initial distance between A and B is D , then the time required for A to catch

up with B (it is given that the speed of A is greater than that of B) is given by: $T = D/(V_a - V_b)$. Similarly, if the two bodies are moving in opposite directions then the time required by both bodies to meet at a point is given by $T = D/(V_a + V_b)$.

4. Boats and Streams

These problems revolve around the movement of bodies in still and moving fluids. If a body swims in still water e.g. in a pond or a swimming pool then the speed with which he moves is the speed of that boy in still water. Now imagine the same boy swimming in a river, then he can either swim with the flow of water or against the flow of water. The movement of the boy in the river with the flow of water is called **downstream movement** whereas his movement against the flow of water is called as an **upstream movement**.

If the speed of the boy is known to be 'x' and that of the flow of water is known to be 'y' then Upstream Speed (u) = x - y and Downstream Speed (v) = x + y.

Upstream downstream average velocity = $(u^2 - v^2)/u$

Important Results

- Two bodies A and B move towards each other with speeds 'a' kmph and 'b' kmph, respectively, from points X and Y. The relative speed is given by $S = a + b$. If the distance between X and Y is D, time taken for A and B to meet, T hours = $D/S = D/(a + b)$. If they meet at a point Z, then

$$XZ = T \times A's \text{ speed} = (D \times a)/(a + b) \text{ and}$$

$$YZ = T \times B's \text{ speed} = (D \times b)/(a + b) \text{ and}$$

- Two bodies A and B move in the same direction with speeds 'a' kmph and 'b' kmph ($a > b$), respectively, from points X and Y. The relative speed is given by $S = a - b$. If the distance between X and Y is D, time taken for A and B to meet, T hours = $D/S = D/(a - b)$. If they meet at a point Z, then

$$XZ = T \times A's \text{ speed} = (D \times a)/(a - b) \text{ and}$$

$$YZ = T \times B's \text{ speed} = (D \times b)/(a - b) \text{ and}$$

- If a body changes its speed in the ratio n:m, then the time taken to cover the same distance changes in the ratio m:n, as speed and time taken share an inverse relationship.
- If the speeds of a bus without stoppages and with stoppages are S_1 and S_2 respectively then the average number of minutes per hour for stoppages = $(\text{difference of speeds}/\text{greater speed}) \times 60$.
- If there are three persons with speeds p, q, r ($p > q > r$) running around a circular track of length of circumference D units then the time they will take to meet for the first time, again at the starting point is LCM of $[D/(p - q)]$ and $[D/(q - r)]$.
- Speed of the boat in still water is $u = (S_{ds} + S_{us})/2$. The speed of the water current or stream is

$$v = (S_{ds} - S_{us})/2.$$

Formulae

We know, if a vehicle (or a person) travels with speed $s \text{ km/hr}$ and takes time $t \text{ hrs}$ to reach the destination, then the distance traveled, d , is given by, $d = s \times t$. That is,

$$\boxed{\text{distance} = \text{speed} \times \text{time}}$$

Remember: For calculation, first check whether all of the three quantities, namely, distance, speed and time have same system of unit. Means, if distance is given in kilometers, then speed and time must be in km/hr and hrs respectively or vice versa.

Or if distance is given in meters, then the units for speed and time must be m/s and seconds respectively.

How to convert: We know,

1 km = 1000 m and 1 hr = 60X60 seconds. Then

$$\boxed{1 \text{ km/hr} = \frac{1000 \text{ m}}{60 \times 60 \text{ sec}} = \frac{5}{18} \text{ m/sec}} \quad \text{And hence,} \quad \boxed{1 \text{ m/s} = \frac{18}{5} \text{ km/hr}}.$$

Useful Formulae:

I] Average Speed:

1) If a vehicle or a person travels two different distances, say $d_1 \text{ km}$ and $d_2 \text{ km}$ with two different speeds, say $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$, then the average speed for the whole journey is given by

$$\boxed{\text{average speed} = \frac{s_1 s_2 (d_1 + d_2)}{s_1 d_2 + s_2 d_1} \text{ km/hr}}$$

2) If in above case, the two distances traveled are same, i.e. if $d_1 = d_2$, then

$$\boxed{\text{average speed} = \frac{2s_1 s_2}{s_1 + s_2} \text{ km/hr}}$$

Note: In case of return journey, a vehicle or a person returns to its initial position. Hence, here also, we take $d_1 = d_2$.

II] Distance:

1) In case of **return journey**, if the forward and the backward speeds of a **vehicle**, namely $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$, and the total time taken, namely $T \text{ hrs}$ for the journey is given, then the distance traveled is given by

$$\boxed{\text{distance} = T \left(\frac{s_1 s_2}{s_1 + s_2} \right) \text{ km}}$$

- 2) Suppose that the **two vehicles** with their speeds $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$ respectively, start towards each other from the stations P and Q . When they meet, it is found that the vehicle which has started from Q has travelled $d \text{ km}$ more distance than the vehicle which has started from P . Then the distance between P and Q is given by

$$\text{distance} = d \left(\frac{s_1 + s_2}{s_2 - s_1} \right) \text{ km}.$$

III] Relative Speed:

- 1) If two vehicles or persons are travelling **in the same direction** with their speeds $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$, then their **relative speed** is $(s_1 - s_2) \text{ km/hr}$, where $s_1 > s_2$. And it is $(s_2 - s_1) \text{ km/hr}$ if $s_2 > s_1$. (**Subtraction of the speeds**)

If two vehicles have their lengths $L_1 \text{ km}$ and $L_2 \text{ km}$ and their speeds are $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$ respectively, then the **faster** vehicle will **pass** the **slower** vehicle in time

$$\left(\frac{L_1 + L_2}{s_1 - s_2} \right) \text{ hrs if } s_1 > s_2 \text{ and in } \left(\frac{L_1 + L_2}{s_2 - s_1} \right) \text{ hrs if } s_2 > s_1.$$

- 2) If two vehicles or persons are travelling **in the opposite direction** with their speeds $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$, then their **relative speed** is $(s_1 + s_2) \text{ km/hr}$. (**Addition of the speeds**)

If two vehicles have their lengths $L_1 \text{ km}$ and $L_2 \text{ km}$ and their speeds are $s_1 \text{ km/hr}$ and

$$s_2 \text{ km/hr} \text{ respectively, then they will } \textbf{cross} \text{ each other in } \left(\frac{L_1 + L_2}{s_2 + s_1} \right) \text{ hrs}.$$

IV] Speeds: If the vehicles of lengths $L_1 \text{ km}$ and $L_2 \text{ km}$, when running in the same direction, the faster one passes the slower one in $T_1 \text{ hrs}$ and when running in the opposite direction, they cross each other in $T_2 \text{ hrs}$, then

i)
$$\text{The speed of the faster vehicle} = \frac{1}{2} (L_1 + L_2) \left(\frac{1}{T_1} + \frac{1}{T_2} \right) \text{ km/hr}$$

ii)
$$\text{The speed of the slower vehicle} = \frac{1}{2} (L_1 + L_2) \left(\frac{1}{T_1} - \frac{1}{T_2} \right) \text{ km/hr}$$

V] (a) Vehicles starting from same destinations: Suppose that a vehicle starts from a place, and another faster vehicle starts from the **same place**, with the speeds $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$ respectively, where $s_1 < s_2$. If the slower vehicle starts from the destination and the faster one starts $T \text{ hrs}$ later from the same destination, then the distance from the starting place, where both the vehicles will meet, is given by

$$\text{distance} = \frac{s_1 s_2 T}{(s_2 - s_1)} \text{ km}$$

And the time **after which** the two vehicles will meet (the time after which the faster vehicle starts) is given by

$$\text{time} = \frac{s_1 T}{(s_2 - s_1)} \text{ km}.$$

(b) Vehicles starting from different destinations: Suppose that the two vehicles with their speeds $s_1 \text{ km/hr}$ and $s_2 \text{ km/hr}$, start from the different destinations, say P and Q which are $d \text{ km}$ apart. Suppose that the vehicle which starts from Q , starts $T \text{ hrs}$ later than the vehicle which starts from P .

Then the distance from P where both the vehicles will meet is given by

$$\text{distance} = s_1 \left(\frac{d + s_2 T}{s_1 + s_2} \right) \text{ km}$$

And the time **after which** the two vehicles will meet (the time after which the faster vehicle starts) is given by

$$\text{time} = \left(\frac{d + s_2 T}{s_1 + s_2} \right) \text{ hr}.$$

Class Work

- 1) How long does a train 100 meters long running at the rate of 40 km/hr take to cross a telegraphic pole?
a) 9 hrs b) 9 min c) 9 sec d) 9.9 hrs
- 2) There is a big square shaped haunted palace in a jungle. A train 1000 meters long passes it with a speed of 120 km/hr in a minute. Find the area of the haunted palace.
a) 1 sq.km b) 800 sq. m. c) 1.5 sq.km d) 1.2 sq.km
- 3) A car travels from P to Q with 15 km/hr and returns with 30 km/hr. What is the average speed of journey?
a) 22.5 km/hr b) 20 km/hr c) 25 km/hr d) 10 km/hr
- 4) Three persons decided to go on a long tour and decided to drive the same distance of 100 km each. First person drove with the speed of 60 km/hr. After a break, second person drove with the speed increased by 10 km/hr. Then at the end, the third person drove again with the speed increased by 10 km/hr. What is the average approximate speed of their journey?
a) 75 km/hr b) 69 km/hr c) 100 km/hr d) none of these
- 5) A cycled from P to Q at 10 km/hr & returned at the rate of 9 km/hr. B cycled both ways at 12 km/hr in the whole journey. B took 10min less than A. find the distance between P & Q.
a) 1.50 km b) 3.75 km c) 4.50 km d) 4.98 km
- 6) A person during his journey travels 40 minutes at a speed of 30 km/hr, another 50 minutes at 60 km/hr and last 1 hr at 30 km/hr. What is his average speed?
a) 38 km/hr b) 40 km/hr c) 60 km/hr d) 35.7 km/hr
- 7) Sohan left his house at 7 am and walked at the rate of 2 km/hr to his college. But he saw that his college was closed and immediately he returned at the rate of 3 km/hr. If he had reached his house at 10 am, what is the distance of his college from his house?
a) 3 km b) 5 km c) 4 km d) 3.6 km
- 8) By walking at $\frac{3}{4}$ th of his usual speed Hemant is 6 minutes late to his office. What is his usual time to cover the distance?
a) 20 min b) 18 min c) 24 min d) 1 hr
- 9) A train 145 meters long is running with a speed of 25 km/hr. In what time will it pass a man who is walking at 4 km/hr in the direction opposite to that of the train?
a) 10 sec b) 8 sec c) 9 sec d) 18 sec
- 10) Two trains of lengths 100 meters and 120 meters are running in the same direction with speeds 30 km/hr and 40 km/hr respectively. In what time (approx.) will they pass each other?
a) 24 sec b) 79 sec c) 80 sec d) 144 sec
- 11) Two persons are running in the same direction with the speeds of 3 km/hr and 6 km/hr. A truck comes running from behind and passes them in 9 and 10 seconds respectively. What is the speed of the truck?
a) 40 km/hr b) 22 km/hr c) 11 km/hr d) 33 km/hr
- 12) A circular playground has an area 616 sq. meters. What time will it take for a runner to run around the circular ground at a speed of 22 km/hr?

- a) 14 hrs b) 14.4 sec c) 16.2 sec d) none of these
- 13) A train goes from P to Q in an hour and another train comes from Q to P in 1.5 hours. After what time will they meet each other?
a) 30 min b) 36 min c) 42 min d) 48 min
- 14) Two trains starting at the same time from two stations 200 km apart, and going in opposite directions cross each other at a distance of 110 km from one of the stations. What is the ratio of their speeds?
a) 9:20 b) 11:20 c) 11:9 d) 1:2
- 15) A cat can climb 6 meters of a round pole in one minute, but slips off 3 meters in the next minute. What time will it take to reach the top of a pole which is 21 meters long?
a) 14 min b) 12 min c) 11 min d) none of these
- 16) A thief steals a car at 2.30 p.m. and drives it at 60 km/hr. The theft is discovered at 3 p.m. and the owner sets off in another car at 75 km/hr. When will he overtake the thief?
a) 4.30 p.m. b) 4.45 p.m. c) 5 p.m. d) 5.15 p.m.
- 17) A and B start from house at 10am. They travel from the same point at 20 km/h and 40 km/h. There is a Junction T on their path. A turns left at T junction at 12:00noon. B reaches T earlier, and turns right. Both of them continue to travel till 2pm. What is the distance between A and B at 2.00 pm?
a) 160 Km b) 120 Km c) 80 Km d) 100 Km
- 18) Two cars travel simultaneously from the cities P and Q towards each other. Their speeds are 20 m/s and 25 m/s respectively. If $d(P, Q) = 900$ km, find at what distance from Q will they meet?
a) 400 km b) 500 km c) 250 km d) 200 km
- 19) Two trains travel toward each other on the same track, beginning 100 miles apart. One train travels at 40 miles per hour; the other travels at 60 miles an hour. A bird starts flight at the same location as the faster train, flying at a speed of 90 miles per hour. When it reaches the slower train, it turns around, flying the other direction at the same speed. When it reaches the faster train again, it turns around and so on. When the trains collide, how far will the bird have flown?
a) 50 miles b) 72 miles c) 118 miles d) 90 miles
- 20) A nonstop bus to Mumbai overtakes an auto also moving towards Mumbai at 10 am. The bus reaches Mumbai at 12.30 pm and starts on the return journey after 1 hr. On the way back it meets the auto at 2 pm. At what time the auto will reach Mumbai?
a) 2.30 pm b) 3 pm c) 3.30 pm d) cannot be determined

Boats and Streams: Class Work

- 21) The speed of a swimmer in still water is 5 km/hr and the speed of the river is 1 km/hr. Find the upstream and downstream speeds of the swimmer.
a) (5, 1) km/hr b) (4, 6) km/hr c) (6, 4) km/hr d) none of these
- 22) A boat is rowed down a river 40 km in 5 hours and up the river 21 km in 7 hours. What is the speed of the boat and the river?
a) (5.5, 2.5) km/hr b) (5, 7) km/hr c) (8, 3) km/hr d) (11, 5) km/hr

- 23) A man rows at a speed of 8 km/hr in still water to a certain distance upstream and back to the starting point in a river flowing at 4 km/hr. What is the average speed of the journey?
a) 6 km/hr b) 4 km/hr c) 32 km/hr d) 8 km/hr
- 24) A man can row 7 km/hr in still water. If the river is flowing at 3 km/hr, it takes 6 hours more in upstream than to o downstream for the same distance. How far is the place?
a) 21 km b) 30 km c) 40 km d) 10 km
- 25) A man can row 6 km/hr in still water. If the river is flowing with 2 km/hr, it takes him 3 hours to row to a place and back. How far is the place?
a) 8 km b) 12 km c) 11 km d) 5 km
- 26) A motorboat covers a certain distance downstream in 6 hours, but takes 8 hours to return upstream to the starting point. If the speed of the river is 6 km/hr, find the speed of the motor boat in still water.
a) 22 km/hr b) 40 km/hr c) 42 km/hr d) 54 km/hr
- 27) A man can row at the rate of 4 km/hr in still water. If the time taken to row a certain distance upstream is 3 times as much as to row the same distance downstream, what is the speed of the current?
a) 4 km/hr b) 6 km/hr c) 1 km/hr d) 2 km/hr
- 28) P, Q and R are the three towns on a river which flows uniformly. Q is equidistant from P and R. A man rows from P to Q and back in 10 hours. He can row from P to R in 4 hours. Find the ratio of the speed of the man in still water to the speed of current.
a) 5:3 b) 3:5 c) 2:5 d) 1:2
- 29) A man who can swim 48 m/min in still water swims 200 m against the current and 200 m with the current. If the difference between those two times is 10 min, find the speed of the current in m/min.
a) 30 m/min b) 31 m/min c) 29 m/min d) 32 m/min
- 30) A swimmer can swim a certain distance in the direction of the current in 5 hours and return the same distance in 7 hours. If the stream flows at the rate of 1 km/hr, find the speed of the swimmer in still water.
a) 4 km/hr b) 5 km/hr c) 6 km/hr d) 7 km/hr

Home Work

- 1) A train running at a speed of 90 km/hr passes a pole in 20 seconds. Find the length of the train.
a) 100 m b) 500 m c) 400 m d) 300 m
- 2) A train meets an accident and travels at $\frac{3}{4}$ of its original speed. Due to this, it is 20 minutes late. Find its original time beyond the point of accident.
a) 40 min b) 50 min c) 60 min d) cannot be determined
- 3) A biker goes from his starting point to his destination with 20 km/hr, but returns with his speed increasing by 50%. What is the average speed of his journey?
a) 25 km/hr b) 30 km/hr c) 28 km/hr d) 24 km/hr
- 4) A jogger wants to save $\frac{1}{4}$ th of his jogging time. By what percentage should he increase his speed?
a) 25% b) 75% c) 33.33% d) 66.66%

- 5) Two cars travel simultaneously from the cities P and Q towards each other. Given that $d(P, Q) = 100$ km. If they meet at a distance of 40 km from P, what is the ratio of their speeds?
a) 1:2 b) 2:3 c) 3:4 d) 4:5
- 6) There is a sparrow sitting on the branch of a tree. The bird is able to see an earthworm and plans to catch it. There is a cat that is watching the movements of the bird and finds that it is going to fly in a straight line to catch the earthworm and thereby decides to hunt the sparrow. If the cat runs at 2m/s, it will reach the earthworm 1 second after the sparrow whereas if the cat runs at 3m/s, it will reach the earthworm 1 second before the sparrow. If the sparrow is flying at 4m/s, what should be the approximate speed of the cat so that it can catch the sparrow?
a) 2.2 m/s b) 2.5 m/s c) 2.8 m/s d) data insufficient
- 7) Two typists are working on an assignment. Typist 1 takes 6 hours to type 32 pages on a computer, while typist 2 takes 5 hours to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?
a) 7 hrs 30 mins b) 8 hrs c) 8 hrs 15 mins d) 8 hrs 25 mins
- 8) In a circular race track of length 100 m, three persons A, B and C start together. A and B start in the same direction at speeds of 10 m/s and 8 m/s respectively. While C runs in the opposite at 15 m/s. When will all the three meet for the first time on the after the start?
a) 4 sec b) 50 sec c) 100 sec d) 200 sec
- 9) A cyclist takes as much time in running 12 meters as a motor-cyclist takes in covering 36 meters. What is the ratio of the speeds of them?
a) 1:3 b) 3:1 c) 2:5 d) 5:2
- 10) A train starts running with the initial speed of 40 km/hr, with its speed increasing every hour by 5 km/hr. How many hours will it take to cover a distance of 460 km?
a) 6 hrs b) 7 hrs c) 8 hrs d) cannot be determined

Answer Keys: Class Work

- 1) c 2) a 3) b 4) b 5) b 6) b 7) d 8) b 9) d 10) b 11) d
12) b 13) b 14) c 15) c 16) c 17) a 18) b 19) d 20) b 21) b 22) a
23) a 24) c 25) a 26) c 27) d 28) a 29) d 30) c

Answer Keys: Home Work

- 1) b 2) c 3) d 4) c 5) b 6) d 7) c 8) c 9) a 10) c