

Chapter 2: HCF LCM

Factors

Factors of a number are all numbers by which it is exactly divisible. Factors of a number are either equal to or less than it.

Multiples

Multiples of a number are all those numbers which are exactly divisible by it. Multiples of a number are either equal to or more than it.

Prime Factorization

We can write any composite number as a product of prime factors. This is called prime factorization.

Least Common Multiple (LCM)

Least common multiple of two or more numbers is the least number which is divisible by each of these numbers without leaving a remainder. It is also known as Lowest Common Dividend.

Methods of Finding LCM of Given Numbers

There are two methods which are usually used to find LCM of two or more numbers.

1. Prime Factorization Method:

Resolve each one of the given numbers into prime factors. The LCM is the product of the highest power of all prime factors.

2. Division Method:

In this method, we divide the given numbers by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. We keep on repeating this process till no two numbers have a common factor. The product of the divisors and the remaining numbers is LCM of the given numbers.

Highest Common Factor (HCF)

Highest Common Factor of two or more given numbers is the largest common factor. It is also known as Greatest Common Factor (GCF) or Greatest Common Divisor (GCD) or Greatest Common Measure (GCM).

Methods of Finding HCF of Given Numbers

There are two methods that are usually used to find the HCF of given numbers.

1. Prime Factorization Method:

In this method, we write prime factors of each of the given numbers in the exponential form. HCF is the product of the common prime factors with least powers.

2. Long Division Method:

In this method, we divide the largest number by smaller number and get a remainder. Then we divide the first divisor by the remainder getting a new remainder and continue this process till the last number is zero. The last divisor in this process is the HCF of the given two numbers.

In order to find the HCF of three or more numbers, follow these steps:

- i. First find the HCF of any two numbers.

- ii. Then find the HCF of the third number and the HCF obtained above.
- iii. HCF obtained in step (ii) is the required HCF of the given three numbers.

Euclid's Algorithm

This is used for large numbers. In this method when $a > b$, $\text{HCF}(a, b) = \text{HCF}(b, a - b)$

HCF and LCM of Fractions

- i) $\text{HCF of Fractions} = \text{HCF of Numerators} / \text{LCM of Denominators}$
- ii) $\text{LCM of Fractions} = \text{LCM of Numerators} / \text{HCF of Denominators}$

Product of Two Numbers

If two numbers are A and B, then $\text{HCF}(A, B) \times \text{LCM}(A, B) = A \times B$. This formula is applicable for two numbers only.

Important

1. For any two digit numbers, HCF is a factor of LCM.
2. Any number which divides each of the two numbers also divides their sum, difference, and also difference of their multiples.

HCF and LCM of Decimals

In given numbers make the same number of places of decimals by annexing zeroes in a number if necessary. Consider these numbers without decimal points, find HCF or LCM as the case may be. Now, in the result, mark off as many decimal places as there are in each of the numbers.

Important Results

1. Lowest number that is divisible by A, B, C leaving same remainder 'r' in each case is $\text{LCM}(A, B, C) + r$.
2. Greatest number that will divide A, B, C leaving remainders r_1, r_2 , and r_3 respectively is HCF of $(A - r_1), (B - r_2)$, and $(C - r_3)$.
3. If a, b and c are such that $a < b < c$ and if H and L are HCF and LCM of a, b and c respectively, then $1 \leq H \leq a$ and $a \leq L \leq abc$.
4. Let H be HCF of a, b and c, then $(a - b), (b - c)$ and $(c - a)$ is also divisible by H.

Class Work

- 1) Find HCF of 70 and 90.
a) 5 b) 10 c) 15 d) 1
- 2) Find HCF of 360 and 132.
a) 2 b) 8 c) 12 d) 16
- 3) Find HCF of 3556 and 3444.
a) 22 b) 26 c) 28 d) 31
- 4) What is the greatest number which divides 852, 1065 and 1491 exactly?
a) 193 b) 183 c) 223 d) 213
- 5) What is the LCM of 25, 30, 35 and 40?
a) 2000 b) 4200 c) 2800 d) 3588
- 6) Find LCM of 12, 15, 20 and 54.
a) 344 b) 540 c) 600 d) 180
- 7) What is the LCM of $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{6}{25}$?
a) $\frac{2}{3}$ b) $\frac{3}{8}$ c) $\frac{6}{5}$ d) $\frac{11}{50}$
- 8) What is the LCM of 1.2, 0.24 and 6?
a) 2 b) 4 c) 6 d) 8
- 9) Find HCF of 6.16 and 13.
a) 0.02 b) 0.04 c) 1 d) none of these
- 10) Which greatest possible length can be used to measure exactly 1 meter 65 cm, 1 meter 95 cm and 75 cm?
a) 45cm b) 15cm c) 125cm d) 55cm
- 11) An oil merchant has three different kinds of oil: 435 liters, 493 liters and 551 liters. Find the least number of casks of equal size required to store all the oil without mixing.
a) 29 b) 41 c) 51 d) 57
- 12) The traffic lights at a T-junction change after every 48 sec, 72 sec and 108 sec respectively. If they all change simultaneously at 8:20:00 hours, then they will again change simultaneously at _____.
a) 8:27:12 hrs b) 8:27:24 hrs c) 8:27:36 hrs d) 8:27:48 hrs

- 13) A room is 4 meters 37 cm long and 3 meters 23 cm broad. It is required to pave the floor with minimum square slabs. Find the number of slabs required for this purpose.
- a) 19 b) 391 c) 441 d) none of these
- 14) Three runners start together to run the same way around a circular track of 11 km in circumference. Their speeds are 4, 5.5 and 8 km/hr respectively. When will they meet at the starting point?
- a) 11 hrs b) 22 hrs c) 22.5 hrs d) 15.5 hrs
- 15) Five bells begin to toll together and toll at intervals of 36, 45, 72, 81 and 108 seconds respectively. After what interval of time will they keep on tolling together?
- a) 1620 sec b) 3240 sec c) 3200 sec d) 3080 sec
- 16) An orchard has 48 apple trees, 60 mango trees, and 96 banana trees. These have to be arranged in rows such that each row has the same number of trees and all are of same type. Find minimum number of such rows that can be formed.
- a) 12 b) 34 c) 17 d) 11
- 17) LCM of two distinct natural numbers is 211. What is their HCF?
- a) 211 b) 37 c) 1 d) data insufficient
- 18) What is the HCF of 2222... 30 times and 3333... 70 times?
- a) 2323... 100 times b) 1111... 10 times
- c) 3232... 10 times d) cannot be determined
- 19) What is the LCM of $2^{300} - 1$ and $8^{250} - 1$?
- a) $8^{250} - 1$ b) $2^{300} - 1$ c) $2^{1500} - 1$ d) $2^{550} - 1$
- 20) The four members of a jazz quartet all play in different rhythms. The pianist plays in 9/8 time, meaning that his downbeats occur every 9/8 of a measure of time. Meanwhile, the saxophonist plays in 7/4 time, the harpist in 5/8 time, and the drummer in plain old 4/4 time (all with respect to the same measure of time as the pianist). If all four musicians start a song together on the same downbeat, how many measures later will all their downbeats occur simultaneously?
- a) 315 b) 630 c) 1260 d) none of these

Home Work

- 1) The manager of newly opened clothing shop wants to arrange stock of 140 Red, 196 Blue and 336 Grey shirts in the display. These shirts are to be arranged in display shelf such that each shelf has shirt of one color only and each shelf must have same number of shirts
How many shirts should be arranged in each shelf?
a) 28 b) 30 c) 32 d) 34
- 2) Find the greatest number which will divide 501 and 353 leaving remainder of 8 and 5 respectively?
a) 28 b) 29 c) 25 d) 27
- 3) Eight different battle tanks fired a round of bomb and continued firing at an interval of 2, 3, 4, 5, 6, 7, 8 and 9 seconds respectively. In 7 hours how many times did they fired together?
a) 11 b) 12 c) 13 d) 10
- 4) A bundle of sticks is to be arranged in into group of 25. But when arranged into groups of 12, 18 and 24, there are always 6 sticks extra. Find the least number of sticks contained in the bundle?
a) 150 b) 175 c) 200 d) 250
- 5) Find HCF of 15/28, 105/117, 260/363
a) 5/396396 b) 5/198198 c) 15/396396 d) 15/198198
- 6) Find LCM of 35/132, 40/231, 44/297
a) 6160/33 b) 6160/11 c) 3080/33 d) 3080/11
- 7) Find the least number which when divided by 8, 15, 20, 21, and 30 in each case leaves remainder of 5.
a) 1685 b) 845 c) 425 d) 215
- 8) Find the least number which when divided by 5, 9, 12 and 20 in each case leaves remainder of 3.
a) 183 b) 93 c) 363 d) 723

- 9) Find the greatest number that will divide 2327, 2677, 4007 and 497 and will leave the remainder of 17, 37, 47 and 57 respectively.
- a) 110 b) 55 c) 11 d) 220
- 10) Find the greatest number that will divide 175, 241, 559, and 1193 and will leave the remainder of 19, 7, 13 and 23.
- a) 78 b) 39 c) 13 d) 156

Answer Keys: Class Work:

- 1) b 2) c 3) c 4) d 5) b 6) b 7) c 8) c 9) b 10) b 11) c
- 12) a 13) b 14) b 15) b 16) c 17) c 18) b 19) c 20) a

Answer Keys: Home Work:

- 1) a 2) b 3) d 4) a 5) a 6) c 7) b 8) a 9) a 10) a