mackeyglass

This script generates a Mackey-Glass time series using the 4th order Runge-Kutta method. The code is a straighforward translation in Matlab of C source code provided by Roger Jang, which is available here

Contents

The theory

Mackey-Glass time series refers to the following, delayed differential equation:

$$\frac{dx(t)}{dt} = \frac{ax(t-\tau)}{1+x(t-\tau)^{10}} - bx(t) \qquad (1)$$

It can be numerically solved using, for example, the 4th order Runge-Kutta method, at discrete, equally spaced time steps:

$$x(t+\Delta t) = mackeyglass\_rk4(x(t), x(t-\tau), \Delta t, a, b)$$

where the function mackeyglass_rk4 numerically solves the Mackey-Glass delayed differential equation using the 4-th order Runge Kutta. This is the RK4 method:

$$k_1 = \Delta t \cdot mackeyglass\_eq(x(t), x(t-\tau), a, b)$$

$$k_2 = \Delta t \cdot mackeyglass\_eq(x(t+\frac{1}{2}k_1), x(t-\tau), a, b)$$

$$k_3 = \Delta t \cdot mackeyglass\_eq(x(t+\frac{1}{2}k_2), x(t-\tau), a, b)$$

$$k_4 = \Delta t \cdot mackeyglass\_eq(x(t+k_3), x(t-\tau), a, b)$$

$$x(t+\Delta t) = x(t) + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{6} + \frac{k_4}{6}$$

where mackeyglass_eq is the function which return the value of the Mackey-Glass delayed differential equation in (1) once its inputs and its parameters (a,b) are provided.

Here is an example:

Input parameters

```
a        = 0.2;     % value for a in eq (1)
b        = 0.1;     % value for b in eq (1)
tau      = 17;          % delay constant in eq (1)
x0       = 1.2;         % initial condition: x(t=0)=x0
deltat   = 0.1;     % time step size (which coincides with the integration step)
sample_n = 12000;       % total no. of samples, excluding the given initial condition
interval = 1;       % output is printed at every 'interval' time steps
```

Main algorithm

- x_t : x at instant t , i.e. x(t) (current value of x)
- x_t_minus_tau : x at instant (t-tau) , i.e. x(t-tau)
- x_t_plus_deltat : x at instant (t+deltat), i.e. x(t+deltat) (next value of x)
- X : the (sample_n+1)-dimensional vector containing x0 plus all other computed values of x
- T : the (sample_n+1)-dimensional vector containing time samples
- x_history : a circular vector storing all computed samples within x(t-tau) and x(t)

```
time = 0;
index = 1;
history_length = floor(tau/deltat);
x_history = zeros(history_length, 1); % here we assume x(t)=0 for -tau <= t < 0
x_t = x0;

X = zeros(sample_n+1, 1); % vector of all generated x samples
T = zeros(sample_n+1, 1); % vector of time samples

for i = 1:sample_n+1,
    X(i) = x_t;
    if (mod(i-1, interval) == 0),
         disp(sprintf('%4d %f', (i-1)/interval, x_t));
    end
    if tau == 0,
        x_t_minus_tau = 0.0;
    else
        x_t_minus_tau = x_history(index);
    end

    x_t_plus_deltat = mackeyglass_rk4(x_t, x_t_minus_tau, deltat, a, b);

    if (tau ~= 0),
        x_history(index) = x_t_plus_deltat;
        index = mod(index, history_length)+1;
    end
    time = time + deltat;
    T(i) = time;
    x_t = x_t_plus_deltat;
end


figure
plot(T, X);
set(gca,'xlim',[0, T(end)]);
xlabel('t');
ylabel('x(t)');
```

```
title(sprintf('A Mackey-Glass time serie (tau=%d)', tau));
```

```
   0 1.200000
   1 1.188060
   2 1.176238
   3 1.164535
   4 1.152947
   5 1.141475
   6 1.130117
   7 1.118873
   8 1.107740
   9 1.096717
  10 1.085805
  11 1.075001
  12 1.064305
  13 1.053715
  14 1.043230
  15 1.032850
  16 1.022573
  17 1.012398
  18 1.002324
  19 0.992351
  20 0.982477
...
```