

Literature Review of PPA

Sawyer Maloney & Drew Bevington

1 Introduction

This paper will talk about ideas of “complete” problems in economics as well as explain to the reader how this completeness is proved. In particular, we will analyze and discuss the findings from “The Complexity of Computing a Nash Equilibrium”.

2 TFNP & sub-classes

In “On the complexity of the parity argument and other inefficient proofs of existence”, Papadimitriou showed that, while there are no problems which are complete for TFNP, there are (at least) four subclasses of TFNP that have complete problems.

TFNP is the total functional non-deterministic polynomial time class. More specifically, it is the collection of functional problems in NP that always have a solution. This requirement of having a solution is enforced through a number of exponentially non-constructive combinatorial lemmas. The term ‘exponentially non-constructive’ means a lemma that does not explicitly give the structure, but only shows that the structure exists, and that trying to construct such a structure would be exponentially difficult.

The most important of these lemmas are defined by Papadimitriou from the above paper:

- Parity argument: if a graph has a node of odd degree, then there must be another odd degree node. This argument creates the PPA subclass of TFNP.
- Parity for directed graphs: If a directed graph has an unbalanced node (a vertex with different in and out degrees), then there must be another unbalanced node. This creates the PPAD subclass.
- Polynomial Local Search: Every directed acyclic graph must have a sink. This argument creates the PLS subclass.
- Pigeonhole Principle: If a function maps n elements to $n-1$ spots, then there must be a collision. This argument creates the PPP subclass.