Answer Key

Algebra I Regents
Course Workbook

with Exam Questions

2019-20 Edition

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Notation

A code next to each Regents Question number states from which Algebra I Common Core Regents exam the question came. For example, AUG '18 [25] means the question appeared on the August 2018 exam as question 25.

2. EQUATIONS AND INEQUALITIES

2.1 Properties of Real Numbers

PRACTICE PROBLEMS

1. (3)	2. (1)
3. associative property of multiplication	4. distributive property
5. (1)	6. (3)
7. $-\frac{2}{3}$	8. $\frac{3}{2}$
9. $-a+b$	10. $-ab$
11. $(2\div 1)\neq(1\div 2)$ or any similar counterexample.	12. No. For example, when we subtract the whole number 5 from the whole number 2, the result is –3, which is *not* a whole number.
13. commutative property of multiplication	14. associative property of addition
15. No. For example, when we divide the integer 1 by the integer 2, the result is $\frac{1}{2}$, which is *not* an integer.	16. If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers and a, b, c, and d are *non-zero* integers, then $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$. Since the set of integers is closed under multiplication, ad and bc are integers, so $\frac{ad}{bc}$ is rational.
17. $5x+25$	18. $4b-16$
19. $-2x+2$	20. $-3a+3b$
21. $-1-y$	22. $a+1$
23. $rs+rt=r(s+t)$	24. $2x+10=2(x+5)$

REGENTS QUESTIONS

1) JUN '14 [1] Ans: 1
2) AUG '17 [1] Ans: 4
3) JAN '18 [1] Ans: 4
4) JAN '19 [8] Ans: 4

2.2 Solve Linear Equations in One Variable

Practice Problems

1. $3(m-2)=18$ $3m-6=18$ $3m=24$ $m=8$	2. $4n-n=-12$ $3n=-12$ $n=-4$
3. $2(x-4)+7=3$ $2x-8+7=3$ $2x-1=3$ $2x=4$ $x=2$	4. $0.2(n-6)=2.8$ $0.2n-1.2=2.8$ $0.2n=4$ $n=20$
5. $-5=-(y+1)-y$ $-5=-y-1-y$ $-5=-2y-1$ $-4=-2y$ $2=y$	6. $15x-3(3x+4)=6$ $15x-9x-12=6$ $6x-12=6$ $6x=18$ $x=3$
7. $3x+8=5x$ $8=2x$ $4=x$	8. $3+2g=5g-9$ $3=3g-9$ $12=3g$ $4=g$
9. $8p+2=4p-10$ $4p+2=-10$ $4p=-12$ $p=-3$	10. $5p-1=2p+20$ $3p-1=20$ $3p=21$ $p=7$
11. $0.06y+200=0.03y+350$ $0.03y+200=350$ $0.03y=150$ $y=5000$	12. $5-2x=-4x-7$ $5+2x=-7$ $2x=-12$ $x=-6$

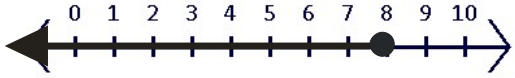
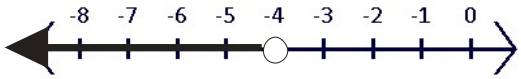
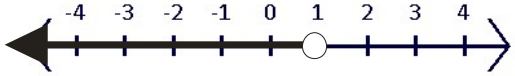
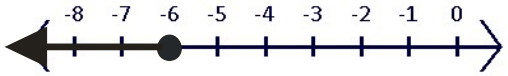
13. $5(2x-7)=15x-10$ $10x-35=15x-10$ $-35=5x-10$ $-25=5x$ $-5=x$	14. $5(x-2)=2(10+x)$ $5x-10=20+2x$ $3x-10=20$ $3x=30$ $x=10$
15. $2(x-4)=4(2x+1)$ $2x-8=8x+4$ $-8=6x+4$ $-12=6x$ $-2=x$	16. $3(x+1)-5x=12-(6x-7)$ $3x+3-5x=12-6x+7$ $3-2x=19-6x$ $3+4x=19$ $4x=16$ $x=4$
17. $-4(y-3)=5(2y-6)$ $-4y+12=10y-30$ $12=14y-30$ $42=14y$ $3=y$	18. $3(x-2)-2(x+1)=5(x-4)$ $3x-6-2x-2=5x-20$ $x-8=5x-20$ $-8=4x-20$ $12=4x$ $3=x$

REGENTS QUESTIONS

1) JUN '17 [19] Ans: 1
2) AUG '18 [4] Ans: 2

2.3 Solve Linear Inequalities in One Variable

Practice Problems

1. $x \le 4$	2. $x > -1$
3. $2x - 5 \le 11$ $2x \le 16$ $x \le 8$ 	4. $-6y + 1 > 25$ $-6y > 24$ $y < -4$ 
5. $-4 > 2(r - 3)$ $-4 > 2r - 6$ $2 > 2r$ $1 > r$ $r < 1$ 	6. $-\frac{4}{3}(x - 3) \ge 12$ $-\frac{4}{3}x + 4 \ge 12$ $-\frac{4}{3}x \ge 8$ $-4x \ge 24$ $x \le -6$ 
7. $-6x - 17 \ge 8x + 25$ $-17 \ge 14x + 25$ $-42 \ge 14x$ $-3 \ge x$ $x \le -3$	8. $-5x + 35 < 15$ $-5x < -20$ $x > 4$
9. $2x - 5 < 3$ $2x < 8$ Graph (1) $x < 4$	10. $3(2m - 1) \le 4m + 7$ $6m - 3 \le 4m + 7$ $2m - 3 \le 7$ $2m \le 10$ $m \le 5$
11. $-4(2m - 6) + m > 3m + 4$ $-8m + 24 + m > 3m + 4$ $-7m + 24 > 3m + 4$ $24 > 10m + 4$ $20 > 10m$ $2 > m$ $m < 2$	12. $-5(p + 1) \ge -p + 11$ $-5p - 5 \ge -p + 11$ $-5 \ge 4p + 11$ $-16 \ge 4p$ $-4 \ge p$ $p \le -4$

Regents Questions

1) JAN '15 [7] Ans: 1
2) JUN '16 [9] Ans: 4
3) AUG '16 [7] Ans: 1
4) JUN '17 [13] Ans: 4
5) AUG '17 [11] Ans: 1
6) JAN '18 [17] Ans: 2
7) JUN '18 [1] Ans: 4
8) JUN '14 [27]

$$2(-1)+a(-1)-7>-12$$
$$-9-a>-12$$
$$-a>-3$$
$$a<3$$

Ans: 2

9) AUG '14 [30]

$$3x+9\le 5x-3$$
$$12\le 2x$$
$$6\le x$$

Ans: 6

10) JUN '15 [30]

$$-8x+7<15$$
$$-8x<8$$
$$x>-1$$

Ans: 0

11) AUG '15 [34]

$$7x-12x+24\le 6x+12-9x$$
$$-5x+24\le -3x+12$$
$$12\le 2x$$
$$6\le x \quad or \quad x\ge 6$$

{6,7,8} is the set of integers that are greater than or equal to 6 in the interval

12) JAN '17 [27]

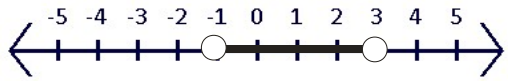
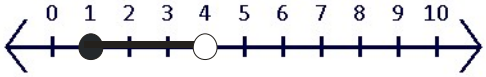
$$1.8-0.4y\ge 2.2-2y$$
$$1.6y\ge 0.4$$
$$y\ge 0.25$$

13) JAN '19 [25]

$$3600+1.02x<2000+1.04x$$
$$1600<0.02x$$
$$80{,}000<x \quad or \quad x>80{,}000$$

2.4 Compound Inequalities

Practice Problems

1. $-4 \le x < 2$	2. $-2 < x \le 3$
3. $-3 < x \le 4$	4. $x < -1$ or $x > 4$
5. 	6. $3 \le 2x + 1 < 9$ $2 \le 2x < 8$ $1 \le x < 4$ 
7. $-2 < 3x + 4 \le 10$ $-6 < 3x \le 6$ $-2 < x \le 2$ There are 2 positive integer solutions, {1, 2}	8. (3)

2.5 Solve Equations with Fractions

Practice Problems

1.

$$\frac{x}{16}+\frac{1}{4}=\frac{1}{2}$$
$$16\left(\frac{x}{16}\right)+16\left(\frac{1}{4}\right)=16\left(\frac{1}{2}\right)$$
$$x+4=8$$
$$x=4$$

2.

$$\frac{x}{2}+\frac{x}{6}=2$$
$$6\left(\frac{x}{2}\right)+6\left(\frac{x}{6}\right)=6(2)$$
$$3x+x=12$$
$$4x=12$$
$$x=3$$

3.

$$\frac{2x}{3}+\frac{x}{6}=5$$
$$6\left(\frac{2x}{3}\right)+6\left(\frac{x}{6}\right)=6(5)$$
$$4x+x=30$$
$$5x=30$$
$$x=6$$

4.

$$\frac{3}{5}x+\frac{2}{5}=4$$
$$5\left(\frac{3}{5}x\right)+5\left(\frac{2}{5}\right)=5(4)$$
$$3x+2=20$$
$$3x=18$$
$$x=6$$

5.

$$\frac{3}{4}x+2=\frac{5}{4}x-6$$
$$4\left(\frac{3}{4}x\right)+4(2)=4\left(\frac{5}{4}x\right)+4(-6)$$
$$3x+8=5x-24$$
$$8=2x-24$$
$$32=2x$$
$$16=x$$

6.

$$\frac{2}{3}x+\frac{1}{2}=\frac{5}{6}$$
$$6\left(\frac{2}{3}x\right)+6\left(\frac{1}{2}\right)=6\left(\frac{5}{6}\right)$$
$$4x+3=5$$
$$4x=2$$
$$x=\tfrac{2}{4}=\tfrac{1}{2}$$

7.

$$\frac{3}{4}x=\frac{1}{3}x+5$$
$$12\left(\frac{3}{4}x\right)=12\left(\frac{1}{3}x\right)+12(5)$$
$$9x=4x+60$$
$$5x=60$$
$$x=12$$

8.

$$\frac{x}{3}+\frac{x+1}{2}=x$$
$$6\left(\frac{x}{3}\right)+6\left(\frac{x+1}{2}\right)=6(x)$$
$$2x+3(x+1)=6x$$
$$2x+3x+3=6x$$
$$5x+3=6x$$
$$3=x$$

9.

$$\frac{2x}{5}+\frac{1}{3}=\frac{7x-2}{15}$$
$$15\left(\frac{2x}{5}\right)+15\left(\frac{1}{3}\right)=15\left(\frac{7x-2}{15}\right)$$
$$6x+5=7x-2$$
$$5=x-2$$
$$7=x$$

10.

$$\frac{1}{7}+\frac{2x}{3}=\frac{15x-3}{21}$$
$$21\left(\frac{1}{7}\right)+21\left(\frac{2x}{3}\right)=21\left(\frac{15x-3}{21}\right)$$
$$3+14x=15x-3$$
$$3=x-3$$
$$6=x$$

11.

$$\frac{3}{4}(x+3)=9$$
$$4\left[\frac{3}{4}(x+3)\right]=4[9]$$
$$3(x+3)=36$$
$$3x+9=36$$
$$3x=27$$
$$x=9$$

12.

$$\frac{3}{5}(x+2)=x-4$$
$$5\left[\frac{3}{5}(x+2)\right]=5[x-4]$$
$$3(x+2)=5x-20$$
$$3x+6=5x-20$$
$$6=2x-20$$
$$26=2x$$
$$13=x$$

13.

$$\frac{1}{2}(18-5x)=\frac{1}{3}(6-4x)$$
$$6\left[\frac{1}{2}(18-5x)\right]=6\left[\frac{1}{3}(6-4x)\right]$$
$$3(18-5x)=2(6-4x)$$
$$54-15x=12-8x$$
$$54=12+7x$$
$$42=7x$$
$$6=x$$

14.

$$\frac{2}{3}\left(2x-\frac{1}{2}\right)=13$$
$$3\left[\frac{2}{3}\left(2x-\frac{1}{2}\right)\right]=3[13]$$
$$2\left(2x-\frac{1}{2}\right)=39$$
$$4x-1=39$$
$$4x=40$$
$$x=10$$

15.

$$\frac{m}{5}+\frac{3(m-1)}{2}=2(m-3)$$
$$\frac{m}{5}+\frac{3m-3}{2}=2m-6$$
$$10\left(\frac{m}{5}\right)+10\left(\frac{3m-3}{2}\right)=10(2m-6)$$
$$2m+15m-15=20m-60$$
$$17m-15=20m-60$$
$$45=3m$$
$$15=m$$

Regents Questions

1) JUN '14 [5] Ans: 1
2) AUG '14 [20] Ans: 1
3) AUG '17 [13] Ans: 2
4) JAN '18 [22] Ans: 4
5) JAN '19 [5] Ans: 2

6) JUN '18 [30]

$$6-\frac{2}{3}(x+5)=4x$$
$$3(6)-3\left[\frac{2}{3}(x+5)\right]=3(4x)$$
$$18-2(x+5)=12x$$
$$18-2x-10=12x$$
$$8-2x=12x$$
$$8=14x$$
$$x=\frac{8}{14}=\frac{4}{7}$$

Ans: $\frac{4}{7}$

2.6 Solve Literal Equations and Inequalities

Practice Problems

1. $2m+2p=16$ $2p=-2m+16$ $p=-m+8$	2. $bx-2=K$ $bx=K+2$ $x=\frac{K+2}{b}$
3. $c=2m+d$ $c-d=2m$ $\frac{c-d}{2}=m$	4. $bx-3a=c$ $bx=3a+c$ $x=\frac{3a+c}{b}$
5. $V=lwh$ $\frac{V}{lh}=w$	6. $A=\frac{bh}{2}$ $2A=bh$ $\frac{2A}{b}=h$
7. $abx-5=0$ $abx=5$ $x=\frac{5}{ab}$	8. $2y+2w=x$ $2w=x-2y$ $w=\frac{x-2y}{2}$
9. $s=\frac{2x+t}{r}$ $sr=2x+t$ $sr-t=2x$ $\frac{sr-t}{2}=x$	10. $V=\frac{1}{3}Bh$ $3V=Bh$ $\frac{3V}{B}=h$
11. $v=\frac{1}{2}at^2$ $2v=at^2$ $\frac{2v}{t^2}=a$	12. $\frac{ey}{n}+k=t$ $\frac{ey}{n}=t-k$ $ey=n(t-k)$ $ey=nt-nk$ $y=\frac{nt-nk}{e}$

13. $3x-ax=b$ $x(3-a)=b$ $x=\frac{b}{3-a}$	14. $bc+ac=ab$ $c(b+a)=ab$ $c=\frac{ab}{b+a}$
15. $k=am+3mx$ $k=m(a+3x)$ $\frac{k}{a+3x}=m$	16. $2ax=-bx+1$ $2ax+bx=1$ $x(2a+b)=1$ $x=\frac{1}{2a+b}$
17. $ax+3=7-bx$ $ax+bx+3=7$ $ax+bx=4$ $x(a+b)=4$ $x=\frac{4}{a+b}$	18. $z+y=x+xy^2$ $z+y=x(1+y^2)$ $\frac{z+y}{1+y^2}=x$

REGENTS QUESTIONS

1) JAN '16 [6] Ans: 3
2) JAN '17 [4] Ans: 3
3) JUN '17 [2] Ans: 2
4) JUN '17 [23] Ans: 3
5) JUN '18 [23] Ans: 4
6) JAN '19 [20] Ans: 2
7) AUG '14 [34]

$2A=h(b_1+b_2)$

$\frac{2A}{h}=b_1+b_2$

$\frac{2A}{h}-b_2=b_1$

$b_1=\frac{2(60)}{6}-12=8\,ft.$

8) JAN '16 [31]

$bx-3b\geq ax+7b$

$-10b\geq(a-b)x$

$\frac{-10b}{a-b}\geq x$

9) JUN '16 [31]

$\frac{S}{180}=n-2$

$\frac{S}{180}+2=n$

10) AUG '16 [32]

$4ax+12-3ax=25+3a$

$ax+12=25+3a$

$ax=13+3a$

$x=\frac{13+3a}{a}$

11) AUG '18 [29]

$\frac{9}{5}K=F+459.67$

$F=\frac{9}{5}K-459.67$

3. VERBAL PROBLEMS

3.1 Translate Expressions

Practice Problems

1. (3)	2. (4)
3. $7x-5$	4. $2(x-8)$
5. $33-g$	6. $20-2d$
7. $4x+10$	8. $\frac{n}{12}$
9. xd	10. $d-2h$
11. $5(x+4)$, or $5x+20$	12. $3x-4$
13. $3d-1200$	14. $280+0.05n$
15. $y+y+1+y+2+y+3$ $4y+6$	16. $x+3+x+5+x+7$ $3x+15$
17. $x+x+2+x+4$ $3x+6$	18. $x(x+1)$ x^2+x
19. t = Tommy's age $t-4$ = Donny's age $t-4+7=t+3$ = Camille's age Sum is $3t-1$	20. h = horse's lifespan $h+70$ = stork's lifespan $4(h+70)=4h+280$ = whale's lifespan Sum is $6h+350$
21. a = number of cookies eaten by Alice $a+4$ = number of cookies eaten by Carl $2(a+4)$ cookies were eaten by Bob	22. x bags of chips $3x$ bags of pretzels $3x-2$ bags of nachos $x+3x+3x-2$ $7x-2$

Regents Questions

1) AUG '15 [3] Ans: 4
2) JAN '17 [18] Ans: 4
3) AUG '17 [12] Ans: 2

3.2 Translate Equations

Practice Problems

1. $9h + 60 = 375$	2. $3(x + 4) = 5x - 2$
3. $\frac{1}{2}l - 4$	4. $x(2x - 3) = 43$
5. $2(3x+2)=22$	6. $0.30(n+4)+0.50n=3.60$
7. $0.05n + 0.10(n + 6) = 1.35$	8. $0.10(72 - q) + 0.25q = 14.70$
9. $x(x + 1) = 20$	10. $x + x + 2 + x + 4 = -3$

Regents Questions

1) MAY '13 [4] Ans: 4
2) JUN '14 [16] Ans: 2
3) JAN '16 [11] Ans: 2
4) AUG '16 [16] Ans: 3

3.3 Translate Inequalities

Practice Problems

1. $3x - 8 > 15$	2. $h \geq 48$
3. $b + (b + 9) < 144$	4. $h + 3h \leq 120$
5. $x + 2x \geq 90$	6. $0.75a + 1.25b \leq 25$
7. $30 + 2w \leq 50$	8. $w(2w - 3) \leq 30$
9. $155 \leq h \leq 190$	10. $1500 \leq c < 1800$
11. $0.75(200) + 1.25x \geq 250$	

Regents Questions

1) AUG '15 [5] Ans: 4
2) JUN '16 [7] Ans: 2
3) JUN '18 [6] Ans: 1

3.4 Linear Model in Two Variables

Practice Problems

1. the number of hours of tutoring	2. the number of miles driven
3. $c = 80x + 75$	4. $P(y) = 5y + 100$
5. $m = 20 - 0.50g$	6. $h(m) = 30{,}000 - 2000m$
7. $c = 2(n-1) + 5$	8. $w(h) = 30(h-40) + 800$

Regents Questions

1) JUN '14 [7] Ans: 3
2) JUN '14 [22] Ans: 4
3) AUG '14 [2] Ans: 2
4) JAN '15 [1] Ans: 2
5) JAN '15 [23] Ans: 4
6) JUN '15 [1] Ans: 3
7) AUG '15 [8] Ans: 4
8) AUG '16 [14] Ans: 3
9) JAN '17 [9] Ans: 2
10) AUG '17 [9] Ans: 4
11) JAN '18 [7] Ans: 4
12) JUN '18 [17] Ans: 3
13) AUG '18 [17] Ans: 2
14) JUN '15 [26]
$f(x) = 6.5x + 4(12)$ or
$f(x) = 6.5x + 48$
15) JAN '17 [30]
$C = 0.99(s-1) + 1.29$
No. $C = 0.99(52-1) + 1.29 = 51.78$
16) JAN '18 [33]
$P(x) = 0.035x + 300$;
$0.035(8250) + 300 = 588.75$

3.5 Word Problems – Linear Equations

Practice Problems

1.
$a+a+5=19$
$2a+5=19$
$2a=14$
$a=7 \qquad a+5=12$
Jamie is 12 years old.

2.
$c+2c=561$
$3c=561$
$c=187 \qquad 2c=374$
There are 187 crickets and 374 grasshoppers.

3.
$c+3c=20$
$4c=20$
$c=5 \quad 3c=15$
There were 15 robins.

4.
$f+2f+4=16$
$3f+4=16$
$3f=12$
$f=4 \qquad 2f+4=12$
There are 4 freshmen and 12 sophomores.

5.
$2x+3=15$
$2x=12$
$x=6$
He bought 6 pizzas last year.

6.
$2(2x)+2x=45$
$4x+2x=45$
$6x=45$
$x=7.50$
Each CD costs $7.50.

7.
$4m-8=28$
$4m=36$
$m=9$
Minnie owns 9 video discs.

8. $7x$ deer, $3x$ foxes
$3x=210$
$x=70$
So, $7x=490$ deer

9.
$b+(2b+3)=42$
$3b+3=42$
$3b=39$
$b=13$
There are 13 black marbles, so there are $42-13=29$ red marbles.

10.
$3x+5=17$
$3x=12$
$x=4$
4 rides

11.
$2.25+3.50(x-1)=44.25$
$2.25+3.50x-3.50=44.25$
$3.50x-1.25=44.25$
$3.50x=45.50$
$x=13$
13 miles

12.
$3x=(x+4)+48$
$3x=x+52$
$2x=52$
$x=26$
26 years old

13. $7x$ boys, $10x$ girls $7x+10x=357$ $17x=357$ $x=21$ So, $7x=147$ boys	14. $4(m+100)+12m=3056$ $4m+400+12m=3056$ $16m+400=3056$ $16m=2656$ $m=166$ $m+100=266$ There were 266 balcony tickets sold.
15. $0.10(3n)+0.25(n+4)+0.05n=4.60$ $0.3n+0.25n+1+0.05n=4.60$ $0.6n+1=4.60$ $0.6n=3.60$ $n=6$ 6 nickels, 18 dimes, 10 quarters	16. $6.50s+9.00(150-s)=1180.00$ $6.5s+1350-9s=1180$ $-2.5s+1350=1180$ $-2.5s=-170$ $s=68$ 68 small and 82 large

REGENTS QUESTIONS

1) AUG '15 [10] Ans: 2

2) MAY '13 [5]

$12x+9(2x)+5(3x)=15$

$45x=15$,

$x=\frac{1}{3}$

$x+2x+3x=6x=6\left(\frac{1}{3}\right)=2lbs.$

3.6 Word Problems – Inequalities

Practice Problems

1. $2n-5>23$ $2n>28$ $n>14$ Smallest integer is 15.	2. $5x<55$ $x<11$ Largest integer is 10
3. $n+7n\le 60$ $8n\le 60$ $n\le 7.5$ Largest two integers are 7 and 49.	4. $375+155w\ge 900$ $155w\ge 525$ $w\ge 3.387...$ He needs to work 4 weeks.
5. $5.95h\ge 215$ $h\ge 36.1344...$ He needs to work 37 hours.	6. $6n>3n+30$ $3n>30$ $n>10$ They need to make 11 toys.
7. $13.95+0.49x\le 50.00$ $0.49x\le 36.05$ $x\le 73.5714...$ She can buy 73 songs.	8. $19.00+0.07x\le 29.50$ $0.07x\le 10.50$ $x\le 150$ She can use 150 minutes.
9. Convert \$1.50 per 30 mins. to \$3/hr. $3(h-1)+5\le 12.50$ $3h-3+5\le 12.50$ $3h+2\le 12.50$ $3h\le 10.50$ $h\le 3.5$ She can park 3.5 hours.	10. $2n-(150+0.50n)\ge 500$ $2n-150-0.50n\ge 500$ $1.5n-150\ge 500$ $1.5n\ge 650$ $n\ge 433\frac{1}{3}$ They must sell 434 programs.

Regents Questions

1) JAN '15 [13] Ans: 3

2) JUN '15 [24] Ans: 4

3) JAN '19 [4] Ans: 1

4) SEP '13 [9]

$8x + 11y \geq 200$;

$8x + 11(15) \geq 200$

$8x + 165 \geq 200$

$8x \geq 35$

$x \geq 4.375$

5 hours

5) AUG '18 [33]

$135 + 72x \geq 580$

$72x \geq 445$

$x \geq 6.2$

Minimum of 7 weeks

3.7 Conversions

Practice Problems

1.
$$20\ in \times \frac{2.54\,cm}{1\ in} = 50.8 \approx 51\,cm$$

2.
$$8900\ ft \times \frac{1\,mi}{5280\ ft} \approx 1.7\,mi$$

3.
$$1680\ oz \times \frac{1\ lb}{16\ oz} \times \frac{1\,bag}{5\ lbs} = 21\,bags$$

4.
$$0.75\ tsp \times \frac{1\,Tbsn}{3\ tsp} \times 5 = 1.25\ Tbsn$$

5.
$$2.625\ in \times \frac{2.54\ cm}{1\ in} \times \frac{10\,mm}{1\ cm} =$$
$$2.625 \times 2.54 \times 10\,mm = 66.675\,mm \approx 67\,mm$$

6.
$$6\ furlongs \times \frac{1\ mile}{8\ furlongs} \times \frac{1.61\,km}{1\ mile} =$$
$$\frac{6 \times 1.61\,km}{8} = 1.2075\,km \approx 1.21\,km$$

7.
$$48\ in. \times \frac{\$3.75}{1\ yd.} \times \frac{1\ yd.}{3\ ft.} \times \frac{1\ ft.}{12\ in.} = \$5.00$$

8.
$$60\ ft \times \frac{12\ in}{1\ ft} \times \frac{2.54\ cm}{1\ in} \times \frac{1m}{100\ cm} =$$
$$\frac{60 \times 12 \times 2.54m}{100} = 18.288m \approx 18.3m$$

9.
$$\frac{150m}{1.5\ min} \times \frac{60\ min}{1hr} = \frac{9000m}{1.5hr}$$
$$= 6000\ m\ per\ hr$$

10.
$$\frac{344m}{1\ s} \times \frac{60\ s}{1\ min} \times \frac{60\ min}{1hr}$$
$$= 1{,}238{,}400\ m\ per\ hr$$

11.
$$\frac{43\ mi}{1\ g} \times \frac{1.61\,km}{1\ mi} \times \frac{1\ g}{3.79l} =$$
$$\frac{43 \times 1.61\,km}{3.79l} \approx 18.3\,km\,per\,liter$$

12.
$$\frac{\$1.50}{2\ l} \times \frac{3.79\ l}{1g} = \frac{\$1.50 \times 3.79}{2g} \approx$$
$\$2.84\,per\,g$
The 1-gallon bottle is the better buy.

13.
$$\frac{8000\ mi}{1\ yr} \times \frac{1760\ yds}{1\ mi} \times \frac{1\ yr}{365 days} =$$
$$\frac{14{,}080{,}000\ yds}{365\ days} \approx 38{,}575\ yds\,per\,day$$

14.
$$\frac{30\ mi}{1\ hr} \times \frac{5280\ ft}{1\ mi} \times \frac{1\ hr}{60\ mins} \times \frac{1\ min}{60s} =$$
$$\frac{30 \times 5280\ ft}{60 \times 60s} = 44\ ft/s$$

15.
$$\frac{100\ yd}{11\ s} \times \frac{3\ ft}{1\ yd} \times \frac{1mi}{5280\ ft} \times \frac{60\ s}{1\ min} \times \frac{60\ mins}{1hr} = \frac{100 \times 3 \times 60 \times 60mi}{11 \times 5280hr} \approx 18.6mph$$

Regents Questions

1) JAN '15 [2] Ans: 2
2) JUN '16 [8] Ans: 1
3) AUG '16 [9] Ans: 3
4) JUN '17 [20] Ans: 4
5) JUN '18 [15] Ans: 1
6) AUG '18 [12] Ans: 3
7) JAN '19 [24] Ans: 4
8) JAN '17 [26]

$$\frac{12\ \cancel{km}}{1\ hr}\times\frac{0.62\ mi}{1\ \cancel{km}}=7.44\ mph$$

$$\frac{7.44\ mi}{1\ hr}=\frac{26.2\ mi}{x\ hrs}\quad x\approx 3.5\ hrs$$

9) AUG '17 [30]
The grasshopper jumps 20 times its height. $5'9" = 69$ inches.
Therefore, the athlete jumps $69\times 20 = 1380$ in per jump.

$$1380\text{ in}\cdot\frac{1\text{ ft}}{12\text{ in}}=115\text{ft}\ \text{ per jump.}$$

1 mile = 5280 ft would take

$$\frac{5280}{115}\approx 46\ \text{ jumps.}$$

10) JAN '18 [27]
The rate of speed is measuring distance over time, so it would be expressed in feet per minute.

4. LINEAR GRAPHS

4.1 Determine Whether a Point is on a Line

PRACTICE PROBLEMS

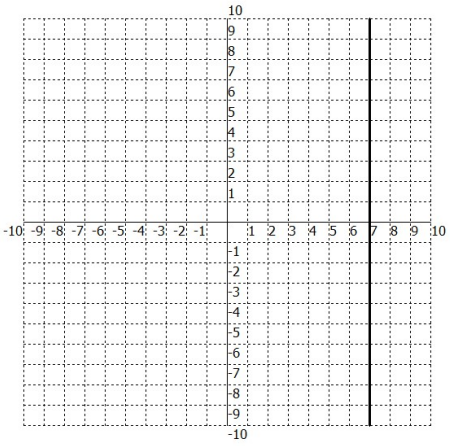
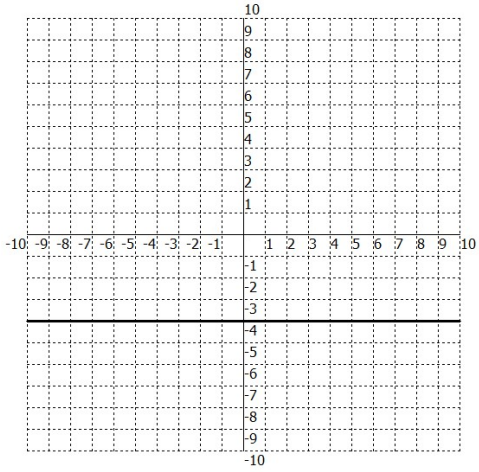
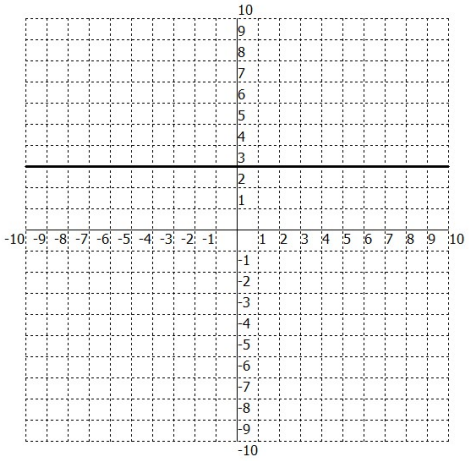
1. Yes. $7 = 3(3) - 2$? $7 = 9 - 2$? $7 = 7$ ✓	2. No. $9 = \frac{1}{2}(4) + 5$? $9 = 2 + 5$? $9 \neq 7$
3. Yes. $0 = 4(0)$? $0 = 0$ ✓	4. Yes. $2(-4) - 3(-2) = -2$? $-8 + 6 = -2$? $-2 = -2$ ✓
5. No. $4(-4) - (3) = -13$? $-16 - 3 = -13$? $-19 \neq -13$	6. Yes. $5(-2) - 2(-4) = -2$? $-10 + 8 = -2$? $-2 = -2$ ✓
7. No. $2(-5) - (-1) = -11$? $-10 + 1 = -11$? $-9 \neq -11$	8. Yes. $4(3) = 3(-2) + 18$? $12 = -6 + 18$? $12 = 12$ ✓
9. $2x + 6(-2) = 4$ $2x - 12 = 4$ $2x = 16$ $x = 8$	10. $4k + (3) = -9$ $4k = -12$ $k = -3$
11. $k - 2(-3) = -2$ $k + 6 = -2$ $k = -8$	12. $2(5) + k = 9$ $10 + k = 9$ $k = -1$

REGENTS QUESTIONS

1) AUG '16 [2] Ans: 3
2) AUG '17 [20] Ans: 3

4.2 Lines Parallel to Axes

Practice Problems

1. (1)	2. (2)
3. $x=9$	4. $y=1$
5. $x=0$	6. $y=0$
7. (5,0)	8. 
9. 	10. $y-4=-1$ $y=3$ 

4.3 Find Intercepts

Practice Problems

1. x-intercept: 2 y-intercept: -4	2. x-intercept: -2 y-intercept: -3
3. x-intercept: $3(0)+2x=6$, $2x=6$, $x=3$ y-intercept: $3y+2(0)=6$, $3y=6$, $y=2$	4. x-intercept: $3x-4(0)=12$, $3x=12$, $x=4$ y-intercept: $3(0)-4y=12$, $-4y=12$, $y=-3$
5. x-intercept: $(0)=-2x+5$, $-5=-2x$, $x=\frac{5}{2}$ y-intercept: $y=-2(0)+5$, $y=5$	6. x-intercept: $9x-6(0)+5=0$, $9x+5=0$, $9x=-5$, $x=-\frac{5}{9}$ y-intercept: $9(0)-6y+5=0$, $-6y+5=0$, $-6y=-5$, $y=\frac{5}{6}$

Regents Questions

1) AUG '14 [8] Ans: 1
2) JAN '15 [9] Ans: 4

4.4 Find Slope Given Two Points

PRACTICE PROBLEMS

1. $m=\frac{4}{8}=\frac{1}{2}$	2. $m=-\frac{2}{6}=-\frac{1}{3}$
3. $m=\frac{4}{3}$	4. $m=-\frac{2}{3}$
5. $m=-\frac{3}{3}=-1$	6. $m=\frac{5}{5}=1$
7. $m=\frac{13-3}{5-1}=\frac{10}{4}=\frac{5}{2}$	8. $m=\frac{8-(-6)}{1-3}=\frac{14}{-2}=-7$
9. $m=\frac{-3-5}{0-4}=\frac{-8}{-4}=2$	10. $m=\frac{-2-(-2)}{2-(-4)}=\frac{0}{6}=0$
11. $m=\frac{3-5}{7-2}=-\frac{2}{5}$	12. $m=\frac{2-5}{-2-3}=\frac{-3}{-5}=\frac{3}{5}$

4.5 Find Slope Given an Equation

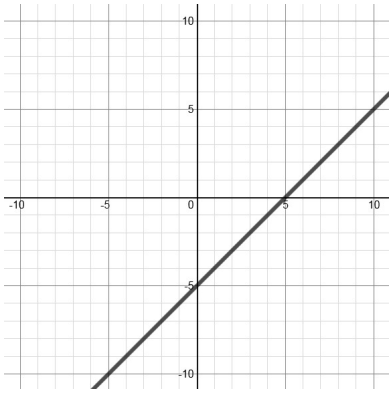
Practice Problems

1. Slope is $\frac{2}{5}$.	2. $y-3x=1$ $y=3x+1$ Slope is 3.
3. $2y=5x+4$ $y=\frac{5}{2}x+2$ Slope is $\frac{5}{2}$.	4. $5y-10x=-15$ $5y=10x-15$ $y=2x-3$ Slope is 2.
5. $4x+3y=12$ $3y=-4x+12$ $y=-\frac{4}{3}x+4$ Slope is $-\frac{4}{3}$	6. $2y=x-4$ $y=\frac{1}{2}x-2$ Slope is $\frac{1}{2}$
7. $3x-2y=12$ $-2y=-3x+12$ $y=\frac{3}{2}x-6$ Slope is $\frac{3}{2}$.	8. $3x-4y-16=0$ $3x-4y=16$ $-4y=-3x+16$ $y=\frac{3}{4}x-4$ Slope is $\frac{3}{4}$.
9. $y=-2x+2$	10. $y=\frac{1}{2}x$
11. (1) Same slope of –3	12. $2x-3y=9$ $-3y=-2x+9$ $y=\frac{2}{3}x-3$ Choice (1)
13. The first equation: $2y+2x=6$ $2y=-2x+6$ $y=-x+3$	14. $4x+6y=5$ $6y=-4x+5$ $y=-\frac{2}{3}x+\frac{5}{6}$ The first equation: $-3y=2x+5$ $y=-\frac{2}{3}x-\frac{5}{3}$

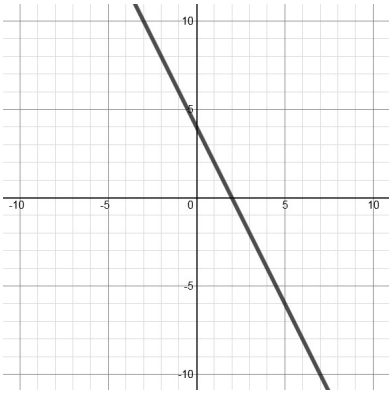
4.6 Graph Linear Equations

Practice Problems

1.



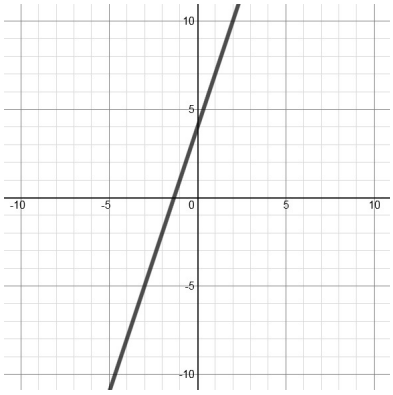
2.



3.

$y-3x=4$

$y=3x+4$

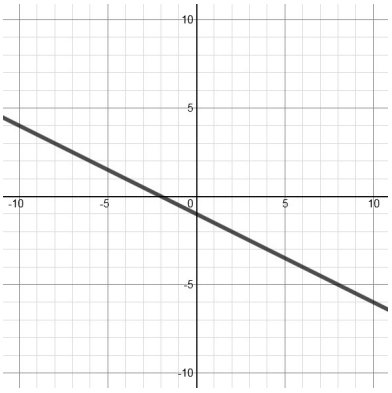


4.

$2y+2x=x-2$

$2y=-x-2$

$y=-\frac{1}{2}x-1$



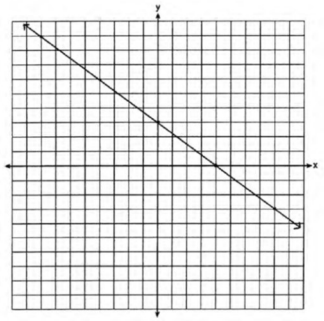
5. (4)

$2y=-4x-10$

$y=-2x-5$

Regents Questions

1) AUG '14 [13] Ans: 2
2) JUN '14 [29]



No, the point is not on the line.

4.7 Write an Equation Given a Point and Slope

Practice Problems

1. $y = mx + b$ $4 = 2(1) + b$ $4 = 2 + b$ $2 = b$	$y = 2x + 2$
2. $y = mx + b$ $5 = 5(-6) + b$ $5 = -30 + b$ $35 = b$	$y = 5x + 35$
3. $y = mx + b$ $2 = \frac{1}{3}(-3) + b$ $2 = -1 + b$ $3 = b$	$y = \frac{1}{3}x + 3$
4. $y = mx + b$ $-3 = \frac{3}{4}(8) + b$ $-3 = 6 + b$ $-9 = b$	$y = \frac{3}{4}x - 9$
5. $y = mx + b$ $4 = \frac{3}{4}(-8) + b$ $4 = -6 + b$ $10 = b$	$y = \frac{3}{4}x + 10$
6. $y = mx + b$ $-7 = -\frac{4}{3}(3) + b$ $-7 = -4 + b$ $-3 = b$	$y = -\frac{4}{3}x - 3$

4.8 Write an Equation Given Two Points

Practice Problems

1.

$$m=\frac{6-2}{5-1}=\frac{4}{4}=1$$

$y=mx+b$

$2=1(1)+b$

$2=1+b$

$1=b$

$y=x+1$

2.

$$m=\frac{4-(-1)}{3-2}=\frac{5}{1}=5$$

$y=mx+b$

$-1=5(2)+b$

$-1=10+b$

$-11=b$

$y=5x-11$

3.

$$m=\frac{-2-0}{3-(-3)}=\frac{-2}{6}=-\frac{1}{3}$$

$y=mx+b$

$0=-\frac{1}{3}(-3)+b$

$0=1+b$

$-1=b$

$y=-\frac{1}{3}x-1$

4.

$$m=\frac{4-4}{2-(-2)}=\frac{0}{4}=0$$

$y=mx+b$

$4=0(-2)+b$

$4=b$

$y=4$

5.

$$m=\frac{5-3}{8-1}=\frac{2}{7}$$

$y-3=\frac{2}{7}(x-1)$

6.

$$m=\frac{0-4}{-5-5}=\frac{2}{5}$$

(a)

$4=\frac{2}{5}(5)+b$

$4=2+b$

$2=b$

$y=\frac{2}{5}x+2$

(b)

$y-4=\frac{2}{5}(x-5)$

Regents Questions

1) JAN '15 [11] Ans: 4

2) JUN '16 [29]

Sue used point-slope form and
Kathy used slope-intercept form.
Both are correct as shown:

$$m = \frac{4-1}{-3-6} = -\frac{1}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\tfrac{1}{3}(x + 3)$$

$$y = mx + b$$

$$4 = -\tfrac{1}{3}(-3) + b$$

$$4 = 1 + b$$

$$3 = b$$

$$y = -\tfrac{1}{3}x + 3$$

4.9 Graph Inequalities

Practice Problems

1. (1)

2. (2)

3. (3)

$$2y + 6 > 4x$$
$$2y > 4x - 6$$
$$y > 2x - 3$$

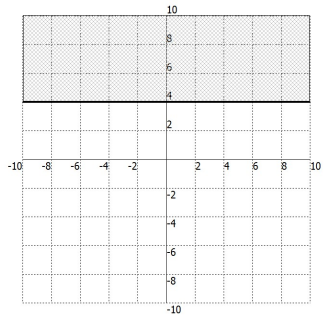
4. $y \leq x - 1$

5. $y \leq \frac{4}{3}x - 4$

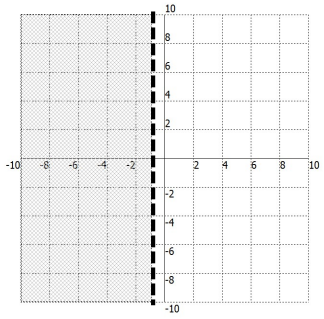
6. $y < 3$

7. $y > \frac{3}{2}x + 2$

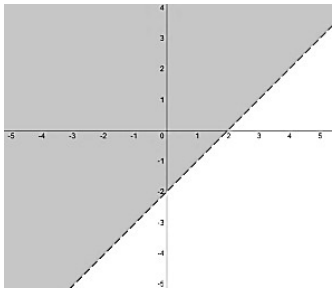
8.



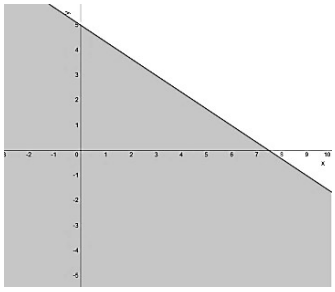
9.



10.

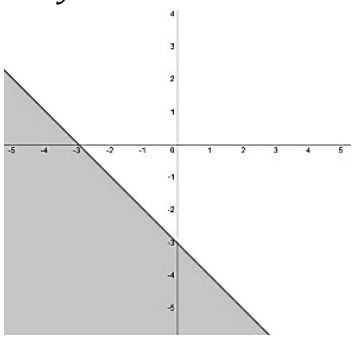


11.



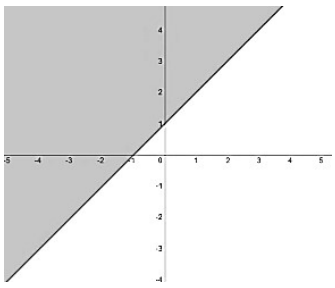
12.

$$x + y \leq -3$$
$$y \leq -x - 3$$



13.

$$x - y \leq -1$$
$$-y \leq -x - 1$$
$$y \geq x + 1$$

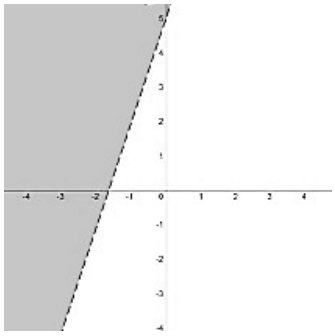


14.

$2y - 6x > 10$

$2y > 6x + 10$

$y > 3x + 5$

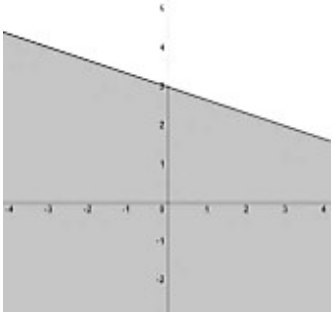


15.

$9 - x \geq 3y$

$3 - \frac{1}{3}x \geq y$

$y \leq -\frac{1}{3}x + 3$



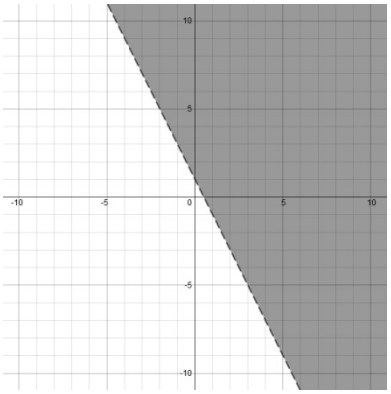
Regents Questions

1) JUN '15 [5] Ans: 1

2) JAN '16 [5] Ans: 2

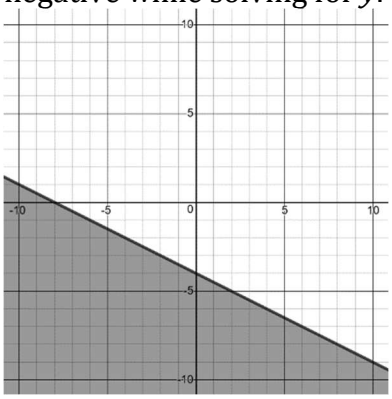
3) AUG '15 [26]

$y > -2x + 1$

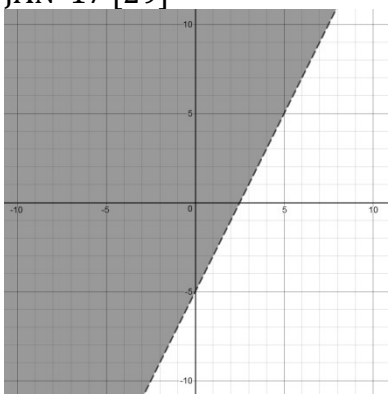


4) AUG '16 [34]

The graph should be shaded below the line. Shawn may have forgotten to flip the sign when dividing by a negative while solving for y.

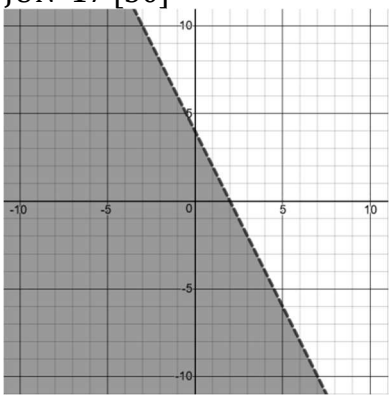


5) JAN '17 [29]



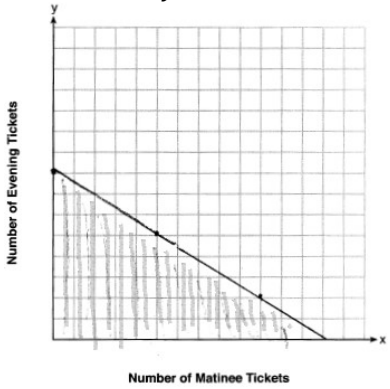
Any point in the shaded area but not on the dashed line, such as (0,0).

6) JUN '17 [30]



7) JAN '19 [35]

$7.5x + 12.5y \leq 100$



$7.5x \leq 100$

$x \leq 13.\overline{3}$, so the maximum is 13

5. LINEAR SYSTEMS

5.1 Solve Linear Systems Algebraically

PRACTICE PROBLEMS

1.

$$\begin{array}{r} 3x-y=8 \\ x+y=4 \\ \hline 4x=12 \\ x=3 \end{array} \qquad \begin{array}{r} x+y=4 \\ 3+y=4 \\ y=1 \end{array}$$

2.

$$\begin{array}{r} 2x-3y=19 \\ 3x+3y=21 \\ \hline 5x=40 \\ x=8 \end{array} \qquad \begin{array}{r} 2(8)-3y=19 \\ 16-3y=19 \\ -3y=3 \\ y=-1 \end{array}$$

3.

$$\begin{array}{r} 3x+2y=12 \\ 5x-2y=4 \\ \hline 8x=16 \\ x=2 \end{array} \qquad \begin{array}{r} 3(2)+2y=12 \\ 6+2y=12 \\ 2y=6 \\ y=3 \end{array}$$

4.

$$\begin{array}{r} 2x-5y=11 \\ -2x+3y=-9 \\ \hline -2y=2 \\ y=-1 \end{array} \qquad \begin{array}{r} 2x-5(-1)=11 \\ 2x+5=11 \\ 2x=6 \\ x=3 \end{array}$$

5.

$$\begin{array}{r} 2x-4y=12 \\ -2x+y=-9 \\ \hline -3y=3 \\ y=-1 \end{array} \qquad \begin{array}{r} -2x-1=-9 \\ -2x=-8 \\ x=4 \end{array}$$

6.

$$\begin{array}{r} 3x+y=0 \\ -x-y=-4 \\ \hline 2x=-4 \\ x=-2 \end{array} \qquad \begin{array}{r} 3(-2)+y=0 \\ -6+y=0 \\ y=6 \end{array}$$

7.

$$\begin{array}{r} 3x+2y=4 \quad \rightarrow \\ -2x+2y=24 \quad \times(-1) \\ \hline \end{array} \qquad \begin{array}{r} 3x+2y=4 \\ 2x-2y=-24 \\ \hline 5x=-20 \\ x=-4 \end{array}$$

$$\begin{array}{r} 3(-4)+2y=4 \\ -12+2y=4 \\ 2y=16 \\ y=8 \end{array}$$

8.

$$\begin{array}{r} 2x+3y=6 \quad \rightarrow \\ 2x+y=-2 \quad \times(-1) \\ \hline \end{array} \qquad \begin{array}{r} 2x+3y=6 \\ -2x-y=2 \\ \hline 2y=8 \\ y=4 \end{array}$$

9.

$$\begin{array}{r} -3x+4y=11 \quad \times 2 \\ 6x-5y=-16 \quad \rightarrow \\ \hline \end{array} \qquad \begin{array}{r} -6x+8y=22 \\ 6x-5y=-16 \\ \hline 3y=6 \\ y=2 \end{array}$$

$$\begin{array}{r} -3x+4(2)=11 \\ -3x+8=11 \\ -3x=3 \\ x=-1 \end{array}$$

10.

$$\begin{array}{r} 2x+3y=7 \quad \rightarrow \\ x+y=3 \quad \times(-3) \\ \hline \end{array} \qquad \begin{array}{r} 2x+3y=7 \\ -3x-3y=-9 \\ \hline -x=-2 \\ x=2 \end{array}$$

$$\begin{array}{r} 2(2)+3y=7 \\ 4+3y=7 \\ 3y=3 \\ y=1 \end{array}$$

11.

$$\begin{array}{rl} 2x+y=8 & \times 3 \\ x-3y=-3 & \rightarrow \end{array} \qquad \begin{array}{r} 6x+3y=24 \\ x-3y=-3 \\ \hline 7x=21 \\ x=3 \end{array}$$

$$\begin{aligned} 2(3)+y&=8 \\ 6+y&=8 \\ y&=2 \end{aligned}$$

12.

$$\begin{array}{rl} x+2y=9 & \rightarrow \\ x-y=3 & \times 2 \end{array} \qquad \begin{array}{r} x+2y=9 \\ 2x-2y=6 \\ \hline 3x=15 \\ x=5 \end{array}$$

$$\begin{aligned} (5)-y&=3 \\ -y&=-2 \\ y&=2 \end{aligned}$$

13.

$$\begin{array}{rl} 3x+2y=4 & \times 3 \\ 4x+3y=7 & \times(-2) \end{array} \qquad \begin{array}{r} 9x+6y=12 \\ -8x-6y=-14 \\ \hline x=-2 \end{array}$$

$$\begin{aligned} 3(-2)+2y&=4 \\ -6+2y&=4 \\ 2y&=10 \\ y&=5 \end{aligned}$$

14.

$$\begin{array}{rl} 3x+4y=9 & \times 3 \\ 5x+6y=21 & \times(-2) \end{array} \qquad \begin{array}{r} 9x+12y=27 \\ -10x-12y=-42 \\ \hline -x=-15 \\ x=15 \end{array}$$

$$\begin{aligned} 3(15)+4y&=9 \\ 45+4y&=9 \\ 4y&=-36 \\ y&=-9 \end{aligned}$$

15.

$$\begin{aligned} 4x-10&=5-x \\ 5x-10&=5 \\ 5x&=15 \\ x&=3 \end{aligned} \qquad \begin{aligned} y&=5-3 \\ y&=2 \end{aligned}$$

16.

$$\begin{aligned} x&=(10-3x)-2 \\ 4x&=8 \\ x&=2 \end{aligned} \qquad \begin{aligned} y&=10-3(2) \\ y&=4 \end{aligned}$$

17.

$$\begin{aligned} 3(9-2x)-2x&=11 \\ 27-6x-2x&=11 \\ 27-8x&=11 \\ -8x&=-16 \\ x&=2 \end{aligned} \qquad \begin{aligned} y&=9-2(2) \\ y&=5 \end{aligned}$$

18.

$$\begin{aligned} x-4y&=-8 \\ x&=4y-8 \end{aligned}$$

$$\begin{aligned} 7(4y-8)+3y&=68 \\ 28y-56+3y&=68 \\ 31y-56&=68 \\ 31y&=124 \\ y&=4 \end{aligned} \qquad \begin{aligned} x-4(4)&=-8 \\ x-16&=-8 \\ x&=8 \end{aligned}$$

19.

$$\begin{aligned} 2\left(\tfrac{1}{2}b-6\right)+3b&=12 \\ b-12+3b&=12 \\ 4b&=24 \\ b&=6 \end{aligned} \qquad \begin{aligned} a&=\tfrac{1}{2}(6)-6 \\ a&=-3 \end{aligned}$$

20.

$$\begin{aligned} (4d-6)+3d&=8 \\ 7d-6&=8 \\ 7d&=14 \\ d&=2 \end{aligned} \qquad \begin{aligned} c&=4(2)-6 \\ c&=2 \end{aligned}$$

21.

$$\begin{aligned} 2x-y&=5 \\ -y&=-2x+5 \\ y&=2x-5 \end{aligned}$$

Choice (1)

Regents Questions

1) JUN '14 [14] Ans: 2
2) JAN '16 [21] Ans: 4
3) AUG '16 [22] Ans: 4
4) AUG '17 [24] Ans: 4
5) JAN '18 [15] Ans: 2
6) AUG '18 [22] Ans: 3
7) JAN '19 [22] Ans: 3

8) JUN '15 [33]

$$\begin{aligned} 3(8x+9y=48) &\rightarrow \quad 24x+27y=144 \\ -2(12x+5y=21) &\rightarrow \underline{-24x-10y=-42} \\ & \qquad\qquad 17y=102 \\ & \qquad\qquad y=6 \end{aligned}$$

$$y=\frac{-51}{-8.5}=6, \qquad \begin{aligned} 8x+9(6)&=48 \\ 8x&=-6, \\ x&=-\tfrac{3}{4} \end{aligned}$$

Yes, $x=-\frac{3}{4}$ and $y=6$ for both

5.2 Solve Linear Systems Graphically

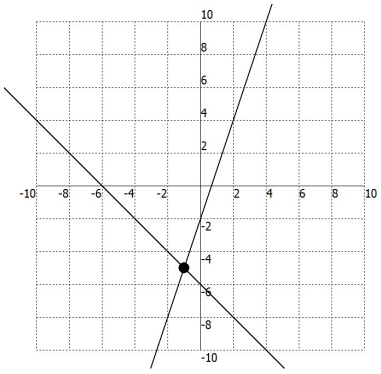
Practice Problems

1. (3)

2. (3)

3. (–2,3)

4.



Solution: (–1,–5)

5.

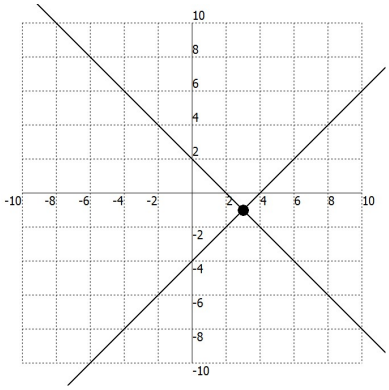
$x + y = 2$

$y = -x + 2$

$x - y = 4$

$-y = -x + 4$

$y = x - 4$



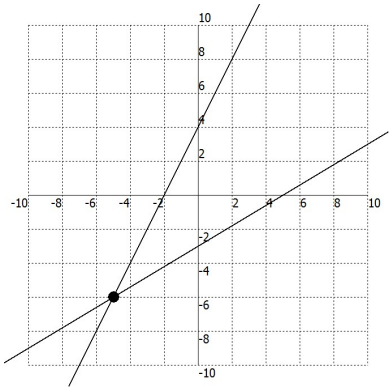
Solution: (3,–1)

6.

$3x - 5y = 15$

$-5y = -3x + 15$

$y = \frac{3}{5}x - 3$



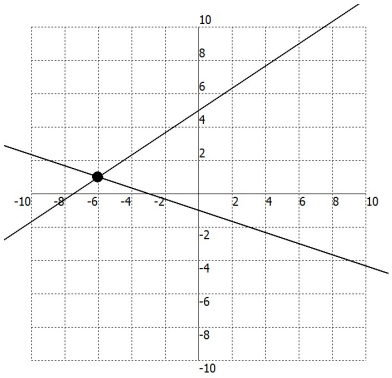
Solution: (–5,–6)

7.

$x + 3y = -3$

$3y = -x - 3$

$y = -\frac{1}{3}x - 1$



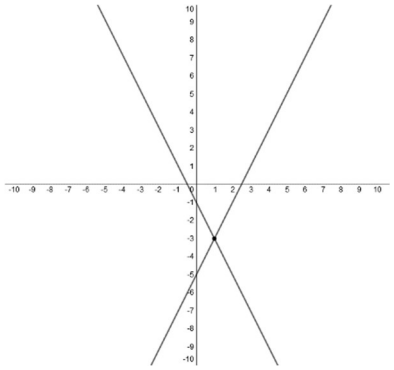
Solution: (–6,1)

8.

$4x-2y=10$

$-2y=-4x+10$

$y=2x-5$

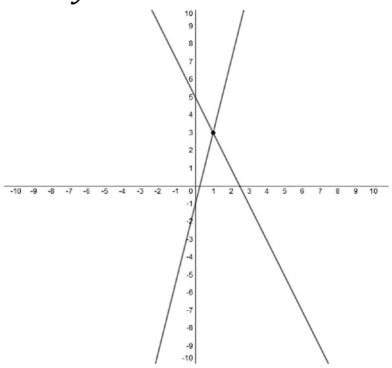


Solution: (1,–3)

9.

$2x+y=5$

$y=-2x+5$

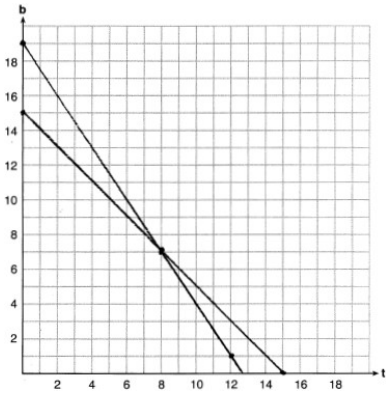


Solution: (1,3)

REGENTS QUESTIONS

1) JAN '15 [18] Ans: 3
2) JUN '16 [18] Ans: 4
3) JUN '17 [8] Ans: 1
4) JAN '17 [25]
   No. The two lines coincide, so there are infinitely many solutions.

5) JAN '19 [37]
   $t+b=15$
   $3t+2b=38$



No, the point of intersection on the graph shows that 8 tricycles were ordered.

5.3 Solutions to Systems of Inequalities

Practice Problems

1. (1) (1,1)	2. (4) (–9,0)
3. (4) (4,0)	4. (2) (2,0)
5. (4) (2,–2)	6. (2) (2,–1)

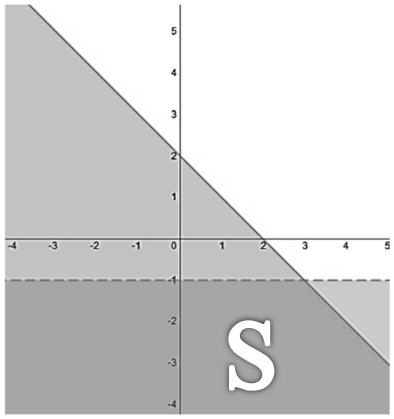
Regents Questions

1) SEP '13 [1] Ans: 2
2) JAN '17 [16] Ans: 4

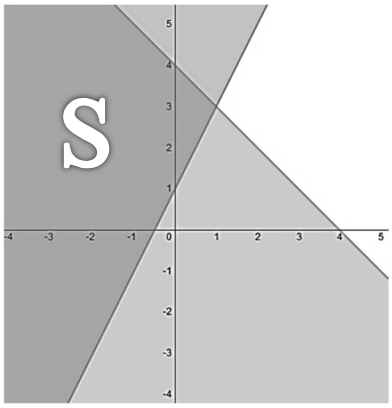
5.4 Solve Systems of Inequalities Graphically

Practice Problems

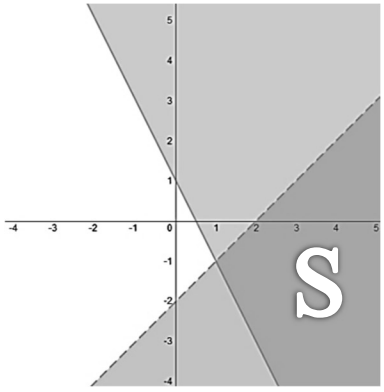
1.



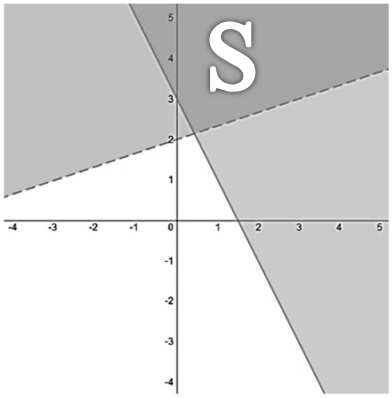
2.



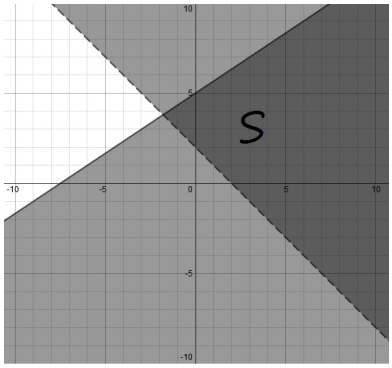
3.



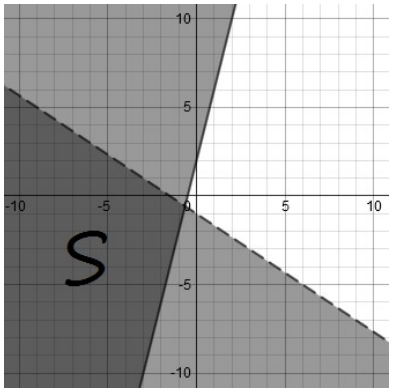
4.

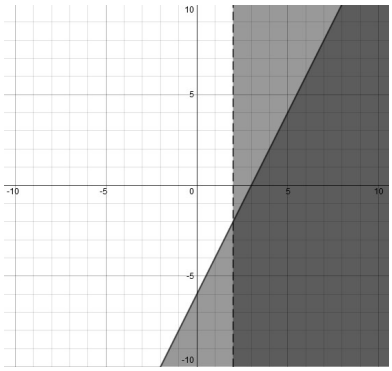
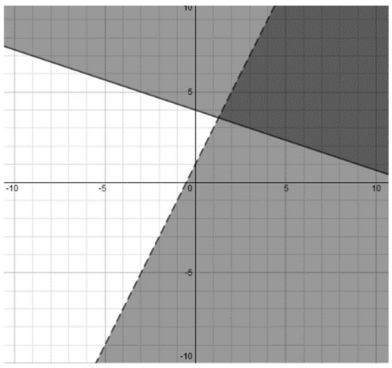
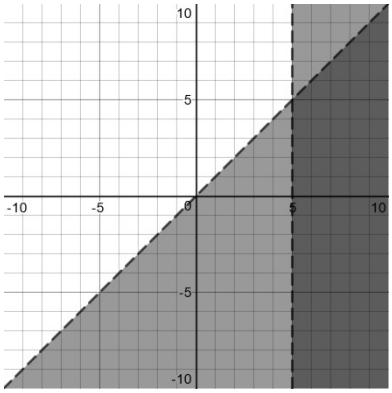
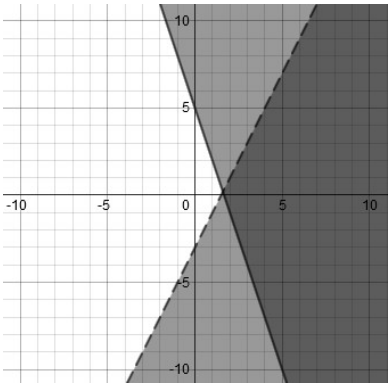
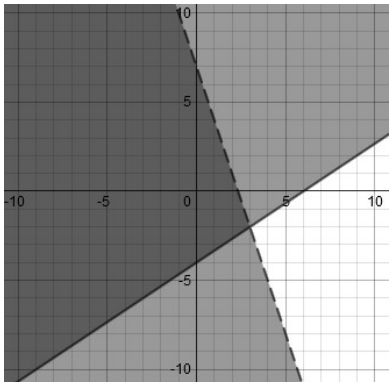
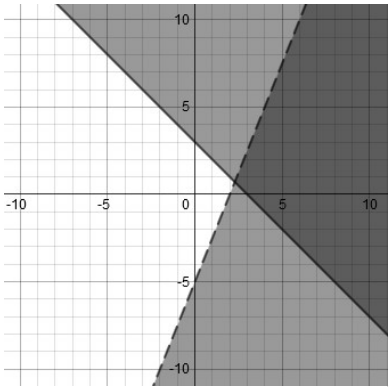


5.



6.

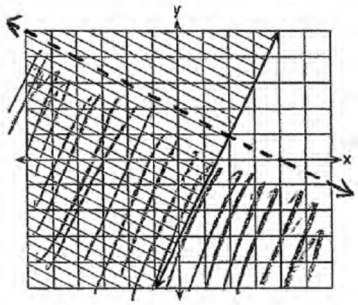


7.  infinitely many solutions such as (3,–3)	8.  infinitely many solutions such as (6, 6)
9.  infinitely many solutions, such as (7,1)	10.  infinitely many solutions, such as (4,0)
11.  infinitely many solutions such as (–1,–1)	12.  infinitely many solutions such as (2,4)
13. Yes	14. No
15. (1) (1,–4)	16. (2) (–2,2)

Regents Questions

1) JUN '14 [4] Ans: 2
2) AUG '14 [7] Ans: 1
3) AUG '15 [6] Ans: 3
4) JAN '18 [20] Ans: 3
5) JAN '15 [34]

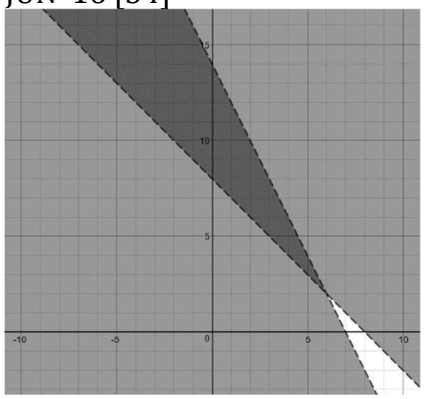
a) $y \geq 2x - 3$,



b) ,

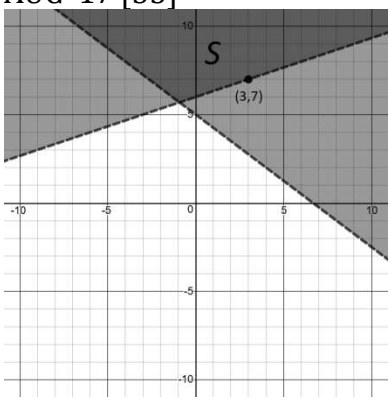
c) disagree since (2,1) is not a solution of $x + 2y < 4$

6) JUN '16 [34]



(6,2) is not a solution as its falls on the edge of each inequality.

7) AUG '17 [35]

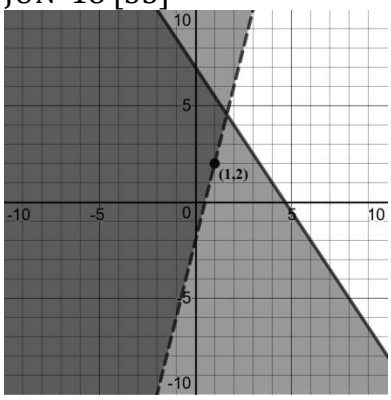


(3,7) is not in the solution set because it is on a dashed boundary.

8) JAN '18 [28]

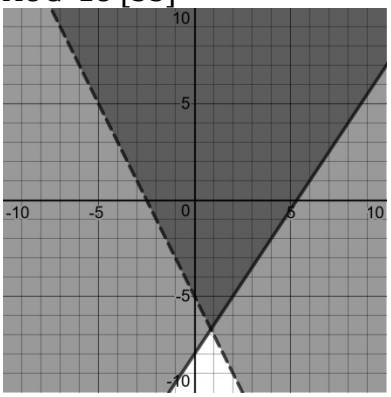
(0,4) is located at the intersection of the two lines. However, since the line for the inequality $y < \frac{1}{2}x + 4$ is dashed, this point is not a solution to the system.

9) JUN '18 [35]



(1,2) is not in the solution set because it lies on the dashed line.

10) AUG ’18 [35]



(6,1) is on a solid line; (–6,7) is on a dashed line.

5.5 Word Problems – Linear Systems

Practice Problems

1. x and y are the two numbers.

$$\begin{array}{r} x-y=5 \\ \underline{x+y=59} \\ 2x=64 \\ x=32 \end{array} \qquad \begin{array}{r} (32)+y=59 \\ y=27 \end{array}$$

The numbers are 32 and 27.

2. a = smaller number
b = larger number

$$\begin{array}{r} b+a=47 \\ \underline{b-a=15} \\ 2b=62 \\ b=31 \end{array}$$

The larger number is 31.

3. p = cost of one bag of popcorn
c = cost of one cookie

$$\begin{array}{rcr} p+2c=5 & \rightarrow & p+2c=5 \\ \underline{p+4c=6 \quad \times(-1)} & & \underline{-p-4c=-6} \\ & & -2c=-1 \\ & & c=0.50 \end{array}$$

One cookie costs \$0.50.

4. d = cost of a doughnut
c = cost of a cookie

$$\begin{array}{r} 2d+3c=3.30 \quad \times 2 \\ \underline{5d+2c=4.95 \quad \times(-3)} \end{array}$$

$$\begin{array}{r} 4d+6c=6.60 \\ \underline{-15d-6c=-14.85} \\ -11d=-8.25 \\ d=0.75 \end{array} \qquad \begin{array}{r} 2(0.75)+3c=3.30 \\ 1.50+3c=3.30 \\ 3c=1.80 \\ c=0.60 \end{array}$$

Doughnuts cost 75¢ and cookies cost 60¢.

5. p = cost of a pizza slice
c = cost of a cola

$$\begin{array}{r} 3p+2c=6.00 \quad \times 3 \\ \underline{2p+3c=5.25 \quad \times(-2)} \end{array}$$

$$\begin{array}{r} 9p+6c=18.00 \\ \underline{-4p-6c=-10.50} \\ 5p=7.50 \\ p=1.50 \end{array} \qquad \begin{array}{r} 3(1.50)+2c=6.00 \\ 4.50+2c=6.00 \\ 2c=1.50 \\ c=0.75 \end{array}$$

Pizzas cost \$1.50 and colas cost \$0.75.

6. s = hourly rate for the sprayer
g = hourly rate for the generator

$$\begin{array}{r} 6s+6g=90 \quad \times 2 \\ \underline{4s+8g=100 \quad \times(-3)} \end{array}$$

$$\begin{array}{r} 12s+12g=180 \\ \underline{-12s-24g=-300} \\ -12g=-120 \\ g=10 \end{array} \qquad \begin{array}{r} 6s+6(10)=90 \\ 6s+60=90 \\ 6s=30 \\ s=5 \end{array}$$

Sprayer costs \$5/hr; generator costs \$10/hr.

7. f = number of fancy shirts bought
p = number of plain shirts bought

$$28f + 15p = 131 \rightarrow$$
$$f + p = 7 \quad \times(-15)$$

$$28f + 15p = 131$$
$$-15f - 15p = -105$$
$$13f = 26$$
$$f = 2$$

$$(2) + p = 7$$
$$p = 5$$

She bought 2 fancy and 5 plain shirts.

8. n = cost of one notebook
p = cost of one pencil

$$-2(3n + 4p = 8.50) \rightarrow -6n - 8p = -17.00$$
$$5n + 8p = 14.50$$
$$-n = -2.50$$
$$n = 2.50$$

$$3(2.50) + 4p = 8.50$$
$$7.50 + 4p = 8.50$$
$$4p = 1.00$$
$$p = 0.25$$

$2.50 per notebook, $0.25 per pencil

9. a = number of apples sold last week
o = number of oranges sold last week

$$-3(a + o = 108) \rightarrow -3a - 3o = -324$$
$$5a + 3o = 452$$
$$2a = 128$$
$$a = 64$$

$$64 + o = 108$$
$$o = 44$$

64 apples and 44 oranges

10. t = tens digit; u = units digit

$$10u + t = 10t + u + 9$$
$$9u + t = 10t + 9$$
$$9u - 9t = 9$$

$$u + t = 7 \quad \times 9$$
$$9u - 9t = 9 \rightarrow$$

$$9u + 9t = 63$$
$$9u - 9t = 9$$
$$18u = 72$$
$$u = 4$$

$$(4) + t = 7$$
$$t = 3$$

The number is 34.

Regents Questions

1) AUG '14 [19] Ans: 4
2) JUN '15 [6] Ans: 3
3) JUN '16 [5] Ans: 1
4) JAN '18 [3] Ans: 1
5) AUG '18 [9] Ans: 2
6) JUN '14 [36]

$2.35c + 5.50d = 89.50$,
No because
$2.35(8) + 5.50(14) = 95.80$,

$$\begin{aligned} c + d &= 22 \\ d &= 22 - c \\ 2.35c + 5.50(22 - c) &= 89.50 \\ 121 - 3.15c &= 89.50 \\ -3.15c &= -31.50 \\ c &= 10 \end{aligned}$$

10 cats

7) JAN '15 [33]

$2p + 3d = 18.25$ and
$4p + 2d = 27.50$,

$$\begin{aligned} -2(2p + 3d = 18.25) \rightarrow -4p - 6d &= -36.50 \\ \underline{4p + 2d = 27.50} \\ -4d &= -9 \\ d &= 2.25 \end{aligned}$$

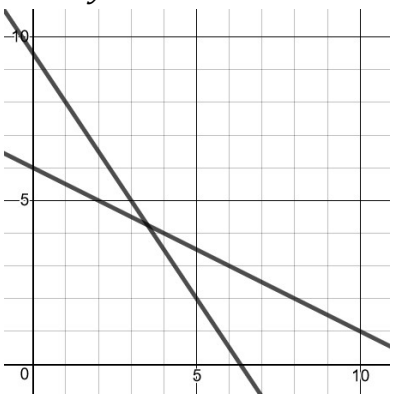
$$\begin{aligned} 2p + 3(2.25) &= 18.25 \\ 2p + 6.75 &= 18.25 \\ 2p &= 11.50 \\ p &= 5.75 \end{aligned}$$

popcorn \$5.75, drink \$2.25

8) JUN '16 [37]

a) $3x + 2y = 19$
$2x + 4y = 24$

b)



c) $\times 2 \Rightarrow \quad 6x + 4y = 38$
$\times(-1) \Rightarrow \underline{-2x - 4y = -24}$

$$\begin{aligned} 4x &= 14 \\ x &= 3.5 \end{aligned}$$

$$\begin{aligned} 2(3.5) + 4y &= 24 \\ 7 + 4y &= 24 \\ 4y &= 17 \\ y &= 4.25 \end{aligned}$$

Cupcakes cost \$3.50 and brownies cost \$4.25 per package.

9) AUG '16 [37]

a) $18j + 32w = 19.92$
$14j + 26w = 15.76$

b) $14(.52) + 26(.33) = 15.86$

c) $18j + 32w = 19.92 \quad \times(-7)$
$14j + 26w = 15.76 \quad \times 9$

$$\begin{aligned} -126j - 224w &= -139.44 \\ \underline{126j + 234w} &= \underline{\quad 141.84} \\ 10w &= 2.4 \\ w &= 0.24 \end{aligned}$$

$$\begin{aligned} 18j + 32(0.24) &= 19.92 \\ 18j + 7.68 &= 19.92 \\ 18j &= 12.24 \\ j &= 0.68 \end{aligned}$$

Each juice box is 68 cents and each water bottle is 24 cents.

10) JAN '17 [34]

$$\begin{array}{r} p+2s=15.95 \ \times(-5) \\ 3p+5s=45.90 \ \times 2 \\ \hline -5p-10s=-79.75 \\ 6p+10s=91.80 \\ \hline p=12.05 \end{array}$$

11) JAN '17 [37]

$$1000-60x=600-20x$$
$$400=40x$$
$$10=x$$

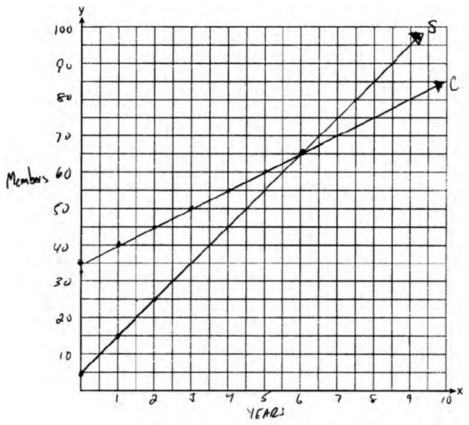
10 months

$1000-60(10)=\$400$

Ian is incorrect because $1000-60(16)=40$, so he would still owe \$40.

12) JUN '17 [37]

$y=10x+5$ and $y=5x+35$.



(6,65) It took 6 years for the two clubs to have the same number of members, at which point they had 65 members each.

13) AUG '17 [28]

For \$50, Dylan can buy 14 games in Plan *B* but only 12 games in Plan *A*. Bobby can buy 20 games for \$65 under both plans, so he can choose either plan.

14) JAN '18 [37]

$d=2c-5$

$$\frac{c+3}{d+3}=\frac{3}{4}$$

No, because it doesn't make each equation true: eg, $20\neq 2(15)-5$;

$$\frac{c+3}{(2c-5)+3}=\frac{3}{4}$$
$$4(c+3)=3(2c-2)$$
$$4c+12=6c-6$$
$$18=2c$$
$$c=9$$

$d=2(9)-5=13$

She had 9 cats and 13 dogs

15) JUN '18 [34]

$A(x)=7+3(x-2)$

$B(x)=3.25x$

$$7+3(x-2)=3.25x$$
$$7+3x-6=3.25x$$
$$1+3x=3.25x$$
$$1=0.25x$$
$$x=4 \text{ hours}$$

16) JUN '18 [37]

$$10d+25q=1755$$
$$d+q=90$$
$$10(90-q)+25q=1755$$
$$900-10q+25q=1755$$
$$15q=855$$
$$q=57$$

No, because $\$20.98\times 1.08>90\times 0.25$

17) AUG '18 [37]

$$b=4s+6$$
$$b-3=7(s-3)$$
$$4s+6-3=7s-21$$
$$3s=24$$
$$s=8$$
$$b=4(8)+6=38$$
$$38+x=3(8+x)$$
$$38+x=24+3x$$
$$2x=14$$
$$x=7$$

5.6 Word Problems – Systems of Inequalities

Practice Problems

1. d = number of dog-walking hours
   c = number of car wash hours

$$d + c \le 20$$
$$7.50d + 6.00c \ge 92.00$$

2. s = number of bags of soil
   p = number of plants

$$4s + 10p \le 100$$
$$p \ge 5$$

3. s = number of boxes of small books
   l = number of boxes of large books

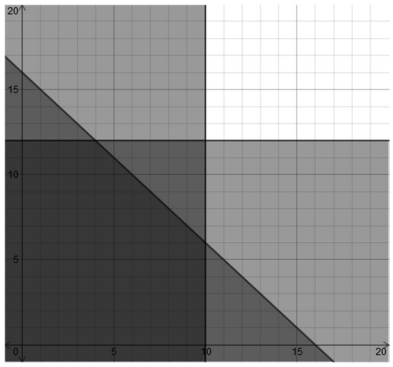
$$15s + 8l \ge 350$$
$$s + l \ge 35$$

4. (a) $t \le 3$
   $d \le 55t$

   Therefore,
   $d \le 55(3)$
   $d \le 165$

   (b) Yes

5. Ans: $x \le 10$, $y \le 12$ and $x + y \le 16$



Regents Questions

1) JUN '17 [11] Ans: 1

2) AUG '18 [10] Ans: 2

3) MAY '13 [6]

$x + y \le 800$, $6x + 9y \ge 5000$; yes,

$6(440) + 9y \ge 5000$

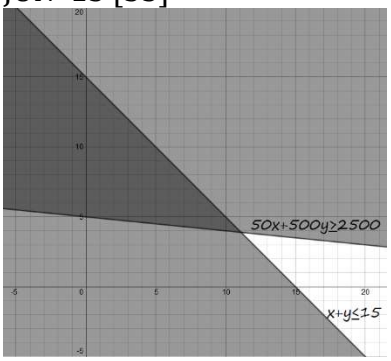
$2640 + 9y \ge 5000$

$9y \ge 2360$

$y \ge 262\frac{2}{9}$

$440 + 263 \le 800$ ✓

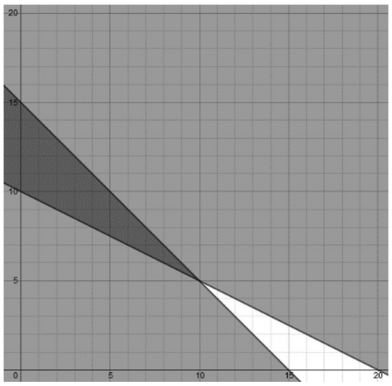
4) JUN '15 [35]



$50x + 500y \ge 2500$ and $x + y \le 15$; any point in the solution set, such as (4,7) for 4 printers and 7 computers

5) AUG '14 [37]

$x + y \le 15$ and $4x + 8y \ge 80$

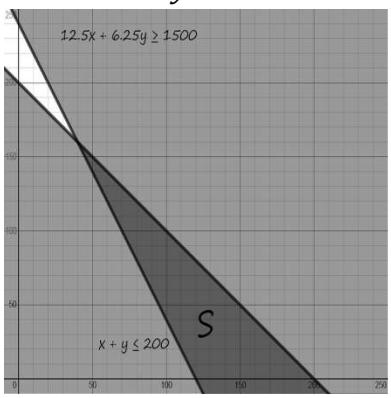


Solution stated such as (3,10) = 3 hrs. of babysitting and 10 hrs. at the library

6) JAN '16 [37]

$x + y \le 200$

$12.5x + 6.25y \ge 1500$



No, because the point (30,80) is not in the shaded area labelled *S*

7) AUG '16 [35]

$x + y \le 200$

$12x + 8.50y \ge 1000$

$12x + 8.50(50) \ge 1000$

$12x + 425 \ge 1000$

$12x \ge 575$

$x \ge \frac{575}{12} \approx 47.9$

Minimum of 48 tickets at the door

8) JAN '18 [35]

$2c + 1.5b \ge 500$

$c + b \le 360$

$2(144) + 1.5b \ge 500$

$288 + 1.5b \ge 500$

$1.5b \ge 212$

$b \ge 141.\overline{3}$

At least 142 bottles must be sold

6. POLYNOMIALS

6.1 Polynomial Expressions

PRACTICE PROBLEMS

1. a) 3 b) 4 c) 3 d) −1	2. a) x^2-2x+3 b) 2 c) 1 d) 3

3.

$$2x-5x^3-2x(x+5)+15-x^2$$
$$=2x-5x^3-2x^2-10x+15-x^2$$
$$=-5x^3-3x^2-8x+15$$

REGENTS QUESTIONS

1) JUN '16 [2] Ans: 4

2) JUN '18 [19] Ans: 3

3) AUG '16 [28]
No, the leading coefficient is the coefficient of the term with the highest power, –2.

4) AUG '17 [31]

$$5x+4x^2(2x+7)-6x^2-9x$$
$$=5x+8x^3+28x^2-6x^2-9x$$
$$=8x^3+22x^2-4x$$

6.2 Add and Subtract Polynomials

Practice Problems

1. $8x^2-1$	2. $4x^2+x-1$
3. $7x^3+9x^2-3x-8$	4. $-8x^2-x+5$
5. $2x^2+8x+3$	6. $5n^2-9n+3$
7. $(3x^2+2xy+7)-(6x^2-4xy+3)=$ $3x^2+2xy+7-6x^2+4xy-3=$ $-3x^2+6xy+4$	8. $(a^2+a-1)-(3a^2-2a+5)=$ $a^2+a-1-3a^2+2a-5=$ $-2a^2+3a-6$
9. $(x^2-3x-2)-(2x^2-x+6)=$ $x^2-3x-2-2x^2+x-6=$ $-x^2-2x-8$	10. $(x^2+1)-(3x^2+4x-1)=$ $x^2+1-3x^2-4x+1=$ $-2x^2-4x+2$
11. $\left(9x^2-2x+3\right)-\left(4x^2+7x-5\right)=$ $9x^2-2x+3-4x^2-7x+5=$ $5x^2-9x+8$	12. $\left(9x^2+3x-4\right)-\left(5x^2-7x-6\right)=$ $9x^2+3x-4-5x^2+7x+6=$ $4x^2+10x+2$
13. $\left(6x^2+3x-2\right)-\left(2x^2-5x+8\right)=$ $6x^2+3x-2-2x^2+5x-8=$ $4x^2+8x-10$	14. $\left(-3x^2+6x+7\right)-\left(6x^2-13x+12\right)=$ $-3x^2+6x+7-6x^2+13x-12=$ $-9x^2+19x-5$
15. $\left(x^3+3x^2-2x\right)-\left(x^2+3x-4\right)=$ $x^3+3x^2-2x-x^2-3x+4=$ x^3+2x^2-5x+4	16. $(5x-4)-(5x+4)=$ $5x-4-5x-4=$ -8

Regents Questions

1) JUN '14 [3] Ans: 2

2) JAN '15 [28]
$-2x^2+6x+4$

3) JUN '17 [25]
$5x^2-10$

6.3 Multiply Polynomials

Practice Problems

1. $7x-7x^4$	2. $6r^3-15r$
3. $-15x^3y^3-3x^3y^2$	4. $4x^3+12x^2+8x$
5. $(3w-7)(w)=$ $3w^2-7w$	6. $(c+8)(c-5)=$ $c^2-5c+8c-40=$ $c^2+3c-40$
7. $(3x+2)(x-7)=$ $3x^2-21x+2x-14=$ $3x^2-19x-14$	8. $(x-7)(2x+3)=$ $2x^2+3x-14x-21=$ $2x^2-11x-21$
9. $a^2+2ab+b^2$	10. $(x-6)(x-6)=$ $x^2-6x-6x+36=$ $x^2-12x+36$
11. <table><tr><td></td><td>x</td><td>3</td></tr><tr><td>x</td><td>x^2</td><td>$3x$</td></tr><tr><td>$-y$</td><td>$-xy$</td><td>$-3y$</td></tr><tr><td>-1</td><td>$-x$</td><td>-3</td></tr></table>	12. (3) $ax+by$
13. $(x-1)(2x^2+x-2)=$ $2x^3+x^2-2x-2x^2-x+2=$ $2x^3-x^2-3x+2$	14. $(x^2+2)(x^2-2x+1)=$ $x^4-2x^3+x^2+2x^2-4x+2=$ $x^4-2x^3+3x^2-4x+2$

Regents Questions

1) JAN '15 [10] Ans: 2
2) AUG '15 [9] Ans: 3
3) AUG '15 [24] Ans: 4
4) JAN '16 [10] Ans: 3
5) JUN '16 [10] Ans: 2
6) AUG '16 [12] Ans: 3
7) JAN '17 [7] Ans: 4
8) JAN '18 [13] Ans: 3
9) JUN '18 [3] Ans: 2
10) AUG '18 [13] Ans: 1
11) AUG '18 [24] Ans: 3
12) JAN '19 [11] Ans: 3

13) AUG '14 [28]

	x	5
$2x^2$	$2x^3$	$10x^2$
$7x$	$7x^2$	$35x$
-10	$-10x$	-50

$2x^3+17x^2+25x-50$

14) JUN '15 [28]

$(2x^2-5x+7)(\frac{1}{2}x^2)=$

$x^4-\frac{5}{2}x^3+\frac{7}{2}x^2$

6.4 Divide a Polynomial by a Monomial

Practice Problems

1. $\frac{2x+4}{2}=\frac{2x}{2}+\frac{4}{2}=x+2$	2. $\frac{x^2+2x}{x}=\frac{x^2}{x}+\frac{2x}{x}=x+2$
3. $\frac{14ab+28b}{14b}=\frac{14ab}{14b}+\frac{28b}{14b}=a+2$	4. $\frac{6x^3+9x^2+3x}{3x}=\frac{6x^3}{3x}+\frac{9x^2}{3x}+\frac{3x}{3x}=$ $2x^2+3x+1$
5. $\frac{12x^3-6x^2+2x}{2x}=\frac{12x^3}{2x}-\frac{6x^2}{2x}+\frac{2x}{2x}=$ $6x^2-3x+1$	6. $\frac{16x^3-12x^2+4x}{4x}=\frac{16x^3}{4x}-\frac{12x^2}{4x}+\frac{4x}{4x}=$ $4x^2-3x+1$
7. $\frac{2x^6-18x^4+2x^2}{2x^2}=\frac{2x^6}{2x^2}-\frac{18x^4}{2x^2}+\frac{2x^2}{2x^2}=$ x^4-9x^2+1	8. $\frac{8x^5-2x^4+4x^3-6x^2}{2x^2}=$ $4x^3-x^2+2x-3$
9. $\frac{45a^4b^3-90a^3b}{15a^2b}=3a^2b^2-6a$	10. $\frac{24x^2y^6-16x^6y^2+4xy^2}{4xy^2}=6xy^4-4x^5+1$

7. IRRATIONAL NUMBERS

7.1 Simplifying Radicals

PRACTICE PROBLEMS

1. $\sqrt{12}=\sqrt{\boxed{2\cdot 2}\cdot 3}=2\sqrt{3}$	2. $\sqrt{50}=\sqrt{2\cdot\boxed{5\cdot 5}}=5\sqrt{2}$
3. $\sqrt{32}=\sqrt{\boxed{2\cdot 2}\cdot\boxed{2\cdot 2}\cdot 2}=2\cdot 2\cdot\sqrt{2}=4\sqrt{2}$	4. $4\sqrt{75}=4\sqrt{3\cdot\boxed{5\cdot 5}}=4\cdot 5\sqrt{3}=20\sqrt{3}$
5. $5\sqrt{20}=5\sqrt{\boxed{2\cdot 2}\cdot 5}=5\cdot 2\sqrt{5}=10\sqrt{5}$	6. $3\sqrt{45}=3\sqrt{\boxed{3\cdot 3}\cdot 5}=3\cdot 3\sqrt{5}=9\sqrt{5}$
7. $5\sqrt{72}=5\sqrt{\boxed{2\cdot 2}\cdot 2\cdot\boxed{3\cdot 3}}=$ $5\cdot 2\cdot 3\sqrt{2}=30\sqrt{2}$	8. $2\sqrt{128}=2\sqrt{\boxed{2\cdot 2}\cdot\boxed{2\cdot 2}\cdot\boxed{2\cdot 2}\cdot 2}=$ $2\cdot 2\cdot 2\cdot 2\sqrt{2}=16\sqrt{2}$
9. $-3\sqrt{48}=-3\sqrt{16\cdot 3}=-3\cdot 4\sqrt{3}=-12\sqrt{3}$	10. $-\sqrt{98}=-\sqrt{49\cdot 2}=-7\sqrt{2}$
11. $2\sqrt{108}=2\sqrt{36\cdot 3}=2\cdot 6\sqrt{3}=12\sqrt{3}$	12. $3\sqrt{250}=3\sqrt{25\cdot 10}=3\cdot 5\sqrt{10}=15\sqrt{10}$
13. $\frac{\sqrt{32}}{4}=\frac{4\sqrt{2}}{4}=\sqrt{2}$	14. $\frac{7\sqrt{18}}{3}=\frac{7\cdot 3\sqrt{2}}{3}=7\sqrt{2}$

7.2 Operations with Radicals

PRACTICE PROBLEMS

1. $\sqrt{75}+\sqrt{3}=5\sqrt{3}+\sqrt{3}=6\sqrt{3}$	2. $\sqrt{27}+\sqrt{12}=3\sqrt{3}+2\sqrt{3}=5\sqrt{3}$
3. $\sqrt{50}+\sqrt{32}=5\sqrt{2}+4\sqrt{2}=9\sqrt{2}$	4. $\sqrt{27}+\sqrt{108}=3\sqrt{3}+6\sqrt{3}=9\sqrt{3}$
5. $\sqrt{28}+\sqrt{63}=2\sqrt{7}+3\sqrt{7}=5\sqrt{7}$	6. $\sqrt{150}+\sqrt{24}=5\sqrt{6}+2\sqrt{6}=7\sqrt{6}$
7. $3\sqrt{2}+\sqrt{8}=3\sqrt{2}+2\sqrt{2}=5\sqrt{2}$	8. $\sqrt{72}-3\sqrt{2}=6\sqrt{2}-3\sqrt{2}=3\sqrt{2}$
9. $5\sqrt{7}+3\sqrt{28}=5\sqrt{7}+6\sqrt{7}=11\sqrt{7}$	10. $2\sqrt{50}-\sqrt{2}=10\sqrt{2}-\sqrt{2}=9\sqrt{2}$
11. $6\sqrt{50}+6\sqrt{2}=30\sqrt{2}+6\sqrt{2}=36\sqrt{2}$	12. $\sqrt{25}-2\sqrt{3}+\sqrt{27}+2\sqrt{9}=$ $5-2\sqrt{3}+3\sqrt{3}+6=11+\sqrt{3}$
13. $\sqrt{6}\cdot\sqrt{15}=\sqrt{90}=3\sqrt{10}$	14. $4\sqrt{2}\cdot 2\sqrt{6}=8\sqrt{12}=16\sqrt{3}$
15. $\sqrt{90}\cdot\sqrt{40}-\sqrt{8}\cdot\sqrt{18}=$ $\sqrt{3600}-\sqrt{144}=$ $60-12=48$	16. $3\sqrt{20}(2\sqrt{5}-7)=$ $6\sqrt{100}-21\sqrt{20}=$ $60-42\sqrt{5}$
17. $3\sqrt{7}\left(\sqrt{14}+4\sqrt{56}\right)=$ $3\sqrt{7}\left(\sqrt{14}+8\sqrt{14}\right)=$ $3\sqrt{98}+24\sqrt{98}=27\sqrt{98}=$ $27\sqrt{36\cdot 2}=189\sqrt{2}$	18. $\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)=$ $9-3\sqrt{5}+3\sqrt{5}-5=$ $9-5=4$
19. $y\sqrt{3}-\left(4\sqrt{2}+3y\sqrt{3}\right)=$ $y\sqrt{3}-4\sqrt{2}-3y\sqrt{3}=$ $-2y\sqrt{3}-4\sqrt{2}$	20. $3\sqrt{8}=6\sqrt{2}$ $P=2\left(6\sqrt{2}+2\right)+2\left(2\sqrt{2}+1\right)$ $=12\sqrt{2}+4+4\sqrt{2}+2=16\sqrt{2}+6$ $A=\left(6\sqrt{2}+2\right)\left(2\sqrt{2}+1\right)$ $=12(2)+6\sqrt{2}+4\sqrt{2}+2=10\sqrt{2}+26$
21. $\frac{\sqrt{65}}{\sqrt{5}}=\sqrt{13}$	22. $\frac{20\sqrt{100}}{4\sqrt{2}}=5\sqrt{50}=25\sqrt{2}$
23. $\frac{\sqrt{84}}{2\sqrt{3}}=\frac{1}{2}\sqrt{28}=\frac{1}{2}\cdot 2\sqrt{7}=\sqrt{7}$	24. $\frac{6\sqrt{20}}{3\sqrt{5}}=2\sqrt{4}=2\cdot 2=4$

25. $\frac{3\sqrt{75}+\sqrt{27}}{3}=\frac{15\sqrt{3}+3\sqrt{3}}{3}=$ $\frac{18\sqrt{3}}{3}=6\sqrt{3}$	26. $\frac{16\sqrt{21}}{2\sqrt{7}}-5\sqrt{12}=$ $8\sqrt{3}-10\sqrt{3}=-2\sqrt{3}$
27. $\frac{\sqrt{48}-5\sqrt{27}+2\sqrt{75}}{\sqrt{3}}=$ $\sqrt{16}-5\sqrt{9}+2\sqrt{25}=$ $4-15+10=-1$	28. $\frac{\sqrt{27}+\sqrt{75}}{\sqrt{12}}=\frac{3\sqrt{3}+5\sqrt{3}}{2\sqrt{3}}=\frac{8\sqrt{3}}{2\sqrt{3}}=4$

7.3 Rationalizing Denominators

Practice Problems

1.

$$\frac{1}{\sqrt{7}}\cdot\left(\frac{\sqrt{7}}{\sqrt{7}}\right)=\frac{\sqrt{7}}{7}$$

2.

$$\frac{6}{\sqrt{2}}\cdot\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{6\sqrt{2}}{2}=3\sqrt{2}$$

3.

$$\frac{5}{\sqrt{10}}\cdot\left(\frac{\sqrt{10}}{\sqrt{10}}\right)=\frac{5\sqrt{10}}{10}=\frac{\sqrt{10}}{2}$$

4.

$$\frac{6}{\sqrt{21}}\cdot\left(\frac{\sqrt{21}}{\sqrt{21}}\right)=\frac{6\sqrt{21}}{21}=\frac{2\sqrt{21}}{7}$$

5.

$$\frac{8}{3\sqrt{6}}\cdot\left(\frac{\sqrt{6}}{\sqrt{6}}\right)=\frac{8\sqrt{6}}{3(6)}=\frac{8\sqrt{6}}{18}=\frac{4\sqrt{6}}{9}$$

6.

$$\frac{10\sqrt{2}}{\sqrt{5}}\cdot\left(\frac{\sqrt{5}}{\sqrt{5}}\right)=\frac{10\sqrt{10}}{5}=2\sqrt{10}$$

7.

$$\frac{2}{\sqrt{3}}\times\frac{\sqrt{2}}{5}=\frac{2\sqrt{2}}{5\sqrt{3}}$$

$$\frac{2\sqrt{2}}{5\sqrt{3}}\cdot\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{2\sqrt{2}\sqrt{3}}{5(3)}=\frac{2\sqrt{6}}{15}$$

8.

$$\sqrt{\frac{16}{3}}=\frac{\sqrt{16}}{\sqrt{3}}=\frac{4}{\sqrt{3}}$$

$$\frac{4}{\sqrt{3}}\cdot\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{4\sqrt{3}}{3}$$

9.

Rationalize denominators of both fractions.

$$\frac{1}{\sqrt{3}}\cdot\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{\sqrt{3}}{3}\qquad\frac{1}{\sqrt{2}}\cdot\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{\sqrt{2}}{2}$$

LCD is 6.

$$\frac{\sqrt{3}}{3}\cdot\left(\frac{2}{2}\right)+\frac{\sqrt{2}}{2}\cdot\left(\frac{3}{3}\right)=\frac{2\sqrt{3}}{6}+\frac{3\sqrt{2}}{6}$$

$$=\frac{2\sqrt{3}+3\sqrt{2}}{6}$$

10.

Rationalize denominators of both fractions.

$$\frac{1}{\sqrt{2}}\cdot\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{\sqrt{2}}{2}\qquad\frac{3}{\sqrt{5}}\cdot\left(\frac{\sqrt{5}}{\sqrt{5}}\right)=\frac{3\sqrt{5}}{5}$$

LCD is 10.

$$\frac{\sqrt{2}}{2}\cdot\left(\frac{5}{5}\right)+\frac{3\sqrt{5}}{5}\cdot\left(\frac{2}{2}\right)=\frac{5\sqrt{2}}{10}+\frac{6\sqrt{5}}{10}$$

$$=\frac{5\sqrt{2}+6\sqrt{5}}{10}$$

11.

Rationalize denominators of both fractions.

$$\frac{3}{\sqrt{5}}\cdot\left(\frac{\sqrt{5}}{\sqrt{5}}\right)=\frac{3\sqrt{5}}{5}$$

$$\frac{4}{\sqrt{6}}\cdot\left(\frac{\sqrt{6}}{\sqrt{6}}\right)=\frac{4\sqrt{6}}{6}=\frac{2\sqrt{6}}{3}$$

LCD is 15.

$$\frac{3\sqrt{5}}{5}\cdot\left(\frac{3}{3}\right)+\frac{2\sqrt{6}}{3}\cdot\left(\frac{5}{5}\right)=\frac{9\sqrt{5}}{15}+\frac{10\sqrt{6}}{15}$$

$$=\frac{9\sqrt{5}+10\sqrt{6}}{15}$$

12.

$$\frac{3-\sqrt{8}}{\sqrt{3}}\cdot\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{\sqrt{3}\left(3-\sqrt{8}\right)}{3}=$$

$$\frac{3\sqrt{3}-\sqrt{24}}{3}=\frac{3\sqrt{3}-2\sqrt{6}}{3}=\sqrt{3}-\tfrac{2}{3}\sqrt{6}$$

13.

$$\sqrt{\frac{4}{3}}-\sqrt{\frac{3}{4}}=\frac{\sqrt{4}}{\sqrt{3}}-\frac{\sqrt{3}}{\sqrt{4}}=\frac{2}{\sqrt{3}}-\frac{\sqrt{3}}{2}=$$

$$\frac{2}{\sqrt{3}}\cdot\left(\frac{\sqrt{3}}{\sqrt{3}}\right)-\frac{\sqrt{3}}{2}=\frac{2\sqrt{3}}{3}-\frac{\sqrt{3}}{2}=$$

$$\frac{2\sqrt{3}}{3}\cdot\left(\frac{2}{2}\right)-\frac{\sqrt{3}}{2}\cdot\left(\frac{3}{3}\right)=\frac{4\sqrt{3}}{6}-\frac{3\sqrt{3}}{6}=\frac{\sqrt{3}}{6}$$

7.4 Closure

Practice Problems

1. (2) $\sqrt{8}$ $-\sqrt{16}=-4$, $\sqrt{64}=8$, $\sqrt{\frac{1}{64}}=\frac{1}{8}$	2. Irrational. 3 is not a perfect square.
3. Irrational. π is irrational, and the quotient of an irrational number and a non-zero rational number is irrational.	4. Irrational. $\sqrt{29}$ is irrational since 29 is not a perfect square. The numerator is the difference of a non-zero rational and irrational, so the numerator is irrational. The fraction is the quotient of an irrational and a non-zero rational, so it is irrational.
5. *x* could be 0, $\sqrt{3}$, $\sqrt{12}$, $\frac{1}{\sqrt{3}}$, etc.	6. $\pi \approx 3.141593$ and $\frac{22}{7} \approx 3.142857$ $\pi - 3.14 \approx 0.001593$ $\frac{22}{7} - \pi \approx 0.001264$ So, $\frac{22}{7}$ is a closer approximation.

Regents Questions

1) JUN '14 [13] Ans: 3
2) AUG '14 [1] Ans: 1
3) JUN '15 [8] Ans: 2
4) AUG '15 [22] Ans: 2
5) JAN '16 [4] Ans: 1
6) JAN '18 [8] Ans: 3
7) JAN '19 [3] Ans: 3
8) JAN '15 [25]
   correct; 4.2 is rational and $\sqrt{2}$ is irrational, and the sum of a rational and irrational is always irrational
9) JUN '16 [26]
   $3\sqrt{2} \cdot 8\sqrt{18} = 24\sqrt{36} = 24(6) = 144$
   144 is an integer and all integers are rational.
10) AUG '16 [29]
    The sum is $7\sqrt{2}$, which is irrational. 7 is rational and $\sqrt{2}$ is irrational, and the product of a rational and irrational is always irrational.
11) JAN '17 [28]
    No. The second fraction is irrational because it is the quotient of an irrational and a rational. The sum is irrational because it is the sum of a rational and an irrational.
12) JUN '17 [27]
    Irrational. 7 is rational and $\sqrt{2}$ is irrational, and the difference of a rational and irrational is always irrational.
13) AUG '17 [25]
    a is irrational. b and c are rational (c = 15). $a + b$ is irrational because the sum of a rational and irrational is always irrational, and $b + c$ is rational because the sum of two rationals is always rational.
14) JUN '18 [31]
    Rational; $\sqrt{16} = 4$, so it is rational, and $\frac{4}{7}$ is a ratio of two integers, so it is also rational. The product of two rationals is always rational. (The product is $\frac{16}{7}$.)

8. UNIVARIATE DATA

8.1 Types of Data

PRACTICE PROBLEMS

1. (4)	2. (3)

8.2 Frequency Tables and Histograms

Practice Problems

1. Add the frequencies:
   2 + 4 + 5 + 4 + 1 = 16

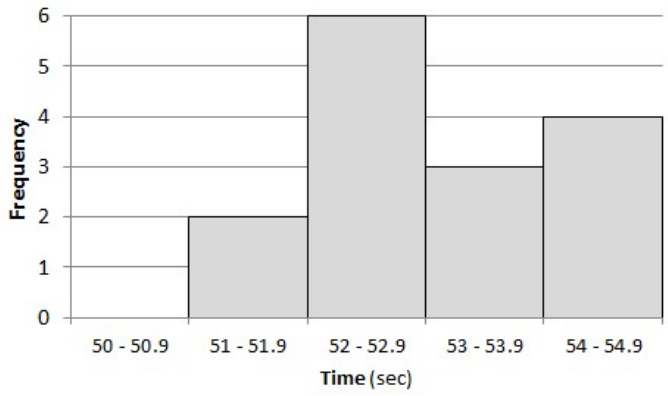
2. Add the frequencies:
   7 + 10 + 3 + 5 = 25

3. 25 – 18 = 7

4. 20

5. 3; 0; 20

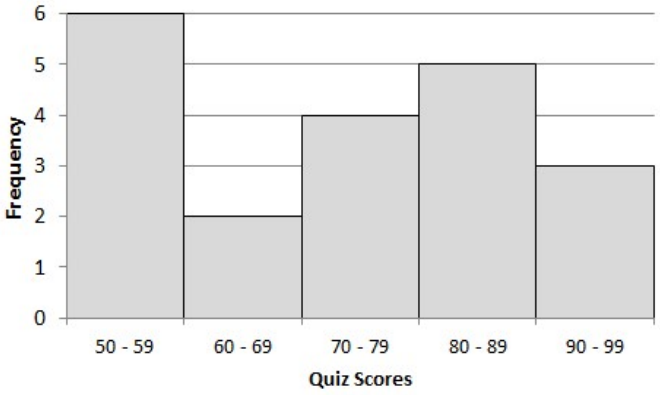
6.



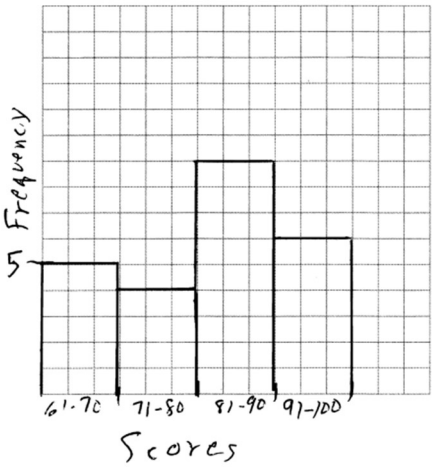
7.

Mathematics Quiz Scores

Interval	Tally	Frequency
50–59	𝍸 I	6
60–69	II	2
70–79	IIII	4
80–89	𝍸	5
90–99	III	3

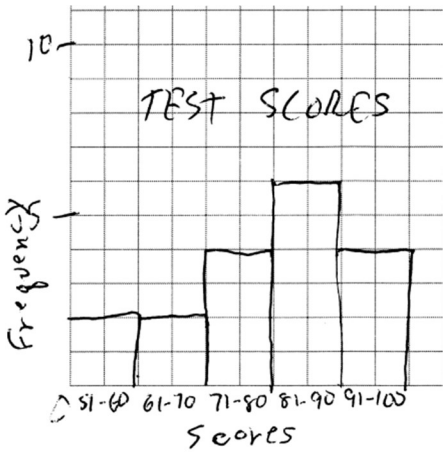


8.



9.

Interval	Tally	Frequency
51–60	II	2
61–70	II	2
71–80	IIII	4
81–90	IIII I	6
91–100	IIII	4



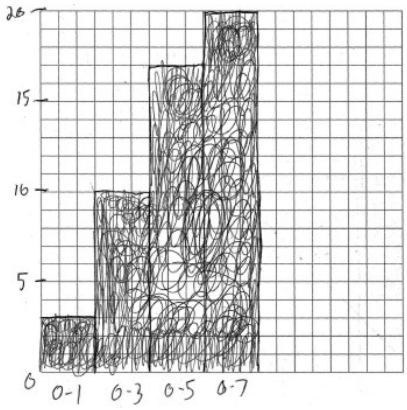
10.

Number of Days Outside

Interval	Tally	Frequency
0–1	III	3
2–3	IIII II	7
4–5	IIII II	7
6–7	III	3

Number of Days Outside

Interval	Cumulative Frequency
0–1	3
0–3	10
0–5	17
0–7	20



8.3 Central Tendency

Practice Problems

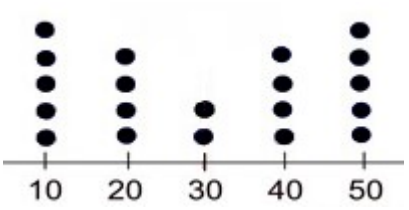
1. mode	2. median
3. (a)	4. They are all divided by two as well.
5. The mean increased by five and the range remained the same.	6. mean ≈ 11.4, median = 12, mode = 7, they all increase by 5
7. City A (22)	8. mean = 79, median = 79, mode = 78
9. (2) mode = median = 6	10. (1) mean = 17, median = 18, mode = 22
11. (3) mean ≈ 85.6, median = 88, mode = 92	12. an outlier such as a very low score could greatly affect the range without affecting the median
13. mean = 22, median = 20, mode = 20	14. 131 – 150 There are 44 total scores, so the median would be the average of the 22nd and 23rd highest scores.
15. mean = 225000, median = 175000, the median because the mean is higher than all but one of the values (an outlier)	16. 71-80; out of 31 students, the 16th lowest value is the median, which is within the 41-80 interval, or 71-80 interval on the related frequency table

Regents Questions

1) AUG '14 [4] Ans: 3
2) JUN '15 [20] Ans: 3
3) JAN '18 [16] Ans: 1

8.4 Distribution

Practice Problems

1. Skewed to the right.	2. 
3. left-skewed; no outliers	4. symmetrical, but with outliers at 9.45

Regents Questions

1) JAN '17 [20] Ans: 4

8.5 Standard Deviation

Practice Problems

1. The population is all the bolts in the shipment. The sample is the 100 selected bolts.	2. The population is all the mall shoppers. The sample is every sixth person within the 3 hour period.
3. (2)	4. (1)
5. The first set, as shown by the smaller SD.	6. McCrane; a larger SD means more variability
5. mean = 66, SD ≈ 30.4	6. mean ≈ 60.7, SD ≈ 15.1
7. SD ≈ 16.8	8. SD ≈ 0.88
7. mean = 9.46; standard deviation = 3.85	8. mean $= \frac{440}{10} = 44$ $(51-44)^2 = 49, (48-44)^2 = 16$, etc. $\frac{49+16+9+4+1+1+9+16+16+25}{10-1} = \frac{146}{9} = 16.\overline{2}$ SD $= \sqrt{16.\overline{2}} \approx 4.0$
9. mean = \$610, SD ≈ 14.7	10. SD ≈ 8.1

Regents Questions

1) AUG '15 [19] Ans: 1
2) JAN '19 [31]
   Los Angeles because the standard deviation for LA (≈3.64) is less than the standard deviation for Miami (≈7.23)

8.6 Percentiles and Quartiles

Practice Problems

1. 25% of 40 = 10 students

2. $\frac{95{,}000}{125{,}000} = 76\%$, so the 76th percentile.

3. $\frac{22}{30} = 73\frac{1}{3}\%$, so the 73rd percentile.

4. $p = \frac{b}{n} = \frac{5}{10} = 0.5$,
so 70 is the 50th percentile.

5. second quartile = median = $\frac{35+45}{2}$ = 40

6.
5, 6, (7), 8, 12, (14), 17, 17, (18), 19, 19

$Q_1 = 7$, $Q_2 = 14$, $Q_3 = 18$

7.
3, 6, 7, | 7, 8, 9, | 9, 9, 10, | 12, 13, 15

$Q_1 = 7$, $Q_2 = 9$, $Q_3 = 11$

8.
21, 28, | 28, 32, (33), 41, 45, | 50, 53

$Q_1 = 28$, $Q_2 = 33$, $Q_3 = 47.5$, IQR = 19.5

9.
71, 71, (72), 74, 74, | 75, 78, (79), 79, 83

$Q_3 = 79$ and $Q_1 = 72$, so IQR = 7

10. $Q_1 = 70$, $Q_2 = 80$, $Q_3 = 90$

11. The corresponding frequency table would show:

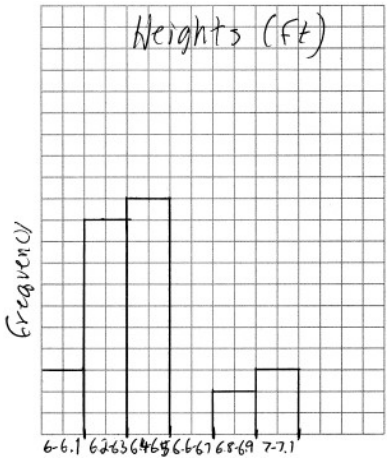
Minutes Used	Frequency
31–40	2
41–50	3
51–60	5
61–70	9
71–80	11

25% of 30 is 7.5, so the first quartile would be between the 7th and 8th smallest values out of 30. This falls within the 51–60 interval.

Regents Questions

1) JUN '14 [19] Ans: 3
2) JUN '16 [20] Ans: 3
3) JUN '17 [15] Ans: 4
4) AUG '17 [34]

Interval	Frequency
6.0 – 6.1	3
6.2 – 6.3	10
6.4 – 6.5	11
6.6 – 6.7	0
6.8 – 6.9	2
7.0 – 7.1	3

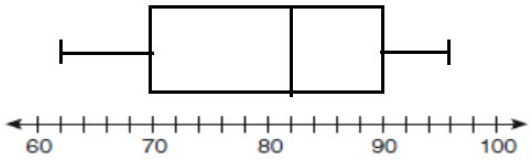
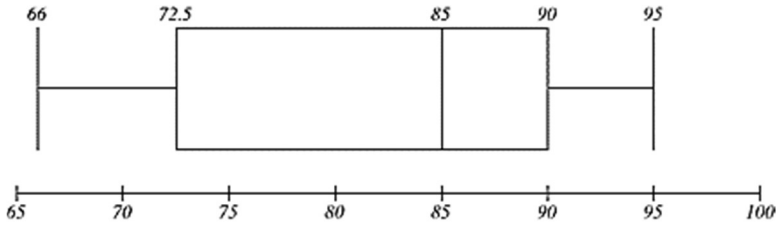
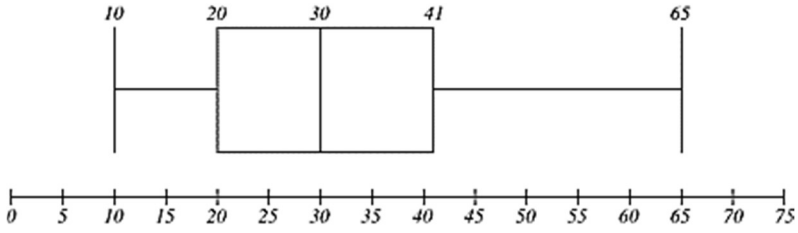


For 29 players, the upper quartile would be taller than 21 heights $(29 \times 0.75 = 21.75)$. The 22nd value is in the 6.4 – 6.5 interval.

5) AUG '18 [31]
4th Period because the IQR and σ_x are greater for 4th Period.

8.7 Box Plots

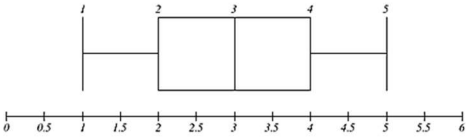
Practice Problems

1. 81	2. 75
3. 10	4. 84
5. 30	6. 4
7. 75 – 15 = 60	8. 25%
9. (4) 75–88	
10. $Q_1 = 77$, $Q_2 = 87$, $Q_3 = 91$ 	11. $Q_1 = 70$, $Q_2 = 82$, $Q_3 = 90$ 
12. 	
13. 	

Regents Questions

1) JAN '15 [14] Ans: 4
2) AUG '16 [3] Ans: 4
3) JUN '18 [5] Ans: 2

4) JUN '14 [32]



9. BIVARIATE DATA

9.1 Two-Way Frequency Tables

Practice Problems

1.
$\frac{15}{113} \approx 13.3\%$ of the students are undecided.
$\frac{31}{60} \approx 51.7\%$ of the 9th graders are watching.

2.

	Fiction	Nonfiction	Total
Hardcover	28	52	80
Paperback	94	36	130
Total	122	88	210

	Fiction	Nonfiction	Total
Hardcover	13.3%	24.8%	38.1%
Paperback	44.8%	17.1%	61.9%
Total	58.1%	41.9%	100%

3. Given data in bold below.

	Coca-Cola	Sprite	Total
Table	16	**14**	30
Garbage	34	8	**42**
Total	**50**	22	**72**

Regents Questions

1) JUN '16 [15] Ans: 4
2) JAN '17 [5] Ans: 2
3) JUN '18 [9] Ans: 1
4) AUG '18 [14] Ans: 2
5) JAN '15 [26]
$\frac{(33+12)}{180} = 25\%$
6) JAN '16 [30]
$\frac{70}{105} = \frac{2}{3}$; $\frac{2}{3} \times 351 = 234$
234 males
7) JUN '17 [29]

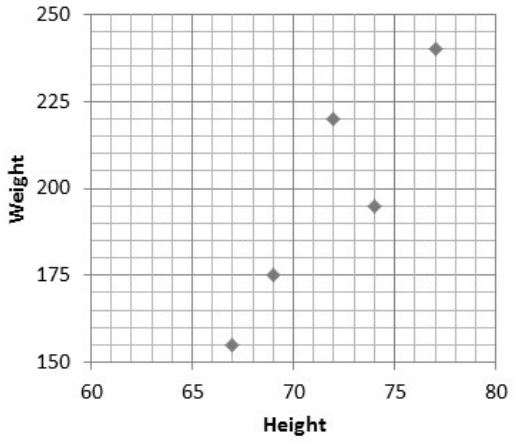
	Watch Sports	Don't Watch Sports	Total
Like Pop	26	28	54
Don't Like Pop	34	12	46
Total	60	40	100

9.2 Scatter Plots

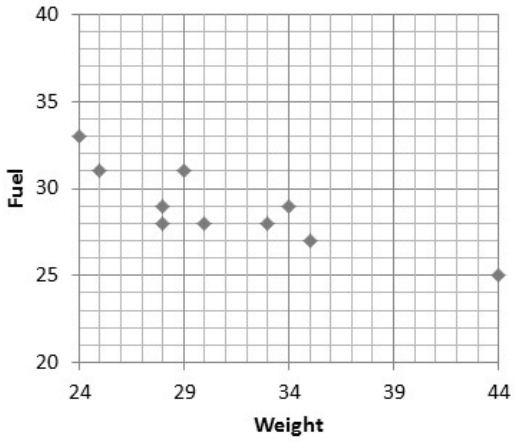
Practice Problems

1. (2)

1.



2.



9.3 Correlation and Causality

Practice Problems

1. (3)	2. (2)
3. A. positive: children usually gain weight as they age and grow B. negative: as the volume of water increases, the remaining space decreases C. none: shoe size and hair length are unrelated D. positive: more people go to the beach when the temperature is higher	4. A. positive, causal B. negative, causal C. positive, not causal, the size and severity of the fire, which results in more firefighters being called D. negative, not causal, the degree of civilization and industrialization over time

Regents Questions

1) JAN '17 [13] Ans: 2
2) AUG '17 [8] Ans: 2
3) AUG '18 [21] Ans: 3

9.4 Identify Correlation in Scatter Plots

Practice Problems

1. (1)	
2. positive correlation	3. negative correlation
4. negative correlation	5. positive correlation
6. no correlation	7. positive correlation

Regents Questions

1) JUN '16 [4] Ans: 2

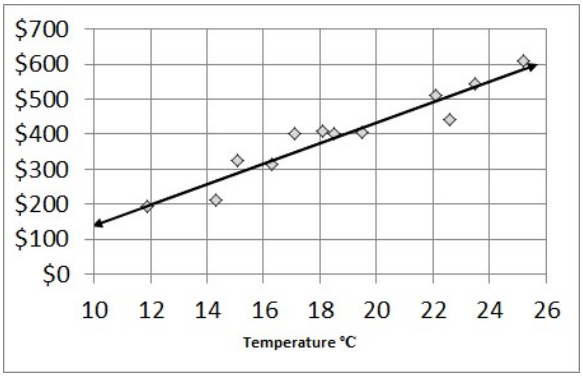
9.5 Lines of Fit

Practice Problems

1. (a) 80 wpm
   (b) 9 wpm

2. Line A. Most of the points are closer to Line A than to Line B.

3.



4. $y = 5x + 25$

5. $y = 0.2x + 7.5$

6. $y = 2x + 5.14$

7. (3) $y = 1{,}000x + 15{,}000$

8. The prediction for the 35 year old is more likely to be accurate, since it is an interpolation rather than an extrapolation.

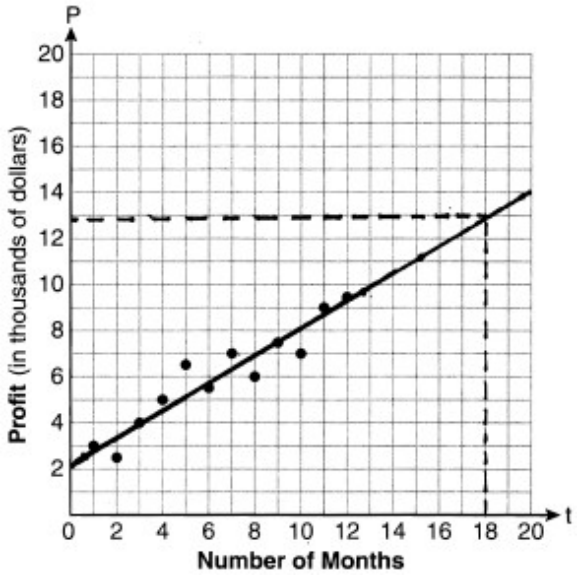
9. (4) $y = \frac{3}{2}x + 1$

10. (2) 72

11. (3) 480

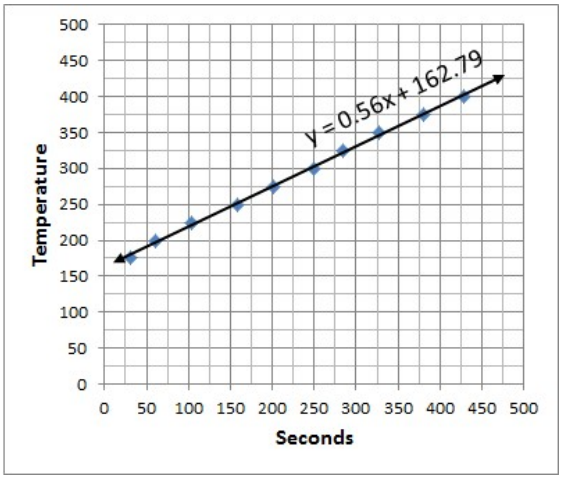
12. (3) $42,500

13.



No, the line crosses near (18,13)

14.



$y = 0.56x + 162.79$

15. $y = -0.112x + 23.448$; -5°C

16. $y = -35.5x + 457.5$; 103

Regents Questions

1) AUG '14 [21] Ans: 4

2) JAN '19 [1] Ans: 2

3) SEP '13 [7]
$y = 0.05x - 0.92$

4) AUG '16 [33]
$y = 17.159x - 2.476$
$y = 17.159(0.65) - 2.476 \approx 8.7$

5) JAN '18 [34]
$y = -8.5x + 99.2$; the y-intercept represents the original length of the rope; the slope represents how much shorter the rope gets (8.5 cm) after each knot.

9.6 Correlation Coefficients

Practice Problems

1. (1) 0.89	2. (4) 0.90
3. (4)	4. (2) –0.24 It is a weak correlation.
5. a. 0.90 b. –0.40 c. 0.99 d. –0.85 e. 0.50 f. 0	6. $r = 1$
7. $r \approx 0.986$	8. $r \approx -0.999$

Regents Questions

1) JUN '14 [11] Ans: 3
2) JUN '15 [16] Ans: 2
3) AUG '16 [6] Ans: 2
4) JAN '17 [3] Ans: 4
5) JUN '17 [14] Ans: 1
6) AUG '17 [22] Ans: 1
7) JAN '15 [35]
a) 0.94, b) it shows a strong positive relationship between the calories and mg of sodium
8) AUG '15 [36]
$y = 0.16x + 8.27$; 0.97, a strong association
9) JAN '16 [35]
$f(t) = -58t + 6182$; –0.94; yes, because it is close to –1
10) JAN '18 [31]
$y = 0.81x + 15.19$; 0.92; there is a high positive correlation between mathematics and physics scores.
11) JUN '18 [36]
$y = 0.96x + 23.95$; 0.92; there is a high positive correlation between scoring 85 or better on the math and English exams.
12) JAN '19 [34]
$y = 1.9x + 29.8$; $r = 0.3$, which represents a weak correlation between a dog's mass and height.

9.7 Residuals

Practice Problems

1. Actual value – Predicted value = 12,550 – 14,050 = –1,500

2.

Study Time in Hours (x)	Test Score (y)	Predicted Test Score	Residual
0.5	63	62.8	0.2
1	67	67.2	-0.2
1.5	72	71.6	0.4
2	76	76.0	0
2.5	80	80.4	-0.4
3	85	84.8	0.2
3.5	89	89.2	-0.2

3.

a) $y = 0.75(22) - 0.25 = 16.25$. (Since scores cannot be fractional, 16 is a valid answer.)

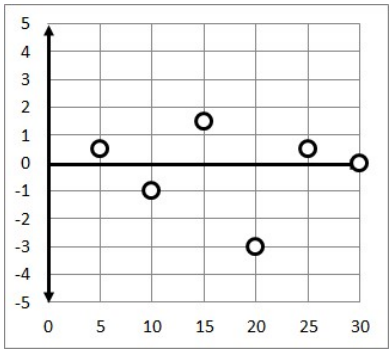
b) $y = 0.75(34) - 0.25 = 25.25$ Residual = 32 – 25.25 = 6.75

c) $y = 0.75(28) - 0.25 = 20.75$ $p - 20.75 = -0.75$

$p = 20$ They scored 20 points.

4.

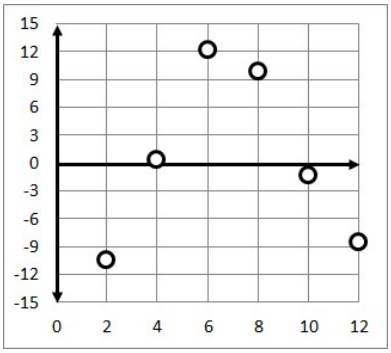
x	y	Predicted Value	Residual
5	3	2.5	0.5
10	4	5.0	-1
15	9	7.5	1.5
20	7	10.0	-3
25	13	12.5	0.5
30	15	15.0	0



Yes. There is no clear pattern in the residual plot.

5.

x	y	Predicted Value	Residual
2	5	15.5	-10.5
4	15	14.7	0.3
6	26	13.9	12.1
8	23	13.1	9.9
10	11	12.3	-1.3
12	3	11.5	-8.5



No. There appears to be a parabola-like pattern in the residual plot.

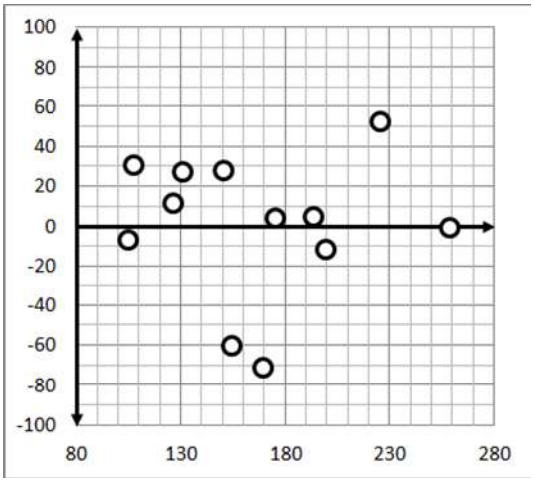
6.

(a) $y = 0.117x + 83.267$

(b) and (c)

Distance (miles)	Airfare ($)	Predicted Price ($)	Residual
576	178	150.7	27.3
370	138	126.6	11.4
612	94	154.9	-60.9
1,216	278	225.5	52.5
409	158	131.1	26.9
1,502	258	259.0	-1.0
946	198	193.9	4.1
998	188	200.0	-12.0
189	98	105.4	-7.4
787	179	175.3	3.7
210	138	107.8	30.2
737	98	169.5	-71.5

(d)



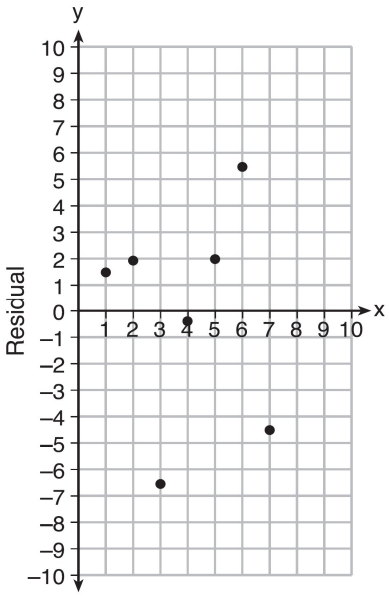
REGENTS QUESTIONS

1) SEP '13 [3] Ans: 2

2) JAN '16 [24] Ans: 3

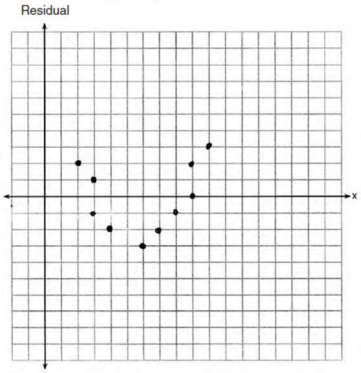
3) SEP '13 [14]

$y = 6.32x + 22.43$



The equation is a good fit for the data because the points on the residual plot are scattered without a pattern.

4) AUG '14 [31]



Poor fit because there is a pattern in the residuals

5) JUN '15 [31]

Graph A; no pattern shows

10. INTRODUCTION TO FUNCTIONS

10.1 Recognize Functions

Practice Problems

1. (2)	2. (2)
3. (1)	4. (2)
5. Yes, each x entry is mapped to a unique y	
6. (1)	7. (2)
8. (1)	
9. (1)	10. (3)

Regents Questions

1) JUN '15 [4] Ans: 3
2) AUG '15 [11] Ans: 2
3) JUN '17 [9] Ans: 3
4) JAN '18 [4] Ans: 2
5) JUN '18 [11] Ans: 4
6) JAN '19 [7] Ans: 4
7) JAN '15 [27]
(−4,1) because the input −4 would lead to two different outputs, which a function cannot have
8) JAN '16 [26]
no, it is not a function because for $x = 2$, there are two different values of y.
9) JAN '17 [32]
Neither is correct. Nora is wrong; a circle is not a function because it fails the vertical line test. Mia's reason is wrong; a circle is not a function because the same x-value maps to multiple values of y.
10) AUG '18 [26]
III and IV are functions. I has two y-values for $x = 6$, and II has two y-values for $x = 1$ and $x = 2$.

10.2 Function Graphs

Practice Problems

1.

x	$f(x)$
0	0
1	3
2	4
3	7
4	2
5	0

2. (a) $f(9)=1$
   (b) $\{\frac{1}{2}, 3\}$

Regents Questions

1) JUN '14 [20] Ans: 1
2) AUG '18 [5] Ans: 1
3) AUG '18 [18] Ans: 1

10.3 Evaluate Functions

Practice Problems

1. $f(3) = -2(3)^2 - 3(3) - 6 =$ $-18 - 9 - 6 = -33$	2. $f(-3) = (-3)^2 - 2(-3) + 1 =$ $9 + 6 + 1 = 16$
3. $f(0) = (0-3)^2 = 9$	4. $f(2) = 0.5^2 = 0.25$
5. $f(3) - g(2) = [3(3) - 4] - [(2)^2] = 1$	6. $h(0) = 2(0) - 1 = -1$ $h(-2) = 2(-2) - 1 = -5$ $h(0) \cdot h(-2) = (-1)(-5) = 5$
7. $-10 = -4x + 2$ $-12 = -4x$ $3 = x$	8. $12 = k(2)^2$ $12 = 4k$ $k = 3$
9. $g(4a) = 2(4a)^2 + 6(4a) - 3 =$ $32a^2 + 24a - 3$	10. $f(a+2) = (a+2)^2 + 2(a+2) - 1 =$ $a^2 + 4a + 4 + 2a + 4 - 1 =$ $a^2 + 6a + 7$
11. $P(125) = 0.0089(125)^2 + 1.1149(125) + 78.4491 \approx 356.9$	

Regents Questions

1) JUN '15 [15] Ans: 1
2) AUG '15 [12] Ans: 3
3) AUG '16 [5] Ans: 2
4) AUG '16 [11] Ans: 4
5) JAN '17 [10] Ans: 1
6) JUN '17 [5] Ans: 1
7) AUG '17 [4] Ans: 3
8) JUN '18 [2] Ans: 4
9) JUN '18 [8] Ans: 4
10) AUG '18 [11] Ans: 4
11) JAN '19 [2] Ans: 1
12) JAN '19 [21] Ans: 4
13) JAN '16 [32]

$f(5) = 8 \cdot 2^5 = 256$

$g(5) = 2^{5+3} = 2^8 = 256$

The functions are equal since they produce the same values for all inputs, *t*. This is shown by the fact that $(8) \cdot 2^t = 2^3 \cdot 2^t = 2^{t+3}$

10.4 Features of Function Graphs

Practice Problems

1. (a) $0 < x < 2$ and $4 < x < 8$ (b) $2 < x < 4$ (c) $8 < x < 10$	2. (a) (–3,5) (b) (1,–3)
3. (a) positive at $x < -2$ and $0 < x < 2.5$; negative at $-2 < x < 0$ and $x > 2.5$ (b) increasing at $-1 < x < 1$; decreasing at $x < -1$ and $x > 1$ (c) relative maximum at (1,2); relative minimum at (–1,–2)	

Regents Questions

1) JUN '14 [9] Ans: 3
2) JAN '17 [21] Ans: 1
3) JUN '17 [1] Ans: 3
4) JUN '18 [20] Ans: 3

10.5 Domain and Range

Practice Problems

1. $\{2, 3, 22, 51\}$	2. $\{x \mid x \neq 0\}$
3. domain: $1 < x \leq 4$ range: $1 < f(x) \leq 7$	4. domain: $-5 \leq x \leq 8$ Range: $-3 \leq y \leq 2$
5. $4 \leq x \leq 13$	6. $0 \leq y \leq 100$
7. $0 \leq x \leq 12$	8. $30 \leq y \leq 80$
9`. the set of counting (natural) numbers	10. $f(5) = 25$ and $f(10) = 40$, so the range is $25 \leq f(x) < 40$
11. (a) $f(x) \geq 0$ (b) $0 \leq f(x) \leq 9$	12. (a) $f(n) = 5n$ (b) whole numbers $n \leq 20$ (c) {0, 5, 10, 15, ... 100}

Regents Questions

1) JUN '14 [2] Ans: 4
2) JUN '14 [17] Ans: 4
3) AUG '14 [11] Ans: 1
4) AUG '14 [23] Ans: 2
5) JAN '15 [6] Ans: 2
6) JUN '15 [9] Ans: 4
7) JAN '16 [15] Ans: 1
8) JAN '16 [19] Ans: 2
9) JUN '16 [23] Ans: 4
10) AUG '16 [20] Ans: 2
11) JAN '17 [19] Ans: 4
12) AUG '17 [10] Ans: 1
13) JAN '18 [12] Ans: 3
14) JUN '18 [16] Ans: 3
15) JUN '18 [21] Ans: 2
16) AUG '18 [6] Ans: 2
17) JAN '19 [14] Ans: 2
18) JAN '19 [17] Ans: 4
19) JUN '14 [30]
yes, because each number in the domain leads to a unique number in the range
20) AUG '17 [29]
Since fractions of cookies may be eaten, the domain is continuous.

11. FUNCTIONS AS MODELS

11.1 Write a Function from a Table

PRACTICE PROBLEMS

1. $f(x)=4x+9$	2. $f(x)=\frac{5}{3}x+10$
3. $f(x)=3(x-1)+7$ $f(x)=3x+4$	4. $f(x)=\frac{1}{2}(x-1)-5$ $f(x)=\frac{1}{2}x-5\frac{1}{2}$
5. $f(x)=-2(x-2)+9$ $f(x)=-2x+13$	6. $f(x)=5(x-11)$ $f(x)=5x-55$

REGENTS QUESTIONS

1) AUG '16 [4] Ans: 4

2) AUG '15 [25]

$h(n)=1.5(n-1)+3$ or

$h(n)=1.5n+1.5$

3) AUG '15 [32]

d	1	2	3	4	5
$T(d)$	30	32	34	36	38

$T(d)=2(d-1)+30$ or

$T(d)=2d+28$;

$T(6)=2(6)+28=40$

4) JAN '17 [35]

$m=\frac{9-7.50}{6-4}=0.75$

$f(x)=0.75(x-4)+7.50$

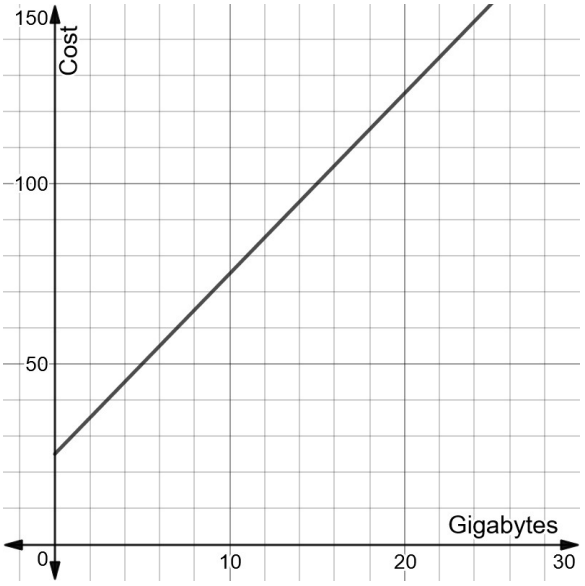
$f(x)=0.75x+4.50$

Each card costs 75¢ and start-up costs were \$4.50.

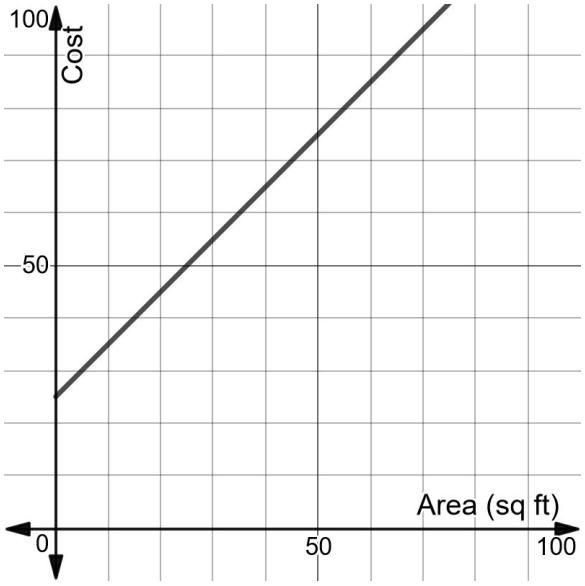
11.2 Graph Linear Functions

Practice Problems

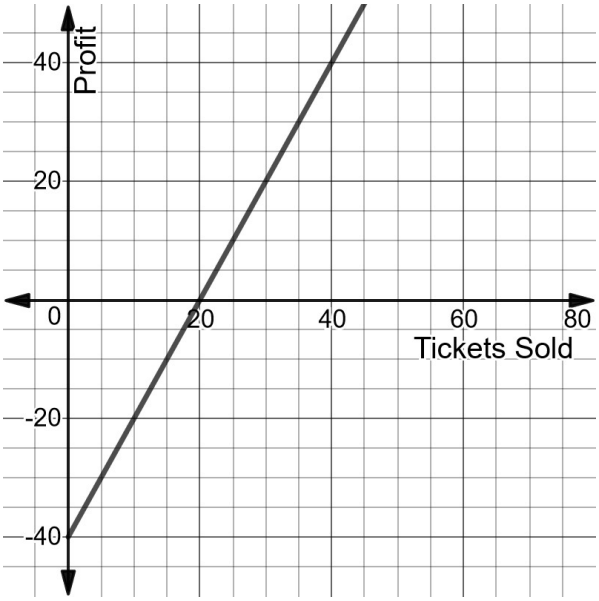
1. $c(g) = 5g + 25$



2. $c(x) = x + 25$

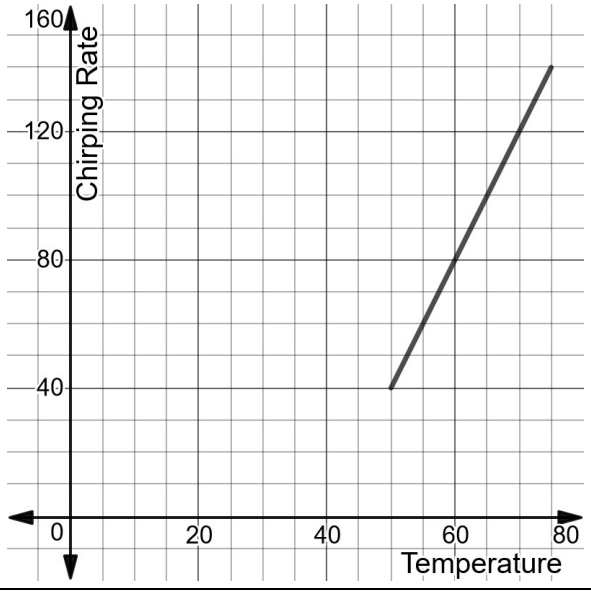


3. $y = 2x - 40$



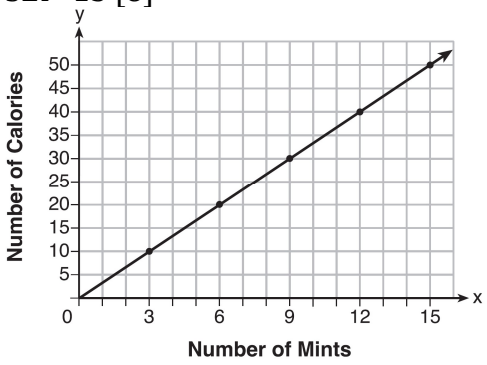
4. Line through points (59,76) and (65,100) has a slope of $\frac{24}{6} = 4$ and y-intercept $b = y - mx = 100 - 4(65) = -160$.

$c(t) = 4t - 160$



Regents Questions

1) SEP '13 [8]



$\frac{10}{3}x = 180$

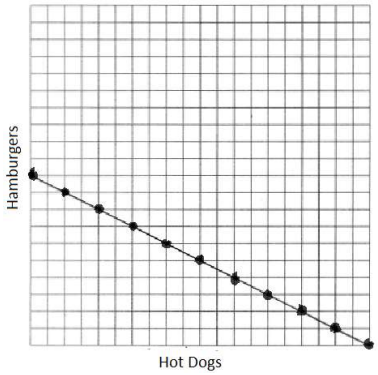
$C(x) = \frac{10}{3}x$; $10x = 540$

$x = 54$

2) AUG '17 [37]

7 sodas cost \$3.50, so they have \$25 left to spend on food.

$1.25x + 2.5y = 25$



There are 11 combinations, represented by the dots in the graph.

11.3 Rate of Change for Linear Functions

Practice Problems

1. negative	2. positive
3. negative	4. positive
5. positive; $m = \frac{348-232}{6-4} = \frac{116}{2} = 58$ mph	6. negative; as the distance travelled increases, the gas in the tank decreases.

Regents Questions

1) AUG '15 [2] Ans: 3
2) JAN '16 [2] Ans: 2
3) AUG '16 [15] Ans: 4
4) JUN '17 [4] Ans: 2
5) JAN '16 [29]
The slope is the amount paid per month and the y-intercept is the initial cost.
6) JUN '16 [30]
There are 2 inches of snow every 4 hours.
7) AUG '17 [33]
a) 10 hrs.

$$55(4) + 65(t-4) = 610$$
$$220 + 65t - 260 = 610$$
$$65t = 650$$
$$t = 10$$

b) 0.3 hrs.

$$55(2) + 65(t-2) = 610$$
$$110 + 65t - 130 = 610$$
$$65t = 630$$
$$t \approx 9.7$$

10 – 9.7 = 0.3

11.4 Average Rate of Change

Practice Problems

1. (a) $m=\frac{9-1}{3-1}=\frac{8}{2}=4$ (b) $m=\frac{4-1}{2-(-1)}=\frac{3}{3}=1$	2. (a) $\frac{5.06-3.91}{1999-1987}=\frac{1.15}{12}\approx 0.096$ (b) $\frac{7.50-5.06}{2009-1999}=\frac{2.44}{10}\approx 0.244$ (b) has the higher average rate of change.
3. $f(5)=5^2+2=27$ $f(15)=15^2+2=227$ $m=\frac{227-27}{15-5}=\frac{200}{10}=20$	4. $f(-3)=(-3)^2+10(-3)+16=-5$ $f(3)=3^2+10(3)+16=55$ $m=\frac{55-(-5)}{3-(-3)}=\frac{60}{6}=10$

Regents Questions

1) MAY '13 [1] Ans: 4
2) JUN '14 [18] Ans: 1
3) AUG '14 [14] Ans: 4
4) JAN '15 [21] Ans: 1
5) JUN '15 [11] Ans: 3
6) AUG '15 [15] Ans: 1
7) JAN '16 [13] Ans: 4
8) JUN '16 [3] Ans: 1
9) AUG '16 [1] Ans: 1
10) AUG '17 [5] Ans: 2
11) JAN '18 [24] Ans: 2
12) JAN '16 [28]
from 1960–1965, because the decrease of 0.15 degrees is the largest change among the intervals (it has the steepest slope)
13) JAN '17 [31]
$\frac{480-140}{7-2}=68 mph$
14) JAN '18 [36]
The domain may include fractions of hours, and the number of hours cannot be negative; $0<t<6$;
$\frac{0-120}{14-6}=\frac{-120}{8}=-15$; the business sold 15 less pairs of shoes per hour
15) AUG '18 [27]
$\frac{3.41-6.26}{9-3}=-0.475$

11.5 Functions of Time

Practice Problems

1. (3)	2. (2)
3. 30 secs.	4. 7 minutes From 7:04 to 7:07 and 7:20 to 7:24
5. a) point B because it is the only point after which her distance from home decreases; b) 5 mins, from point D to point E	6. (a) Spencer starts at (0,20) and McKenna starts at (0,0). (b) McKenna speeds up, as the graph curves upward. The average rate of change increases. (c) At about 3.2 hours. They traveled about 41 miles.

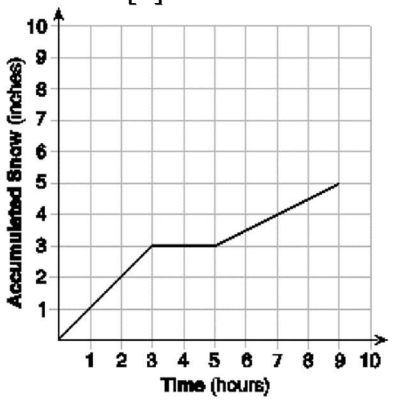
7.

Characteristic of Graph	Interpretation in Terms of the Race
y-intercepts	At 11 A.M. Runner *A* is 10 miles from the finish line and Runner *B* is 7 miles from the finish line.
Slopes	Runner A is decreasing the distance to the finish line at 8 mph and Runner B is running at 3.5 mph.
Point of intersection	The two runners meet after about 2/3 hour at about $4\frac{2}{3}$ miles from the finish line.
x-intercepts	Runner A finishes at 12:15 and Runner B finishes at 1:00.

Regents Questions

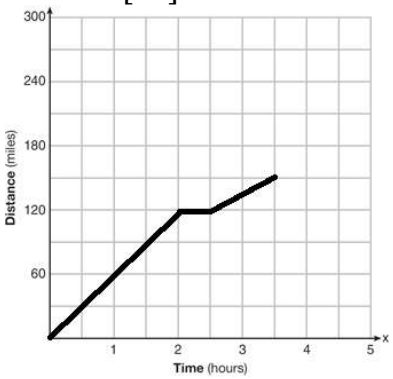
1) JUN '15 [2] Ans: 4

2) MAY '13 [7]



3.5

3) AUG '15 [28]



4) JUN '16 [35]

a) 762 – 192 = 570 miles

92 – 32 = 60 minutes

$\frac{570}{60} = 9.5$ miles per minute

b) $y = 9.5x$

c) $192 + 9.5(120 - 32) = 1028$ miles

5) JUN '17 [34]

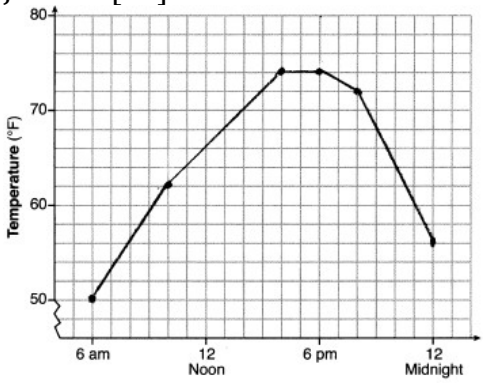
D to *E* because his speed was slower; Craig may have stopped to eat;

$\frac{230}{7} \approx 32.9$ mph

6) JAN '19 [28]

$2 < t < 6$ and $14 < t < 15$ because horizontal lines have a slope of zero.

7) JAN '19 [36]



6 am to 4 pm; $\frac{74-56}{6-12} = -3$

11.6 Systems of Functions

Practice Problems

1. (a) \$50 (b) 5 months; \$125 (c) slope = $\frac{125-75}{5-0}=\frac{50}{5}=\$10$	2. Tasha: $A(x)=60+5x$ Tyson: $B(x)=135-10x$ $60+5x=135-10x$ $15x=75$ $x=5$ 5 weeks
3. (a) $R(x)=25x$ (b) $C(x)=20x+50000$ (c) $25x=20x+50000$ $5x=50000$ $x=10000$ 10,000 widgets	4. (a) $f(h)=36h+50$ (b) $g(h)=39h+35$ (c) $36h+50=39h+35$ $50=3h+35$ $15=3h$ $5=h$ 5 hours

Regents Questions

1) JAN '16 [17] Ans: 1

2) AUG '18 [19] Ans: 3

3) SEP '13 [15]
$f(x)=120x$ and
$g(x)=70x+1600$;
$120x=70x+1600$
$50x=1600$;
$x=32$
$f(35)=4200$, $g(35)=4050$, so Green Thumb is less expensive

4) MAY '13 [8]
a) $A(x)=1.50x+6$ and
$B(x)=2x+2.50$; b)
$1.50x+6=2x+2.50$
$3.50=0.50x$
$7=x$
7 rides; c)
$A(5)=1.50(5)+6=13.50$ and
$B(5)=2(5)+2.50=12.50$ so B has a lower cost.

5) AUG '14 [27]
$185+0.03x=275+0.025x$
$0.005x=90$
$x=18{,}000$

6) JAN '15 [31]
$36+15x=48+10x$
$5x=12$
$x=2.4$

7) AUG '16 [30]
–3 and 1, because the two functions intersect at (–3,4) and (1,3).

11.7 Combine Functions

Practice Problems

1. $h(x)=(x^2+x+1)+(x-5)=$ x^2+2x-4	2. $h(x)=(2x+1)(x-2)=$ $2x^2-3x-2$
3. (a) $R(c)=20c+500$ (b) $E(c)=6c$ (c) $P(c)=R(c)-E(c)=(20c+500)-(6c)=14c+500$	

Regents Questions

1) AUG '14 [6] Ans: 2

2) JUN '16 [25]

$$g(x)=2(2x+1)^2-1$$
$$=2(4x^2+4x+1)-1$$
$$=8x^2+8x+2-1$$
$$=8x^2+8x+1$$

12. EXPONENTIAL FUNCTIONS

12.1 Exponential Growth and Decay

PRACTICE PROBLEMS

1. $x(1.1)^{20}$	2. $x(0.98)^{n}$
3. $2500(1.03)^{4}$	4. $10{,}000(1.2)^{t}$
5. $\$1500(1.05)^{6} \approx \$2010.14$	6. $1000(1.03)^{5} \approx 1159.27$
7. $2000(1.035)^{4} \approx \$2295$	8. $30{,}000(0.95)^{4} \approx 24{,}435.19$
9. $25{,}000(0.8)^{3} = 12{,}800$	10. $3{,}810(1.035)^{5} \approx 4{,}525$
11. $20{,}000(0.88)^{3} = 13{,}629.44$	12. $\$11{,}900(0.87)^{3} \approx \$7{,}800$
13. $\$1.39(1.005)^{12} \approx \$1.48$	14. $256(0.25)^{3} = 4$

Regents Questions

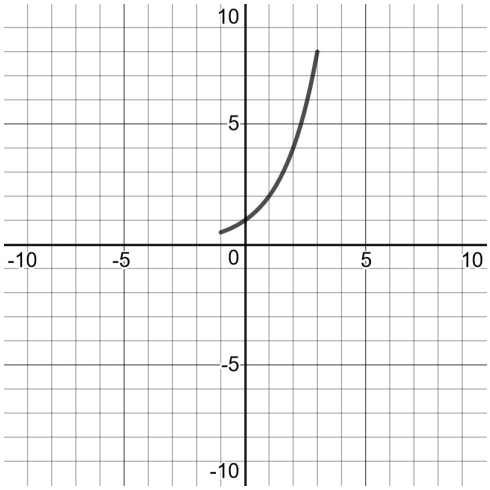
1) JAN '15 [4] Ans: 1
2) JAN '15 [8] Ans: 1
3) JUN '15 [17] Ans: 2
4) AUG '15 [7] Ans: 3
5) JAN '16 [3] Ans: 3
6) JAN '16 [8] Ans: 4
7) JUN '16 [17] Ans: 2
8) AUG '16 [17] Ans: 1
9) AUG '16 [24] Ans: 2
10) JAN '17 [24] Ans: 3
11) JUN '17 [12] Ans: 2
12) AUG '17 [14] Ans: 2
13) AUG '17 [16] Ans: 2
14) AUG '17 [21] Ans: 3
15) JAN '18 [2] Ans: 3
16) JAN '19 [12] Ans: 4
17) JUN '14 [26]
rate of decay; number of milligrams of the substance at the start
18) AUG '14 [26]
$B = 3000(1.042)^t$
19) JUN '15 [29]
$600(1.016)^2 \approx 619.35$
20) AUG '15 [30]
5%; in the decay function $y = a(1-r)^x$, r represents the percent of change, $1-r = 0.95$, so $r = 0.05 = 5\%$
21) JUN '17 [28]
15%; in the decay function $y = a(1-r)^x$, r represents the percent of change, $1-r = 0.85$, so $r = 0.15 = 15\%$
22) JUN '18 [33]
There are 20 rabbits at the start, and their population is growing at 1.4% per day.
$\frac{p(100)-p(50)}{100-50} \approx 0.8$
23) AUG '18 [34]
$V(t) = 25000(0.815)^t$
$V(3) - V(4) \approx 2503.71$
24) JAN '19 [33]
$V = 450(1.025)^t$
No, because $(1.025)^{20} < 2$.

12.2 Graphs of Exponential Functions

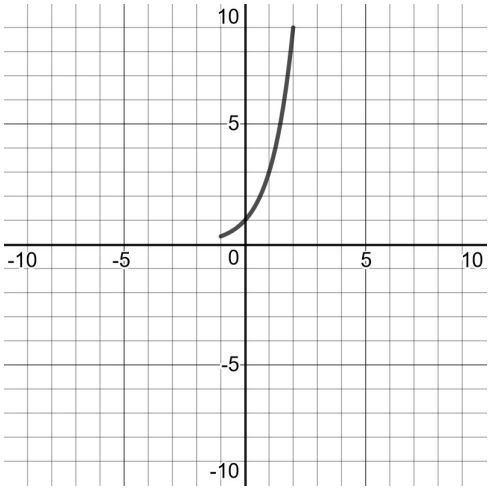
Practice Problems

1. (4)

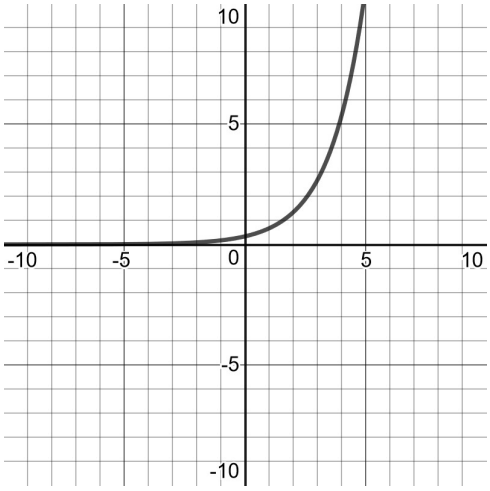
2.



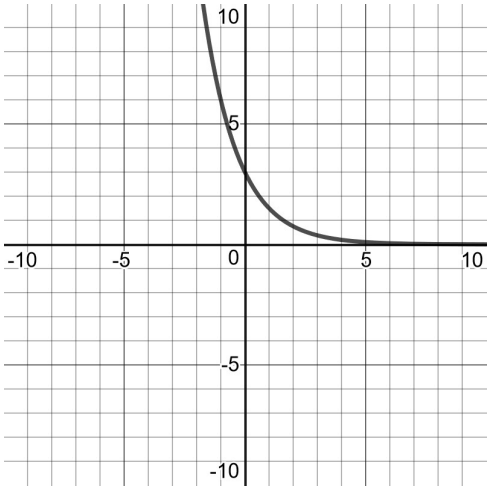
3.



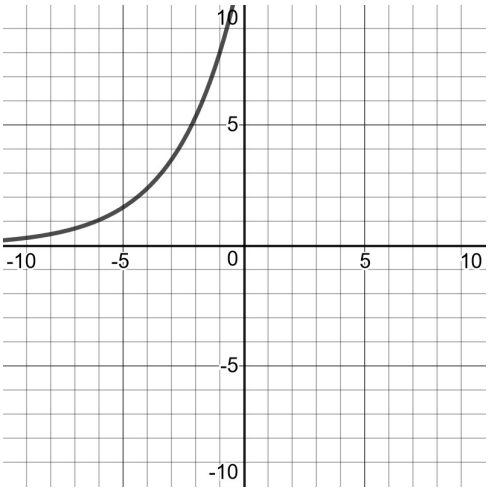
4.



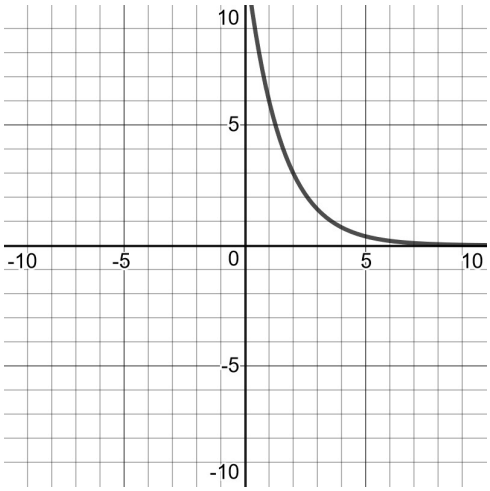
5.



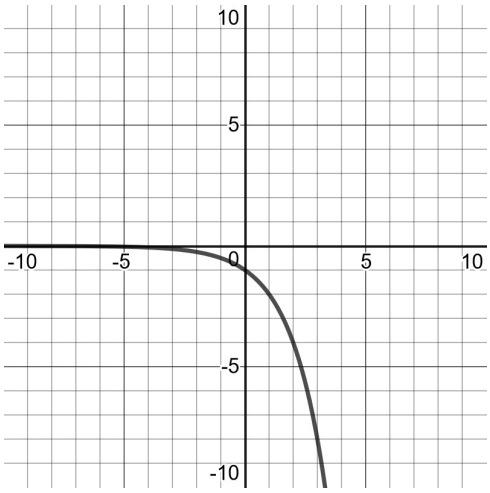
6.



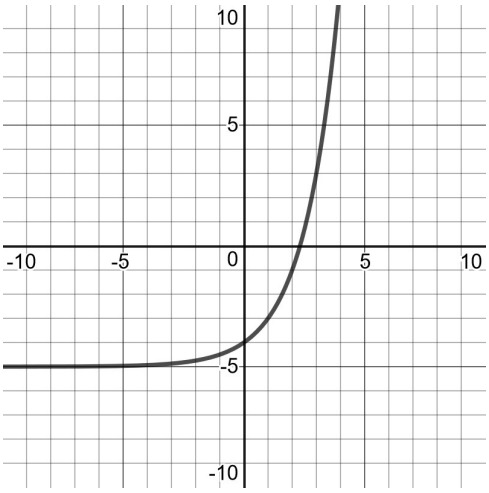
7.



8.



9.



It is shifted (translated) down by 5 units.

10. $y = 0.1(4)^x$

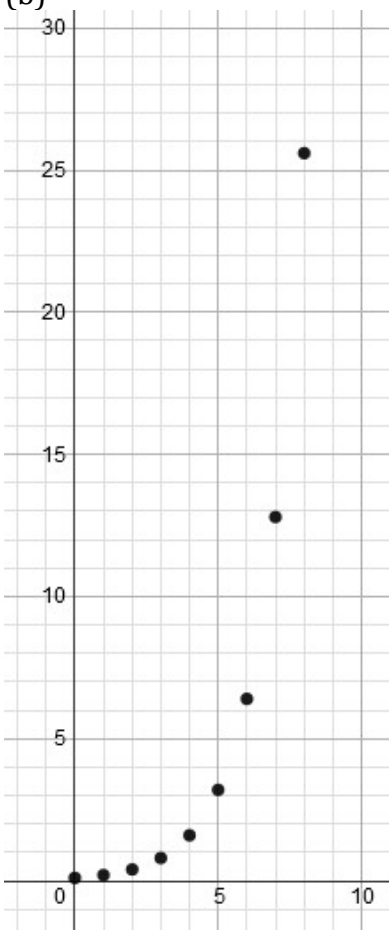
11. $y = \left(\frac{1}{4}\right)^x$

12. $y = 0.488(1.116)^x$

13. $y = 733.646(0.786)^x$; for $x = 12$, $y \approx 41$

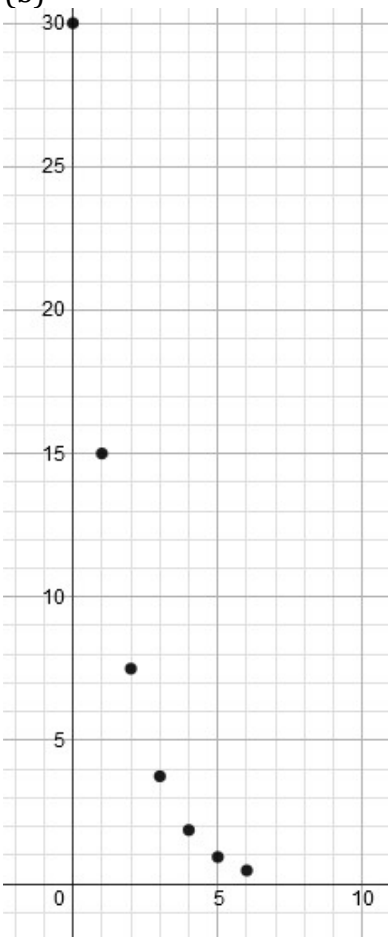
14. (a) $t(n) = 0.1(2)^n$

(b)



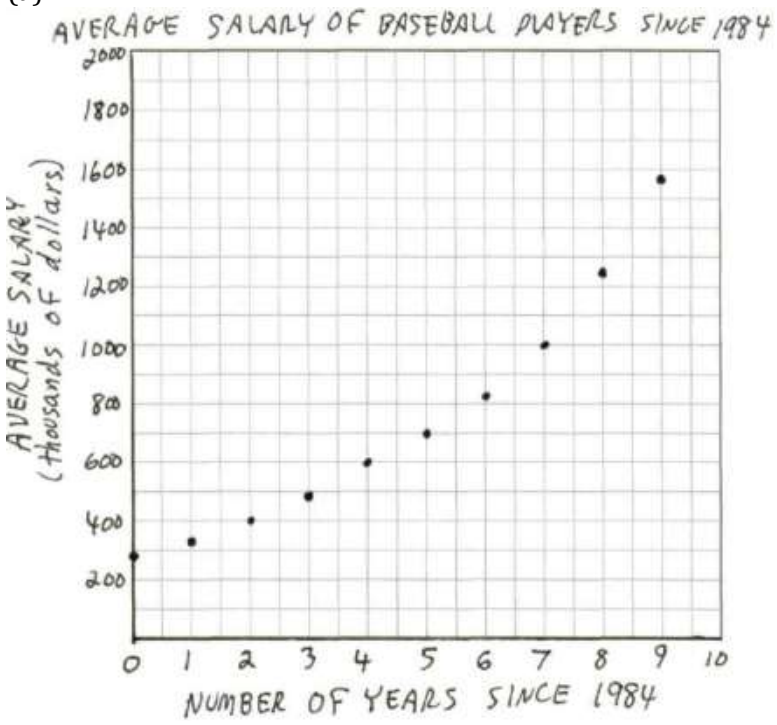
15. (a) $h(n) = 30\left(\frac{1}{2}\right)^n$

(b)



16.

(a)



(b) $y = 276.67(1.21)^x$

(c) $y = 276.67(1.21)^{21} \approx 15{,}151$, or \$15,151,000

REGENTS QUESTIONS

1) AUG '14 [10] Ans: 3

2) JAN '15 [15] Ans: 3

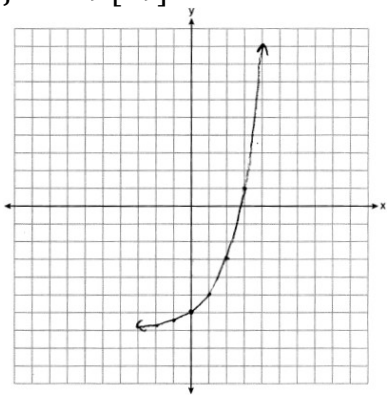
3) JAN '15 [32]

$y = 0.25(2)^x$, by entering (x,y) coordinates, for integer values of x from 2 to 5, into calculator, then STAT / ExpReg

4) JUN '15 [36]

$y = 80(1.5)^x$; 3,030,140; No, because the number would grow so large it would be more than the number of potential customers

5) JAN '19 [29]



Yes, $f(4) > g(4)$ because $2^4 - 7 > 1.5(4) - 3$.

12.3 Rewrite Exponential Expressions

Practice Problems

1. $5^{2x} = \left(5^2\right)^x = 25^x$	2. $10(1.1)^{5x} = 10\left(1.1^5\right)^x = 10(1.61051)^x$
3. $2^{3x+2} = \left(2^3\right)^x \cdot \left(2^2\right) = 4(8)^x$	4. $4(3)^{x+1} = 4(3)^x\left(3^1\right) = 12(3)^x$

Regents Questions

1) JAN '15 [19] Ans: 4
2) JUN '15 [13] Ans: 2
3) JUN '16 [14] Ans: 3
4) JAN '17 [14] Ans: 2
5) JAN '18 [21] Ans: 4
6) AUG '18 [1] Ans: 2
7) JAN '19 [23] Ans: 4

12.4 Compare Linear and Exponential Functions

Practice Problems

1. (a) linear (c) exponential (b) exponential (d) neither	
2. $f(n)=3n$	3. $f(n)=3^n$

Regents Questions

1) JUN '14 [6] Ans: 4
2) JUN '14 [15] Ans: 3
3) AUG '14 [12] Ans: 3
4) JAN '15 [5] Ans: 3
5) AUG '15 [18] Ans: 3
6) JAN '16 [16] Ans: 4
7) JAN '16 [23] Ans: 1
8) JUN '16 [6] Ans: 1
9) JUN '16 [21] Ans: 3
10) AUG '16 [18] Ans: 1
11) JAN '17 [11] Ans: 3
12) JUN '17 [7] Ans: 1
13) JUN '17 [21] Ans: 3
14) AUG '17 [17] Ans: 1
15) JAN '18 [5] Ans: 1
16) JUN '18 [14] Ans: 4
17) AUG '18 [2] Ans: 1
18) AUG '18 [23] Ans: 4
19) JAN '19 [16] Ans: 3
20) SEP '13 [13]
(a) $y=836.47(2.05)^x$; The data appear to grow at an exponential rate; (b) $y=836.47(2.05)^2\approx 3515$
21) JUN '14 [35]
$A(n)=175-2.75n$
$175-2.75n=0$
$n=63.\overline{63}$
63 weeks; she won't have enough money for 64 rentals
22) AUG '15 [27]
exponential; the function does not grow at a constant rate, it is close to a function with a common ratio of 1.25
23) JAN '16 [25]
linear; there is a constant rate of change (a slope of –1.25).
24) AUG '16 [27]
exponential, because the function does not grow at a constant rate.
25) JUN '17 [36]
$f(x)=100x+10$, $g(x)=10(2)^x$
Both, since
$f(7)=100(7)+10=710$ and
$g(7)=10(2)^7=1280$.
26) JUN '18 [26]
Yes, because $f(x)$ does not have a constant rate of change.
27) JAN '19 [26]
Linear, because the function grows at a constant rate ($d = 87$).

13. SEQUENCES

13.1 Arithmetic Sequences

Practice Problems

1. 3	2. –4
3. $a_n = a_1 + (n-1)d$ $a_n = 15 + (n-1)\cdot 5$ $a_n = 15 + 5n - 5$ $a_n = 5n + 10$	4. $a_n = a_1 + (n-1)d$ $a_n = 10 + (n-1)\cdot 2$ $a_n = 10 + 2n - 2$ $a_n = 2n + 8$
5. $a_n = a_1 + (n-1)d$ $a_8 = 21 + (8-1)\cdot 9$ $a_8 = 84$	6. $a_n = a_1 + (n-1)d$ $a_{27} = 5 + (27-1)\cdot 3$ $a_{27} = 83$
7. Find *d* by calculating the slope using (6,10) and (21,55) as two points: $d = \frac{55-10}{21-6} = \frac{45}{15} = 3$ Solve for a_1: $a_6 = a_1 + (6-1)d$ $10 = a_1 + 5(3)$ $-5 = a_1$ Write the rule: $a_n = a_1 + (n-1)d$ $a_n = -5 + (n-1)3$ $a_n = -5 + 3n - 3$ $a_n = -8 + 3n$	8. Find *d* by calculating the slope using (4,–23) and (22,49) as two points: $d = \frac{49-(-23)}{22-4} = \frac{72}{18} = 4$ Solve for a_1: $a_4 = a_1 + (4-1)d$ $-23 = a_1 + 3(4)$ $-35 = a_1$ Write the rule: $a_n = a_1 + (n-1)d$ $a_n = -35 + (n-1)4$ $a_n = -35 + 4n - 4$ $a_n = -39 + 4n$

Regents Questions

1) JUN '14 [24] Ans: 2
2) AUG '14 [16] Ans: 2
3) JUN '16 [13] Ans: 3
4) AUG '16 [10] Ans: 1
5) JUN '18 [7] Ans: 1
6) AUG '18 [20] Ans: 4

13.2 Geometric Sequences

Practice Problems

1. $\frac{1}{2}$	2. -2
3. -4	4. $a_n = a_1 r^{n-1}$ $a_n = 4(2.5)^{n-1}$
5. $a_n = a_1 r^{n-1}$ $a_n = (-1)(-2)^{n-1} = -(-2)^{n-1}$	6. $a_n = a_1 r^{n-1}$ $a_{15} = 5(-2)^{15-1} = 5(-2)^{14}$ $= 5(16{,}384) = 81{,}920$
7. $a_n = a_1 r^{n-1}$ $a_7 = 6\left(-\frac{1}{2}\right)^{7-1} = 6\left(-\frac{1}{2}\right)^{6}$ $= 6\left(\frac{1}{64}\right) = 0.09375$	8. $a_{10} = a_1 r^9 = 512$ $a_{15} = a_1 r^{14} = 16384$ $\frac{a_1 r^{14}}{a_1 r^9} = \frac{16384}{512}$, so $r^5 = 32$, or $r = 2$. Using $a_{10} = a_1(2)^9 = 512$, so $a_1 = 1$. $a_{30} = 1(2)^{29} = 536{,}870{,}912$

Regents Questions

1) AUG '17 [26]
Yes, because the sequence has a common ratio, 3.

13.3 Recursively Defined Sequences

Practice Problems

1. $a_1 = 3$ $a_2 = a_1 + n = 3 + 2 = 5$ $a_3 = a_2 + n = 5 + 3 = 8$ $a_4 = a_3 + n = 8 + 4 = 12$ neither; there is no common difference nor common ratio	2. $a_1 = -4$ $a_2 = a_1 - 2n = -4 - 2 \cdot 2 = -8$ $a_3 = a_2 - 2n = -8 - 2 \cdot 3 = -14$ $a_4 = a_3 - 2n = -14 - 2 \cdot 4 = -22$
3. $a_1 = 3$ $a_2 = 2a_1 - 1 = 2(3) - 1 = 5$ $a_3 = 2a_2 - 1 = 2(5) - 1 = 9$	4. (a) $a_1 = 40$ $a_2 = 8$ $a_n = \frac{a_{n-2} + a_{n-1}}{2}$ for $n > 2$ (b) $a_7 = 19$

Regents Questions

1) JUN '14 [21] Ans: 4
2) AUG '14 [24] Ans: 4
3) JAN '15 [20] Ans: 3
4) JUN '15 [22] Ans: 3
5) AUG '15 [14] Ans: 1
6) JAN '16 [18] Ans: 3
7) JAN '17 [8] Ans: 1
8) JUN '17 [18] Ans: 2
9) AUG '17 [15] Ans: 3
10) JAN '18 [18] Ans: 3
11) JUN '18 [24] Ans: 3
12) JAN '19 [19] Ans: 2
13) AUG '18 [32]
0, –1, 1, 1, 1

14. FACTORING

14.1 Factor Out the Greatest Common Factor

PRACTICE PROBLEMS

1. $4x^2 - 6x = 2 \cdot 2 \cdot x \cdot x - 2 \cdot 3 \cdot x$ $= 2x(2x - 3)$	2. $5a^2 - 10a = 5 \cdot a \cdot a - 2 \cdot 5 \cdot a$ $= 5a(a - 2)$
3. $7x(2x^2 + 1)$	4. $x(x^2 + x - 1)$
5. $6xy(2x^2 + 3y)$	6. $2y(y^2 - 2y + 1)$
7. $3x(x^2 - 2x + 2)$	8. $-2(x + y)$
9. $3mn(m + 4n)$	10. $2x^2y^2(3xz - 2)$

14.2 Factor a Trinomial

Practice Problems

1. $(x+7)(x+2)$	2. $(x-9)(x-2)$
3. $(x-9)(x+3)$	4. $(a-15)(a+14)$
5. $(x+8)(x-3)$	6. $(x+5)(x-3)$
7. $(x-12)(x+2)$	8. $(x-3)(x-2)$
9. yes, it is prime	10. $(x+2)(x+5)=x^2+7x+10$, so $b=7$
11. $(-3x^2+x-2)+(4x^2+3x-10)=x^2+4x-12=(x+6)(x-2)$	

Regents Questions

1) AUG '14 [15] Ans: 1
2) JUN '18 [10] Ans: 1
3) AUG '18 [3] Ans: 3
4) AUG '14 [25] $(x+6)(x+4)$ or $(x+4)(x+6)$, so 4 and 6

14.3 Factor the Difference of Perfect Squares

Practice Problems

1. $(x+6)(x-6)$	2. $(2x+3)(2x-3)$
3. $(3+x)(3-x)$	4. $(a+1)(a-1)$
5. $(7x+y)(7x-y)$	6. $(2a+3b)(2a-3b)$
7. $(xy+4)(xy-4)$	8. $(x^5+10)(x^5-10)$
9. $(10n+1)(10n-1)$	10. $(11+x)(11-x)$
11. $(3a+8b)(3a-8b)$	12. 10

Regents Questions

1) JUN '15 [3] Ans: 2
2) JUN '16 [1] Ans: 3
3) JUN '17 [6] Ans: 3
4) AUG '17 [3] Ans: 3
5) JAN '18 [9] Ans: 3
6) AUG '18 [7] Ans: 3

14.4 Factor Trinomials with a≠1

Practice Problems

1. $6x^2+x-2=$ $6x^2-3x+4x-2=$ $3x(2x-1)+2(2x-1)=$ $(3x+2)(2x-1)$	2. $12x^2+5x-2=$ $12x^2-3x+8x-2=$ $3x(4x-1)+2(4x-1)=$ $(3x+2)(4x-1)$
3. $12x^2-29x+15=$ $12x^2-9x-20x+15=$ $3x(4x-3)-5(4x-3)=$ $(3x-5)(4x-3)$	4. $6x^2-11x+4=$ $6x^2-3x-8x+4=$ $3x(2x-1)-4(2x-1)=$ $(3x-4)(2x-1)$
5. $15x^2+14x-8=$ $15x^2+20x-6x-8=$ $5x(3x+4)-2(3x+4)=$ $(5x-2)(3x+4)$	6. $-10x^2-29x-10=$ $-10x^2-25x-4x-10=$ $-5x(2x+5)-2(2x+5)=$ $(-5x-2)(2x+5)$
7. $4x^2+12x+9=$ $4x^2+6x+6x+9=$ $2x(2x+3)+3(2x+3)=$ $(2x+3)(2x+3)$ The square root is $2x+3$.	8. First, write in standard form: $10x^2+11x-6=$ $10x^2+15x-4x-6=$ $5x(2x+3)-2(2x+3)=$ $(5x-2)(2x+3)$
9. $2x^2+4x-3x-6=$ $2x(x+2)-3(x+2)=$ $(2x-3)(x+2)$ So, $(2x-3)$	

Regents Questions

1) JAN '19 [6] Ans: 1

14.5 Factor Completely

Practice Problems

1. $2y^2+12y-54=$ $2(y^2+6y-27)=$ $2(y+9)(y-3)$	2. $3x^2+15x-42=$ $3(x^2+5x-14)=$ $3(x+7)(x-2)$
3. $3x^2-27=$ $3(x^2-9)=$ $3(x+3)(x-3)$	4. $2x^2-50=$ $2(x^2-25)=$ $2(x+5)(x-5)$
5. $2a^2-10a-28=$ $2(a^2-5a-14)=$ $2(a-7)(a+2)$	6. $x^3+8x^2+7x=$ $x(x^2+8x+7)=$ $x(x+7)(x+1)$
7. $2x^8+16x^7+32x^6=$ $2x^6(x^2+8x+16)=$ $2x^6(x+4)(x+4)$	8. $3ax^2-27a=$ $3a(x^2-9)=$ $3a(x+3)(x-3)$
9. $5x^2y^3-180y=$ $5y(x^2y^2-36)=$ $5y(xy+6)(xy-6)$	10. $2x^5-32x=$ $2x(x^4-16)=$ $2x(x^2+4)(x^2-4)=$ $2x(x^2+4)(x+2)(x-2)$
11. $2x^2+10x-12=$ $2\left(x^2+5x-6\right)=$ $2(x+6)(x-1)$	12. $a^3-4a=$ $a\left(a^2-4\right)=$ $a(a+2)(a-2)$
13. $3x^3-33x^2+90x=$ $3x\left(x^2-11x+30\right)=$ $3x(x-6)(x-5)$	14. $36x^2-100y^6=$ $4\left(9x^2-25y^6\right)=$ $4\left(3x+5y^3\right)\left(3x-5y^3\right)$

15.	16.
$4x^3y^3 - 36xy =$ $4xy\left(x^2y^2 - 9\right) =$ $4xy(xy+3)(xy-3)$	$-x^3 - x^2 + 6x =$ $-x\left(x^2 + x - 6\right) =$ $-x(x+3)(x-2)$

REGENTS QUESTIONS

1) JAN '15 [22] Ans: 3
2) JAN '16 [12] Ans: 3
3) AUG '16 [8] Ans: 2
4) JAN '17 [1] Ans: 2

5) JUN '14 [31]
$(x^2+7)(x^2-1) =$
$(x^2+7)(x+1)(x-1)$

15. QUADRATIC FUNCTIONS

15.1 Solve Simple Quadratic Equations

PRACTICE PROBLEMS

1. $x = \pm\sqrt{81} = \pm 9$ $\{9,-9\}$	2. $y = \pm\sqrt{20} = \pm 2\sqrt{5}$ $\left\{2\sqrt{5},-2\sqrt{5}\right\}$
3. $3x^2 = 75$ $x^2 = 25$ $x = \pm\sqrt{25}$ $\{5,-5\}$	4. $4x^2 - 36 = 0$ $4x^2 = 36$ $x^2 = 9$ $x = \pm\sqrt{9}$ $\{3,-3\}$
5. $2x^2 = 12$ $x^2 = 6$ $x = \pm\sqrt{6}$ $\left\{\sqrt{6},-\sqrt{6}\right\}$	6. $9x^2 = 4$ $x^2 = \frac{4}{9}$ $x = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$ $\left\{\frac{2}{3},-\frac{2}{3}\right\}$
7. $5x^2 - 5 = 0$ $5x^2 = 5$ $x^2 = 1$ $x = \pm\sqrt{1} = \pm 1$ $\{1,-1\}$	8. $3m^2 = 0$ $m^2 = 0$ $m = 0$ $\{0\}$
9. $h^2 = 2s$ $h = \pm\sqrt{2s}$	10. $a^2 = c^2 - b^2$ $a = \sqrt{c^2 - b^2}$

REGENTS QUESTIONS

1) JUN '14 [23] Ans: 1
2) AUG '14 [3] Ans: 3
3) JAN '15 [16] Ans: 1
4) JUN '15 [19] Ans: 2
5) JAN '17 [15] Ans: 4
6) SEP '13 [6]

$\frac{1}{2}x^2 - 4 = 0$

$x^2 - 8 = 0$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

7) AUG '15 [35]

$r = \sqrt{\frac{V}{\pi h}}$;

$d = 2r = 2\sqrt{\frac{V}{\pi h}} = 2\sqrt{\frac{66}{3.3\pi}} \approx 5$

8) JUN '16 [33]

a)

$H(1) = -16(1)^2 + 144 = 128$

$H(2) = -16(2)^2 + 144 = 80$

$128 - 80 = 48\ ft.$

b)

$0 = -16t^2 + 144$

$16t^2 = 144$

$t^2 = 9$

$t = \pm 3$

3 *secs* (reject –3)

9) AUG '17 [27]

$V = \frac{1}{3}\pi r^2 h$

$3V = \pi r^2 h$

$\frac{3V}{\pi h} = r^2$

$r = \sqrt{\frac{3V}{\pi h}}$

(reject negative radius)

10) JAN '18 [30]

$r^2 = \frac{GM_1M_2}{F_g}$

$r = \sqrt{\frac{GM_1M_2}{F_g}}$

11) JAN '19 [32]

$4x^2 = 80$

$x^2 = 20$

$x = \pm\sqrt{20}$

15.2 Solve Quadratic Equations by Factoring

Practice Problems

1. $x^2-5x=0$ $x(x-5)=0$ $x=0$ *or* $x-5=0$ $\{0,5\}$	2. $x^2+3x-18=0$ $(x+6)(x-3)=0$ $x+6=0$ *or* $x-3=0$ $\{-6,3\}$
3. $4x^2-36=0$ $4(x^2-9)=0$ $4(x+3)(x-3)=0$ $x+3=0$ *or* $x-3=0$ $\{-3,3\}$	4. $x^2-4x-32=0$ $(x-8)(x+4)=0$ $x-8=0$ *or* $x+4=0$ $\{8,-4\}$
5. $x^2-5x=6$ $x^2-5x-6=0$ $(x-6)(x+1)=0$ $x-6=0$ *or* $x+1=0$ $\{6,-1\}$	6. $x^2-3=2x$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x-3=0$ *or* $x+1=0$ $\{3,-1\}$
7. $x^2-x=6$ $x^2-x-6=0$ $(x-3)(x+2)=0$ $x-3=0 \mid x+2=0$ $\{3,-2\}$	8. $x^2=30-13x$ $x^2+13x-30=0$ $(x+15)(x-2)=0$ $x+15=0 \mid x-2=0$ $\{-15,2\}$
9. $x^2-4x=x+24$ $x^2-5x-24=0$ $(x-8)(x+3)=0$ $x-8=0$ *or* $x+3=0$ $\{8,-3\}$	10. $2x^2+10x=12$ $2x^2+10x-12=0$ $2(x^2+5x-6)=0$ $2(x+6)(x-1)=0$ $x+6=0$ *or* $x-1=0$ $\{-6,1\}$

11.	12.
$x(x+2)=3$ $x^2+2x=3$ $x^2+2x-3=0$ $(x+3)(x-1)=0$ $x+3=0$ *or* $x-1=0$ $\{-3,1\}$	$(x+2)(x+3)=12$ $x^2+5x+6=12$ $x^2+5x-6=0$ $(x+6)(x-1)=0$ $x+6=0$ *or* $x-1=0$ $\{-6,1\}$

Regents Questions

1) JUN '15 [10] Ans: 4
2) AUG '15 [13] Ans: 4
3) JAN '16 [9] Ans: 1
4) JUN '16 [12] Ans: 1
5) JAN '17 [2] Ans: 3
6) JUN '18 [4] Ans: 3
7) JUN '14 [33]

$m(x)=9x-3x^2-3+x+4x^2+19$

$m(x)=x^2+10x+16$

$x^2+10x+16=0$

$(x+8)(x+2)=0$

$\{-8,-2\}$

8) JAN '16 [27]

$y^2-6y+9=4y-12$

$y^2-10y+21=0$

$(y-3)(y-7)=0$

$\{3,7\}$

9) AUG '16 [36]

$0=(B+3)(B-1)$

$0=(8x+3)(8x-1)$

$\left\{-\frac{3}{8},\frac{1}{8}\right\}$

Janice substituted B for $8x$, resulting in a simpler quadratic to factor. Once factored, she could replace the $8x$ for B.

10) JAN '18 [26]

$x^2-4x+3=0$

$(x-1)(x-3)=0$

$\{1,3\}$

11) JAN '19 [27]

$x^2-8x-9=0$

$(x-9)(x+1)=0$

$\{9,-1\}$;

I factored the trinomial.

15.3 Solve Quadratics for Trinomials with $a \neq 1$

Practice Problems

1. $ac = 20$ and $b = 9$, so use 5 and 4

$$10x^2 + 5x + 4x + 2 = 0$$
$$5x(2x+1) + 2(2x+1) = 0$$
$$(5x+2)(2x+1) = 0$$
$$5x + 2 = 0 \qquad 2x + 1 = 0$$
$$x = -\frac{2}{5} \qquad x = -\frac{1}{2}$$
$$\left\{-\frac{1}{2}, -\frac{2}{5}\right\}$$

2. $ac = 24$ and $b = -14$, so use –12 and –2

$$3x^2 - 12x - 2x + 8 = 0$$
$$3x(x-4) - 2(x-4) = 0$$
$$(3x-2)(x-4) = 0$$
$$3x - 2 = 0 \qquad x - 4 = 0$$
$$x = \frac{2}{3} \qquad x = 4$$
$$\left\{\frac{2}{3}, 4\right\}$$

3. $ac = -4$ and $b = -3$, so use –4 and 1

$$2x^2 - 4x + x - 2 = 0$$
$$2x(x-2) + 1(x-2) = 0$$
$$(2x+1)(x-2) = 0$$
$$2x + 1 = 0 \qquad x - 2 = 0$$
$$x = -\frac{1}{2} \qquad x = 2$$
$$\left\{-\frac{1}{2}, 2\right\}$$

4.

$$4x(x-1) = 15$$
$$4x^2 - 4x = 15$$
$$4x^2 - 4x - 15 = 0$$

$ac = -60$ and $b = -4$, so use 6 and –10

$$4x^2 + 6x - 10x - 15 = 0$$
$$2x(2x+3) - 5(2x+3) = 0$$
$$(2x-5)(2x+3) = 0$$
$$2x - 5 = 0 \qquad 2x + 3 = 0$$
$$x = \frac{5}{2} \qquad x = -\frac{3}{2}$$
$$\left\{-\frac{3}{2}, \frac{5}{2}\right\}$$

Regents Questions

1) JAN '15 [3] Ans: 4

2) AUG '16 [19] Ans: 1

3) AUG '18 [16] Ans: 2

4) SEP '13 [5]

$$8m^2 + 20m - 12 = 0$$
$$4(2m^2 + 5m - 3) = 0$$
$$4(2m^2 - m + 6m - 3) = 0$$
$$4[m(2m-1) + 3(2m-1)] = 0$$
$$4(m+3)(2m-1) = 0$$
$$\{-3, \tfrac{1}{2}\}$$

5) JAN '15 [29]

$$4x^2 - 12x - 7 = 0$$
$$4x^2 + 2x - 14x - 7 = 0$$
$$2x(2x+1) - 7(2x+1) = 0$$
$$(2x-7)(2x+1) = 0$$
$$\left\{\tfrac{7}{2}, -\tfrac{1}{2}\right\}$$

6) JUN '16 [28]

$$2x^2 + 5x - 42 = 0$$
$$2x^2 + 12x - 7x - 42 = 0$$
$$2x(x+6) - 7(x+6) = 0$$
$$(2x-7)(x+6) = 0$$
$$\left\{\tfrac{7}{2}, -6\right\}$$

Yes, as shown by solving the equation.

15.4 Find Quadratic Equations from Given Roots

Practice Problems

1. $x = 10 \text{ or } x = -2$ $x - 10 = 0 \text{ or } x + 2 = 0$ $(x-10)(x+2) = 0$ $x^2 - 8x - 20 = 0$	2. $x = 0 \text{ or } x = 3$ $x = 0 \text{ or } x - 3 = 0$ $x(x-3) = 0$ $x^2 - 3x = 0$
3. $x = -12 \text{ or } x = 2$ $x + 12 = 0 \text{ or } x - 2 = 0$ $(x+12)(x-2) = 0$ $x^2 + 10x - 24 = 0$	4. $x = -3 \text{ or } x = 5$ $x + 3 = 0 \text{ or } x - 5 = 0$ $(x+3)(x-5) = 0$ $x^2 - 2x - 15 = 0$
5. $x = -5 \text{or } x = 2$ $x + 5 = 0 \text{ or } x - 2 = 0$ $(x+5)(x-2) = 0$ $x^2 + 3x - 10 = 0$	6. $x = 1 \text{ or } x = 3$ $x - 1 = 0 \text{ or } x - 3 = 0$ $(x-1)(x-3) = 0$ $x^2 - 4x + 3 = 0$
7. $\Downarrow$ multiply both sides by 2 $x - \frac{3}{2} = 0 \text{ or } x - 2 = 0$ $2x - 3 = 0 \text{ or } x - 2 = 0$ $(2x-3)(x-2) = 0$ $2x^2 - 7x + 6 = 0$	8. $(x-1)^2 = 0$ $x^2 - 2x + 1 = 0$
9. $(x-4)(x+4) = 0$ $x^2 - 16 = 0$	10. $x = 0 \text{ or } x = 1 \text{ or } x = -1$ $x(x-1)(x+1) = 0$ $x(x^2 - 1) = 0$ $x^3 - x = 0$

Regents Questions

1) MAY '13 [3] Ans: 2
2) JUN '14 [12] Ans: 3

15.5 Equations with the Square of a Binomial

Practice Problems

1. $x+5=\pm\sqrt{16}=\pm 4$ $x=-5\pm 4$ $\{-1,-9\}$	2. $x-4=\pm\sqrt{10}$ $x=4\pm\sqrt{10}$ $\left\{4+\sqrt{10},4-\sqrt{10}\right\}$
3. $(b-1)^2=8$ $b-1=\pm\sqrt{8}=\pm 2\sqrt{2}$ $b=1\pm 2\sqrt{2}$ $\left\{1+2\sqrt{2},1-2\sqrt{2}\right\}$	4. $-(m+1)^2=-30$ $(m+1)^2=30$ $m+1=\pm\sqrt{30}$ $m=-1\pm\sqrt{30}$ $\left\{-1+\sqrt{30},-1-\sqrt{30}\right\}$
5. $(x-2)^2=0$ $x-2=0$ $x=2$ $\{2\}$	6. $2(x+5)^2-50=0$ $2(x+5)^2=50$ $(x+5)^2=25$ $x+5=\pm\sqrt{25}=\pm 5$ $x=-5\pm 5$ $\{0,-10\}$

Regents Questions

1) AUG '14 [18] Ans: 4
2) JUN '15 [21] Ans: 1
3) AUG '15 [23] Ans: 3
4) JUN '16 [19] Ans: 3
5) JAN '18 [14] Ans: 1

6) AUG '16 [31]

$(x-3)^2-49=0$
$(x-3)^2=49$
$x-3=\pm 7$
$x=3\pm 7$
$\{-4,10\}$

15.6 Complete the Square

Practice Problems

1.

$$\left(\frac{b}{2}\right)^2=\left(\frac{10}{2}\right)^2=25$$

$$x^2+10x-11=0$$
$$x^2+10x=11$$
$$x^2+10x+25=11+25$$
$$(x+5)^2=36$$
$$x+5=\sqrt{36}$$
$$x+5=\pm 6$$
$$x=-5\pm 6$$

Solution: {1, –11}

2.

$$\left(\frac{b}{2}\right)^2=\left(\frac{-8}{2}\right)^2=16$$

$$x^2-8x+16=0$$
$$x^2-8x=-16$$
$$x^2-8x+16=-16+16$$
$$(x-4)^2=0$$
$$x-4=0$$

Solution: {4}

3.

$$\left(\frac{b}{2}\right)^2=\left(\frac{4}{2}\right)^2=4$$

$$x^2+4x+2=0$$
$$x^2+4x=-2$$
$$x^2+4x+4=-2+4$$
$$(x+2)^2=2$$
$$x+2=\pm\sqrt{2}$$
$$x=-2\pm\sqrt{2}$$

Solution: $\left\{-2+\sqrt{2},-2-\sqrt{2}\right\}$

4.

$$\left(\frac{b}{2}\right)^2=\left(\frac{-4}{2}\right)^2=4$$

$$x^2-4x-8=0$$
$$x^2-4x=8$$
$$x^2-4x+4=8+4$$
$$(x-2)^2=12$$
$$x-2=\pm\sqrt{12}=\pm 2\sqrt{3}$$
$$x=2\pm 2\sqrt{3}$$

Solution: $\left\{2+2\sqrt{3},2-2\sqrt{3}\right\}$

5.

$$2(x^2-6x+2)=0$$
$$x^2-6x+2=0$$

$$\left(\frac{b}{2}\right)^2=\left(\frac{-6}{2}\right)^2=9$$

$$x^2-6x=-2$$
$$x^2-6x+9=-2+9$$
$$(x-3)^2=7$$
$$x-3=\pm\sqrt{7}$$

Solution: $\left\{3+\sqrt{7},3-\sqrt{7}\right\}$

6.

$$\left(\frac{b}{2}\right)^2=\left(\frac{-2}{2}\right)^2=1$$

$$x^2-2x+3=0$$
$$x^2-2x=-3$$
$$x^2-2x+1=-3+1$$
$$(x-1)^2=-2$$
$$x-1=\boxed{\pm\sqrt{-2}}$$

No real solutions, since the square root of a negative number is not a real number.

7.

$w(w+10)=880$ $\qquad \left(\frac{b}{2}\right)^2=\left(\frac{10}{2}\right)^2=25$

$w^2+10w=880$

$w^2+10w+25=880+25$

$(w+5)^2=905$

$w+5=\pm\sqrt{905}$

$w=-5\pm\sqrt{905}\approx -5\pm 30.1$

$w=25.1$ (*reject negative w*)

width ≈ 25.1 ft. and length ≈ 35.1 ft.

REGENTS QUESTIONS

1) JUN '14 [8] Ans: 2
2) JUN '14 [10] Ans: 2
3) JAN '15 [17] Ans: 4
4) JUN '15 [23] Ans: 1
5) AUG '15 [20] Ans: 1
6) JAN '16 [14] Ans: 2
7) JAN '17 [22] Ans: 1
8) JUN '17 [22] Ans: 2
9) JUN '18 [12] Ans: 3
10) JAN '19 [15] Ans: 1
11) AUG '14 [32]

9, $c=\left(\frac{b}{2}\right)^2=\left(\frac{6}{2}\right)^2=9$

12) AUG '17 [32]

$x^2-6x+9=15+9$

$(x-3)^2=24$

$x-3=\pm\sqrt{24}$

$x=3\pm 2\sqrt{6}$

$\left\{3+2\sqrt{6},3-2\sqrt{6}\right\}$

13) AUG '18 [30]

$x^2+4x+4=2+4$

$(x+2)^2=6$

$x+2=\pm\sqrt{6}$

$x=-2\pm\sqrt{6}$

15.7 Complete the Square with $a \neq 1$

Practice Problems

1.

$$a\left(\frac{b}{2a}\right)^2 = 4\left(\frac{8}{2\cdot 4}\right)^2 = 4$$

$$4x^2 + 8x = 45$$
$$4x^2 + 8x + 4 = 45 + 4$$
$$x^2 + 2x + 1 = \frac{49}{4}$$
$$(x+1)^2 = \frac{49}{4}$$
$$x + 1 = \pm\sqrt{\frac{49}{4}} = \pm\frac{7}{2}$$
$$x = -1 \pm \frac{7}{2} = -\frac{2}{2} \pm \frac{7}{2}$$

Solution: $\left\{\frac{5}{2}, -\frac{9}{2}\right\}$

2.

$$a\left(\frac{b}{2a}\right)^2 = 3\left(\frac{-2}{2\cdot 3}\right)^2 = \frac{1}{3}$$

$$3x^2 - 2x - 1 = 0$$
$$3x^2 - 2x = 1$$
$$3x^2 - 2x + \tfrac{1}{3} = \tfrac{4}{3}$$
$$x^2 - \tfrac{2}{3}x + \tfrac{1}{9} = \tfrac{4}{9}$$
$$\left(x - \tfrac{1}{3}\right)^2 = \tfrac{4}{9}$$
$$x - \tfrac{1}{3} = \pm\sqrt{\tfrac{4}{9}} = \pm\tfrac{2}{3}$$
$$x = \tfrac{1}{3} \pm \tfrac{2}{3}$$

Solution: $\left\{1, -\frac{1}{3}\right\}$

15.8 Quadratic Formula and the Discriminant

Practice Problems

1. (2) 2

2. $b^2 - 4ac = 4^2 - 4(1)(7) = -12$, so the answer is (a) not real

3. $b^2 - 4ac = 3^2 - 4(9)(-4) = 153$, so (4) real, irrational, and unequal

4. $b^2 - 4ac = (-9)^2 - 4(2)(4) = 49$, so (2) real, rational, and unequal

5.
$$6x^2 - 2x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$
$$\frac{2 \pm \sqrt{(-2)^2 - 4(6)(-3)}}{2(6)} =$$
$$\frac{2 \pm \sqrt{76}}{12} = \frac{2 \pm 2\sqrt{19}}{12} =$$
$$\frac{1 \pm \sqrt{19}}{6}$$
Solution: $\left\{\frac{1+\sqrt{19}}{6}, \frac{1-\sqrt{19}}{6}\right\}$

6.
$$2x^2 + 7x - 3 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$
$$\frac{-7 \pm \sqrt{(7)^2 - 4(2)(-3)}}{2(2)} =$$
$$\frac{-7 \pm \sqrt{73}}{4}$$
Solution: $\left\{\frac{-7+\sqrt{73}}{4}, \frac{-7-\sqrt{73}}{4}\right\}$

7.
$$x^2 + 7x + 8 = 0$$
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$
$$\frac{-(7) \pm \sqrt{(7)^2 - 4(1)(8)}}{2(1)} =$$
$$\frac{-7 \pm \sqrt{17}}{2}$$
Solution: $\left\{\frac{-7+\sqrt{17}}{2}, \frac{-7-\sqrt{17}}{2}\right\}$

8.
$$2x^2 - 8x + 3 = 0$$
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$
$$\frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)} =$$
$$\frac{8 \pm \sqrt{40}}{4} = \frac{8 \pm 2\sqrt{10}}{4} =$$
$$2 \pm \frac{\sqrt{10}}{2}$$
Solution: $\left\{2+\frac{\sqrt{10}}{2}, 2-\frac{\sqrt{10}}{2}\right\}$

Regents Questions

1) SEP '13 [2] Ans: 1

2) JUN '18 [22] Ans: 3

3) AUG '15 [29]
no real solutions; the discriminant $b^2-4ac=(-2)^2-4(1)(5)=-16$ is negative

4) JAN '16 [34]
a) completing the square or quadratic formula;
b)
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=$$
$$\frac{-16\pm\sqrt{16^2-4(4)(9)}}{2(4)}=$$
$$\frac{-16\pm\sqrt{112}}{8}\approx\frac{-16\pm10.6}{8}$$
$\{-0.7,-3.3\}$

5) JUN '17 [35]
$$2x^2+3x+10=4x+32$$
$$2x^2-x-22=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=$$
$$\frac{1\pm\sqrt{(-1)^2-4(2)(-22)}}{2(2)}=$$
$$\frac{1\pm\sqrt{177}}{4}\approx0.25\pm3.33\approx\{-3.1,3.6\}$$
Chose the quadratic formula because the equation could not be solved by factoring, and completing the square would require fractions.

6) JUN '18 [27]
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=$$
$$\frac{-1\pm\sqrt{1^2-4(1)(-5)}}{2(1)}=$$
$$\frac{-1\pm\sqrt{21}}{2}\approx0.5\pm2.29\approx\{-2.8,1.8\}$$

7) AUG '18 [28]
Irrational, since the discriminant $b^2-4ac=3^2-4(2)(-10)=89$ is not a perfect square.

15.9 Word Problems – Quadratic Equations

Practice Problems

1. $x^2 - 2x = 48$ $x^2 - 2x - 48 = 0$ $(x-8)(x+6) = 0$ $x - 8 = 0$ *or* $x + 6 = 0$ $x = 8$ (*reject* $x = -6$) The number is 8.	2. $x^2 + (x+8)^2 = 104$ $x^2 + x^2 + 16x + 64 = 104$ $2x^2 + 16x - 40 = 0$ $2(x^2 + 8x - 20) = 0$ $2(x+10)(x-2) = 0$ $x = 2$ (*reject* $x = -10$) Numbers are 2 and 10.
3. $x^2 - 36 = 5x$ $x^2 - 5x - 36 = 0$ $(x-9)(x+4) = 0$ $x - 9 = 0$ *or* $x + 4 = 0$ $x = 9$ (*reject* $x = -4$) Positive solution is 9.	4. $x^2 = 5x + 24$ $x^2 - 5x - 24 = 0$ $(x-8)(x+3) = 0$ $x - 8 = 0$ *or* $x + 3 = 0$ $x = 8$ (*reject* $x = -3$) The positive number is 8.
5. $w(w+5) = 500$ $w^2 + 5w - 500 = 0$ $(w+25)(w-20) = 0$ $w = 20$ (*reject* $w = -25$) Width is 20 and length is 25.	6. $(t+7)(t-3) = 24$ $t^2 + 4t - 21 = 24$ $t^2 + 4t - 45 = 0$ $(t+9)(t-5) = 0$ $t = 5$ (*reject* $t = -9$) Tamara is 5 years old.
7. $x(x+2) = 63$ $x^2 + 2x - 63 = 0$ $(x+9)(x-7) = 0$ $x = -9$ (*reject* $x = 7$) Numbers are –9 and –7.	8. $x(x+2) = (x+4) + 8$ $x^2 + 2x = x + 12$ $x^2 + x - 12 = 0$ $(x+4)(x-3) = 0$ $x = 3$ (*reject* $x = -4$) Numbers are 3, 5, and 7.
9. $(x+1)(x+10) = 90$ $x^2 + 11x + 10 = 90$ $x^2 + 11x - 80 = 0$ $(x+16)(x-5) = 0$ $x = 5$ (*reject* $x = -16$) Numbers are 5 and 6.	10. $x(x+4) = 2(x+2) + 20$ $x^2 + 4x = 2x + 4 + 20$ $x^2 + 2x - 24 = 0$ $(x+6)(x-4) = 0$ $x = 4$ (*reject* $x = -6$) Ages are 4, 6, and 8

11. $x^2+(x+1)^2=6^2$ $x^2+x^2+2x+1=36$ $2x^2+2x-35=0$ $x=\frac{-2\pm\sqrt{284}}{4}\approx\{\cancel{-4.71},3.71\}$ 3.71 and 4.71	12. $-16x^2+32x=0$ $-16x(x-2)=0$ $-16x=0$ *or* $x-2=0$ $x=2$ (*reject* $x=0$) 2 seconds.
13. (a) $w(0)=120$ gallons (b) $t=\frac{8\pm\sqrt{2464}}{-10}\approx\{\cancel{-5.8},4.2\}$ 4.2 mins.	14. (a) $(x+40)(x+60)$ (b) 2400, $60x$, x^2, $40x$ (c) Both equal $x^2+100x+2400$

REGENTS QUESTIONS

1) AUG '14 [9] Ans: 3

2) JUN '16 [24] Ans: 2

3) AUG '17 [23] Ans: 4

4) JUN '14 [34]

$(2x+16)(2x+12)=396$, the length of the garden plus walkway times the width of the garden plus walkway,

$4x^2+56x-204=0$

$x^2+14x-51=0$, width = 3 m

$(x+17)(x-3)=0$

5) AUG '14 [36]

$x(x+40)=6000$

$x^2+40x-6000=0$

~~$(x+100)$~~$(x-60)=0$

$x=60$

60 and 100

6) JAN '15 [37]

$(2x)(x-3)=1.25x^2$; the product of the new length and width give an area that is 1.25 larger;

$2x^2-6x=1.25x^2$

$0.75x^2-6x=0$

$x(0.75x-6)=0$

~~$x=0$~~ *or* $x=8$

$1.25(8)^2=80$

80 square meters

7) JUN '15 [27]

$60+5x=x^2+46$

$0=x^2-5x-14$

$0=(x-7)(x+2)$

$x=7$ *or* ~~$x=-2$~~

Set the equations equal and solve for a positive value of *x*.

8) JUN '15 [32]

$w(2w)=34$

$2w^2=34$

$w^2=17$

$w=\sqrt{17}\approx 4.1$

9) AUG '15 [31]

$0\le t\le 4$, the roots 0 and 4 represent the start time and when it hits the ground in 4 seconds

10) AUG '15 [37]

$(2x+8)(2x+6)=100$ or

$4x^2+28x-52=0$ or

$x^2+7x-13=0$; if *x* is the width of the frame, add 2*x* to each dimension to account for both sides, then multiply to find the area; $x=\dfrac{-7\pm\sqrt{7^2-4(1)(-13)}}{2(1)}$

$=1.5$ *or* ~~-8.5~~

11) JAN '16 [36]

width = $\dfrac{48-2x}{2}=24-x$

$x(24-x)=108$

$24x-x^2=108$

$-x^2+24x-108=0$

$x^2-24x+108=0$

$(x-6)(x-18)=0$

$\{6,18\}$

dimensions are 6 and 18 meters

12) JUN '18 [29]

$-16t^2+256=0$

$16t^2=256$

$t^2=16$

$t=4$

16. PARABOLAS

16.1 Find Roots Given a Parabolic Graph

Practice Problems

1. (c)	2. (a)
3. 2 and 4	4. 1 and 5
5. –2 and 3	6. –4 and 2
7. –1 and 4	8. –1 and 5
9. –6 and 3	10. 0 (only)

Regents Questions

1) MAY '13 [2] Ans: 3
2) AUG '14 [5] Ans: 4
3) JAN '17 [6] Ans: 4
4) JAN '19 [9] Ans: 3
5) JUN '17 [33]

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$\{-6, 3\}$

The zeros are the x-intercepts on the graph of $r(x)$.

6) JAN '18 [32]
yes, because the x-intercepts (roots) are –2 and 3.

16.2 Find Vertex and Axis Graphically

Practice Problems

1. Vertex is (3,–1); axis of symmetry is $x = 3$.	2. Vertex is (1,–5); axis of symmetry is $x = 1$.
3. Vertex is (–1,7); axis of symmetry is $x = -1$.	4. Vertex is (–2,–3); axis of symmetry is $x = -2$.
5. Vertex is (3,23); axis of symmetry is $x = 3$.	6. Vertex is (3,8); axis of symmetry is $x = 3$.
7. Vertex is (0,0); axis of symmetry is $x = 0$.	8. (3) (3,0) and (1,0)

16.3 Finding Vertex and Axis Algebraically

Practice Problems

1. $x = \frac{-b}{2a} = \frac{-4}{-2} = 2$ $y = -x^2 + 4x - 8$ $y = -(2)^2 + 4(2) - 8 = -4$ Vertex is (2,–4); axis of symmetry is $x = 2$.	2. $x = \frac{-b}{2a} = \frac{6}{2} = 3$ $y = x^2 - 6x + 10$ $y = (3)^2 - 6(3) + 10 = 1$ Vertex is (3,1); axis of symmetry is $x = 3$.
3. $x = \frac{-b}{2a} = \frac{-6}{6} = -1$ $y = 3(-1)^2 + 6(-1) - 1 = -4$ Vertex is (–1,–4).	4. $x = \frac{-b}{2a} = \frac{-8}{4} = -2$ $y = 2(-2)^2 + 8(-2) + 9 = 1$ Vertex (minimum) is (–2,1).
5. $x = \frac{-b}{2a} = \frac{-2}{2} = -1$ $y = (-1)^2 + 2(-1) = -1$ Vertex is (–1,–1); axis of symmetry is $x = -1$.	6. $x = \frac{-b}{2a} = \frac{0}{6} = 0$ $y = 3(0)^2 + 1 = 1$ Vertex is (0,1); axis of symmetry is $x = 0$.
7. $x = \frac{-(-8)}{2(-2)} = -2$ $y = -2(-2)^2 - 8(-2) + 3 = 11$ $x = -2$ and (–2,11)	8. $x = \frac{-(-2)}{2(-1)} = -1$ $y = -(-1)^2 - 2(-1) + 1 = 2$ $x = -1$ and $(-1,2)$
9. $x = \frac{-b}{2a} = \frac{8}{0.5} = 16$ $y = 0.25(16)^2 - 8(16) + 800 = 736$ Vertex is (16,736). So, producing 16 units will result in the minimum cost of \$736.	10. $w = \frac{-b}{2a} = \frac{-160}{2(-4)} = 20$ workers

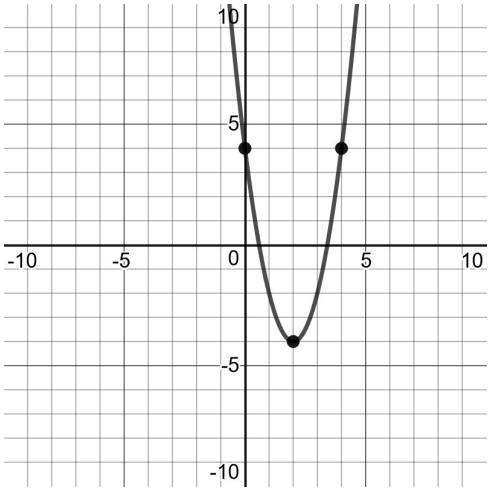
REGENTS QUESTIONS

1) JUN '15 [14] Ans: 3
2) AUG '15 [21] Ans: 4
3) JAN '16 [22] Ans: 3
4) JUN '16 [11] Ans: 2
5) JAN '18 [23] Ans: 2
6) JUN '18 [13] Ans: 2
7) AUG '14 [29]
the vertex for f is (1,6) so the maximum is 6;
for the vertex of g,
$$x = \frac{-4}{2(-\frac{1}{2})} = 4$$
$$y = -\frac{1}{2}(4)^2 + 4(4) + 3 = 11$$
the vertex is (4,11) so the maximum is 11; therefore, $g(x)$ has the larger maximum
8) JAN '16 [33]
$$t = \frac{-64}{2(-16)} = 2$$
maximum at the vertex where $t = 2$ seconds;
$$-16t^2 + 64t + 80 = 0$$
$$-16(t^2 - 4t - 5) = 0$$
$$-16(t+1)(t-5) = 0$$
$$\{\cancel{-1}, 5\}$$
decreases from the vertex until it hits the ground at $h(t) = 0$, or when $2 < t < 5$
9) AUG '17 [36]
$$x = \frac{-128}{2(-16)} = 4$$
$$h(4) = -16(4)^2 + 128(4) + 9000$$
$$= 9256$$
(4, 9256) The y-coordinate is the pilot's maximum height above the ground after being ejected.
9256 – 9000 = 256, so she was 256 feet above the aircraft.
10) JAN '18 [29]
$$(x+3)(x-5) = 0$$
$$x^2 - 2x - 15 = 0$$
$$x = \frac{-(-2)}{2(1)} \quad x = 1$$
(Alternate solution: $x = \frac{-3+5}{2} = 1$)

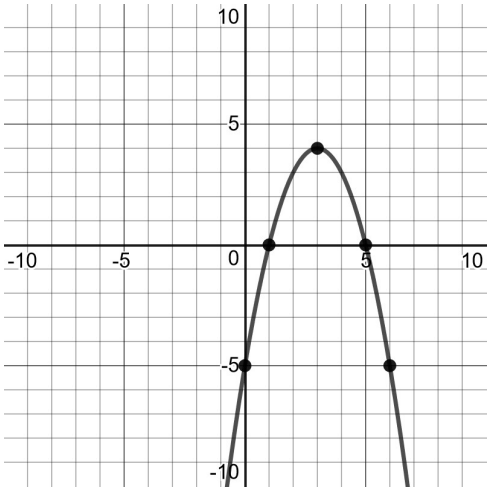
16.4 Graph Parabolas

Practice Problems

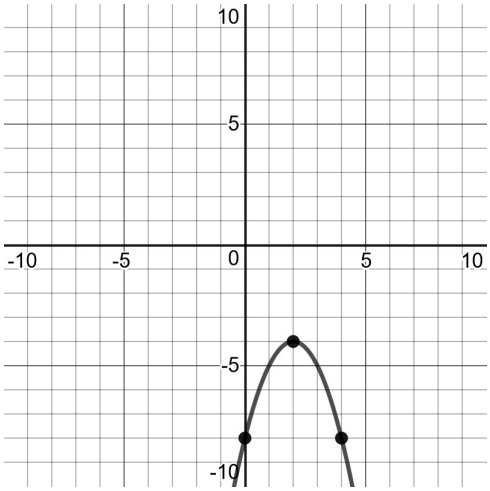
1.



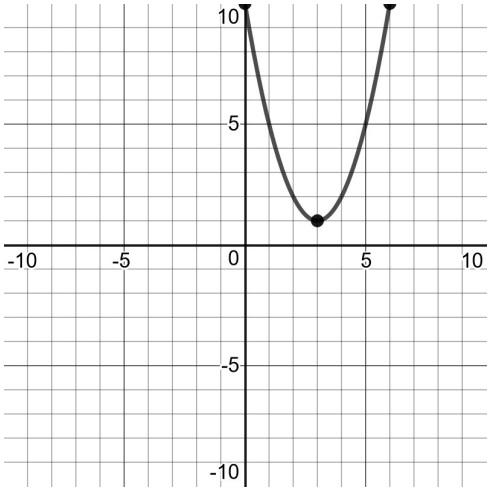
2.



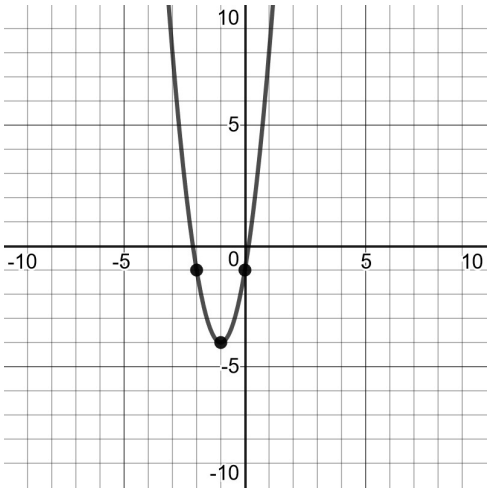
3.



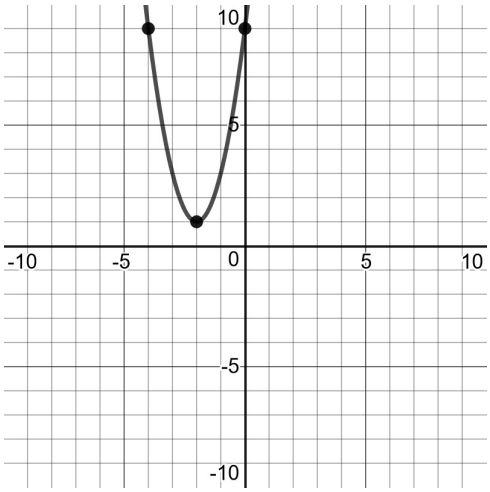
4.



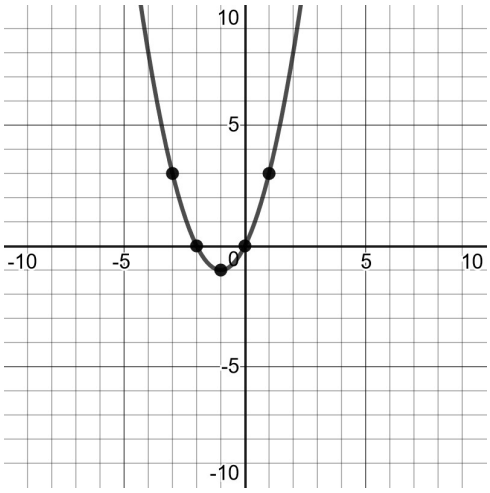
5.



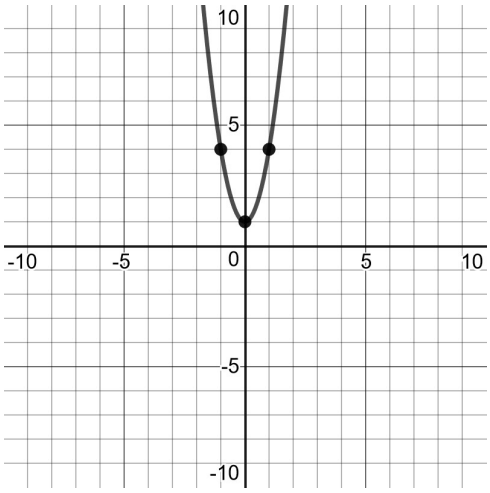
6.



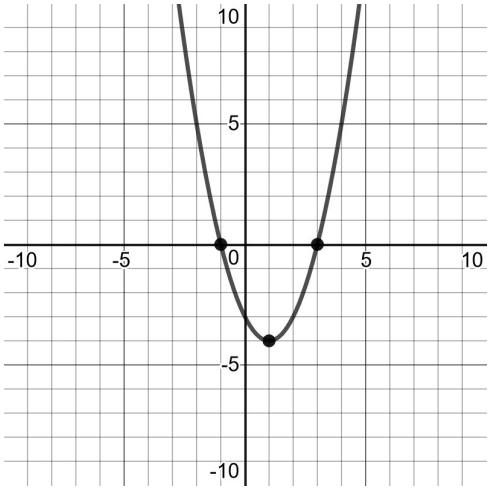
7.



8.

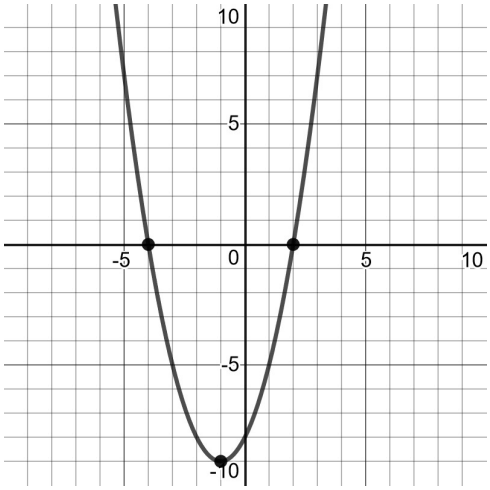


9.



Roots are –1 and 3.

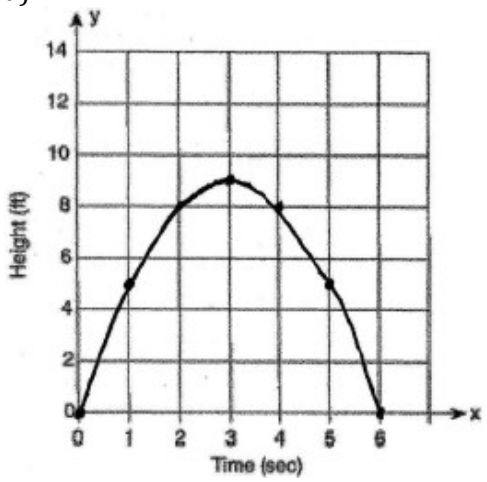
10.



Roots are –4 and 2.

11.

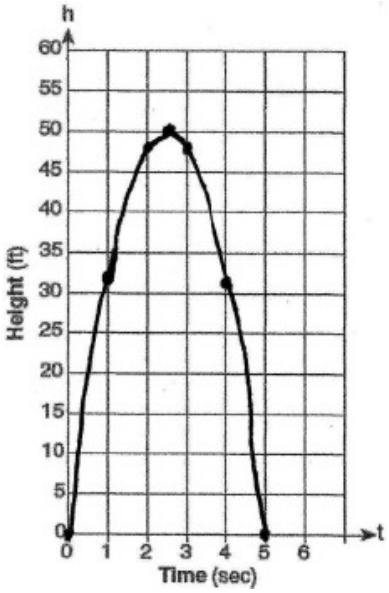
a)



$x = \frac{-b}{2a} = \frac{-6}{2(-1)} = 3$ seconds

12.

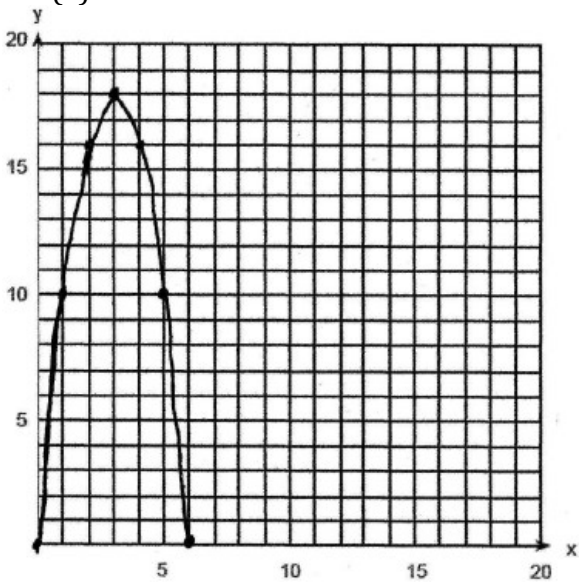
a)



$t = \frac{-b}{2a} = \frac{-40}{2(-8)} = 2.5$ seconds

13.

(a)



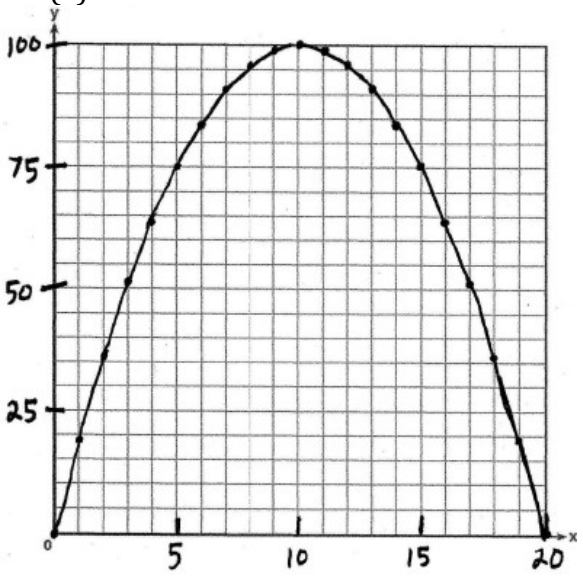
(b)

$x = \frac{-b}{2a} = \frac{-12}{2(-2)} = 3$

$y = -2(3)^2 + 12(3) = 18$ feet

14.

(a)



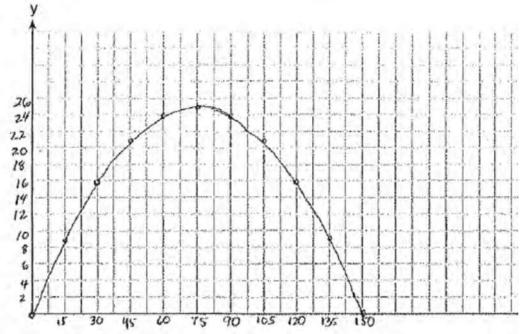
(b)

$x = \frac{-b}{2a} = \frac{-20}{2(-1)} = 10$

$y = -10^2 + 20(10) = 100$ feet

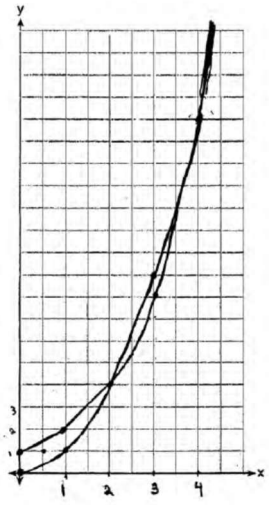
Regents Questions

1) AUG '16 [13] Ans: 3
2) JUN '17 [24] Ans: 1
3) JAN '18 [11] Ans: 2
4) JAN '19 [13] Ans: 1
5) JUN '15 [37]



(75,25), maximum height of 25 feet at a distance of 75 feet; no, 45 yds = 135 ft and $h(135)=9$.

6) AUG '15 [33]

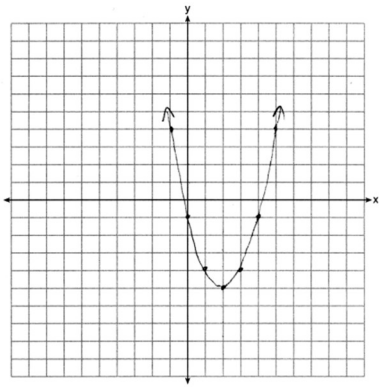


$f(20)=400$ and $g(20)=1{,}048{,}576$, so $g(x)$ is greater

7) JUN '16 [27]

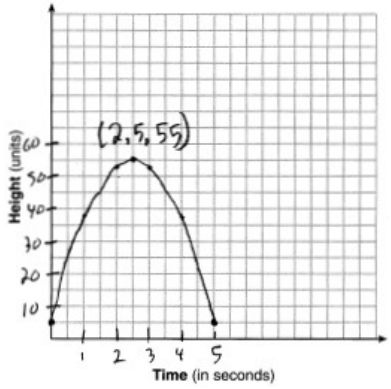
$$x=\frac{-b}{2a}=\frac{-(-4)}{2(1)}=2$$

$$y=(2)^2-4(2)-1=-5$$



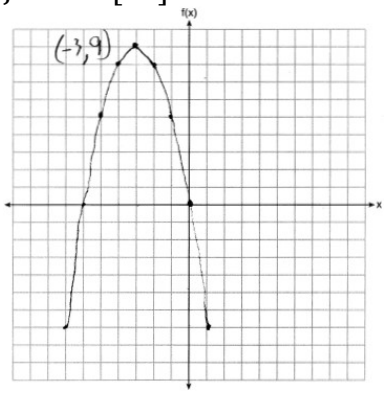
$x=2$

8) JAN '17 [36]



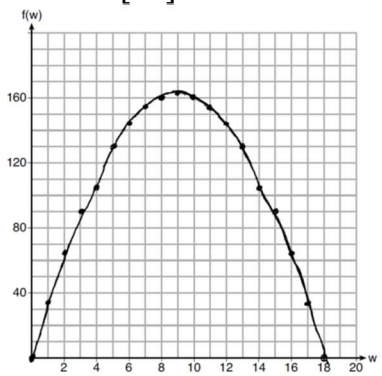
(2.5,55) The ball reaches a maximum height of 55 units at 2.5 seconds.

9) JUN '17 [26]



(−3,9)

10) AUG ’18 [36]



If the width is 9 ft, its area is 162 ft^2.

16.5 Vertex Form

Practice Problems

1.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$$

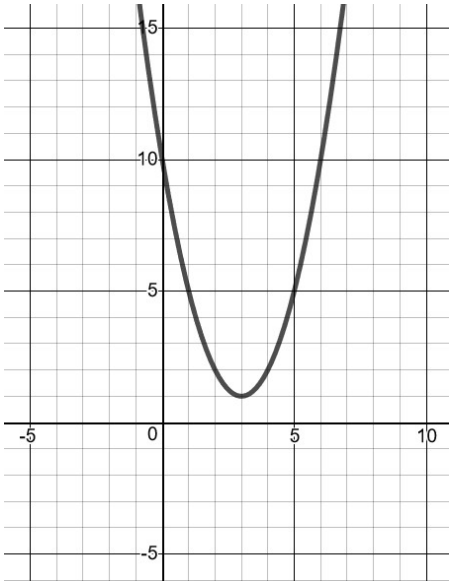
$y = x^2 + 6x + 10$
$y - 10 = x^2 + 6x$
$y - 1 = x^2 + 6x + 9$
$y - 1 = (x+3)^2$
$y = (x+3)^2 + 1$
vertex: (–3,1)

2.

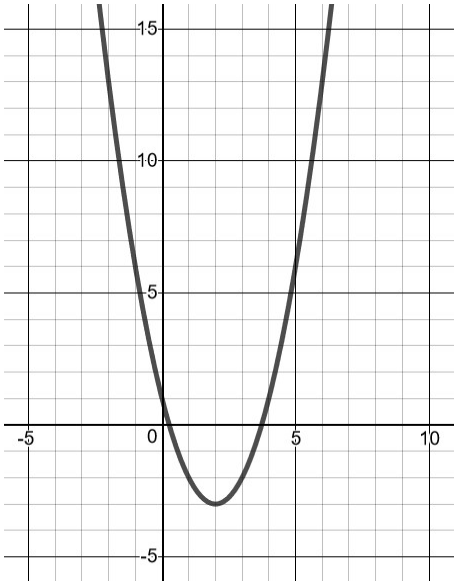
$$\left(\frac{b}{2}\right)^2 = \left(\frac{10}{2}\right)^2 = 25$$

$y = x^2 + 10x + 21$
$y - 21 = x^2 + 10x$
$y + 4 = x^2 + 10x + 25$
$y + 4 = (x+5)^2$
$y = (x+5)^2 - 4$
vertex: (–5,–4)

3.



4.



Regents Questions

1) JAN '16 [1] Ans: 2
2) JAN '16 [7] Ans: 4
3) JUN '16 [16] Ans: 3
4) JUN '17 [17] Ans: 3

16.6 Vertex Form with $a \neq 1$

Practice Problems

1.

$$a\left(\frac{b}{2a}\right)^2 = 2\left(\frac{4}{2\cdot 2}\right)^2 = 2$$

$y = 2x^2 + 4x + 6$
$y - 6 = 2x^2 + 4x$
$y - 4 = 2x^2 + 4x + 2$
$y - 4 = 2(x^2 + 2x + 1)$
$y - 4 = 2(x+1)^2$
$y = 2(x+1)^2 + 4$
vertex: $(-1,4)$

2.

$$a\left(\frac{b}{2a}\right)^2 = 3\left(\frac{18}{2\cdot 3}\right)^2 = 27$$

$y = 3x^2 + 18x + 26$
$y - 26 = 3x^2 + 18x$
$y + 1 = 3x^2 + 18x + 27$
$y + 1 = 3(x^2 + 6x + 9)$
$y + 1 = 3(x+3)^2$
$y = 3(x+3)^2 - 1$
vertex: $(-3,-1)$

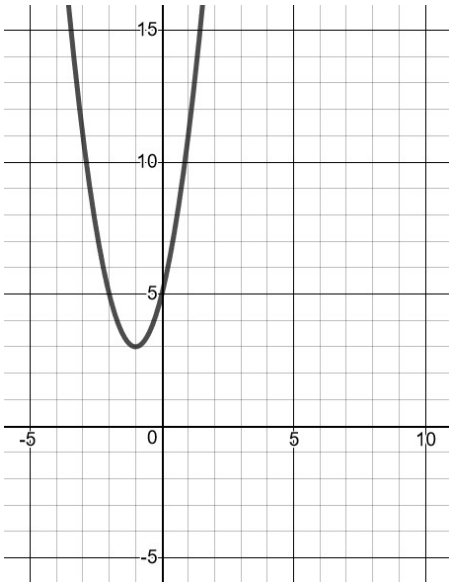
3. $(h,k) = (60,150)$ is the vertex.
So, the maximum profit is \$150.

4.

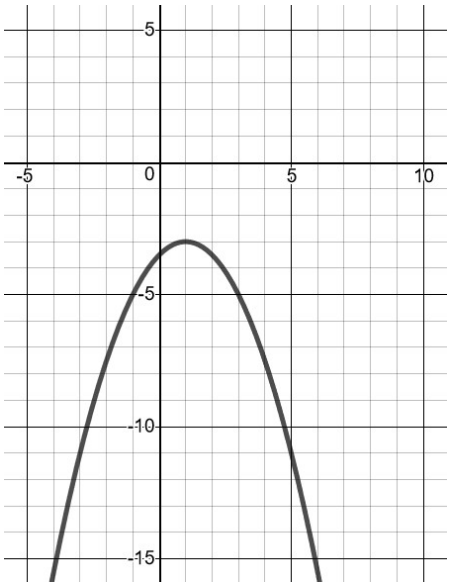
$$a\left(\frac{b}{2a}\right)^2 = -1\left(\frac{60}{2(-1)}\right)^2 = -900$$

$P(n) = -n^2 + 60n - 400$
$P(n) + 400 = -n^2 + 60n$
$P(n) - 500 = -n^2 + 60n - 900$
$P(n) - 500 = -(n^2 - 60n + 900)$
$P(n) - 500 = -(n-30)^2$
$P(n) = -(n-30)^2 + 500$
vertex: (30,500)
maximum daily profit: \$500

5.



6.



REGENTS QUESTIONS

1) AUG '16 [21] Ans: 3

2) JUN '17 [16] Ans: 4

3) JAN '15 [36]

maximum because of the negative coefficient of x^2;

$y - 9 = -x^2 + 8x$

$a\left(\frac{b}{2a}\right)^2 = -\left(\frac{8}{-2}\right)^2 = -16$

$y - 25 = -x^2 + 8x - 16$

$y - 25 = -(x^2 - 8x + 16)$

$y = -(x-4)^2 + 25$

17. QUADRATIC-LINEAR SYSTEMS

17.1 Solve Quadratic-Linear Systems Algebraically

PRACTICE PROBLEMS

1.

$$x^2-5=-4x$$
$$x^2+4x-5=0$$
$$(x+5)(x-1)=0$$
$$x=\{-5,1\}$$

When $x=-5, y=-4(-5)=20$

When $x=1, y=-4(1)=-4$

Solutions: (−5,20) and (1,−4)

2.

$$x^2+4x+1=5x+3$$
$$x^2-x-2=0$$
$$(x-2)(x+1)=0$$
$$x=\{2,-1\}$$

When $x=2, y=5(2)+3=13$

When $x=-1, y=5(-1)+3=-2$

Solutions are (2,13) and (−1,−2)

3.

$$x^2+2x-1=3x+5$$
$$x^2-x-6=0$$
$$(x-3)(x+2)=0$$
$$x=\{3,-2\}$$

When $x=3, y=3(3)+5=14$

When $x=-2, y=3(-2)+5=-1$

Solutions: (3,14) and (−2,−1)

4.

$$x^2+4x-2=2x+1$$
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x=\{-3,1\}$$

When $x=-3, y=2(-3)+1=-5$

When $x=1, y=2(1)+1=3$

Solutions: (−3,−5) and (1,3)

5.

$$y+3x=1 \rightarrow y=-3x+1$$
$$x^2+7x+22=-3x+1$$
$$x^2+10x+21=0$$
$$(x+7)(x+3)=0$$
$$x=\{-7,-3\}$$

When $x=-7, y=-3(-7)+1=22$

When $x=-3, y=-3(-3)+1=10$

Solutions: (−7,22) and (−3,10)

6.

$$y+3x=6 \rightarrow y=-3x+6$$
$$x^2=y+2x+6 \rightarrow y=x^2-2x-6$$
$$x^2-2x-6=-3x+6$$
$$x^2+x-12=0$$
$$(x+4)(x-3)=0$$
$$x=\{-4,3\}$$

When $x=-4, y=-3(-4)+6=18$

When $x=3, y=-3(3)+6=-3$

Solutions: (−4,18) and (3,−3)

7.

$$x^2+2x-8=2x+1$$
$$x^2=9$$
$$x=\pm 3$$
$$y=2(3)+1=7$$
$$y=2(-3)+1=-5$$

(3,7) and (−3,−5)

8.

$$x^2-6x+9=-9x+19$$
$$x^2+3x-10=0$$
$$(x+5)(x-2)=0$$
$$\{-5,2\}$$
$$y=-9(-5)+19=64$$
$$y=-9(2)+19=1$$

(−5,64) and (2,1)

9. $x^2+5x-17=x-5$ $x^2+4x-12=0$ $(x+6)(x-2)=0$ $\{-6,2\}$ $y=(-6)-5=-11$ $y=(2)-5=-3$ (–6,–11) and (2,–3)	10. $x^2-x-6=3x-6$ $x^2-4x=0$ $x(x-4)=0$ $\{0,4\}$ $y=3(0)-6=-6$ $y=3(4)-6=6$ (0,–6) and (4,6)

REGENTS QUESTIONS

1) AUG '15 [17] Ans: 2
2) JAN '19 [18] Ans: 3
3) JUN '17 [31]

$x^2=x$
$x^2-x=0$
$x(x-1)=0$
$\{0,1\}$

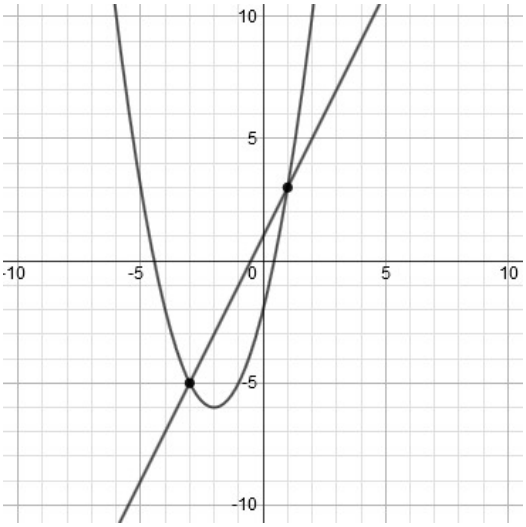
17.2 Solve Quadratic-Linear Systems Graphically

Practice Problems

1. (2) (–3, 5)

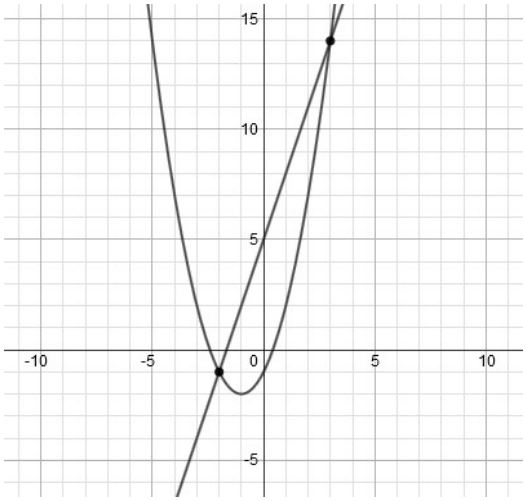
2. (1) (8, 9)

3.



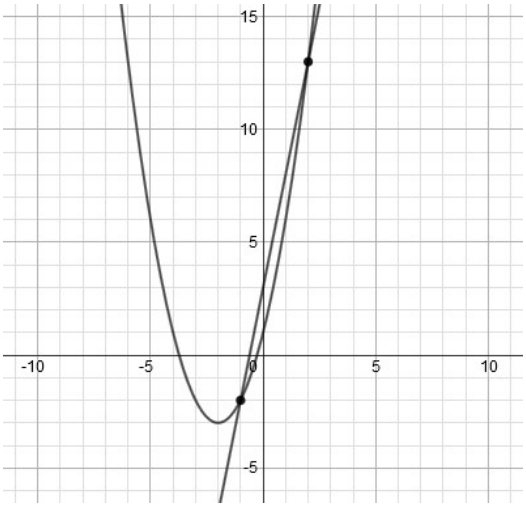
Solutions: (1,3) and (–3,–5)

4.



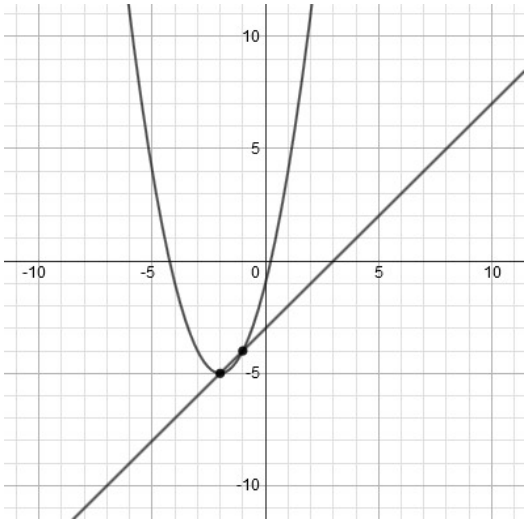
Solutions: (3,14) and (–2,–1)

5.



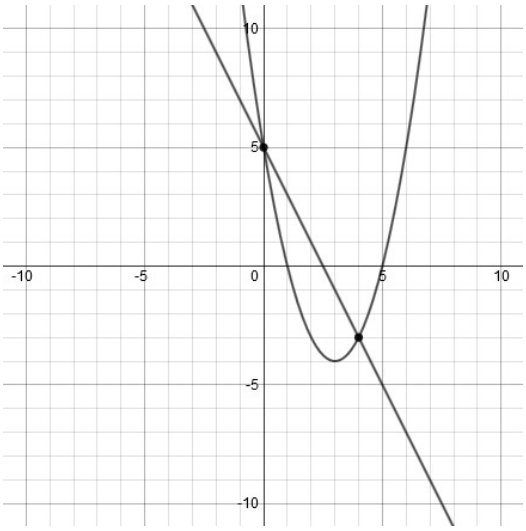
Solutions: (2,13) and (–1,–2)

6.



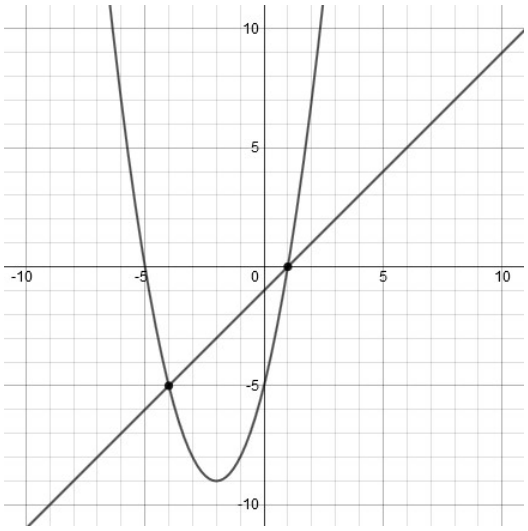
Solutions: (–1,–4) and (–2,–5)

7.



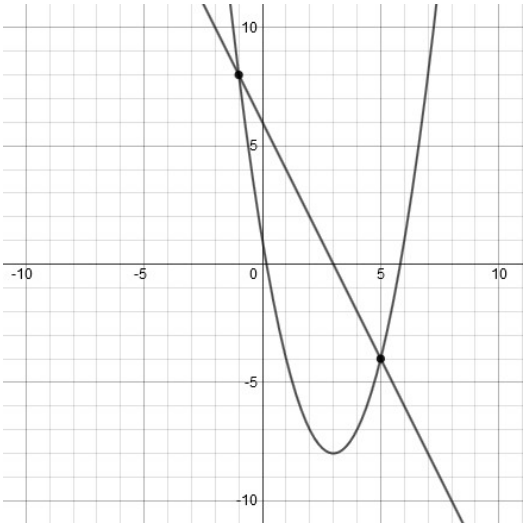
Solutions: (0,5) and (4,–3)

8.



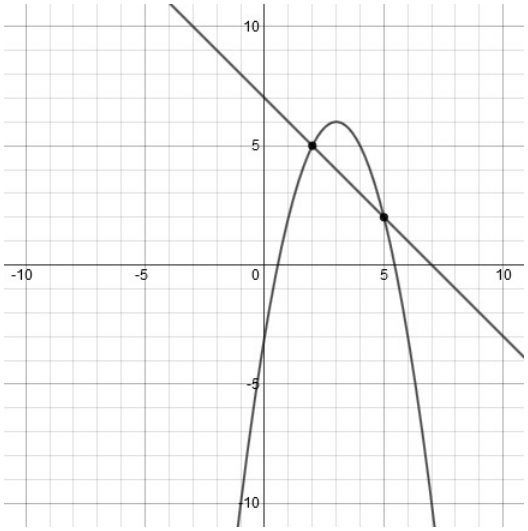
Solutions: (–4,–5) and (1,0)

9.



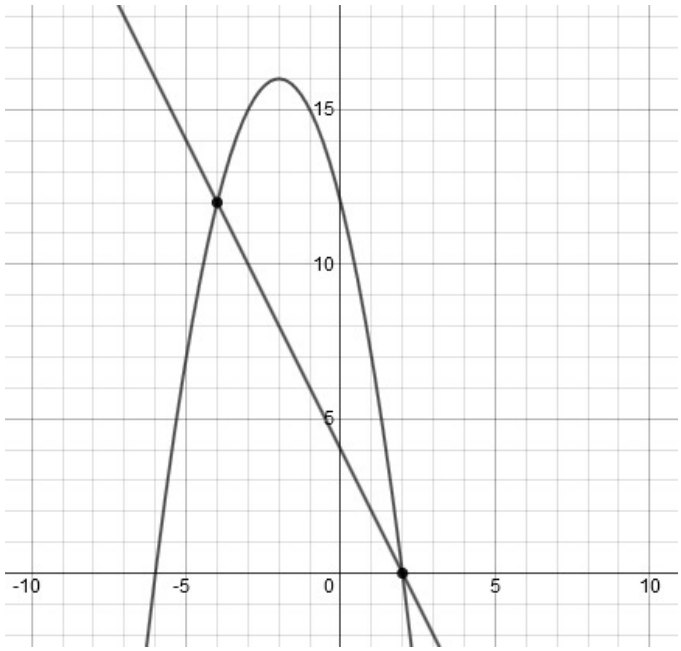
Solutions: (–1,8) and (5,–4)

10.



Solutions: (2,5) and (5,2)

11.



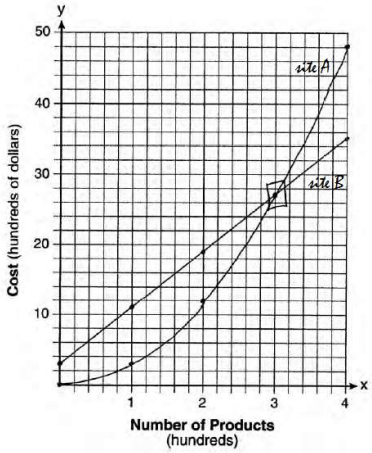
Solutions: (–4,12) and (2,0)

REGENTS QUESTIONS

1) JAN '17 [23] Ans: 2

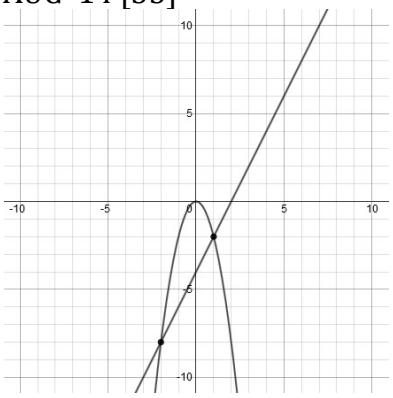
2) JAN '18 [10] Ans: 3

3) JUN '14 [37]



3 because the graphs intersect where $x = 3$; site A because the costs $A(x) < B(x)$ at $x = 2$

4) AUG '14 [35]



–2 and 1

18. OTHER FUNCTIONS AND TRANSFORMATIONS

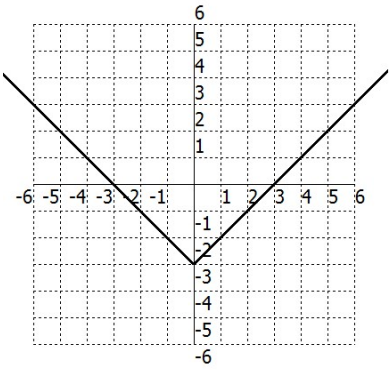
18.1 Absolute Value Functions

PRACTICE PROBLEMS

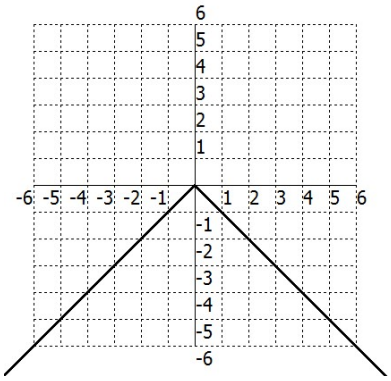
1. (4)

2. (1)

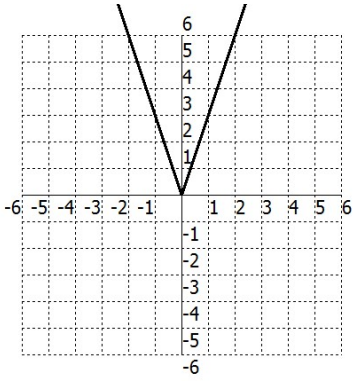
3.



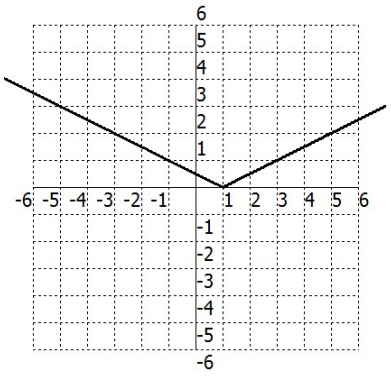
4.



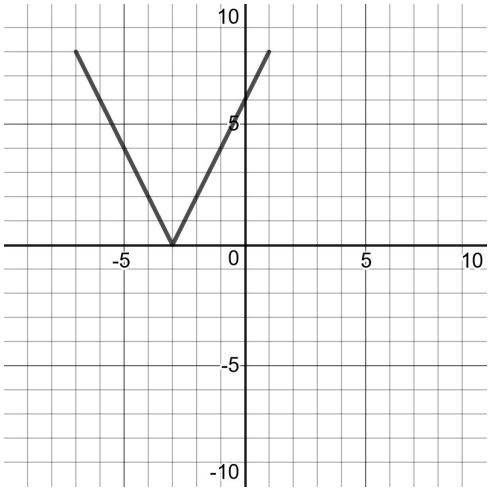
5.



6.

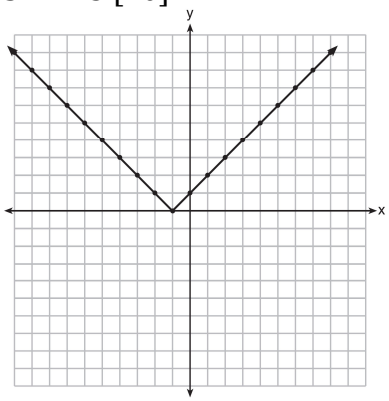


7.



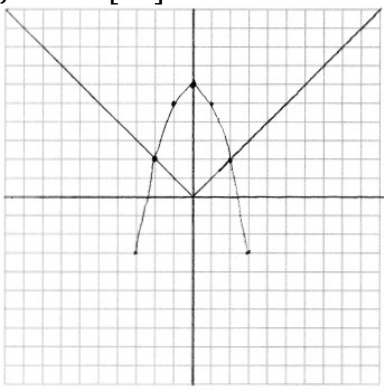
Regents Questions

1) JUN '16 [22] Ans: 2
2) JAN '17 [12] Ans: 1
3) AUG '17 [2] Ans: 2
4) AUG '17 [18] Ans: 2
5) SEP '13 [10]



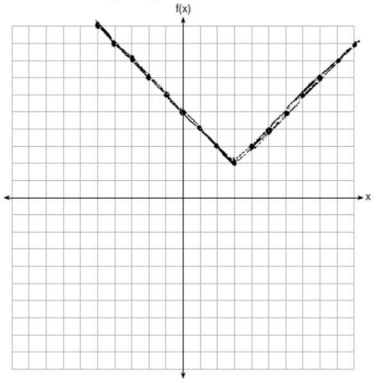
range $y \geq 0$; increasing for $x > -1$

6) JAN '17 [33]



Yes, because the graph of $f(x)$ intersects the graph of $g(x)$ at $x = -2$.

7) JAN '18 [25]

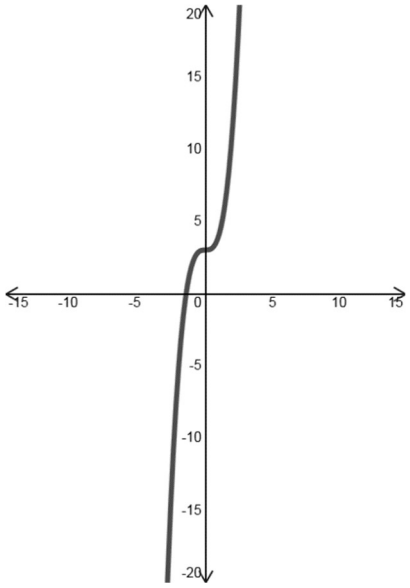


18.2 Cubic Functions

Practice Problems

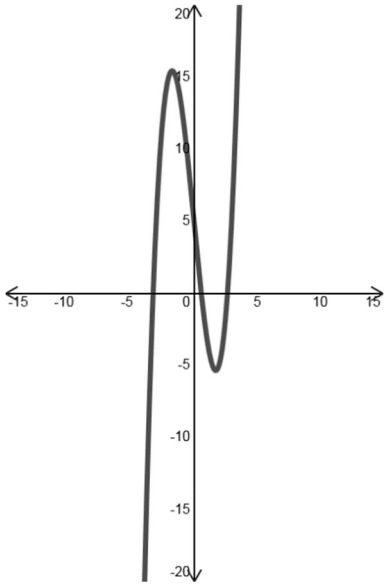
1. The function has one root.

x	y
–2	–5
–1	2
0	3
1	4
2	11



2. The function has three roots.

x	y
–3	5
–2	15
–1	13
0	5
1	–3
2	–5
3	5



Regents Questions

1) JAN '15 [24] Ans: 1
2) JUN '15 [12] Ans: 2
3) AUG '15 [4] Ans: 1
4) AUG '16 [23] Ans: 1
5) JUN '17 [10] Ans: 3
6) AUG '17 [7] Ans: 1
7) AUG '17 [19] Ans: 3
8) JAN '18 [6] Ans: 4
9) JUN '18 [18] Ans: 2
10) AUG '18 [25]
Set each factor equal to zero and solve for x, or graph the function and find the x-intercepts. −3, 1, 8.

11) JAN '19 [30]

$$3x^3 + 21x^2 + 36x = 0$$
$$3x\left(x^2 + 7x + 12\right) = 0$$
$$3x(x+4)(x+3) = 0$$
$$\{0, -4, -3\}$$

18.3 Square Root and Cube Root Functions

Practice Problems

1. (A)

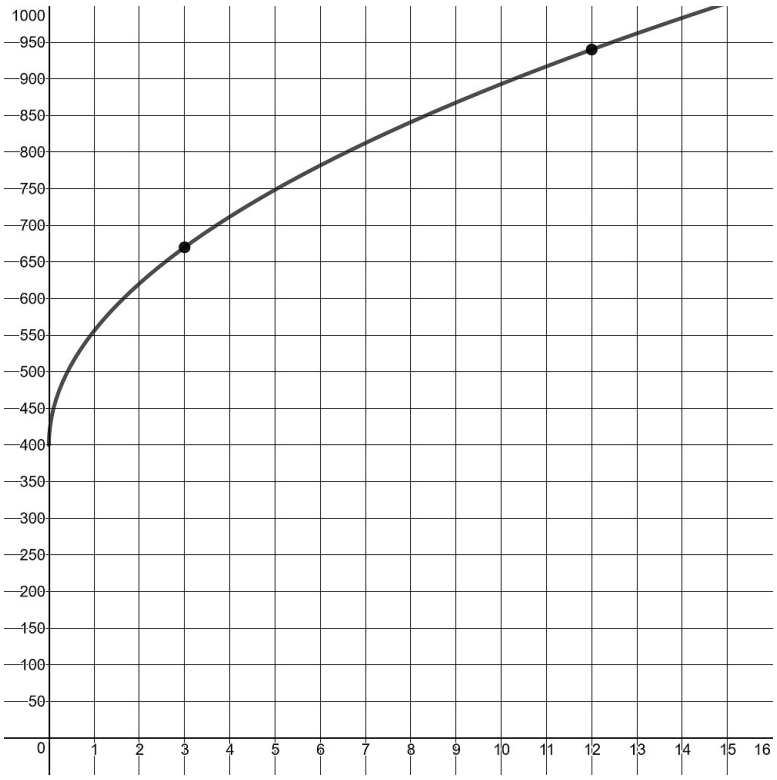
2. (A)

3.

(a)

x	y
0	400
3	670
6	781.8
9	867.7
12	940
15	1003.7

(b)



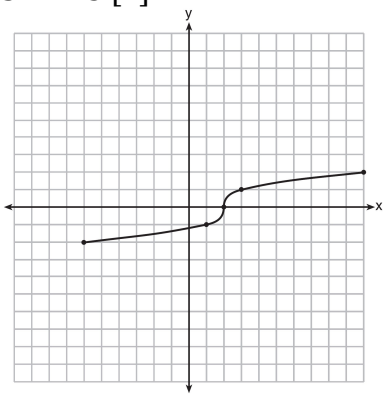
(c) 670

(d) 12

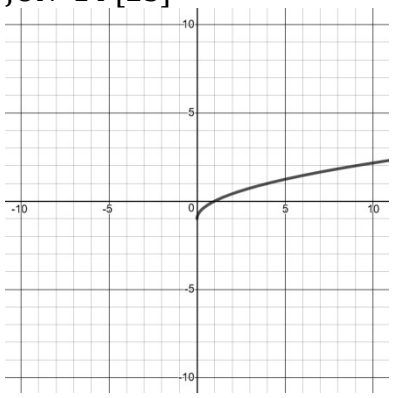
Regents Questions

1) JUN '17 [3] Ans: 4

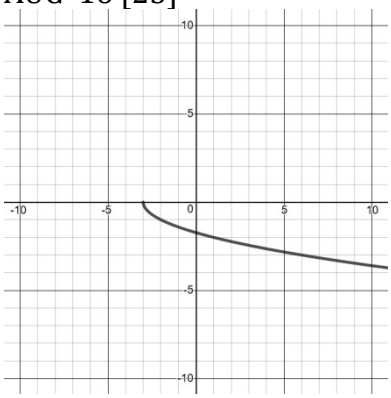
2) SEP '13 [4]



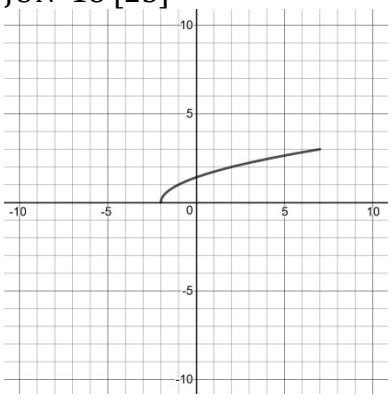
3) JUN '14 [25]



4) AUG '16 [25]

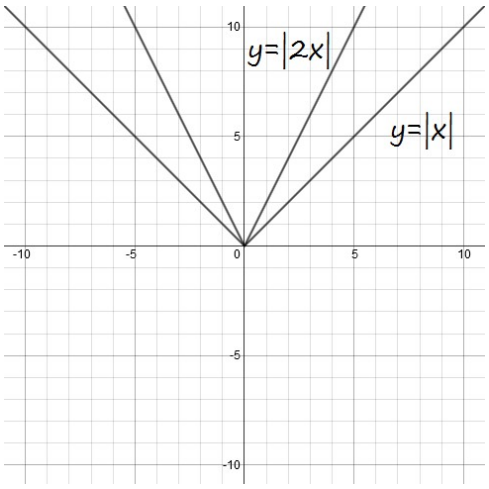
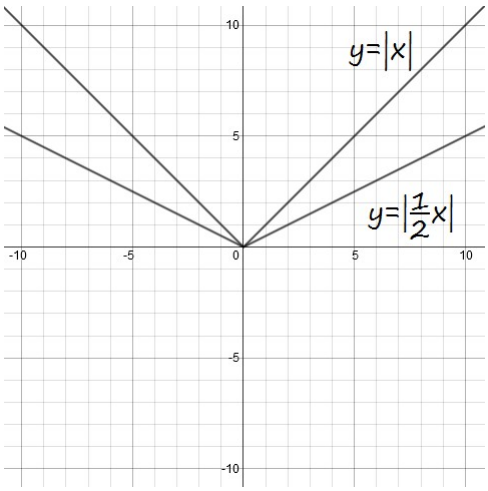
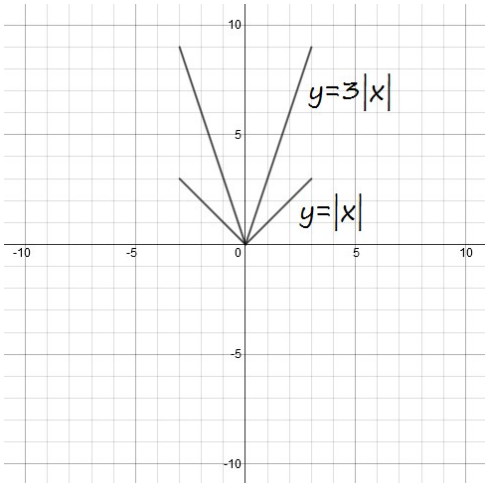


5) JUN '18 [25]



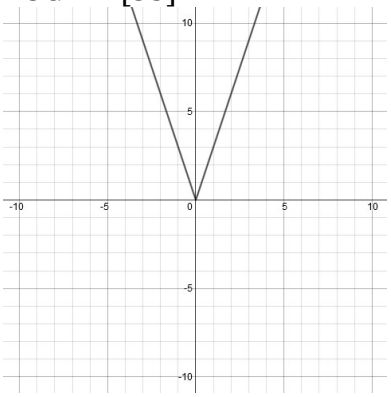
18.4 Transformations of Functions

Practice Problems

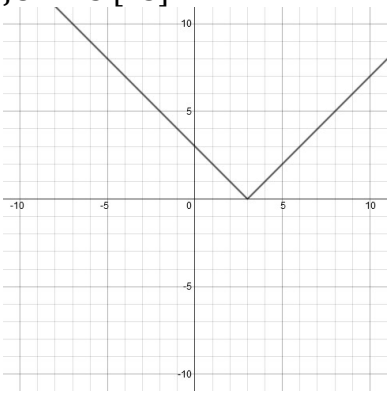
1. (4)	2. (2)
3. (3)	4. (1)
5. (2)	6. (2)
7. (4)	8. (4)
9. (1)	10. (3)
11. $y=\frac{1}{2}x^2$; wider	12. $y=\lvert x\rvert-1$
13. $y=\lvert x+4\rvert$	14. $y=-(x-1)^2$
15.  becomes narrower (horizontally shrinks)	16.  becomes wider (horizontally stretches)
17.  becomes narrower (vertically stretches)	

Regents Questions

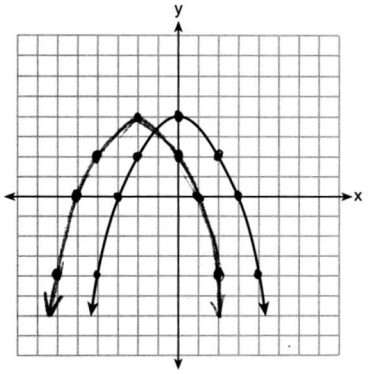
1) AUG '14 [17] Ans: 1
2) JAN '15 [12] Ans: 2
3) AUG '15 [1] Ans: 2
4) JAN '16 [20] Ans: 1
5) JAN '17 [17] Ans: 2
6) AUG '17 [6] Ans: 1
7) JAN '18 [19] Ans: 2
8) AUG '18 [8] Ans: 3
9) JAN '19 [10] Ans: 3
10) JUN '14 [28]
(4,–1), function is shifted 2 to the right
11) AUG '14 [33]



shifted 2 units down; shifted 4 units to the right
12) JUN '15 [25]



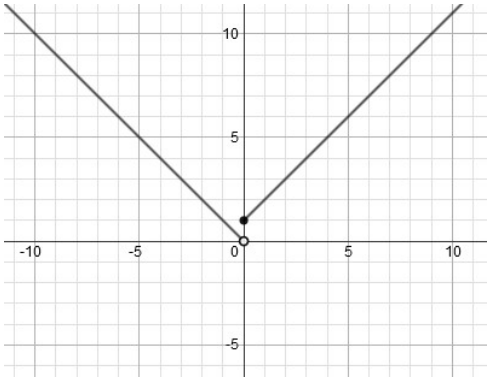
shifted 3 units to the right
13) JUN '16 [32]
$g(x) = x^3 + 2x^2 - 4$ because $g(x)$ is a translation 4 units down.
14) AUG '16 [26]
Translated 2 units right and 3 units down
15) JUN '17 [32]
$g(x)$ is a translation a units right and $h(x)$ is a translation a units down.
16) JUN '18 [28]



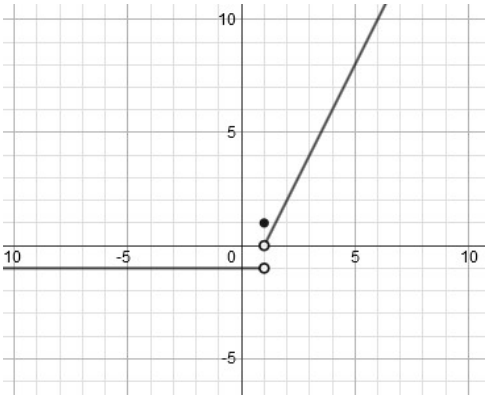
18.5 Piecewise-Defined Functions

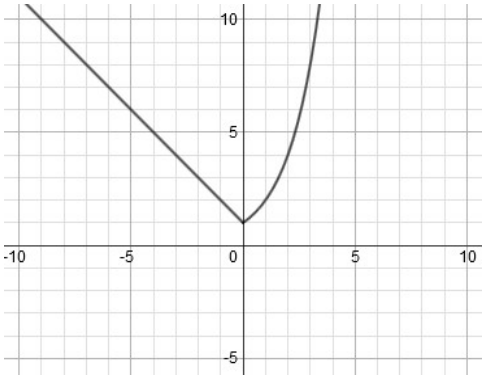
Practice Problems

1. $f(-3)=3,\ f(0)=1,\ f(2)=3$

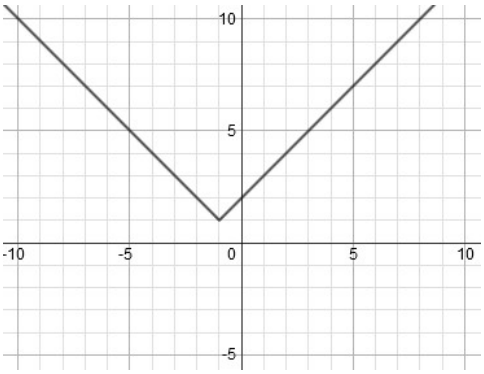
2. 

Not a continuous function.

3. 

4. 

This is a continuous function.

5. 

$$f(x)=\begin{cases}-x & x\le -1\\ x+2 & x>-1\end{cases}$$

6. $$c(t)=\begin{cases}4t & 0<t\le 2\\ 2(t-2)+8 & 2<t\le 6\\ 16 & 6<t\le 8\end{cases}$$

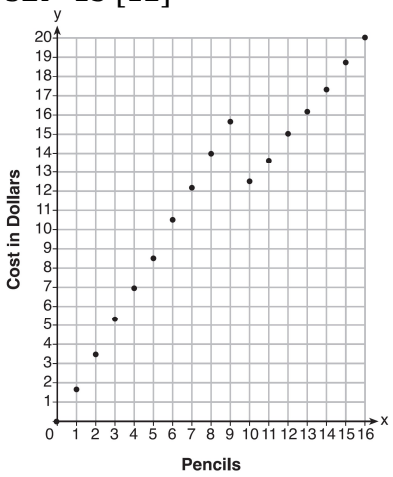
Regents Questions

1) AUG '14 [22] Ans: 2

2) AUG '15 [16] Ans: 2

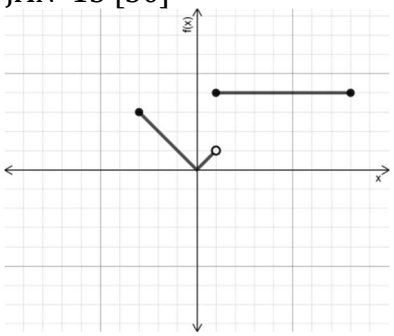
3) AUG '18 [15] Ans: 4

4) SEP '13 [12]



Since according to the graph, 8 pencils cost \$14 and 10 pencils cost \$12.50, the cashier is correct.

5) JAN '15 [30]



6) JUN '15 [34]

$15(52-40)+400=580$

$10(38)=380$

$580-380=\$200$

$15(x-40)+400=445$

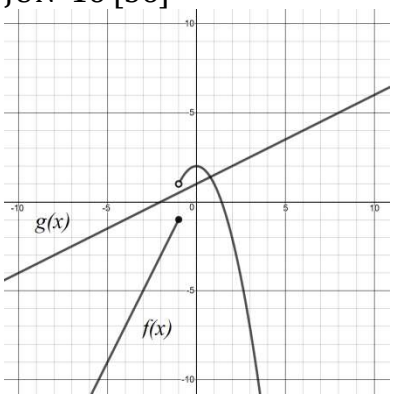
$15x-600+400=445$

$15x=645$

$x=43$

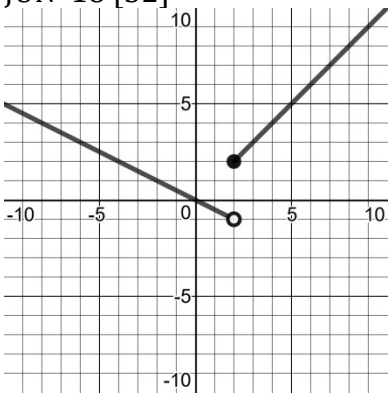
Solve $15(x-40)+400=445$ for x

7) JUN '16 [36]



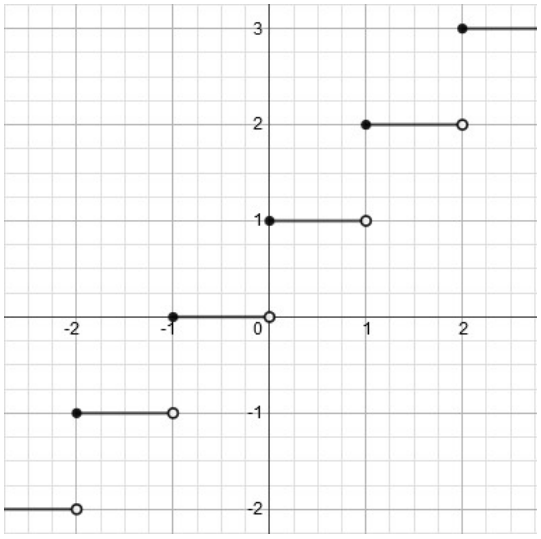
1, because there is one point of intersection

8) JUN '18 [32]



18.6 Step Functions

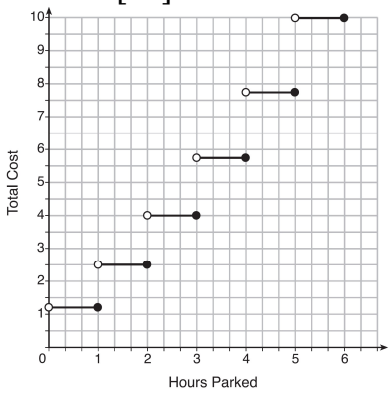
Practice Problems

1. $f(6.25) = 3\lceil 6.25 \rceil + 5 = 3(7) + 5 = 26$	2. 

Regents Questions

1) JUN '15 [7] Ans: 1

2) SEP '13 [11]



The cost for each additional hour increases after the first 2 hours.