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Effets d'écoulements non-uniformes sur la transmission et la réflexion d'ondes acoustiques

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The most exciting phrase to hear in science, the one that heralds new discoveries, is not “Eureka!” (I found it!) but “That’s funny . . .”

Isaac Asimov

LISTE DES SYMBOLES ET NOTATIONS

Notations

Dans ce manuscrit, les grandeurs notées en gras (\mathbf{v}) sont des vecteurs et celles notées en sans serif (M) sont des matrices.

Symboles de grandeurs

- A, B, C, D : coefficients de la matrice de transfert
- c_0 : célérité du son
- c_p, c_v : chaleurs spécifiques
- d : décalage latéral d'un faisceau (Chapitre 3), périodicité (Chapitres 4 et 6)
- D : décalage du cylindre de l'axe du guide d'onde
- \mathcal{D} : terme de dissipation
- e : énergie potentielle spécifique par unité de masse
- \mathbf{f}_e : forces externes
- h : taille d'un intervalle pour la résolution numérique
- I : flux d'énergie
- k : nombre d'onde ($k = \omega/c_0$)
- l : longueur des portions de tube vides dans le système périodique
- L : longueur de la portion de tube discrétisée
- \dot{m} : sources de masse

-
- M : nombre de Mach
 - \mathbf{M} : matrice de transfert
 - M_0 : nombre de Mach dans les domaines où l'écoulement moyen est uniforme
 - p : pression
 - P : composante de la pression dépendant de y
 - q : description de la périodicité ($q = 2\pi/d$)
 - \mathbf{q} : flux de chaleur
 - Q : admittance équivalente ($Q = P'/kP$)
 - Q_h : chaleur générée par unité de volume
 - R : rayon du cylindre
 - $R_{L,R}$: coefficient de réflexion à gauche/droite
 - \mathbf{S} : matrice de transmission / réflexion
 - $T_{L,R}$: coefficient de transmission à gauche / droite
 - T_e : température
 - u : vitesse
 - U : écoulement moyen
 - \mathbf{x} : vecteur des coordonnées spatiales
 - Y_0 : admittance de paroi
 - α : composante horizontale du vecteur d'onde
 - β : composante verticale du vecteur d'onde
 - δ : épaisseur de la couche de cisaillement
 - ϵ : taille du "détour" pour l'intégration dans le plan complexe

-
- Γ : circulation de l'écoulement
 - λ_ν, μ_ν : coefficients de Lamé
 - ω : fréquence angulaire ($\omega = 2\pi f$)
 - θ : angle entre le vecteur d'onde et la normale à la paroi
 - Π : tenseur des contraintes visqueuses de Cauchy
 - ρ : densité
 - τ : contrainte visqueuse

Opérateurs

- D/Dt : dérivée convective $D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla$
- ∇ : gradient
- $\nabla \cdot$: divergence
- $\nabla \wedge$: rotationnel
- Δ : Laplacien

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INTRODUCTION

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1.1 Introduction générale

La propagation du son dans des écoulements fluides est un sujet bien connu et son étude remonte à déjà plus de 200 ans [20]. L'effet de l'écoulement est intéressant en acoustique puisqu'il est, par nature, asymétrique (il provient d'une direction et se dirige vers une autre). Ainsi, une onde plane rencontrant un écoulement verra son comportement modifié de manière différente selon qu'elle se propage dans la direction de l'écoulement ou contre celui-ci. La propagation d'un son dans un écoulement uniforme permet déjà de constater ce type de comportement. En effet, la présence d'un écoulement de vitesse U modifie la longueur d'onde d'un son se propageant dans ce milieu d'un facteur $1 \pm M$, avec M le nombre de Mach, $M = U/c$. L'onde incidente verra ainsi sa longueur d'onde augmenter si elle se propage dans la direction d'un écoulement et diminuer si elle se propage contre un écoulement (Fig. 1.1).

Dans le cas d'un écoulement cisaillé, le gradient de vitesse moyenne pourra mener à une modification de la direction de propagation. Ce comportement, déjà décrit par Lord Rayleigh [63], est facilement observé par temps venteux. En effet, un son émis "contre" le vent sera dévié vers le ciel alors qu'un son émis "avec" le vent sera dévié vers le sol (Fig.1.2).

Plus récemment, de nombreuses publications ont fait usage de l'asymétrie inhérente aux écoulements pour des applications aussi diverses que de la transmission unilatérale [45,

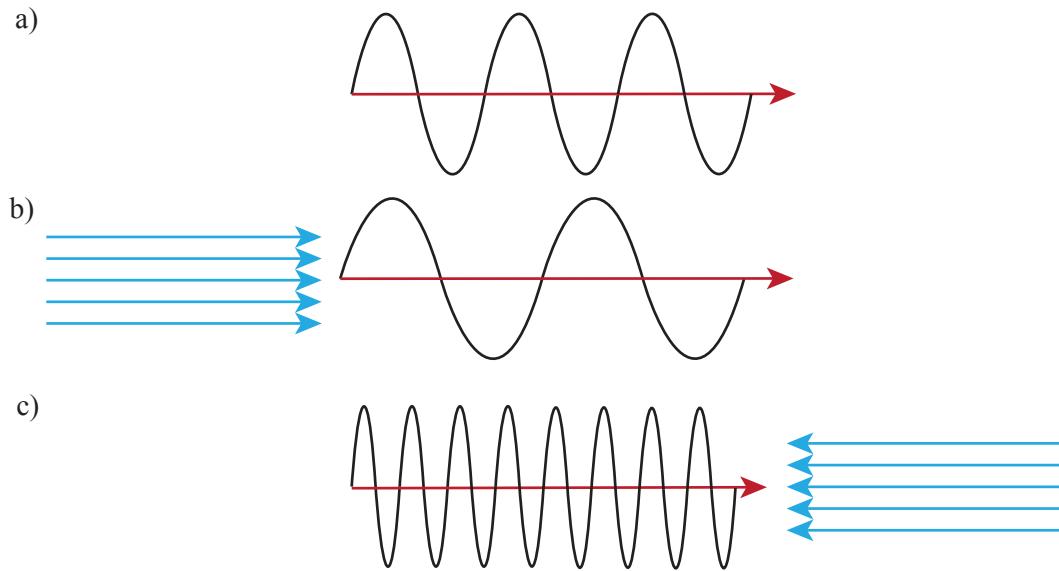


FIGURE 1.1 – Propagation d'une onde plane a) dans un milieu au repos, b) dans la direction d'un écoulement uniforme, c) contre un écoulement uniforme.

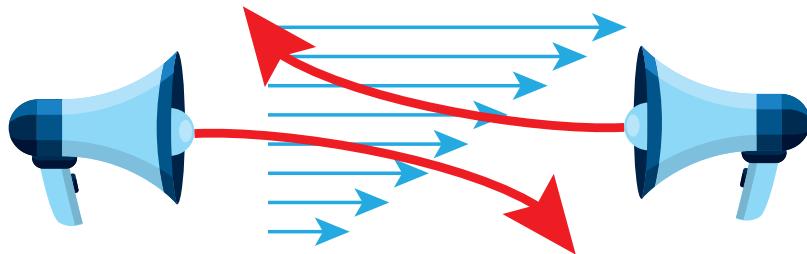


FIGURE 1.2 – Illustration de l'asymétrie de la déflection du son dans un écoulement.

52, 26], la conception de systèmes \mathcal{PT} -symétriques [4] ou encore de trous noirs analogues [3].

1.2 Description mathématique de la propagation du son en écoulement

Plusieurs modèles permettent de décrire la propagation d'ondes acoustiques en écoulement. En partant des équations de Navier-Stokes, le fait de négliger la viscosité de l'écoulement permettra par exemple de poser les équations d'Euler linéarisées, si l'écoulement est rotationnel on pourra utiliser le modèle de Möhring, s'il est parallèle on

aboutira à une équation de Pridmore-Brown et s'il est potentiel, on pourra écrire une LPE (*Linearized Potential Equation*). Dans ce manuscrit, seuls ces deux derniers cas de figure seront abordés. Dans tous les cas, ces modèles supposent la séparation d'une grandeur $\tilde{\phi}$ en une partie stationnaire liée à l'écoulement notée ϕ_0 et en une partie décrivant l'acoustique du problème notée ϕ . L'établissement de ces modèles suppose les lois de conservation listées ci-après.

- La conservation de la quantité de mouvement :

$$\frac{\partial \tilde{\rho} \tilde{\mathbf{u}}}{\partial t} + \nabla \cdot (\tilde{\rho} \tilde{\mathbf{u}} \wedge \tilde{\mathbf{u}}) = \nabla \cdot \Pi + \mathbf{f}_e + \dot{m} \tilde{\mathbf{u}}, \quad (1.1)$$

où $\tilde{\rho}$ est la densité, $\tilde{\mathbf{u}}$ est le champs de vitesse, Π est le tenseur des contraintes visqueuses de Cauchy, \mathbf{f}_e est le vecteur des forces de volumes externes et \dot{m} est un terme regroupant les sources de masses. Les termes de Π sont donnés par $\Pi_{ij} = -\tilde{p}\delta_i^j + \tau_{ij}$, avec δ_i^j le symbole de Kronecker et τ_{ij} la contrainte visqueuse, donnée pour un fluide Newtonien en fonction des coefficients de Lamé μ_ν et λ_ν par :

$$\tau_{ij} = \mu_\nu \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) + \lambda_\nu \frac{\partial \tilde{u}_k}{\partial \tilde{x}_k} \delta_i^j. \quad (1.2)$$

- La conservation de la masse :

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot \tilde{\rho} \tilde{\mathbf{u}} = \dot{m}. \quad (1.3)$$

- La conservation de l'énergie :

$$\frac{\partial \tilde{\rho}(e + \frac{|\tilde{\mathbf{u}}|^2}{2})}{\partial t} + \nabla \cdot [\tilde{\rho}(e + \frac{|\tilde{\mathbf{u}}|^2}{2})] \tilde{\mathbf{u}} = \nabla \cdot (\Pi \cdot \tilde{\mathbf{u}}) + \mathbf{f}_e \cdot \tilde{\mathbf{u}} - \nabla \cdot \mathbf{q} + Q_h + \dot{m}(e + \frac{|\tilde{\mathbf{u}}|^2}{2}), \quad (1.4)$$

où e est l'énergie potentielle spécifique par unité de masse, \mathbf{q} est le flux de chaleur et Q_h est la chaleur générée par unité de volume.

- Le premier principe de la thermodynamique :

$$T_e ds = de - \frac{\tilde{p}}{\tilde{\rho}^2} d\tilde{\rho}, \quad (1.5)$$

où T_e est la température thermodynamique.

- Le second principe de la thermodynamique :

$$d\tilde{\rho} = \left(\frac{\partial \tilde{\rho}}{\partial \tilde{p}} \right)_s d\tilde{p} + \left(\frac{\partial \tilde{\rho}}{\partial s} \right)_{\tilde{p}} ds. \quad (1.6)$$

Ces propriétés se simplifient sous l'hypothèse d'un problème isentropique ($ds = 0$) en gaz parfait et avec un écoulement non-visqueux et non-conductif. De plus, dans notre cas le problème ne comporte pas de sources de masse, de chaleur ou de quantité de mouvement, si bien que les champs de pression, de vitesse et de densité peuvent être décrits par un système de trois équations (les équations d'Euler) :

$$\begin{cases} \frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\tilde{\rho} \tilde{\mathbf{u}}) = 0 \\ \frac{\partial \tilde{\rho} \tilde{\mathbf{u}}}{\partial t} + \nabla \cdot (\tilde{\rho} \tilde{\mathbf{u}} \wedge \tilde{\mathbf{u}}) = 0 \\ \frac{1}{c_0^2} \frac{\partial \tilde{p}}{\partial t} + \tilde{\rho} \nabla \cdot \mathbf{u} = 0, \end{cases} \quad (1.7)$$

où c_0 , la célérité du son, est définie par le rapport des chaleurs spécifiques c_p/c_v .

En supposant à présent que \tilde{p} , $\tilde{\rho}$ et $\tilde{\mathbf{u}}$ sont composées d'une partie stationnaire (liée à l'écoulement) et d'une partie oscillante, on a :

$$\begin{cases} \tilde{p}(\mathbf{x}, t) = p_0(\mathbf{x}) + p(\mathbf{x}, t) \\ \tilde{\rho}(\mathbf{x}, t) = \rho_0(\mathbf{x}) + \rho(\mathbf{x}, t) \\ \tilde{\mathbf{u}}(\mathbf{x}, t) = \mathbf{u}_0(\mathbf{x}) + \mathbf{u}(\mathbf{x}, t), \end{cases} \quad (1.8)$$

où \mathbf{x} est le vecteur des coordonnées spatiales. Ainsi, l'écoulement vérifiera l'ensemble d'équations suivantes :

$$\begin{cases} \nabla \cdot (\rho_0 \mathbf{u}_0) = 0 \\ \rho_0 \mathbf{u}_0 \cdot (\nabla \cdot \mathbf{u}_0) + \nabla p = 0 \\ \frac{1}{c_0^2} \mathbf{u}_0 \cdot \nabla p_0 + \rho_0 \nabla \cdot \mathbf{u}_0 = 0, \end{cases} \quad (1.9)$$

et les équations d'Euler, une fois linéarisées, deviennent :

$$\begin{cases} \frac{D\rho}{Dt} + \rho_0 \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla \rho_0 + \rho \nabla \cdot \mathbf{u}_0 = 0, \\ \rho_0 \frac{D}{Dt} \mathbf{u} + \nabla p + \rho_0 (\mathbf{u} \cdot \nabla) \mathbf{u}_0 + \rho (\mathbf{u}_0 \cdot \nabla) \mathbf{u}_0 = 0 \\ \frac{Dp}{Dt} + \rho_0 c_0^2 \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{p}_0 + c_0^2 \rho \nabla \cdot \mathbf{u}_0 - \mathbf{u}_0 \cdot \nabla p_0 \left(\frac{p}{p_0} - \frac{\rho}{\rho_0} \right) = 0, \end{cases} \quad (1.10)$$

où $D/Dt = \partial/\partial t + \mathbf{u}_0 \cdot \nabla$ désigne la dérivée convective qui porte déjà le caractère non-réiproque de la propagation acoustique en écoulement. Finalement, des hypothèses sur l'écoulement (uniforme, parallèle, potentiel...) permettront d'aboutir aux équations de propagation que nous utiliserons dans la suite de ce manuscrit.

1.3 Structure du manuscrit

Ici, la propagation d'ondes acoustique est étudiée dans plusieurs types d'écoulements, comme illustré en Fig.1.3. Pour chacune de ces configurations, la non-réciprocité du système sera utilisée pour déclencher des phénomènes pouvant exister dans d'autres domaines de la physique ou dans d'autres systèmes.

Par exemple, dans le Chapitre 2, la réfraction négative (Fig.1.4 a)) d'un faisceau gaussien est réalisée à l'aide d'un écoulement cisaillé, comme elle pourrait l'être à l'aide d'un changement d'indice optique pour un faisceau d'ondes électromagnétiques. Dans le Chapitre 3, l'écoulement est utilisé pour atteindre l'absorption parfaite par une paroi absorbante, mais aussi pour décaler latéralement un faisceau réfléchi. Ce phénomène est connu en optique sous le nom de décalage de Goos-Hänchen [31], ou en ultrasonique sous le nom d'effet de Schoch [18] et a déjà pu être observé en géophysique [41].

Les Chapitres 4, 5 et 6, finalement, concernent l'influence de l'écoulement sur des systèmes aux propriétés particulières. Celui-ci sera alors utilisé comme un outil pour modifier voire contrôler des comportements particulier de ces systèmes. Ainsi, l'écoulement permettra dans le Chapitre 4 de contrôler la direction principale de la réflexion sur une métasurface. Dans le Chapitre 5, c'est le caractère piégé ou fuyant d'un mode de résonance qui se verra modifié par la présence d'un écoulement. Finalement, le chapitre 6 permettra de mettre en lumière la présence d'un mode d'interface dans un système périodique, qui s'exprimera plus ou moins selon l'écoulement.

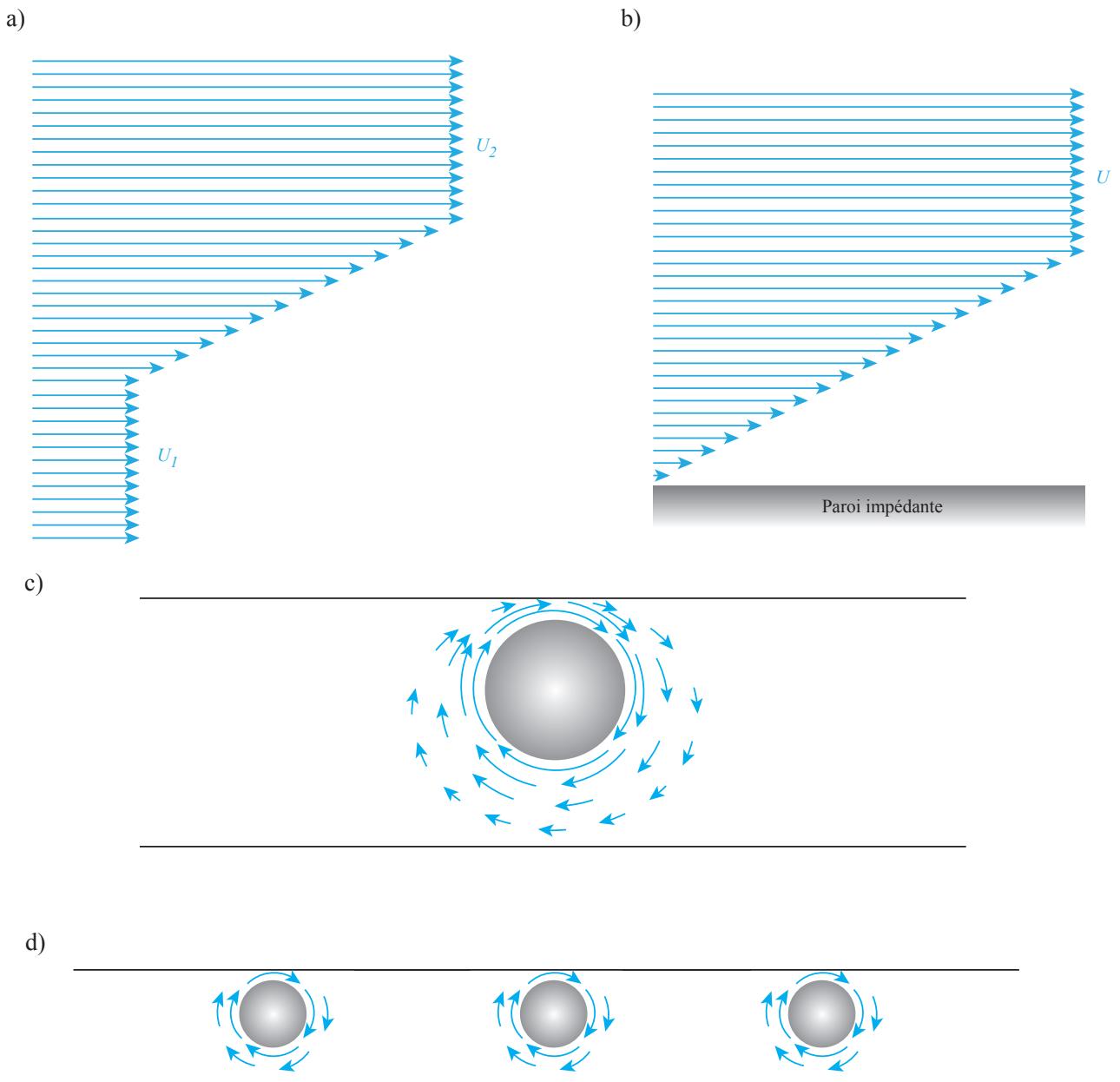


FIGURE 1.3 – Types d’écoulements considérés dans ce manuscrit a) dans le Chapitre 2 ; b) dans les Chapitres 3 et 4 ; c) dans le chapitre 5 ; d) dans le Chapitre 6.

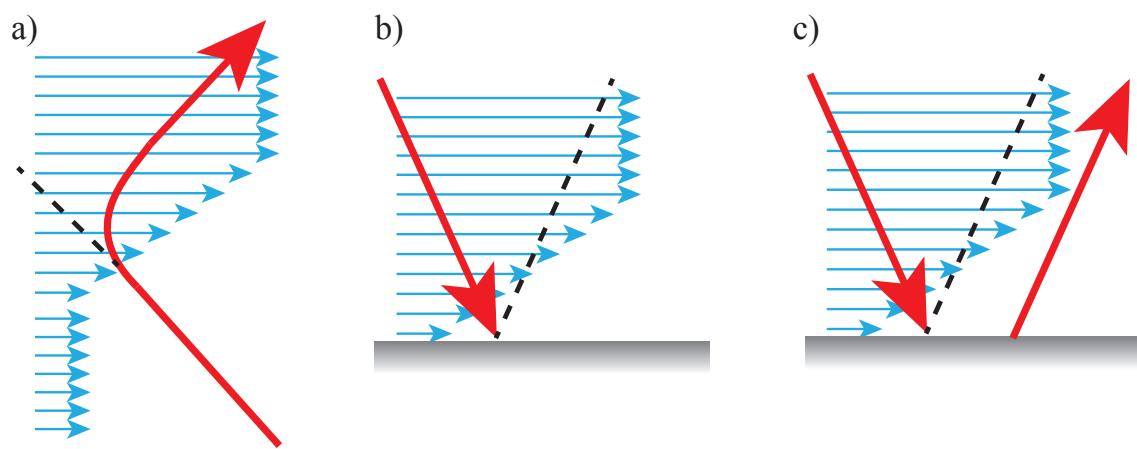


FIGURE 1.4 – Phénomènes non-réciroques observés dans les Chapitres 2 et 3. a) Réfraction négative, b) Absorption parfaite , c) Décalage latéral.

NON-RECIPROCAL SCATTERING IN SHEAR FLOW : TRANSMISSION / REFLECTION BETWEEN TWO MEDIA IN UNIFORM MOTION

Ce premier chapitre est tiré de la première partie de l'article publié en Août 2019 dans le *Journal of the Acoustical Society of America* [60]. Il expose des phénomènes remarquables lors de la transmission/réflexion d'ondes planes acoustiques à travers une couche de cisaillement linéaire entre deux milieux au repos. Une relation de conservation de l'énergie est également dérivée.

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2.1 Introduction

Adding flow to an acoustic system is one of the simplest ways to make it non-reciprocal. Recently, it has been used to obtain sound isolation [27] or asymmetric

propagation [52], in the broad context of research on unidirectional transmission devices for acoustic waves [45, 26]. It appears that sound-flow interactions have been studied for a very long time [54], early works being motivated by the study of the deflection of sound in the wind [20]. For instance, in the *Theory of Sound*, Rayleigh [63] uses geometric acoustics to show in which direction the sound is deflected according to the direction of the wind. Subsequently, varieties of studies were then led on the effect of shear flows on acoustic propagation, from a simple flow velocity discontinuity [46, 47, 58], to piece-wise constant profiles [66, 1] or continuous parallel shear flows [32, 39, 37]. Many other papers deal with the propagation of sound in parallel shear flow in the presence of walls, either for ducts [57] or for reflection problems [35, 48, 30, 53, 15, 44, 51, 11]. For impedance walls, it can be noted that analytic expressions exist for very small boundary layers [9, 28], which tend to the one given by Ingard [35] and Myers [49] for vanishing boundary layers.

Though rarely mentioned as such, non-reciprocity is obviously an existing feature in most of the previously cited works. In this paper, we will inspect the effect of basic parallel shear flows on the non-reciprocity properties. In the model we use, both the mean Mach number and the height of the shear layer can be varied independently. Though not so realistic, this will allow us to highlight some key aspects of the influence of shear. We consider two specific scattering problems : i) reflection and transmission, when the fluid is moving at two different velocities above and under a layer of shear flow with a linear profile, ii) reflection¹, when the fluid is flowing above a lined wall with a shear layer. The first problem we consider is similar to the ones studied in jet-like configurations by Amiet[1] and Jones [37], except we consider it as a scattering problem. This enables us to get a unitary relation on the full scattering matrix. In the second problem, we highlight other non-reciprocal effects such as unilateral perfect absorption or asymmetric lateral displacement of Gaussian beams. The latter phenomenon is known as Goos-Hänchen shift [31, 2] in optics [12, 18] or Schoch effect in ultrasounds [64, 22, 33]. In the presence of shear flow, we show it is possible to reach large lateral displacement due to non-reciprocity.

2.2 General scattering properties of a shear flow

We consider the problem of the scattering of a plane wave by a layer of shear flow between two fluids moving at constant Mach number M_1 and M_2 , as described in Fig.

1. dans ce manuscrit, cette seconde partie est traitée dans le chapitre 3

2.1. We study this problem using a scattering matrix S linking the incident amplitudes $A_{1,2}$ to the scattered amplitudes $B_{1,2}$.

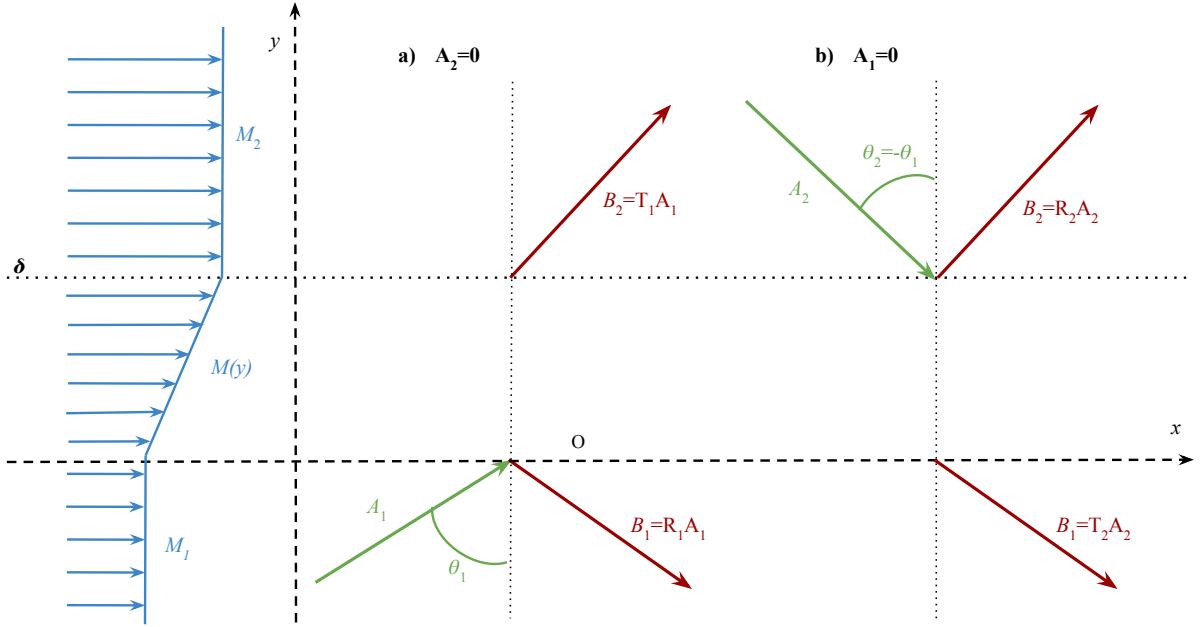


FIGURE 2.1 – Scheme of the problem of scattering by a layer of shear flow a) in the case when the incident wave comes from below the shear layer, b) when it comes from above the shear layer.

2.2.1 Energy conservation

The propagation equation for a wave in a parallel flow is given by [57] :

$$D_t[(\partial_x^2 + \partial_y^2)p - \frac{1}{c_0^2} D_t^2 p] - 2\partial_y U(y) \partial_{x,y}^2 p = 0, \quad (2.1)$$

where D_t is a convective derivative term, $D_t = \partial_t + U\partial_x$, and $U(y)$ is the mean velocity of the flow, such that $M(y) = U(y)/c_0$, with c_0 the constant sound speed. To study the energy conservation, non-trivial in shear flow, we consider the pressure p to be a plane harmonic wave, such that $p(x, y, t) = \Re[P(y)e^{-i(\omega t - \alpha x)}]$, where $\omega = 2\pi f$ is the circular frequency, $\alpha = K \sin(\theta_i)/(1 + M_i \sin(\theta_i))$ is the horizontal component of the wave wave number, $K = \omega/c_0$ is the natural wave number, $M_i = M_{1,2}$ the mean Mach in the medium of incidence and $\theta_i = \theta_{1,2}$, the angle of incidence. Then, we obtain the Pridmore-Brown

[57] equation for P :

$$P'' + \frac{2\alpha M'(y)}{K - M(y)\alpha} P' + [(K - M(y)\alpha)^2 - \alpha^2]P = 0, \quad (2.2)$$

where the prime notation stands for a derivation with respect to y . The time-averaged energy equation, as given by Myers [50] is :

$$\nabla \cdot \langle \mathbf{I} \rangle = - \langle \mathcal{D} \rangle, \quad (2.3)$$

with \mathbf{I} the energy flux given by $\mathbf{I} = (\rho_0 \mathbf{v} + \rho \mathbf{v}_0)(c_0^2 \rho / \rho_0 + \mathbf{v}_0 \cdot \mathbf{v})$, and \mathcal{D} the dissipation, given by $\mathcal{D} = \rho_0 \mathbf{v}_0 \cdot ((\nabla \times \mathbf{v}) \times \mathbf{v}) - \rho \mathbf{v} \cdot ((\nabla \times \mathbf{v}_0) \times \mathbf{v}_0)$. In these expressions, ρ_0 is the mean density, ρ is the density variation which in adiabatic conditions is equal to p/c_0^2 , \mathbf{v}_0 is the mean velocity field and \mathbf{v} is the acoustic particle velocity which we can evaluate using linearized Euler equations with flow as :

$$\mathbf{v} = \begin{cases} V_x(y) e^{-i(\omega t - \alpha x)}, \\ V_y(y) e^{-i(\omega t - \alpha x)}, \end{cases} \quad (2.4)$$

with :

$$\begin{cases} \rho_0 c_0 V_x(y) = [\frac{\alpha P}{K - M\alpha} - \frac{M' P'}{(K - M\alpha)^2}], \\ \rho_0 c_0 V_y(y) = \frac{-i P'}{K - M\alpha}. \end{cases} \quad (2.5)$$

Substituting these values in the definition of \mathcal{D} , we find :

$$\langle \mathcal{D} \rangle = \frac{MM'}{2(K - M\alpha)^3} \Im \left[(P'' + \frac{2\alpha M'}{K - M\alpha} P' + [(K - M\alpha)^2 - \alpha^2]P) P'^* \right]. \quad (2.6)$$

Using Eq. 2.2, $\langle \mathcal{D} \rangle = 0$ and Eq. 2.3 can now be evaluated to :

$$\left[\frac{\Im(PP'^*)}{(K - M(y)\alpha)^2} \right]_{y_1}^{y_2} = 0, \quad (2.7)$$

meaning that the energy flux of plane waves is conserved across the shear layer.

2.2.2 Scattering matrix

In the domains of constant velocity, the pressure is given by :

$$\begin{cases} P(y) = A_1 e^{i\beta_1 y} + B_1 e^{-i\beta_1 y} & \text{for } y < 0, \\ P(y) = A_2 e^{-i\beta_2 y} + B_2 e^{i\beta_2 y} & \text{for } y > \delta, \end{cases} \quad (2.8)$$

where $\beta_j^2 = (K - M_j \alpha)^2 - \alpha^2$ ($j = 1, 2$). Using Eq. 2.7 and the formalism shown in Fig. 2.1 we obtain :

$$\frac{\beta_1}{(K - M_1 \alpha)^2} |A_1|^2 + \frac{\beta_2}{(K - M_2 \alpha)^2} |A_2|^2 = \frac{\beta_1}{(K - M_1 \alpha)^2} |B_1|^2 + \frac{\beta_2}{(K - M_2 \alpha)^2} |B_2|^2. \quad (2.9)$$

We define a scattering matrix S by :

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = S \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} R_1 & T_2 \\ T_1 & R_2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \quad (2.10)$$

where R_1 (respectively R_2) is the reflection coefficient for a wave coming from medium 1 (resp. 2); T_1 (respectively T_2) is the transmission coefficient for a wave coming from medium 1 (resp. 2). With the correct normalization on the amplitudes, one can re-write equation 2.9 as $\tilde{S}^\dagger \tilde{S} = I$, where the \dagger exponent stands for Hermitian transposition, \tilde{S} is the normalized scattering matrix and I is the 4-by-4 identity matrix. This last relation is exactly the same relation as in the no-flow case. The scattering coefficients then satisfy the four relations :

$$\begin{cases} |R_1|^2 + \frac{\beta_2}{\beta_1} \left(\frac{K - M_1 \alpha}{K - M_2 \alpha} \right)^2 |T_1|^2 = 1, \\ |R_2|^2 + \frac{\beta_1}{\beta_2} \left(\frac{K - M_2 \alpha}{K - M_1 \alpha} \right)^2 |T_2|^2 = 1, \\ |R_1| = |R_2|, \\ |T_1| = \frac{\beta_1}{\beta_2} \left(\frac{K - M_2 \alpha}{K - M_1 \alpha} \right)^2 |T_2|. \end{cases} \quad (2.11)$$

Equations 2.11 show that the relation between the scattering coefficients is independent of the shear layer thickness. However, their values do depend on it (see appendix 2.3). In the case when one of the incident waves has zero-amplitude, Eq. 2.11 verifies the relation found by Jones [37].

2.2.3 Dispersion relation and anomalous refraction properties

A useful representation of scattering between two media of different characteristics is to represent dispersion relations in both media in the (a, b) plane (a and b being respectively the horizontal and vertical component of the wave number, normalized by K). In our case, we have the following dispersion relation :

$$a_j^2 + b_j^2 = (1 - M_j a_j)^2, \quad (2.12)$$

with j the index of the considered medium. In this formulation, we recognize a conic equation, with an eccentricity directly given by $|M_j|$. Therefore, we will have a circle of radius 1 in the (a, b) plane if $M_j = 0$; an ellipse if $|M_j| < 1$; a parabola if $|M_j| = 1$; an hyperbola if $|M_j| > 1$. This property is illustrated in Fig. 2.2. Plotting such a relation gives access to both the group and the phase velocity directions for the refracted wave (see Fig. 2.3). As we study Gaussian beams, the direction of the group velocity, given by the direction of the normal to the ellipse at the point $(a(\theta), b(\theta))$ will have a particular interest as it will give the apparent direction of the beam.

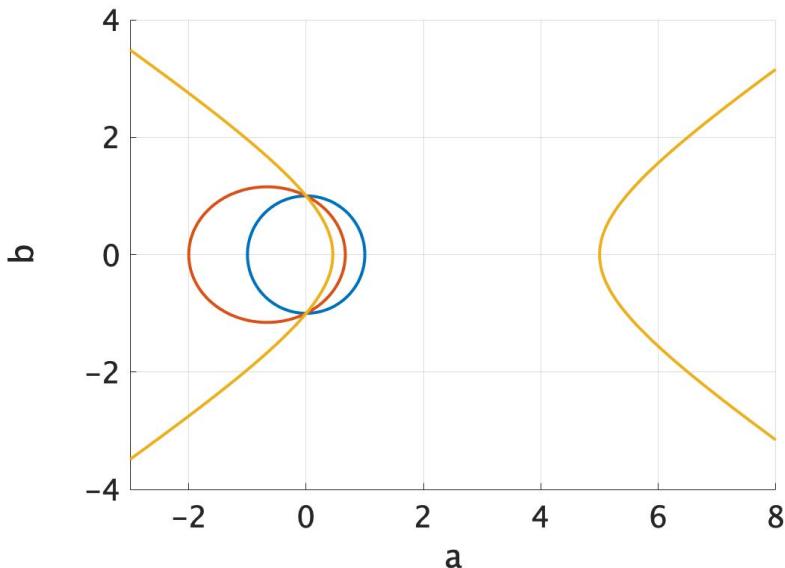


FIGURE 2.2 – Dispersion relation curve in the $(a; b)$ plane (a and b being the dimensionless horizontal and vertical wave numbers), when $M_0 = 0$ (blue circle); $M_0 = 0.5$ (red ellipse) and $M_0 = 1.2$ (yellow hyperbola).

The beams are built by summing weighed plane waves as follows :

$$p(x, y) = \int_{-\infty}^{+\infty} e^{-(\alpha - \alpha_0)^2/w^2} p(\alpha, x, y) d\alpha, \quad (2.13)$$

with α_0 the horizontal wave number in the (imposed) main direction of the beam, and w the waist of the beam. In Fig. 2.3, these beams are represented for the propagation from a medium 1 at rest to a medium 2 where $M_2 = 0.4$. By looking at the ellipses in the (a, b) plane, and since α is constant between the two media, we can see that for some incidence, θ_1 and θ_2 can be of opposite sign. This result can be interpreted as an equivalent negative refractive index, analog to what is observed for electromagnetic waves in left handed materials [62] and in hyperbolic meta-materials [29, 16, 38]. The shear layer thickness will influence the value of the moduli of R and T but not the direction of the transmitted beam which only depends on the Mach numbers.

2.3 Appendix A : Numerical determination of the scattering coefficients in a transmission problem

To find the scattering coefficients, we need to solve Eq. 2.2. For this, we define Q :

$$Q(y) = \frac{P'(y)}{KP(y)}. \quad (2.14)$$

Then Eq. 2.2 can be re-written as a first-order differential equation for Q , and becomes a Riccati equation. Moreover, to make its integration easier, a dimensionless variable s is introduced, such that $s = \frac{y}{\delta}$. The integration will then be done on $[0, 1]$. Finally, the equation to solve is :

$$Q'(s) = -K\delta[Q(s)^2 + (1 - M_0as)^2 - a^2] - \frac{2M_0a}{1 - M_0as}Q(s), \quad (2.15)$$

where $M_0 = M_2 - M_1$ and $a = \frac{\alpha}{K}$. Solving for the whole scattering matrix requires the study of two cases :

- if the incident wave comes from medium 1, A_2 is null, and Eq. 2.8 gives us the following boundary condition in $s = 1$:

$$Q(1) = ib_2, \quad (2.16)$$

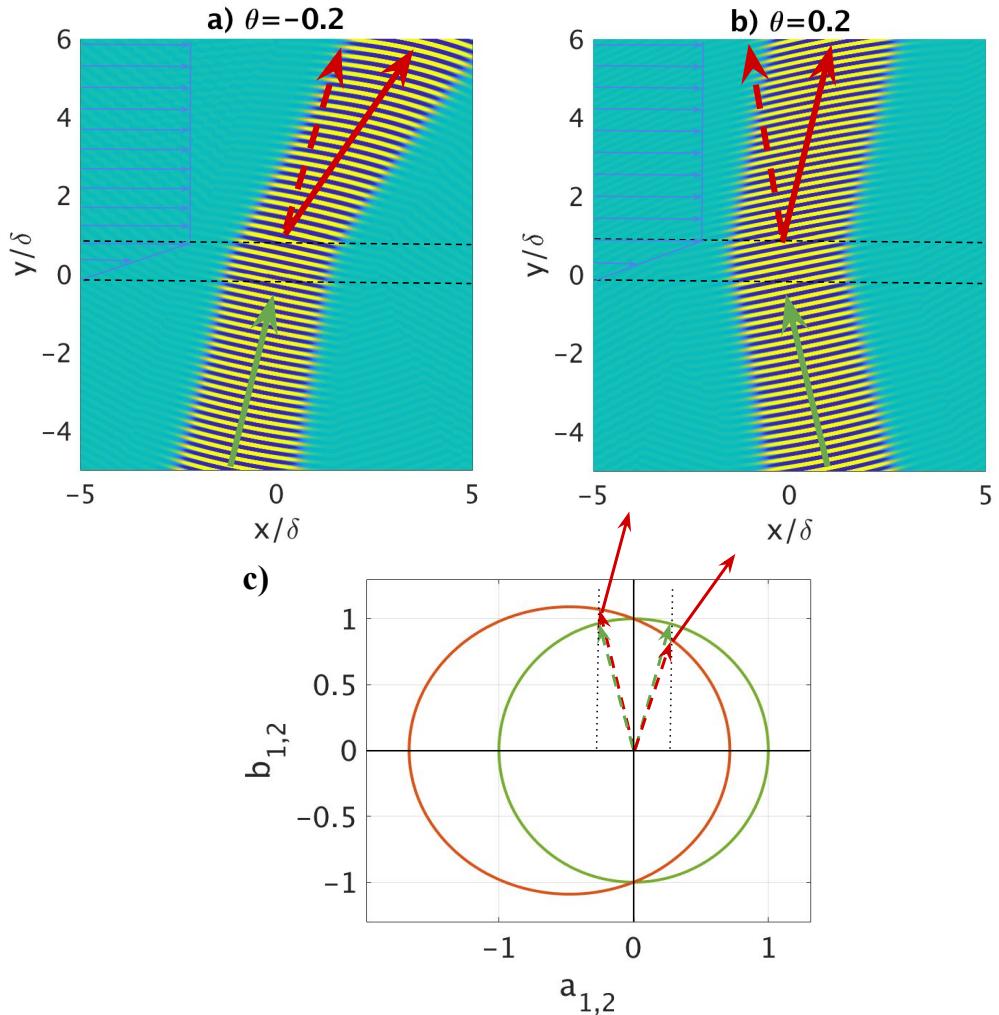


FIGURE 2.3 – Beams coming from medium 1 with an incident angle a) $\theta = -0.2$ or b) $\theta = 0.2$. c) represents the dispersion relations in the plane (a_i, b_i) , with Mach numbers $M_1 = 0$ and $M_2 = 0.4$. Conservation of a at the interfaces allows us to find the direction of the phase velocity by finding the point of the ellipse with abscissa a . The normal to the ellipse in this point then gives the direction of the group velocity (i.e. the visible direction of the beam). Green items are linked to medium 1 and red items to medium 2. Solid arrows stand for group velocity direction and dashed arrows for phase velocity. In medium 1, only one arrow is represented as phase and group velocity are co-linear in a medium at rest.

where $b_2 = \beta_2/K$. Equation 2.15 could have been solved with a common numerical method (such as a Runge-Kutta algorithm), but large heights of shear layer resulted in divergences in the result. Thus, we implemented a Magnus-Möbius scheme (as described in [55]) for s going from 1 to 0. In order to do so, it is necessary to go back to Eq. 2.2 which, when written on s , becomes :

$$P'' + \frac{2a(M_2 - M_1)}{1 - [M_1 + (M_2 - M_1)s]a} P' + (K\delta)^2[((1 - M_1 + (M_2 - M_1)s)a)^2 - a^2]P = 0. \quad (2.17)$$

When put under a matrix form, Eq. 2.17 becomes :

$$\mathbf{d}_s \begin{pmatrix} P \\ P' \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -(K\delta)^2[((1 - M_1 + (M_2 - M_1)s)a)^2 - a^2] & -\frac{2a(M_2 - M_1)}{1 - [M_1 + (M_2 - M_1)s]a} \end{bmatrix} \begin{pmatrix} P \\ P' \end{pmatrix}. \quad (2.18)$$

The matrix involved in Eq. 2.18 will later be denoted $\mathbf{M}(s)$. Discretizing the domain in $N+1$ points (intervals of size $h = 1/N$) allows us to apply the following scheme :

$$\begin{pmatrix} P(nh) \\ P'(nh) \end{pmatrix} = e^{\mathbf{H}_n} \begin{pmatrix} P((n+1)h) \\ P'((n+1)h) \end{pmatrix}, \quad (2.19)$$

where $\mathbf{H}_n = -h\mathbf{M}[(n+1/2)h]$. Note that the exponential in Eq. 2.19 is a matrix exponential. At each iteration, it is possible to compute $Q(nh)$ from $Q((n+1)h)$:

$$Q(nh) = \frac{\frac{E_{2,1}}{K\delta} + E_{2,2}Q((n+1)h)}{E_{1,1} + K\delta E_{1,2}Q((n+1)h)}, \quad (2.20)$$

where the $E_{i,j}$ are the coefficients of $e^{\mathbf{H}_n}$. After N iterations, we can access R_1 via Eq. 2.8, and we find :

$$R_1 = \frac{ib_1 - Q(0)}{ib_1 + Q(0)}, \quad (2.21)$$

where $b_1 = \frac{\beta_1}{K}$.

- if the incident wave comes from medium 2, A_1 is null and we can now write a boundary condition in $s = 0$:

$$Q(0) = -ib_1. \quad (2.22)$$

Once again, Eq. 2.18 is solved iteratively for s going from 0 to 1. This time, we

have :

$$\begin{pmatrix} P((n+1)h) \\ P'((n+1)h) \end{pmatrix} = e^{-H_n} \begin{pmatrix} P(nh) \\ P'(nh) \end{pmatrix}. \quad (2.23)$$

Thus :

$$Q((n+1)h) = \frac{\frac{E_{2,1}}{K\delta} + E_{2,2}Q(nh)}{E_{1,1} + K\delta E_{1,2}Q(nh)}, \quad (2.24)$$

and we can access R_2 using :

$$R_2 = \frac{ib_2 + Q(1)}{ib_2 - Q(1)} e^{-2iK\delta b_2}. \quad (2.25)$$

The determination of transmission coefficients T_1 and T_2 , now that Q is known in $[0, 1]$, requires to integrate Eq. 2.17 one more time, but in the opposite direction. Here also, we will need to study the two cases of upward and downward traveling incident waves.

- if the incident wave comes from medium 1, we know from Eq. 2.8 that the boundary condition in $s = 0$ is :

$$\tilde{P}(0) = 1 + R_1, \quad (2.26)$$

where \tilde{P} is the y -dependant component of the pressure field normalized by the incident amplitude A_1 . Then, by solving for \tilde{P} on $[0, 1]$ using the scheme in Eq. 2.23, we can access T_1 using :

$$T_1 = \tilde{P}(1)e^{-iK\delta b_2}. \quad (2.27)$$

- if the incident wave comes from medium 2, then this time the boundary condition is :

$$\tilde{P}(1) = e^{-iK\delta b_2} + R_2 e^{iK\delta b_2}, \quad (2.28)$$

where $\tilde{P}(s)$ is normalized by A_2 . Then, by solving for \tilde{P} on $[1, 0]$ using the scheme in Eq. 2.19, we can access T_2 using :

$$T_2 = \tilde{P}(0). \quad (2.29)$$

The whole matrix can then be reconstituted. It turns out from Eq. 2.15 that three parameters will influence the values of the scattering coefficients : the dimensionless width of the shear layer $K\delta$, the relative Mach number $M_2 - M_1$ and the direction of the incident

wave given by the horizontal wave number a .

NON-RECIPROCAL SCATTERING IN SHEAR FLOW : REFLECTION ON AN IMPEDANCE WALL

Dans ce chapitre est étudiée la réflexion d'une onde plane sur une paroi d'admittance donnée plongée dans un écoulement. Il s'agit de la seconde partie de l'article publié en 2019 dans le *Journal of Acoustical Society of America* [60]. La conservation de l'énergie dérivée au chapitre précédent est d'abord utilisée pour lier la valeur du coefficient de réflexion à celle de l'impédance de paroi. Ensuite, l'effet de l'écoulement sur des phénomènes de réflexion remarquables (absorption parfaite, décalage latéral de faisceau gaussien) est discuté.

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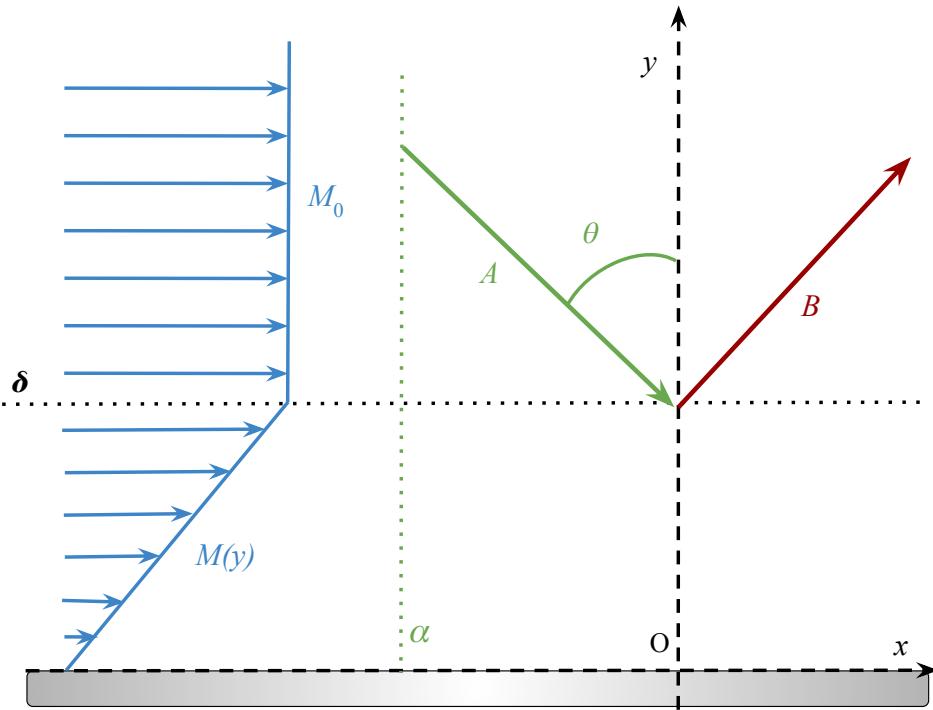


FIGURE 3.1 – Geometry of the problem for the reflection of a plane on an admittance wall.

3.1 Reflection on an impedance wall

We now consider an acoustically treated wall with uniform admittance Y_0 placed in $y = 0$ with a shear flow (see Fig. 3.1). Y_0 is such that its imaginary part stands for the reactance of the wall, and its real part stands for the resistance of the wall ($\Re(Y_0) < 0$ means that there are losses at the wall, $\Re(Y_0) > 0$ means that there is gain at the wall).

3.1.1 Energy conservation

We know that Eq. 2.7 is still valid, and that the pressure field for $y > \delta$ is such that :

$$P(y) = Ae^{-i\beta y} + Be^{+i\beta y}, \quad (3.1)$$

where $\beta^2 = (K - M_0\alpha)^2 - \alpha^2$. Now, due to the impedance wall, there is a mixed boundary condition involving the admittance Y_0 at $y = 0$:

$$P'(0) = iKY_0P(0). \quad (3.2)$$

The scattering problem that we consider is only with reflection and the energy flux conservation 2.7 applied between $y_1 = 0$ and $y_2 = \delta$ leads to :

$$|R|^2 = 1 + \frac{(\alpha^2 + \beta^2)}{K\beta} |P(0)|^2 \Re(Y). \quad (3.3)$$

As long as the flow is subsonic, one can check that β is positive. Therefore, the sign of $|R|^2 - 1$ will only depend on the sign of $\Re(Y)$, *whatever the mean flow velocity or the shear layer thickness*. In particular, as noted by Campos [14], if the wall is purely reactive ($\Re(Y) = 0$) then the reflection will always be perfect i.e. $|R| = 1$.

3.1.2 Non-reciprocal perfect absorption

The computation of R will be achieved numerically using the method detailed in appendix 3.3. The addition of flow leads to non-reciprocity testified in the value of the absorption coefficient, defined as $A = 1 - |R|^2$. Indeed, an incoming plane wave will not be absorbed to the same extend at the wall depending on whether it comes in the direction of the flow or against the flow. This non-reciprocal property is highlighted in Fig. 3.2, for a liner of admittance $Y = 0.1i - 0.1$. This figure shows clearly that varying $K\delta$ allows to reach high absorption for one incidence. This is a typical behaviour as long as the shear layer thickness is large enough. One can also remark on this figure that the higher the Mach number and the height of the shear layer, the more non-reciprocal the system becomes. For a clearer view of this asymmetry, we plot in Fig. 3.3 the value of the absorption coefficient as a function of both θ and $K\delta$ and θ and M_0 . A clear difference can be noted whether incident waves come from the left (in the direction of the flow) or from the right (against the flow). In particular, what is striking on Fig. 3.3 is that, above a certain thickness of shear layer, waves coming against the flow with a wide angle are fully reflected. This could be explained by a strong refraction of the rays, such that the incident wave does not see the effects of the liner before being pushed back towards the "sky". This corresponds to the "turning points", evoqued by Rienstra [59]. On the other side, waves coming with the flow ($\theta > 0$) with an almost grazing incidence can either

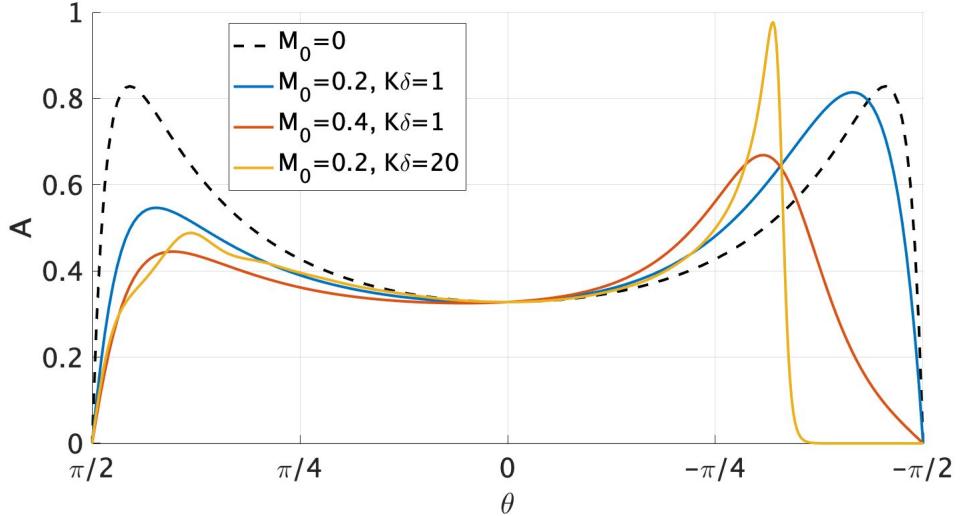


FIGURE 3.2 – Value of the absorption coefficient A as a function of θ for several flows, described by their mean Mach number M_0 and the height of their shear layer δ for a liner of admittance $Y = 0.1i - 0.1$.

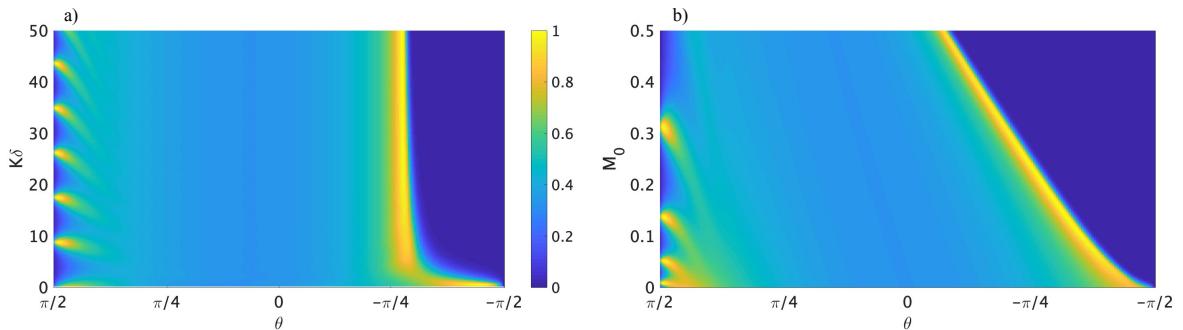


FIGURE 3.3 – (color online) Value of the absorption coefficient A for a liner of admittance $Y = 0.1i - 0.1$ as a function of a) θ and $K\delta$ for $M_0 = 0.1$, b) θ and M for $K\delta = 30$.

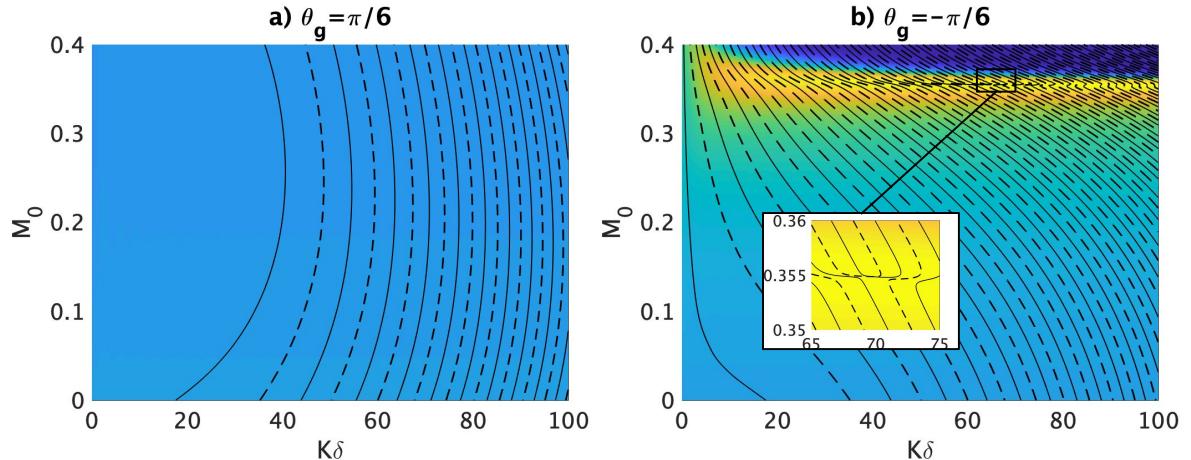


FIGURE 3.4 – Value of A as a function of M_0 and $K\delta$ for waves coming either in the direction of the flow (case a) or counter-flow (case b). Black lines show the values of M_0 and $K\delta$ for which $\Re(R) = 0$ (dashed lines) or $\Im(R) = 0$ (solid lines). It appears that we can find one crossing point, corresponding to $|R| = 0$ (i.e. $A = 1$) for waves coming counter-flow only, for $K\delta = 69.68$ and $M_0 = 0.355$.

be perfectly reflected or perfectly absorbed depending on the shear layer thickness. The quasi-periodic phenomenon which appears is related to resonance phenomena inside the shear layer. For the previous Mach number, for small incidence angles, the absorption does not vary a lot. Therefore, we look at the variation of A with respect to both θ and M_0 to see if the peak absorption can be displaced towards normal incidence for higher Mach numbers (see Fig. 3.3). The position of the near-to-perfect absorption for negative θ then depends linearly on M_0 . Using numerical simulations, we want to design a system which would enhance the absorption of a given liner by adding a specific flow above it, so that it reaches perfect absorption, or zero reflection. The admittance of the liner is fixed at $Y_0 = -0.1 + 0.1i$, and the incident angle such that $|\theta| = \frac{\pi}{6}$. Without flow, in such a situation $A \approx 0.36$ (see Fig. 3.5). In order to get a physically consistent simulation, it is not the direction of the phase velocity which is held constant, but the direction of the group velocity θ_g . A relation between θ and θ_g can be found using the dispersion relation (as shown in Fig. 2.2) :

$$\theta_g = \arctan[\tan(\theta) + \frac{M_0}{\cos(\theta)}], \quad (3.4)$$

or $\theta = \arcsin[\sin(\theta_g) \cos(\theta_g) \sqrt{\frac{1}{\cos^2(\theta)} - M_0^2} - M_0 \cos^2(\theta_g)]$. We now look at the variations of $|R|$ as both M_0 and $K\delta$ vary for $\theta_g = \pi/6$. In Fig. 3.4, an optimum is found, leading to no reflection when $\theta_g = -\pi/6$ (counter-flow propagation). We precisely read it as being

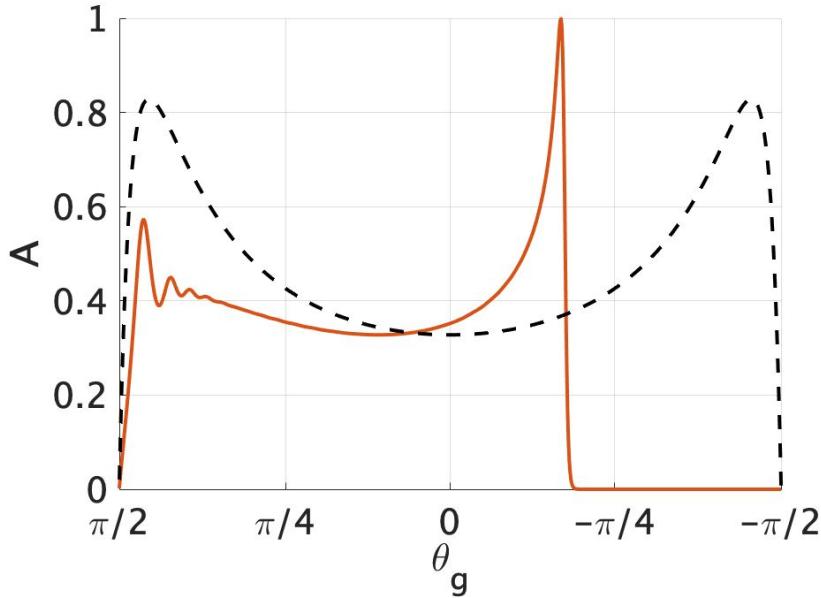


FIGURE 3.5 – Value of A as a function of θ_g with and without flow ($M_0 = 0.355$), for $Y = -0.1 + 0.1i$ and $K\delta = 69.68$.

linked to the values $M_0 = 0.355$ and $K\delta = 69.68$. Non-reciprocity can also be pointed out here, as there is no way to reach perfect absorption if the opposite incidence is considered. We can check that these values are also associated with an optimal absorption in terms of incidence by plotting A as a function of θ without flow or with a flow corresponding to the optimum previously identified. Figure 3.5 gives us this confirmation : the absorption (the absorption coefficient is 0.36 without flow) can be enhanced all the way to perfect absorption with the right flow parameters.

Let us now illustrate the non-reciprocal perfect absorption with Gaussian beams following either upstream or downstream propagation. The results are plotted in Fig. 3.6. The reflection becomes highly non-reciprocal when flow is added : perfect absorption here is only achieved for waves coming counter-flow. Waves traveling in the direction of the flow actually encounter more reflection than they did in a medium at rest on the same liner (as pointed in Fig. 3.5). This behaviour is in agreement with the refraction of acoustic rays in shear flow : rays are pushed towards the regions of low local wave speed, consequently they are trapped near the wall when propagating upstream and pushed towards the sky when propagating downstream.

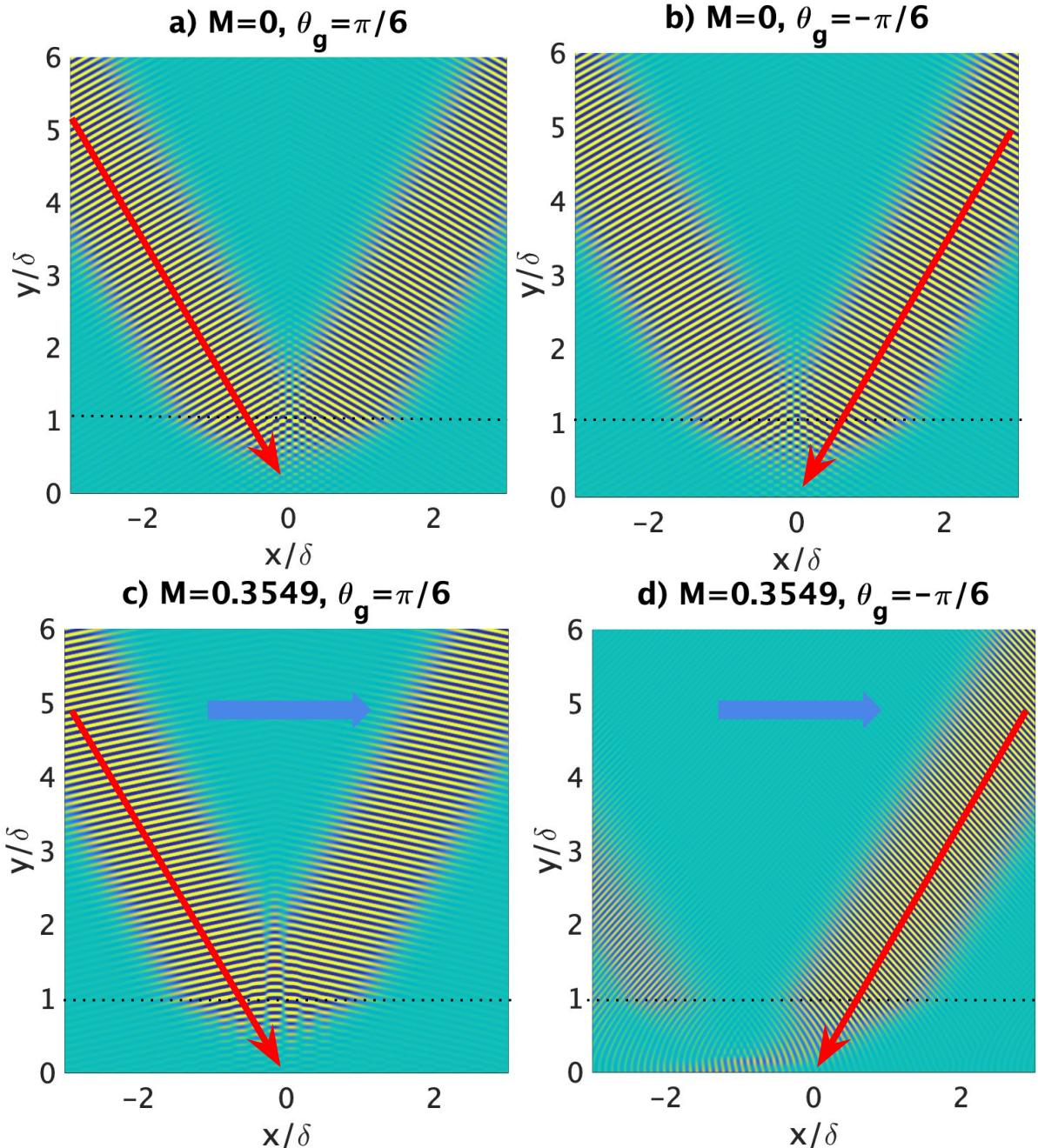


FIGURE 3.6 – Pressure field for an incident Gaussian beam coming with a constant incident angle $\theta_g = \frac{\pi}{6}$ between its group velocity and the normal to the wall, either from the left (cases a and c) or from the right (cases b and d) on a liner of admittance $Y_0 = -0.1 + 0.1i$ in a medium at rest (cases a and b) or in which we impose a flow with Mach number $M_0 = 0.355$ and a boundary layer such that $K\delta = 69.68$. Red arrows show the direction of the group velocity, blue arrows show the direction of the flow.

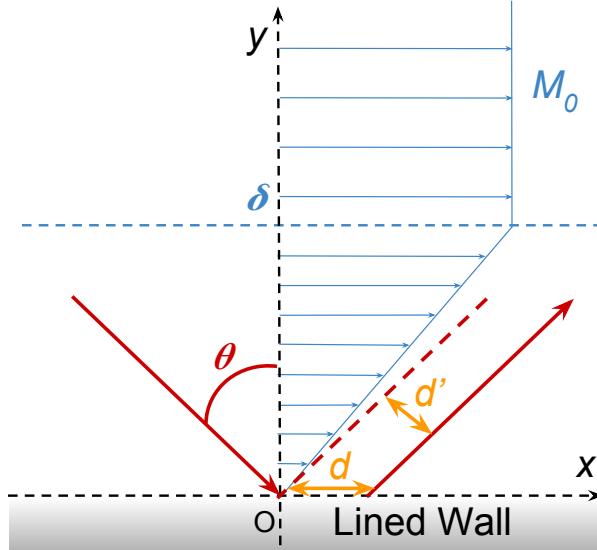


FIGURE 3.7 – Scheme of the lateral shift phenomenon for an incident beam. d is the shift along the wall, d' is the shift perpendicularly to the direction of the beam.

3.1.3 Non-reciprocal Goos-Hänchen shift

When the reflection implies an incidence-depending phase shift, Gaussian beams are shifted along the wall. This phenomenon is called Goos-Hänchen shift [31], and can be quantified using Artmann's formula [2] :

$$d = -\frac{\partial \phi_R}{\partial \alpha} = -\frac{(1 + M \sin(\theta))^2}{K \cos(\theta)} \frac{\partial \phi_R}{\partial \theta}, \quad (3.5)$$

where ϕ_R is the angle of the reflection coefficient, and d is the lateral shift, as displayed in Fig. 3.7. We also define the displacement of the beam perpendicularly to its direction of propagation : $d' = \cos(\theta)d$.

Without flow

We know that, without flow, the reflection coefficient is given by :

$$R = \frac{\cos(\theta) + Y_0}{\cos(\theta) - Y_0}. \quad (3.6)$$

In the following, $Y_0 = Y_1 + iY_2$. Then its angle ϕ_R is given by :

$$\phi_R = \arctan\left(\frac{Y_2}{\cos(\theta) + Y_1}\right) + \arctan\left(\frac{Y_2}{\cos(\theta) - Y_1}\right). \quad (3.7)$$

In the case of a lossless wall, $Y_1 = 0$, Eq. 3.7 reduces to

$$\phi_R = 2 \arctan\left(\frac{Y_2}{\cos(\theta)}\right). \quad (3.8)$$

We can then compute the value of d' , and we get :

$$Kd' = -\frac{2Y_2 \sin(\theta)}{\cos^2(\theta) + Y_2^2}. \quad (3.9)$$

By differentiating Eq. 3.9 with respect to Y_2 , we find a maximum displacement of the beam $Kd'_{max} = \pm \tan(\theta)$ for $Y_2 = \pm \cos(\theta)$. Thus, it appears that, for lossless walls, the value of Kd'_{max} is small for small incident angles. Nevertheless, it is possible to make this limitation disappear by adding losses to the liner. Indeed, in the case when $Y_1 \neq 0$, we find that Kd' can take much larger values. With losses ($Y_1 \neq 0$), Eq. 3.9 takes the form :

$$Kd' = -Y_2 \sin(\theta) \frac{\cos^2(\theta) + |Y|^2}{[(\cos(\theta) + Y_1)^2 + Y_2^2][(cos(\theta) - Y_1)^2 + Y_2^2]} \quad (3.10)$$

where, for $Y_1 = \pm \cos(\theta)$ and Y_2 is close to zero, the denominator can go to zero leading to large Kd' . This result is also the one leading to perfect absorption. Therefore, large lateral shift is associated with large absorption coefficient. This phenomenon is easily understandable if one is to look at the phase portrait of the reflection coefficient when it varies with θ (see Fig. 3.8) : phase variation of R is enhanced near zero reflection.

With Flow

When flow is added, the Goos-Hänchen displacement has to be computed numerically. It is then interesting to study the impact of both the flow and the wall admittance on the beam displacement. Results shown in Fig. 3.9 illustrate that the effect of the flow is predominant. Indeed, the two maximal values of displacement visible without flow for $\theta \approx \pm\pi/6$ are still visible when the flow is added but are negligible compared to the effect of the flow with a much larger displacement at $\theta \approx -\pi/4$. Remark that this displacement is comparable to the large displacement of beams observed in the case of

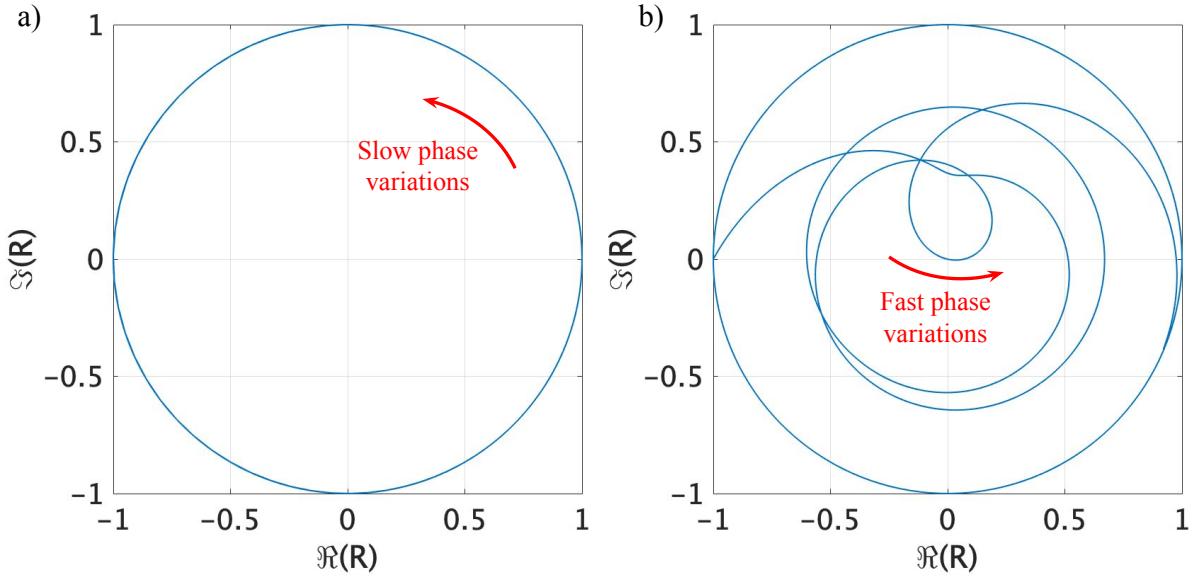


FIGURE 3.8 – Phase portrait of R as θ varies from $-\pi/2$ to $+\pi/2$, with $M_0 = 0.23$, $K\delta = 15.04$ and a) $Y_0 = 0.1i$, b) $Y_0 = 0.1i - 0.2$. When losses are added on the wall and in a situation where perfect absorption can be achieved, the reflection coefficient can cross the zero point, and therefore experience a sudden phase jump of amplitude π , resulting in a high phase gradient and therefore a large lateral displacement of the beam.

reflection on multi-layered materials in optics [64, 19, 24].

As in the previous section, where we illustrated non-reciprocal perfect absorption, Gaussian beams can nicely illustrate the Goos-Hänchen effect (Fig. 3.10). By choosing the angle for which the flow has the most visible effect ($\theta = -0.89$), we see a large lateral shift. Besides, a strong non-reciprocal effect is demonstrated by taking the opposite angle.

3.2 Conclusion

In the global context of acoustics with flow we have shown that the presence of a shear layer leads to non-reciprocal effects whose origin can be found in the known effects due to convection. We have illustrated some of these phenomena by considering diffraction by a shear layer that can lead to negative refraction, as well as absorption by an acoustic material and lateral displacement of a beam along this material which both depend on the direction of propagation relatively to the flow. In this paper, we have only considered homogeneous problems along the direction of flow. Interesting new effects appear as soon as inhomogeneities are considered along the flow direction.

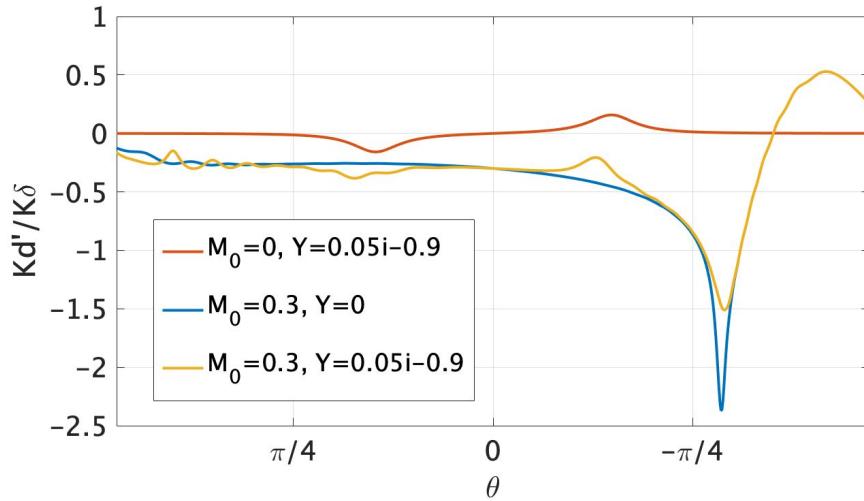


FIGURE 3.9 – Value of d'/δ as a function of θ_g for a reflection either on a hard wall or with an acoustic treatment, with and without flow. The flow used here is such that $M_0 = 0.3$ and $K\delta = 50$.

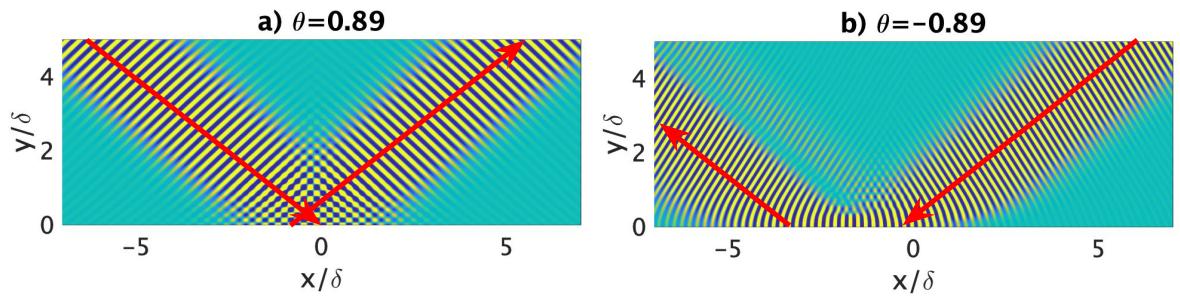


FIGURE 3.10 – Pressure field for Gaussian beam with a constant main direction $\theta_g = \pm 0.877$, reflecting on a hard wall, with $M_0 = 0.3$ and $K\delta = 30$. In case a), the beam comes in the direction of the flow and is slightly displaced backwards in case b), the beam comes against the direction of the flow and is widely displaced forwards. The black dashed line shows the height of the shear layer.

3.3 Appendix B : Numerical determination of R in a reflection problem

In order to determine R , we apply the same method as for the transmission problem : solving Eq. 2.15 for s in $[0; 1]$, but this time the boundary condition will be given by the wall admittance :

$$Q(0) = iY_0. \quad (3.11)$$

Here also, the resolution is led to its end using a Magnus-Möbius scheme [42] of the second order as described in [55]. In order to compute the solution, we go back to Eq. 2.18, and after writing P and P' in discrete forms, we get :

$$\begin{pmatrix} P((n+1)h) \\ P'((n+1)h) \end{pmatrix} = e^{\mathbf{H}_n} \begin{pmatrix} P(nh) \\ P'(nh) \end{pmatrix} = \begin{bmatrix} E_{1,1} & E_{1,2} \\ E_{2,1} & E_{2,2} \end{bmatrix} \begin{pmatrix} P(nh) \\ P'(nh) \end{pmatrix}, \quad (3.12)$$

where $\mathbf{H}_n = h\mathbf{M}[(n+1/2)h]$. Thus :

$$Q((n+1)h) = \frac{\frac{E_{2,1}}{K\delta} + E_{2,2}Q(nh)}{E_{1,1} + K\delta E_{1,2}Q(nh)}. \quad (3.13)$$

R is finally given by :

$$R = \frac{ib + Q(1)}{ib - Q(1)} e^{-2ibK\delta}. \quad (3.14)$$

ACOUSTIC METASURFACES IN NON-UNIFORM FLOWS

Ce chapitre, rédigé en langue anglaise, est une ébauche d'article portant sur la réflexion d'ondes planes sur une paroi d'admittance périodique au-dessus de laquelle est imposé un écoulement parallèle avec une couche de cisaillement. Cette étude a été motivée par le constat dressé dans le chapitre précédent selon lequel l'énergie acoustique était conservée à travers une couche de cisaillement quand le système est uniforme en x . Elle est construite autour de deux axes principaux : l'impact de l'écoulement sur le champ réfléchi et l'effet des couches critiques sur la conservation de l'énergie.

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4.1 Introduction

Metasurfaces are used in several domains, mostly because they offer unique opportunities to manipulate reflected wavefronts as wanted [21] while remaining some rather compact, sub-wavelength-sized, equipment. Indeed, over the last years, occurrences of metasurfaces in the literature have multiplied with several purposes including lensing, shielding, negative refraction or cloaking [56, 25]. Here the particular property of metasurfaces that we will be interested in is their abilities to reflect an incident waves in an infinity of directions. Some of these directions will correspond to propagative waves and others to evanescent ones, explaining the aspect of the field near the wall in Fig.4.1. This behavior is the one that could lead to the breaking of the energy conservation as, for some of these directions, the convected wave number can vanish for a specific Mach number $M(y_c)$. These points y_c , called *critical layers*, can become problematic in the solving of the governing equations of the problem as some of their coefficients become singular.

As a result, this chapter focuses on these critical layers. First of all, we will present the problem that we want to solve and its governing equations, as well as the numerical methods that were employed to maneuver around their singularities. We also derive some asymptotic solutions around the critical layer by performing a Frobenius-Fuchs expansion around the critical layers, before concluding on the energy conservation in the system.

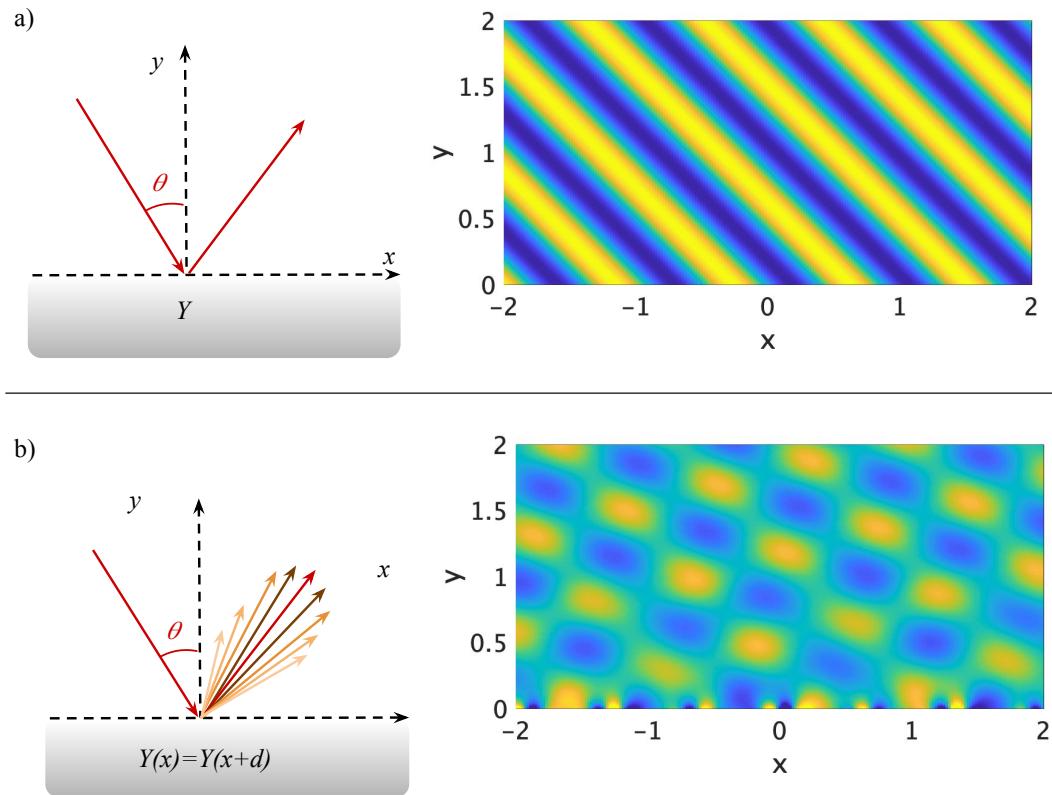


FIGURE 4.1 – Example of reflected pressure field a) by a uniform wall b) by a periodic wall (in a still medium).

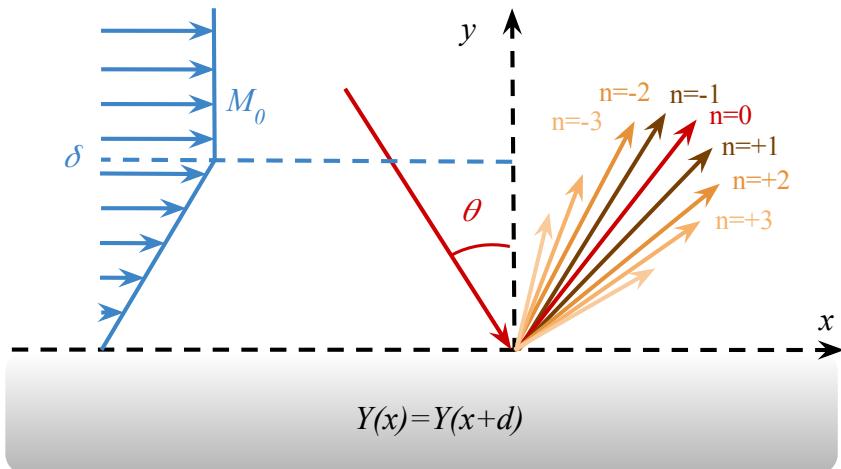


FIGURE 4.2 – Geometry of the problem.

4.2 Geometry and governing equations of the problem

In order to potentially excite critical layers, we find a way to diffract an incident plane wave into a discrete spectrum of reflected waves. To this extend, we study the reflection of such a plane wave on an admittance wall, whose admittance varies along x . The admittance $Y(x)$ is periodic with a spatial period d , so that the reflected field is a sum of components of higher order. Its value varies around a mean admittance Y_0 as :

$$Y(x) = Y_0[1 + 2a \cos(\frac{2\pi x}{d})]. \quad (4.1)$$

In the half-infinite space above the wall, we define a parallel flow such that it has a constant velocity above a distance δ and a linear profile below δ (the point of zero velocity is on the wall). The mean flow velocity $U(y)$ is given by :

$$U(y) = \begin{cases} U_0 \frac{y}{\delta} & \text{for } y \leq \delta, \\ U_0 & \text{for } y > \delta. \end{cases} \quad (4.2)$$

In the following, we will rather consider the Mach number of the flow, $M(y)$, defined as $U(y)/c$, with c the sound speed in the medium. Similarly, M_0 , the constant Mach number above δ will be used. The total pressure field $p(x, y, t)$ in the domain verifies the propagation equation in parallel flows [57] :

$$D_t \left(\nabla^2 p - \frac{1}{c_0^2} D_t^2 p \right) = 2U' \partial_{xy}^2 p, \quad (4.3)$$

with D_t the convective derivative such that $D_t = \partial_t + U(y)\partial_x$. We assume the pressure field to be harmonic, with frequency ω . Moreover, the scattering into multiple directions create orders of reflection, denoted n , and represented on Fig. 4.2. The specular reflection is of order 0. A reflection with a wider angle will be of order $n > 0$, whereas one with a smaller angle will be of order $n < 0$. The total pressure field can be written as an infinite sum over the orders of reflection :

$$p(x, y, t) = \sum_{n=-\infty}^{+\infty} P_n(y) e^{-i(\omega t - \alpha_n x)}, \quad (4.4)$$

with α_n the horizontal component of the wave number defined by the grating relation :

$$\alpha_n = \alpha_0 + nq = \frac{k \sin(\theta)}{1 + M_0 \sin(\theta)} + nq, \quad (4.5)$$

where θ is the angle between the normal to the wall and the wave vector of the incident wave, k is the wave number of the incident wave ($k = \omega/c$) and $q = 2\pi/d$. Eq. 4.4 can be written in a vector form by defining a vector $\psi(x)$ containing the terms $e^{i\alpha_n x}$ and a vector $\mathbf{P}(y)$ containing the $P_n(y)$:

$$p(x, y, t) = \psi^T(x) \mathbf{P}(y) e^{-i\omega t} \quad (4.6)$$

Inserting Eq.4.6 into Eq. 4.3 gives :

$$\mathbf{P}'' + 2M'(k\mathbf{Id} - M\mathbf{A})^{-1}\mathbf{P}' + [(k\mathbf{Id} - M\mathbf{A})^2 - \mathbf{A}^2]\mathbf{P} = \mathbf{0}, \quad (4.7)$$

where \mathbf{A} is the diagonal matrix whose diagonal components are the α_n and the prime notation stands for a derivation with respect to y . Therefore, each component $P_n(y)$ of vector \mathbf{P} verifies a Pridmore-Brown equation :

$$P_n'' + \frac{2\alpha_n M'}{k - M\alpha_n} P_n' + [(k - M\alpha_n)^2 - \alpha_n^2] P_n = 0. \quad (4.8)$$

Equation 4.8 has a simple solution in the domain $y > \delta$ since the term in $M'(y)$ vanishes :

$$P_n(y > \delta) = e^{-i\beta_n y} I_n + e^{i\beta_n y} R_n, \quad (4.9)$$

where $\beta_n = \sqrt{(k - M_0\alpha_n)^2 - \alpha_n^2}$. This solution can be written in vector form as $\mathbf{P} = \mathbf{M}\mathbf{I} + \mathbf{M}^{-1}\mathbf{R}$, with \mathbf{M} the diagonal matrix containing the propagation terms $e^{-i\beta_n y}$. The R_n coefficients will be determined by solving Eq. 4.8 on the lower part of the domain. However, we should note that Eq.4.8 can present regular singularities where the term in $k - M(y)\alpha_n$ vanishes. These singularities are the critical layers, located in :

$$y_{c,n} = \frac{k\delta}{M_0\alpha_n}. \quad (4.10)$$

In addition, we know that $k - M(y_{c,n})\alpha_n = 0$ and that $M(y_{c,n}) < M_0$ (by definition). Therefore if an order of reflection possesses a critical layer, it can be shown that it is

evanescent ($\beta_n^2 < 0$).

These singularities can become particularly problematic when it comes to numerical resolution of the Pridmore-Brown equation. Therefore, the next section will detail the choices made to achieve this resolution.

4.3 Impact of critical layers on the numerical resolution and ways to avoid singularities

4.3.1 Numerical integration scheme

To simplify the numerical integration of the problem, we define the non-dimensional value $s = y/\delta$. The domain of integration is now $[0, 1]$. Therefore, equation 4.8 becomes :

$$P_n''(s) + \frac{2M_0a_n}{1 - M_0a_n s} P_n'(s) + (k\delta^2)[(1 - M_0a_n s)^2 - a_n^2] P_n(s) = 0, \quad (4.11)$$

where $a_n = \alpha_n/k$ is the non-dimensional horizontal component of the wave number. This relation can be written under matrix-form on vectors \mathbf{P} and \mathbf{P}' , leading to :

$$\partial_s \begin{pmatrix} \mathbf{P} \\ \mathbf{P}' \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -(k\delta)^2 \mathbf{B} & k\mathbf{Id} - M\mathbf{A} \end{bmatrix} \begin{pmatrix} \mathbf{P} \\ \mathbf{P}' \end{pmatrix} = \mathbf{F}(s) \begin{pmatrix} \mathbf{P} \\ \mathbf{P}' \end{pmatrix} \quad (4.12)$$

where \mathbf{A} and \mathbf{B} are diagonal matrices containing respectively the α_n and β_n . The domain of integration is discretized in N parts and we define the discrete variable $s_j = jh$, where j is a integer in $[0, N]$ and $h = 1/N$. We write \mathbf{P}_j (resp. \mathbf{P}'_j) the value of $\mathbf{P}(s_j)$ (resp. $\mathbf{P}'(s_j)$). A Magnus-Moebius scheme is applied, giving us access to \mathbf{P}_{j+1} and \mathbf{P}'_{j+1} as functions of \mathbf{P}_j and \mathbf{P}'_j :

$$\begin{pmatrix} \mathbf{P}_{j+1} \\ \mathbf{P}'_{j+1} \end{pmatrix} = e^{h\mathbf{F}(s_{mid})} \begin{pmatrix} \mathbf{P}_j \\ \mathbf{P}'_j \end{pmatrix} = \begin{bmatrix} \mathbf{E}_1 & \mathbf{E}_2 \\ \mathbf{E}_3 & \mathbf{E}_4 \end{bmatrix} \begin{pmatrix} \mathbf{P}_j \\ \mathbf{P}'_j \end{pmatrix}, \quad (4.13)$$

where $s_{mid} = (s_j + s_{j+1})/2$. Note that the exponential operator in Eq. 4.13 is a matrix exponential. The admittance matrix \mathbf{Q} is then introduced such that $\mathbf{P}' = k\delta\mathbf{Q}\mathbf{P}$. By introducing it in Eq. 4.8, we notice that \mathbf{Q} follows the following Riccati equation :

$$\mathbf{Q}' + k\delta\mathbf{Q}^2 - 2M_0\mathbf{a}(\mathbf{Id} - M_0\mathbf{a}s)^{-1}\mathbf{Q} + k\delta\mathbf{b} = 0, \quad (4.14)$$

with $\mathbf{a} = \mathbf{A}/k$ and $\mathbf{b} = \mathbf{B}/k$ the diagonal matrices containing respectively the a_n and the $\sqrt{(1 - M_0 a_n s)^2 - a_n^2}$. After some manipulation, Eq. 4.13 gives a relation on \mathbf{Q} , such that :

$$\mathbf{Q}(s_{j+1}) = \frac{1}{k\delta} ([\mathbf{E}_3 + k\delta \mathbf{E}_4 \mathbf{Q}(s_j)][\mathbf{E}_1 + k\delta \mathbf{E}_2 \mathbf{Q}(s_j)]^{-1}. \quad (4.15)$$

Equation 4.15 allows us to determine the value of the admittance matrix \mathbf{Q} at every position inside the shear layer, given it is known on the wall. A boundary condition on \mathbf{Q} is then needed.

4.3.2 Boundary condition

The wall admittance $Y(x)$ in $s = 0$ is known, and it links the pressure field to its derivative in $s = 0$ as $p'(x, 0) = jk\delta Y(x)p(x, 0)$. $Y(x)$ being periodic, it can be broken down into the sum of the terms of its Fourier series :

$$Y(x) = \sum_{p=-\infty}^{+\infty} Y_p e^{j \frac{p 2 \pi x}{d}}. \quad (4.16)$$

In which we recognize $2\pi p/d = qp = \alpha_p - \alpha_0$. Moreover, according to Eq. 4.4, $p(x, y)$ can be written as a sum of reflections. This leads to :

$$\sum_{n=-\infty}^{+\infty} P'_n(0) e^{j \alpha_n x} = jk\delta \sum_{p=-\infty}^{+\infty} Y_p e^{j(\alpha_p - \alpha_0)} \sum_{n=-\infty}^{+\infty} P_n(0) e^{j \alpha_n x}. \quad (4.17)$$

After some manipulation, Eq. 4.17 can be written :

$$\sum_{n=-\infty}^{+\infty} P'_n(0) e^{j \alpha_n x} = jk\delta \sum_{p=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} Y_p P_n(0) e^{j \alpha_{n-p}}. \quad (4.18)$$

After a change of variable in the right hand side of Eq. 4.18 as detailed in [34] we get :

$$\mathbf{P}'(0) = jk\delta \mathbf{Y} \mathbf{P}(0), \quad (4.19)$$

where \mathbf{Y} is the matrix whose all terms in the p -th diagonal are equal to Y_p . Finally, the boundary condition on \mathbf{Q} is :

$$\mathbf{Q}(0) = j\mathbf{Y}. \quad (4.20)$$

4.3.3 Path of integration in the complex plan

To avoid the integration at the singular points $y = y_c$, the chosen integration path is rerouted into the complex plane when approaching the critical layer. This path is denoted $z(y)$ and is such that $z(y) = y + jw(y)$. In our case, the integration path will describe some half circles of radius ϵ around y_c , as shown in Fig. 4.3. This means that w is defined as :

$$w(y) = \sqrt{\epsilon^2 - (y - y_{c,n})^2} \text{ for } |y - y_{c,n}| < \epsilon, \quad (4.21)$$

where $y_{c,n}$ is the position of the critical layer for the n -th order of reflection. In the case when two semi-circles are crossing each other, the largest value of w is chosen so that the integration path is never closer to a critical layer than a distance ϵ . The only exception will be when the critical layer is close to the limits of the domain (0 or δ). Indeed, we want to keep both ends of the integration domain real so that the integral stays non-dependant on the followed path. Therefore, $w(0) = w(\delta) = 0$ and a linear ramp is defined between 0 and the value of $w(y)$ (given by Eq. 4.21) at a distance $\epsilon/2$ from the end of the domain.

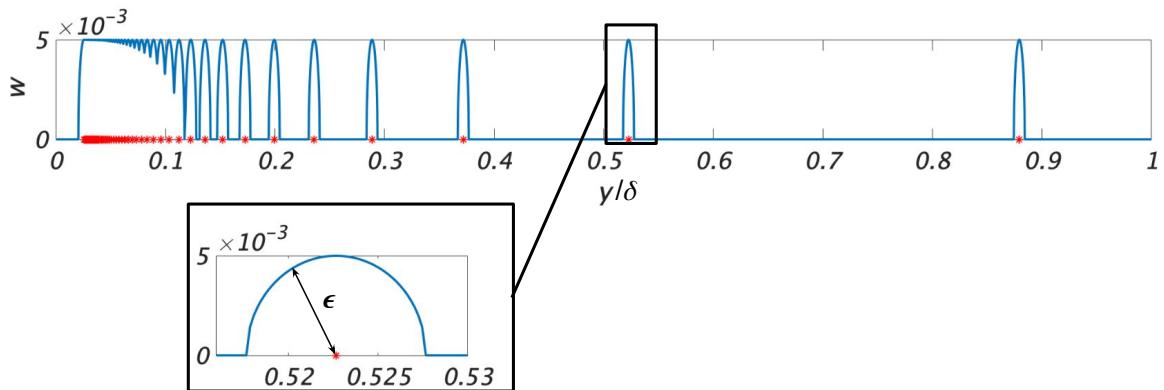


FIGURE 4.3 – Example of integration path in the complex plane. The red crosses stand for the position of the critical layers.

Then we have the following relation between the integration of $f(y)$ and $f(z)$:

$$\int f(z)dz = \int (1 + jw'(y))f(y)dy. \quad (4.22)$$

The idea is to have ϵ large enough to avoid the influence of the critical layer, but small enough for the solution to be correct. In the following, ϵ is fixed at 0.005. This method of integration, applied to Magnus-Moebius scheme, allows us to prevent the computation

from stopping at singularities because of exploding coefficients.

4.4 Effects of the flow on the reflection by a metasurface

4.4.1 Reminder : metasurfaces in a medium at rest

In the absence of flow, we have a direct relation between the scattering matrix S and the admittance matrix Y :

$$S = (\text{Id} - \delta Y)^{-1}(\text{Id} + \delta Y). \quad (4.23)$$

We remind that, if $Y(x)$ is non-uniform, then Y is non-diagonal and relation 4.23 results in the scattering of the incident wave into multiple directions given by the α_n . The resulting reflected pressure field can, for example, look like the one in Fig. 4.4. In the configuration

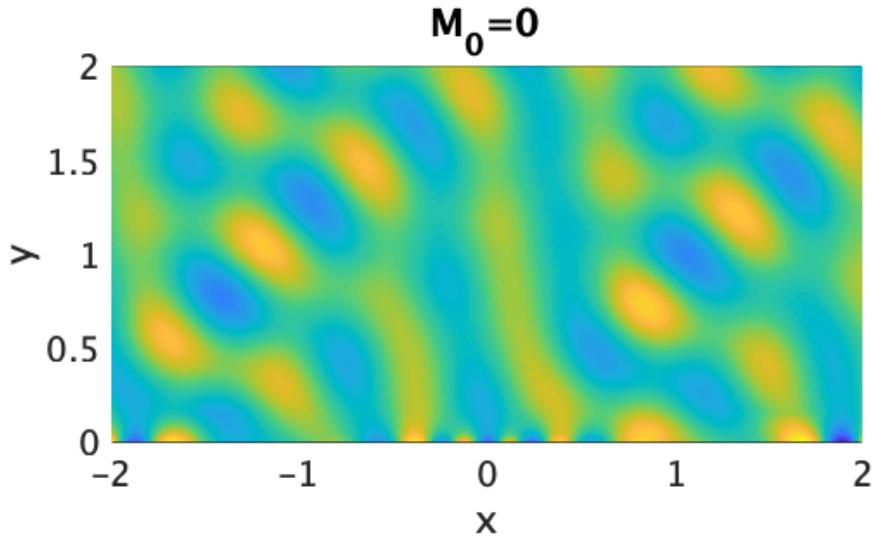


FIGURE 4.4 – Reflected pressure field for a metasurface with $Y_0 = i$, $q = 2.7$, $a = 0.9$ and an incident plane wave with wave number $k = 10$ and angle $\theta_0 = \pi/4$ rad.

presented in this figure, the scattered field is composed of seven propagating modes whose

amplitudes can be calculated. Indeed, we have the reflected amplitudes R_n in a vector \mathbf{R} given by $\mathbf{R} = \mathbf{S}\mathbf{I}$, with \mathbf{I} the incident amplitudes (in all of our simulations, only mode 0 is sent, with unit amplitude). In Fig.4.5 are represented the amplitudes of the reflected modes. It is interesting to note that modes 1 and -1 have higher amplitudes than the

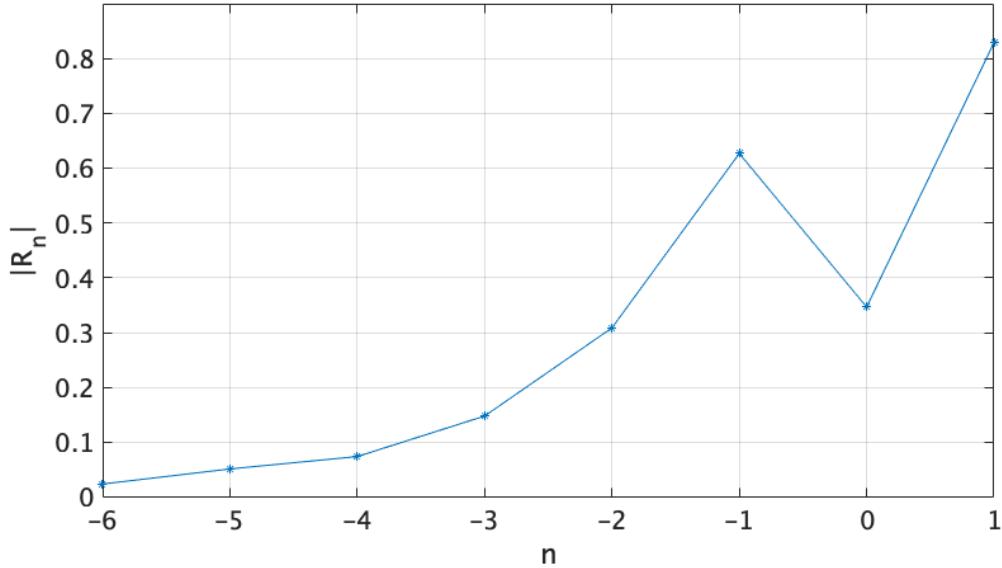


FIGURE 4.5 – Magnitude of the R_n in the same configuration as in Fig.4.4.

specular reflection $n = 0$. In the simulations, we also consider 20 evanescent modes, which allows us to model the pressure field close to the wall and its fast variations due to the short periodicity of the wall admittance.

4.4.2 Effect of the flow

The flow will have an influence on both the number of propagative modes and their amplitudes. For example, by adding a uniform flow with Mach number $M_0 = 0.4$ with a linear boundary layer of height $\delta = 1$ above the wall, the scattered pressure field becomes the one shown in Fig.4.6 and mode 1 (which was the one of highest amplitude without flow) is now evanescent.

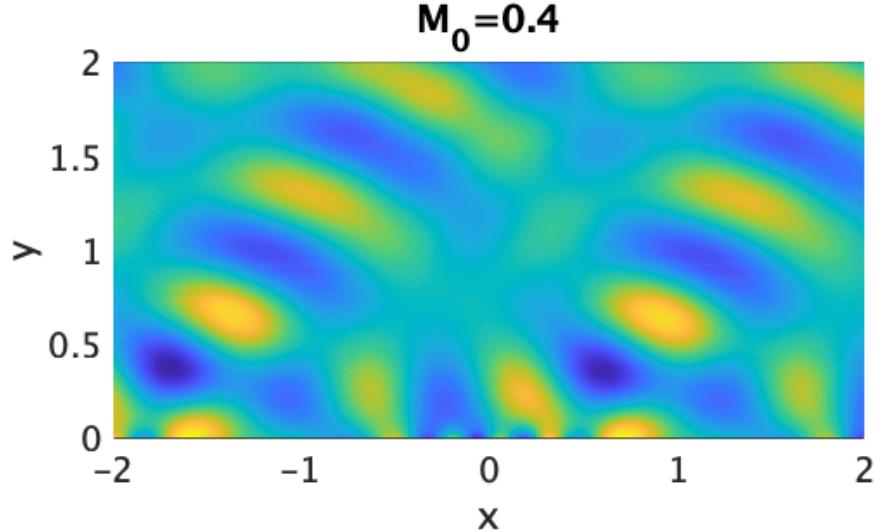


FIGURE 4.6 – Scattered pressure field for the configuration of Fig. 4.4 with $M_0 = 0.4$ and $\delta = 1$.

4.4.3 Control of the magnitude of the orders of reflection by addition of flow

In the following, we will consider a given metasurface for which, without flow, only modes -1 and 0 are propagative. To this extend, we choose $q = 9.7$. The admittance is such that $Y_0 = i$ and $a = 0.8$. We can plot the amplitude of each order of reflection as a function of M_0 (we keep $\delta = 1$). In this configuration, we can see in Fig. 4.7 that the fact of adding flow results in a decrease of the specular reflection and an increase of the order -1 . The resulting scattered fields are plotted in Fig.4.8. In conclusion, we can say that a parallel flow above a given metasurface can change the direction of the main reflection. This behavior might need to be taken into account in certain uses of metasurfaces, but can also be a tool to adjust this direction without changing the metasurface.

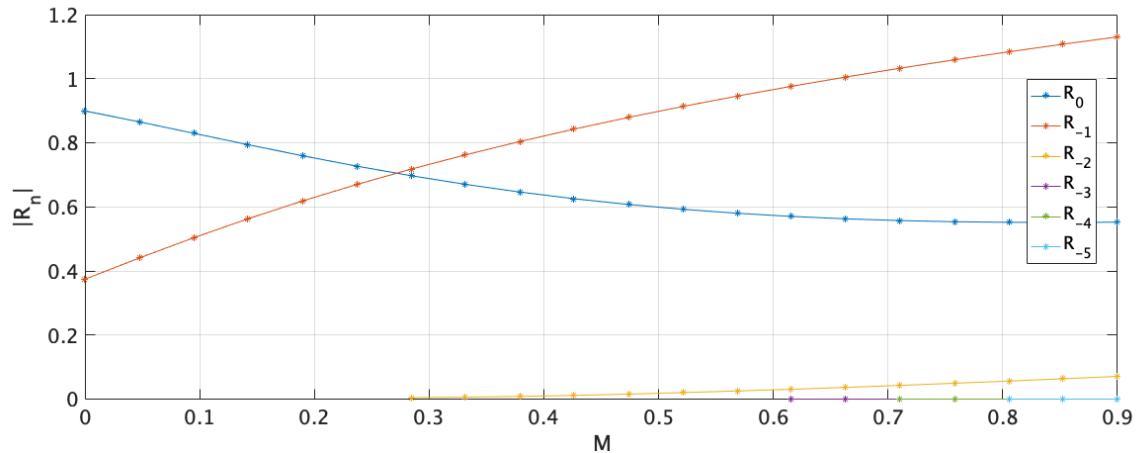


FIGURE 4.7 – Amplitude of the reflection coefficient for each order of reflection for the given admittance wall.

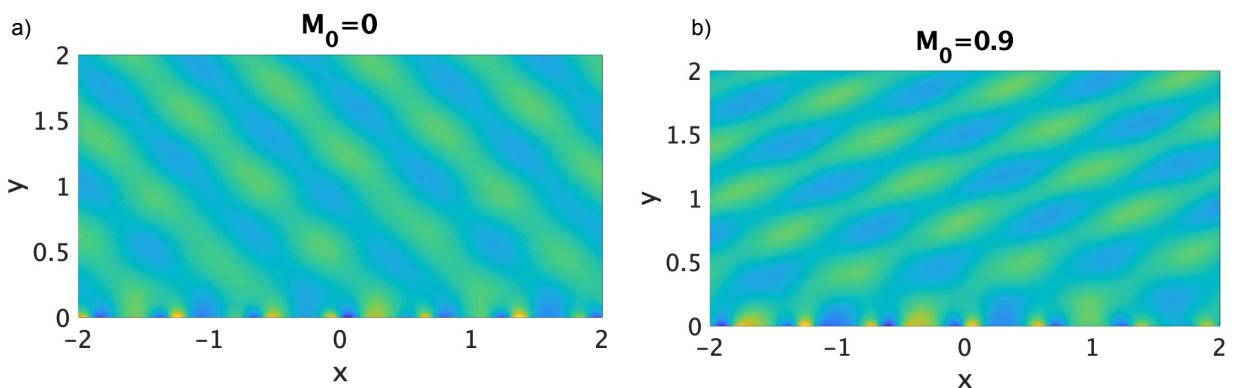


FIGURE 4.8 – Scattered pressure fields for the given metasurface a) without flow b) with flow

4.4.4 Energy conservation

If the critical layers have no effect on the energy conservation, then it can be shown (Appendix 4.7) that the scattering matrix \mathbf{S} linking the incident amplitudes to the reflected ones should verify :

$$\mathbf{S}^\dagger \mathbf{B} \mathbf{D}^2 \mathbf{S} = \mathbf{B} \mathbf{D}^2, \quad (4.24)$$

where we remind that \mathbf{B} and \mathbf{D} are diagonal matrices containing, respectively, the β_n and the $1/(k - M_0 \alpha_n)$. When checking this result numerically, the norm of $\mathbf{S}^\dagger \mathbf{B} \mathbf{D}^2 \mathbf{S} - \mathbf{B} \mathbf{D}^2$ goes to zero without flow. But when $M_0 \neq 0$, this norm is of order 10^{-6} . We were not able to conclude here on whether this is caused by an amplification of numerical error or this invalidates Eq.4.24. This second hypothesis could come from singularities at the critical layers which were neglected in Appendix 4.7. To try to answer this question, asymptotic solutions around the critical layers are sought for.

4.5 Asymptotic solution around the critical layer

In order to reach a conclusion on the conservation of energy in the system, we approximate the solution of the wave equation locally around the critical layers by expanding it in Fuchs-Frobenius series [13, 10]. The goal here is to find approximations of $P_n(y)$ around its critical layer $y_{c,n}$. We show that two solutions exist : one is a polynomial approximation and the other one describes the eventual singularities at the critical layer.

To begin with, we remind that the Pridmore-Brown equation on each order of reflection P_n is given by :

$$P_n''(y) + \frac{2\alpha_n M'(y)}{k - M(y)\alpha_n} P_n'(y) + [(k - M(y)\alpha_n)^2 - \alpha_n^2] P_n(y) = 0 \quad (4.25)$$

with $\alpha_n = k \sin(\theta)/(1 + M_0 \sin(\theta) + nq)$ the horizontal component of the wavenumber. **For now, we make no assumptions on the profile of $M(y)$,** therefore, around the critical layer $y_{c,n}$, such that $k - M(y_{c,n})\alpha_n = 0$, we can write :

$$M(y) = M(y_{c,n}) + M'(y_{c,n})(y - y_{c,n}) + \frac{M''(y_{c,n})}{2}(y - y_{c,n})^2 + \dots = \sum_{p=0}^{+\infty} \frac{M^{(p)}(y_{c,n})}{p!}(y - y_{c,n})^p. \quad (4.26)$$

This Taylor expansion leads to :

$$\frac{2\alpha_n M'(y)}{k - M(y)\alpha_n} = \frac{\sum_{p=0}^{+\infty} \frac{2M^{(p+1)}(y_{c,n})}{p!}(y - y_{c,n})^p}{\sum_{p=0}^{+\infty} \frac{M^{(p+1)}(y_{c,n})}{(p+1)!}(y - y_{c,n})^{p+1}} = \frac{\sum_{p=0}^{+\infty} \alpha_n (y - y_{c,n})^p}{\sum_{p=0}^{+\infty} \beta_n (y - y_{c,n})^{p+1}} = \sum_{p=0}^{+\infty} f_p (y - y_{c,n})^{p-1}. \quad (4.27)$$

and

$$(k - M(y)\alpha_c, n)^2 - \alpha_n^2 = \alpha_n^2 \left[\left(\sum_{p=0}^{+\infty} \frac{M^{(p)}(y_{c,n})}{p!} (y - y_{c,n})^p \right)^2 - 1 \right] = \sum_{p=0}^{+\infty} g_p (y - y_{c,n})^{p-2}. \quad (4.28)$$

We re-write Eq. 4.8 as :

$$P_n''(y) + \frac{f(y)}{y - y_{c,n}} P_n'(y) + \frac{g(y)}{(y - y_{c,n})^2} P_n(y) = 0, \quad (4.29)$$

with :

$$\begin{cases} f(y) = \sum_{p=0}^{+\infty} f_p (y - y_{c,n})^p = \frac{\sum_{p=0}^{+\infty} \frac{2M^{(p+1)}(y_{c,n})}{p!} (y - y_{c,n})^p}{\sum_{p=0}^{+\infty} \frac{-M^{(p+1)}(y_{c,n})}{(p+1)!} (y - y_{c,n})^p} = \frac{\sum_{p=0}^{+\infty} A_p (y - y_{c,n})^p}{\sum_{p=0}^{+\infty} B_p (y - y_{c,n})^p}, \\ g(y) = \sum_{p=0}^{+\infty} g_p (y - y_{c,n})^p = \alpha_n^2 (y - y_{c,n})^2 \left[\left(\sum_{p=0}^{+\infty} \frac{M^{(p)}(y_{c,n})}{p!} (y - y_{c,n})^p \right)^2 - 1 \right]. \end{cases} \quad (4.30)$$

4.5.1 General form of the solution

Following the Fuchs-Frobenius method, we will be looking for solutions to Eq.4.29 under the form :

$$P_n(y) = (y - y_{c,n})^\zeta \sum_{p=0}^{+\infty} a_p (y - y_{c,n})^{\zeta} = \sum_{p=0}^{+\infty} a_p (y - y_{c,n})^{p+\zeta}, \quad (4.31)$$

with ζ a number to be found. By injecting this solution into Eq.4.29, we get :

$$\begin{aligned} & \sum_{p=0}^{+\infty} a_p (p + \zeta)(p + \zeta - 1)(y - y_{c,n})^{p+\zeta-2} + \sum_{p=0}^{+\infty} \sum_{q=0}^{+\infty} a_p f_q (p + \zeta)(y - y_{c,n})^{p+q+\zeta-2} \\ & + \sum_{p=0}^{+\infty} \sum_{q=0}^{+\infty} a_p f_q (y - y_{c,n})^{p+q+\zeta-2} = 0. \end{aligned} \quad (4.32)$$

Isolating the terms in $(y - y_{c,n})^{\zeta-2}$ leads to with the relation :

$$a_0 (\zeta^2 + \zeta(f_0 - 1) + g_0) = 0. \quad (4.33)$$

Let ψ be a polynomial function of α such that :

$$\psi(\zeta) = \zeta^2 + (f_0 - 1)\zeta + g_0. \quad (4.34)$$

Using the definitions of f and g in Eq.4.30 we notice that f_0 (resp. g_0) is equal to $f(0)$ respectively $g(0)$. We get :

$$\begin{cases} f_0 = f(0) = -2 \frac{M'(y_{c,n})}{M''(y_{c,n})} = -2, \\ g_0 = g(0) = 0. \end{cases} \quad (4.35)$$

Therefore, $\psi(\zeta) = \zeta^2 - 3\zeta$. If we assume a_0 to be non-zero (it will take an arbitrary value), we want $\psi(\zeta) = 0$. Then, ζ can only take two values (the roots of ψ) : $\zeta_1 = 3$ and $\zeta_2 = 0$. The terms in $(y - y_{c,n})^{p+\zeta-2}$ give us a relation on a_p when $p \neq 0$:

$$[(\zeta + p)^2 - 3(\zeta + p)]a_p = - \sum_{q=0}^{p-1} [(\zeta + q)f_{p-q} + g_{p-q}]a_q, \quad (4.36)$$

in which the left hand side can actually be written as $\psi(\zeta + p)$. One can notice that when $\zeta = \zeta_2 = 0$ and $p = 3$, we get $\psi(\zeta_2 + 3) = \psi(3) = \psi(\zeta_1) = 0$. Therefore a_3 will be undefined and another form of solution will be necessary.

4.5.2 First solution under the Frobenius form

The first solution, for $\zeta = \zeta_1 = 3$, lets us determine the a_p coefficients using relation 4.36.

Determination of a_1 : Relation 4.36 for $p = 1$ gives us $[(3 + 1)^2 - 3(3 + 1)]a_1 = -[(3 + 0)f_1 + g_1]a_0$, or in other words :

$$a_1 = \frac{-(3f_1 + g_1)}{4}a_0. \quad (4.37)$$

Using Eq. 4.30, we identify $f_1 = -M''(y_{c,n})/M'(y_{c,n})$ and $g_1 = 0$. Therefore :

$$a_1 = \frac{3M''(y_{c,n})}{4M''(y_{c,n})}a_0. \quad (4.38)$$

Determination of a_2 : Relation 4.36 for $n = 2$ gives us $[(3+2)^2 - 3(3+2)]a_2 = -[(3+0)f_2 + g_2]a_0 - (4f_1 + g_1)a_1$, or in other words :

$$a_2 = \frac{1}{40}[12f_2 + 4g_2 - 12f_1^2 - 7f_1g_1 - g_1^2]a_0 \quad (4.39)$$

Using Eq. 4.30, we identify $f_1 = -M''(y_{c,n})/M'(y_{c,n})$, $g_1 = 0$, $f_2 = M''(y_{c,n})^2/2M'(y_{c,n})^2$ and $g_2 = \zeta^2(M(y_{c,n})^2 - 1)$. Therefore :

$$a_2 = \frac{-1}{20}[3\frac{M''(y_{c,n})^2}{M'(y_{c,n})^2} + 4\frac{M'''(y_{c,n})}{M'(y_{c,n})} - 2\alpha^2(M(y_{c,n})^2 - 1)]a_0. \quad (4.40)$$

Final solution : the first solution therefore takes the form :

$$P_{n,1}(y) = a_0(y - y_{c,n})^3[1 + \frac{3M''(y_{c,n})}{M'(y_{c,n})}(y - y_{c,n}) - \frac{1}{20}[\frac{4M'''(y_{c,n})}{M'(y_{c,n})} + \frac{3M''(y_{c,n})^2}{M'(y_{c,n})^2} - 2\zeta^2(M(y_{c,n})^2 - 1)](y - y_{c,n})^2] + \mathcal{O}(y - y_{c,n})^6 \quad (4.41)$$

where a_0 has an arbitrary value, set to 1 in the following.

4.5.3 Seeking for a second solution

The Pridmore-Brown equation being of second order, we seek for a second solution, independent of the first one. It will turn out that this solution is the one carrying the singularity.

Let \mathcal{L} be the differential operator :

$$\mathcal{L} = \frac{d^2}{dy^2} + \frac{f(y)}{y - y_{c,n}} \frac{d}{dy} + \frac{q(y)}{(y - y_{c,n})^2}. \quad (4.42)$$

Then, when $\zeta \neq \zeta_1$, we have : $\mathcal{L}P_{n,1}(y, \zeta) = a_0(y - y_{c,n})^{\zeta-2}\psi(\zeta)$. $P_{n,1}(y, \zeta)$ is solution only if $\zeta = \zeta_1$. Let us check if the derivative of $P_{n,1}(y, \zeta)$ with respect to ζ in $\zeta = \zeta_1$ is solution of $\mathcal{L}P_n(y) = 0$. We get :

$$\mathcal{L} \frac{\partial P_{n,1}(y, \zeta)}{\partial \zeta}|_{\zeta=\zeta_1} = a_0\psi'(\zeta_1)(y - y_{c,n})^{\zeta_1-2} = 3a_0(y - y_{c,n})^{\zeta_2-2+N}, \quad (4.43)$$

where $N = \zeta_1 - \zeta_2$. Here, $\partial_\zeta P_{n,1}(y, \zeta)|_{\zeta=\zeta_1}$ is then solution of an inhomogeneous problem. Let us look for another particular solution to this problem under the form : $P_{n,p}(y) =$

$\sum_{p=0}^{+\infty} c_p (y - y_{c,n})^{\zeta_2+p}$. We now want to solve :

$$\begin{aligned} \sum_{p=0}^{+\infty} c_p (y - y_{c,n})^{\zeta_2+p-2} [(\zeta_2 + p)(\zeta_2 + p - 1) + \sum_{q=0}^{+\infty} (\zeta_2 + p) f_q (y - y_{c,n})^q \\ + \sum_{r=0}^{+\infty} g_r (y - y_{c,n})^r] = 3a_0 (y - y_{c,n})^{\zeta_2-2+N}. \end{aligned} \quad (4.44)$$

Terms of power $\zeta_2 - 2$: for $p = 0, q = 0$ and $r = 0$ we get $c_0 \psi(\zeta_2) = 0$. Therefore, c_0 takes an arbitrary value a priory.

Terms of power $\zeta_2 - 2 + p$ with $p \neq N$: these terms give a recurrence relation between c_p and the previous coefficients :

$$\psi(\zeta_2 + p) c_p + \sum_{q=0}^{p-1} [(\zeta_2 + q) f_{p-q} + g_{p-q}] c_q = 0. \quad (4.45)$$

Terms of power $\zeta_2 - 2 + N$: here, we get the following relation :

$$\psi(\zeta_2 + N) c_N + \sum_{q=0}^{N-1} [(\zeta_2 + q) f_{N-q} + g_{N-q}] c_q = 3a_0. \quad (4.46)$$

Several things can be noted about Eq.4.46. First of all, one can notice that $\psi(\zeta_2 + N) = \psi(\zeta_1) = 0$. Therefore, we get one more degree of freedom on c_N . However, the rest of the equation actually offers us a relation between c_0, c_1, c_2 and a_0 , allowing us to now determine these coefficients.

Determination of c_1 : using Eq.4.45, we can determine that :

$$c_1 = -\frac{\zeta_2 f_1 + g_1}{\psi(\zeta_2 + 1)} c_0 = 0, \quad (4.47)$$

because both ζ_2 and g_1 are null.

Determination of the relation between c_0 and c_2 : using Eq.4.45 we get :

$$c_2 = \frac{g_2}{2} c_0 = \frac{\alpha^2 (M(y_{c,n})^2 - 1)}{2} c_0. \quad (4.48)$$

Determination of c_0 : thanks to Eq. 4.46, we can write :

$$(\zeta_2 f_3 + g_3) c_0 + [(\zeta_2 + 1) f_2 + g_2] c_1 + [(\zeta_2 + 2) f_1 + g_1] c_2 = 3a_0. \quad (4.49)$$

Using the previous results, we can now establish a relation between c_0 and a_0 :

$$c_0 = \frac{-3M'(y_{c,n})}{M''(y_{c,n})\alpha^2(M(y_{c,n})^2 - 1)}. \quad (4.50)$$

Inserting this last result in Eq. 4.48, we find :

$$c_2 = \frac{-3M'(y_{c,n})}{2M''(y_{c,n})} a_0. \quad (4.51)$$

Particular solution to the non-homogeneous equation : A particular solution to the non-homogeneous equation can therefore be written :

$$P_{n,p}(y) = -\frac{3M'(y_{c,n})}{M''(y_{c,n})\alpha^2(M(y_{c,n})^2 - 1)} - \frac{3M'(y_{c,n})}{2M''(y_{c,n})}(y - y_{c,n})^2 + c_N(y - y_{c,n})^3 + \mathcal{O}(y - y_{c,n})^4. \quad (4.52)$$

General solution : We now have two solutions to the non-homogeneous problem ($\partial_\alpha P_1 n, p(y, \zeta)|_{\zeta=\zeta_1}$ and $P_{n,p}(y)$). Therefore, a second solution to the homogeneous problem is $P_2(y) = P_1(y) \ln(y - y_c) + P_p(y)$ which writes :

$$P_{n,2}(y) = \frac{-3M'(y_{c,n})}{M''(y_{c,n})\alpha^2(M(y_{c,n})^2 - 1)} - \frac{3M'(y_{c,n})}{2M''(y_{c,n})}(y - y_{c,n})^2 + [\ln(y - y_{c,n}) + c_N](y - y_{c,n})^3 + \mathcal{O}[(y - y_{c,n})^4], \quad (4.53)$$

with c_N a constant. It can be noted here that this result highly resembles the one obtained in the literature for circular waveguides [10].

The case of a linear shear profile, meaning that $M''(y) = 0$, cannot be described by this second solution. It requires to go back to the Pridmore-Brown equation with linear shear flow to derive the solutions. In that case, the asymptotic solutions behave purely as one odd and one even polynomial and no singularity appears in y_c , as shown in [14].

This last observation can back up the hypothesis that the energy is indeed conserved, but only in the case of a linear flow profile in the shear layer.

4.6 Conclusion

Some effects of a parallel flow on the reflection by a metasurface have been discussed here. The ability of a periodic admittance wall to reflect a plane wave in an infinity of directions can be altered by the presence of flow. As a result, the velocity of the flow becomes a tool to select the main direction of the reflected field. However, some of the evanescent waves created at the wall can see their convected wave number vanish at a position called critical layer, resulting in singularities in the wave equation. The effect of the critical layers on the conservation of energy remains to be determined. However the derivation of asymptotic solutions around these singularities suggests that the solution is singular only for a non-linear flow profile. This last result seems to show that the energy remains conserved in the system we studied here.

4.7 Appendix : energy conservation relation

We derive a relation on the scattering matrix assuming that the critical layers have no influence. The Pridmore-Brown equation, governing the magnitude of each order of reflection can be written as a Sturm-Liouville problem. We get :

$$\left[\frac{P'_n}{(k - M\alpha)^2} \right]' + \left[1 - \frac{\alpha^2}{(k - M\alpha)^2} \right] P_n = 0. \quad (4.54)$$

After multiplying by P_n^* (the star notation standing for complex conjugation) :

$$\int_{y_1}^{y_2} \left(\left[\frac{P'_n}{(k - M\alpha)^2} \right]' P_n^* + \left[1 - \frac{\alpha^2}{(k - M\alpha)^2} \right] |P_n|^2 \right) dy = 0. \quad (4.55)$$

After integration by part, it comes :

$$\left[\frac{P'_n P_n^*}{(k - M(y)\alpha_n)^2} \right]_{y_1}^{y_2} - \int_{y_1}^{y_2} \frac{|P'_n|^2}{(k - M(y)\alpha_n)^2} dy + \int_{y_1}^{y_2} \left[1 - \frac{\alpha_n^2}{(k - M(y)\alpha_n)^2} \right] |P_n|^2 dy = 0, \quad (4.56)$$

and since $|P_n|^2$ and $|P'_n|^2$ are real :

$$\Im \left[\frac{P'_n P_n^*}{(k - M(y)\alpha_n)^2} \right]_{y_1}^{y_2} = 0. \quad (4.57)$$

In the shear layer ($y_1 = 0$ and $y_2 = \delta$) :

$$\Im \left[\frac{P'_n(\delta)P_n^*(\delta)}{(k - M_0\alpha_n)^2} - \frac{P'_n(0)P_n^*(0)}{k^2} \right] = 0. \quad (4.58)$$

$P_n(\delta)$ and $P'_n(\delta)$ are known and depend on the reflection coefficients contained in the scattering matrix \mathbf{S} . $P_n(0)$ and $P'_n(0)$ are linked by the admittance matrix on the wall \mathbf{Y} by $\mathbf{P}'(0) = jk\delta\mathbf{Y}\mathbf{P}(0)$. For a lossless wall, \mathbf{Y} is purely imaginary and therefore $P'_n(0)P_n^*(0)$ is real. The conservation of energy then depends solely on the validity of the following relation :

$$\Re[\mathbf{I}^\dagger(\mathbf{M}^\dagger\mathbf{B}\mathbf{D}^2\mathbf{M} - \mathbf{S}^\dagger(\mathbf{M}^{-1})^\dagger\mathbf{B}\mathbf{D}^2\mathbf{M}^{-1}\mathbf{S})\mathbf{I}] = 0. \quad (4.59)$$

The latter result being true for any incident vector \mathbf{I} , it can be reduced to :

$$\mathbf{S}^\dagger\mathbf{B}\mathbf{D}^2\mathbf{S} = \mathbf{B}\mathbf{D}^2. \quad (4.60)$$

With the right modal normalization, Eq. 4.60 can be written :

$$\tilde{\mathbf{S}}^\dagger\tilde{\mathbf{S}} = \text{Id}. \quad (4.61)$$

INFLUENCE OF FLOW ON TRAPPED MODES

Ce Chapitre, rédigé en langue anglaise, est une ébauche d'article concernant l'évolution de modes piégés ou quasi-piéges en guide d'onde lorsqu'un écoulement est imposé autour de l'obstacle. L'influence de l'écoulement sur la réflexion et la transmission dans le guide est également abordée. La résolution numérique est effectuée via un code éléments finis écrit par Gwénaël Gabard.

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5.1 Introduction : considered problem

In the following, we study the impact of adding flow in a configuration which is known to create some trapped modes : an infinite acoustic waveguide in which an obstacle is placed. Indeed, in such a system, there can exist a resonant phenomenon for which the acoustic energy stays confined around the obstacle, as illustrated in Fig.5.1.

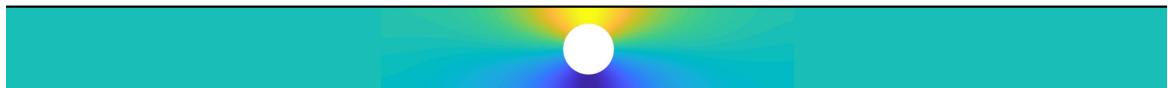


FIGURE 5.1 – Pressure field of a trapped mode in an acoustic waveguide.

As in Fig.5.1, we will study the case of a semi-2D waveguide, in which a sideway cylinder is placed. This problem will be solved thanks to the Finite Element Method.

5.2 Numerical resolution

5.2.1 Solved Equations

In the finite elements code that we use here, the equations solved are the Linearized Euler Equations [43]. In the following, we will consider a velocity potential ϕ , linked to the velocity by :

$$\nabla \phi(x, y, t) = \mathbf{u}(x, y, t) \quad (5.1)$$

This velocity potential can actually be split between a time-dependent and a time-independent part, respectively $\hat{\phi}$ and ϕ_0 . The background mean flow is therefore given by $\mathbf{U}(x, y) = \nabla \phi_0$. Linearizing the equation of an isentropic flow gives :

$$\frac{\rho_0}{\rho} = (\gamma - 1) \frac{c_0^2}{c^2}, \quad (5.2)$$

with γ the specific heat ratio. This result can be substituted in the Bernoulli equation :

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \frac{c^2}{\gamma - 1} = \text{cst}, \quad (5.3)$$

leading to :

$$\rho = -\frac{\rho_0}{c_0^2} \left(\frac{\partial \hat{\phi}}{\partial t} + \mathbf{U} \cdot \nabla \hat{\phi} \right). \quad (5.4)$$

Combining this result with the energy conservation equation given in the introduction by Eq.1.4 leads to the so-called Linearized Potential Equation (LPE) :

$$\frac{1}{c_0^2} \frac{D^2}{Dt^2} \hat{\phi} + \frac{1}{c_0^2} \frac{D}{Dt} \hat{\phi} \nabla \cdot \mathbf{U} - \Delta \hat{\phi} = 0, \quad (5.5)$$

where $D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla$ is the convected time-derivative operator. If we suppose the solution to be time harmonic in $e^{+i\omega t}$, then Eq.5.5 reduces to :

$$[-\Delta + (\mathbf{M} \cdot \nabla)^2 + (2ik + \nabla \cdot \mathbf{M})(\mathbf{M} \cdot \nabla) - k^2 + ik\nabla \cdot \mathbf{M}] \hat{\phi} = 0 \quad (5.6)$$

with $k = \omega/c_0$ and $\mathbf{M} = \mathbf{U}/c_0$. The numerical resolution of Eq. 5.6 will allow us to access the pressure field via $p(x, y, t) = i\rho_0 c_0 k \hat{\phi}$. Note that in the following, $\rho_0 = 1$ and $c_0 = 1$.

5.2.2 Mesh of the domain

The domain is meshed according to the mesh presented in Fig. 5.2. The dimensions of the waveguide are scaled by its height H . The portion of the wave guide that is meshed has a length $L = 5$ and occupies a domain $\Omega = \{x \in [-L/2, L/2], y \in [0, 1]\}$. On each side of the meshed portion, we define some Perfectly Matched Layers (PML) [5]

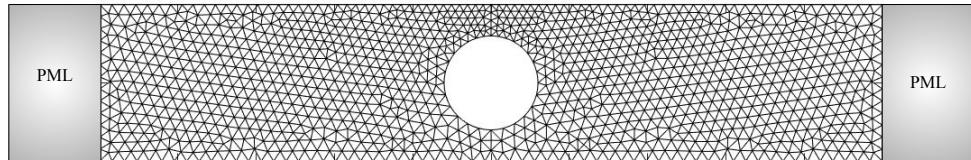


FIGURE 5.2 – Mesh for the numerical resolution

to fulfill a Sommerfeld condition, ensuring no reflection coming from infinity. The whole domain is meshed with triangular elements, for a total of about 2500 elements. The shape functions used to approximate the solution on each element are second-order polynomials.

5.3 Reminder on trapped and quasi-trapped modes

Before putting the fluid in rotation, we want to give the reader a reminder on trapped and quasi-trapped modes. To do so, we will study two types of geometry, that we will later refer to as symmetrical and non-symmetrical, schemed on Fig.5.3.

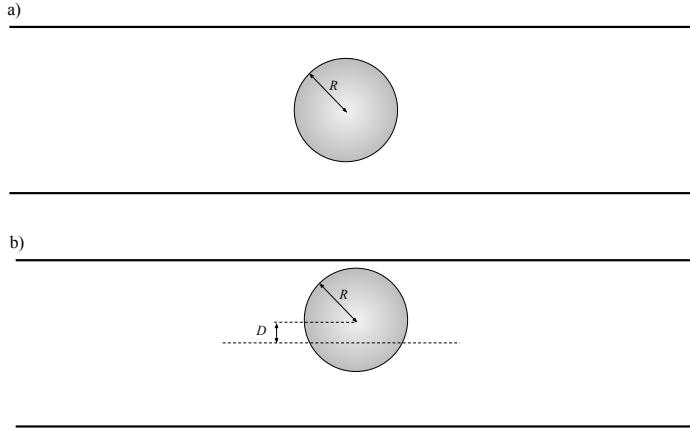


FIGURE 5.3 – a) Geometry of the symmetric configuration, b) geometry of the non-symmetric configuration.

5.3.1 Numerical identification of the trapped mode

Eigenmodes in an open geometry can be calculated thanks to PMLs. These modes can either be *leaky modes* (radiating energy towards infinity) or trapped modes. In the following, we calculate the values of the eigenfrequencies thanks to a finite element code. These are spread in the complex plane as illustrated in Fig. 5.4. It can be shown [8]

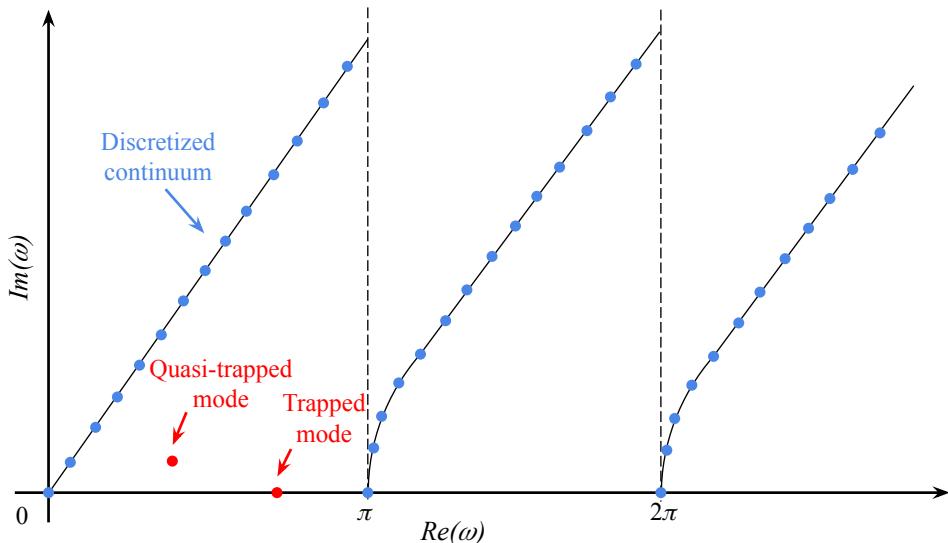


FIGURE 5.4 – Illustration of the arrangement of the eigenfrequencies in the complex space.

that a trapped mode will stand out in this arrangement by being real-valued. The presence

of a trapped mode requires several symmetries in the system [56] which are not always present. In that case, this mode can become leaky and will be referred to as quasi-trapped.

5.3.2 In a symmetrical guide

Trapped modes can appear in waveguides comporting either a defect or an obstacle if the global geometry of the waveguide remains symmetrical [56]. Applying the method detailed above, it is possible to extract the frequency of this mode as it is real-valued. As pointed in Fig. 5.5, this makes this mode stand out when plotting the eigenfrequencies of the system in the complex plane. The points which are out of the expected curves might be a result of the number of elements chosen for the calculation as less and less of them appear when more elements are considered. Figure 5.6 shows

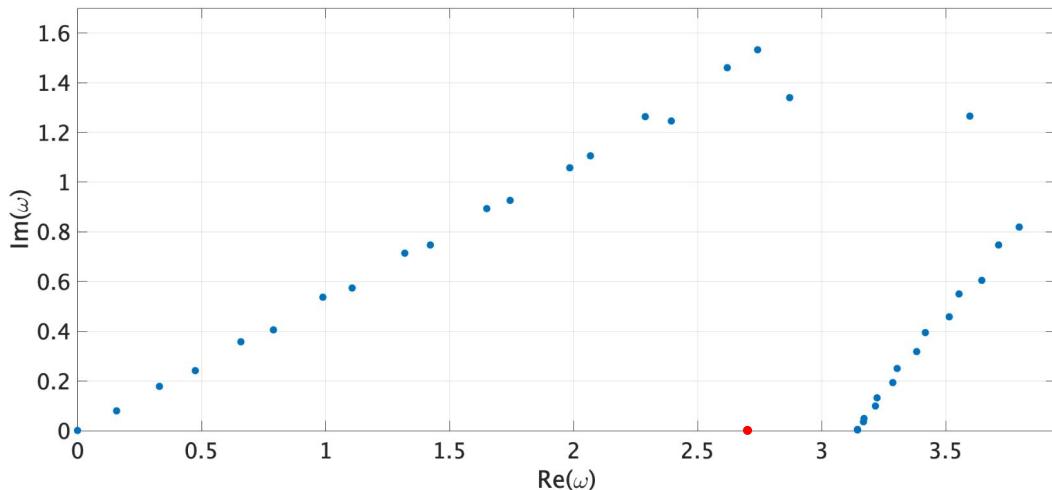


FIGURE 5.5 – Eigenmodes of the system when the cylinder is on the axis of the guide. The trapped mode (in red) stands out for being real-valued.

the pressure field of the trapped mode for a waveguide of height 1, in which a cylinder of radius $R = 0.3$ is placed. One can notice that this trapped mode is characterized by a nodal line along the axis of the waveguide and an anti-symmetrical repartition of the pressure field. As expected, all of the energy is concentrated around the cylinder and is null away from it. The size of the obstacle, or in our case the radius of the cylinder, will have an influence on the frequency of the trapped mode. Indeed, the smaller the obstacle, the closer the frequency of the trapped mode will be to the cut-off frequency of the first transverse mode ($\omega = \pi$). One can understand this behavior by noticing that, for an infinitesimally small obstacle, the first mode presenting this horizontal anti-symmetry

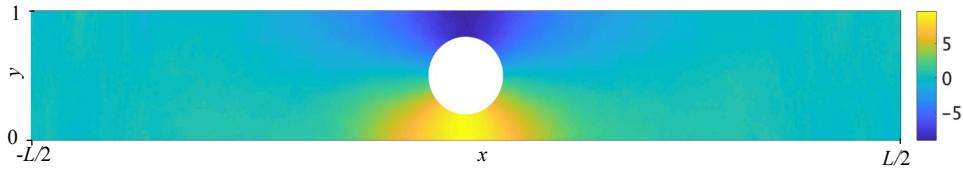
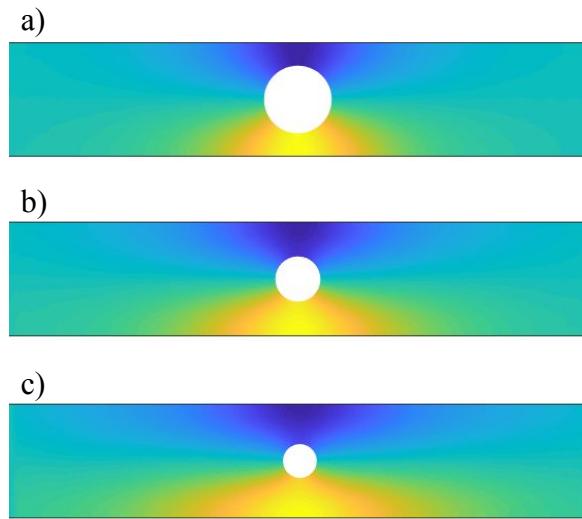


FIGURE 5.6 – Pressure field of the trapped mode.


 FIGURE 5.7 – Pressure field of the trapped mode for a) $R = 0.3$, $\omega_T = 2.70$ b) $R = 0.2$, $\omega_T = 2.90$ c) $R = 0.15$, $\omega_T = 3.01$.

will be the first transverse mode. As plotted in Fig. 5.7, the reduction of the obstacle will also result in a larger spread of the zone of trapped energy, tending towards infinity for an infinitesimally small obstacle.

5.3.3 In a non-symmetrical guide

When the waveguide is not exactly symmetrical, the obstacle does not coincide with the node line of the first anti-symmetric axial mode anymore. Therefore, part of the energy leaks away from it, resulting in a so-called quasi-trapped mode. Consequently, the associated eigenfrequency of this mode is no longer purely real-valued but has a small imaginary part (Fig. 5.8). Figure 5.9 shows the pressure field of the trapped mode for a waveguide of height 1, in which a cylinder of radius $R = 0.3$ is placed and shifted from the axis of a distance $D = 0.1$. When comparing it to Fig. 5.6, one can notice that the pressure field spreads more along the length of the waveguide. As a matter of fact, the

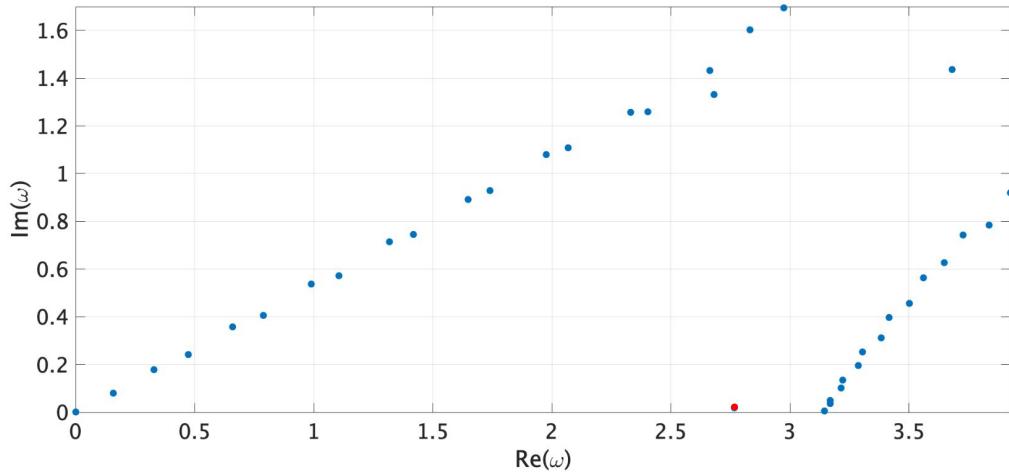


FIGURE 5.8 – Eigenmodes of the system when the cylinder is off the axis of the guide. The quasitrapped (in red) mode has a very little imaginary part.

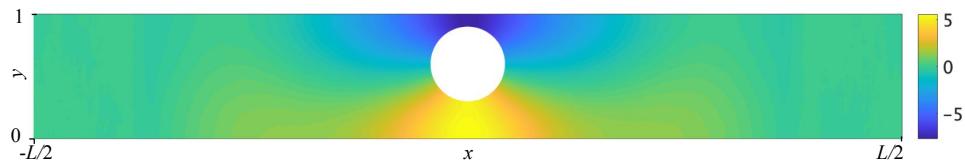


FIGURE 5.9 – Pressure field of the quasitrapped mode.

further the cylinder will sit from the axis of the cylinder, the more energy will be leaked away from it.

5.4 Effect of flow on the modes

The vortex flow around the cylinder is such that it decays exponentially away from it. Numerically, we approximate this hypothesis as a null flow in the PMLs. In order to calculate this flow, we impose a jump in the mean flow potential ϕ_0 along a line situated above the cylinder. The value of this jump is denoted Γ , the flow circulation. The flow has an impact on the frequency of the trapped (or quasi-trapped) mode. The first observation to be made is that the real part of the frequency is very slightly impacted by the presence of the flow, whereas its imaginary part grows bigger and bigger as the flow velocity is increased. This results in a more and more leaky mode. In Figure 5.11, we plot the imaginary part of the eigenfrequency of the trapped mode ω_t as a function of the vortex circulation Γ . It appears that the imaginary shift of the eigenvalue from the

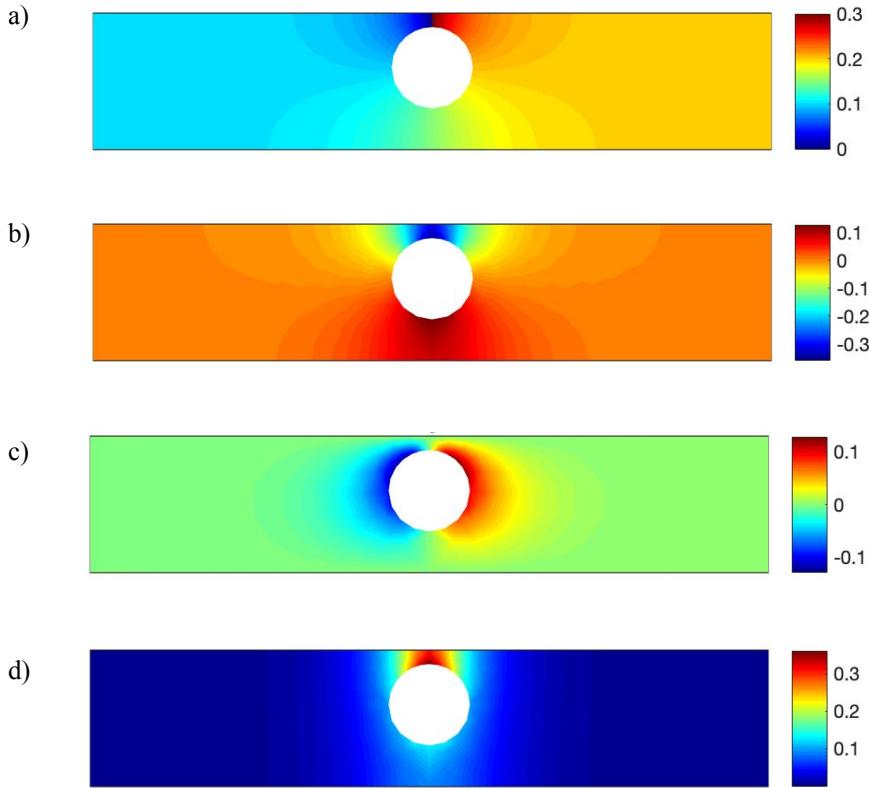


FIGURE 5.10 – Spatial distribution, for $D = 0.1$ and $\Gamma = 0.3$, of a) the mean flow velocity potential (the imposed jump Γ is arbitrarily positioned above the cylinder here), b) v_x , the horizontal component of the velocity field, c) v_y , the vertical component of the velocity field, d) the mean Mach number $M = |\mathbf{U}(x, y)|/c_0$.

real axis increases Γ increases. Moreover, the growth of the imaginary part is faster for the symmetric case, which is something we did not expect. As a result, for $\Gamma = 0.5$, the quasi-trapped mode will actually radiate more energy towards infinity in the symmetric case than in the non-symmetric case.

5.5 Effect of flow on the scattering of the system and link to the trapped mode

In addition to looking at the eigenmodes of the system, we can also investigate how such a system will reflect or transmit an incident wave. To do so, we define a scattering matrix containing the reflection and transmission coefficients $R_{L,R}$ and $T_{L,R}$ linking the

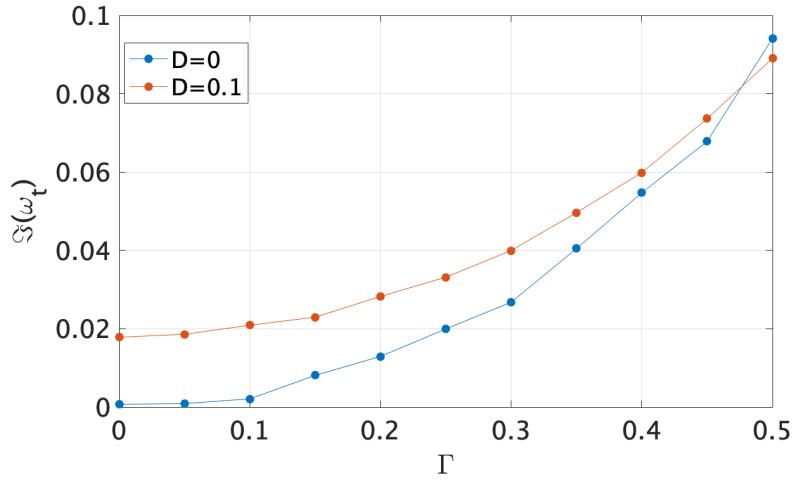


FIGURE 5.11 – Imaginary part of the eigenfrequency of the (quasi)trapped mode for a symmetrical or non-symmetrical waveguide as a function of Γ .

incoming and out-going waves (defined on Fig. 5.12) as :

$$\begin{pmatrix} p_L^- \\ p_R^+ \end{pmatrix} = S \begin{pmatrix} p_L^+ \\ p_R^- \end{pmatrix} = \begin{bmatrix} R_L & T_R \\ T_L & R_R \end{bmatrix} \begin{pmatrix} p_L^+ \\ p_R^- \end{pmatrix} \quad (5.7)$$

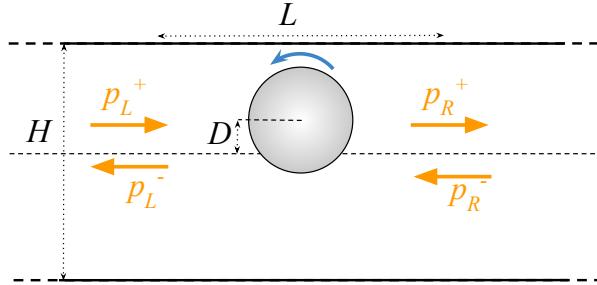


FIGURE 5.12 – Nomenclature of incoming and outgoing waves on the system.

5.5.1 Symmetric guide without flow

When the obstacle is on the axis of the waveguide, then the trapped mode has no impact on the scattering of the system. It is then perfectly symmetrical, as shown in Fig.5.13 with $R_L = R_R$ and $T_L = T_R$.

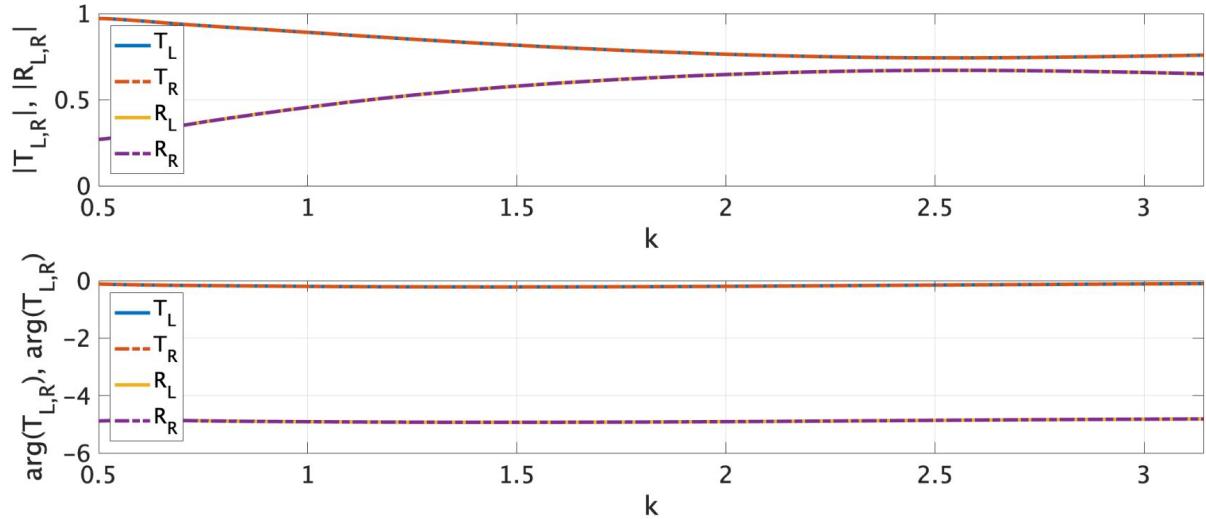


FIGURE 5.13 – Modulus and argument of the coefficients of the scattering matrix for $D = 0$, $\Gamma = 0$.

5.5.2 Non-symmetric guide without flow

When the cylinder is slightly shifted from the axis, a very quick change in the value of the moduli of the scattering coefficient, characteristic of a Fano resonance [7, 23, 40] happens (see Fig. 5.14). In this case, it is possible to achieve perfect reflection or perfect transmission for two very close frequencies. A 2π phase jumps accompanies this fast change in modulus. It can be noticed that this specific phenomenon happens very close to the frequency of the quasi-trapped mode for such a geometry.

Calculations were also led for $D = 0.01$ to check if there exists a threshold in the position of the cylinder to excite the Fano-resonance(Fig. 5.15). It appears that the Fano resonance is visible even for a very little shift from the axis of the cylinder, but it is located in a very narrow frequency domain to describe it correctly. Note that in Fig. 5.15, the coefficients do not go all the way to 0 nor 1 because the phenomenon is so sharp, it would require a huge amount of points in the numerical simulation.

5.5.3 Symmetric guide with flow

When adding flow to the symmetrical case, a Fano resonance also appears (Fig.5.16), however it is inverted in comparison to the one of the non-symmetric case : this time, the modulus of the transmission coefficients go to zero before quickly going

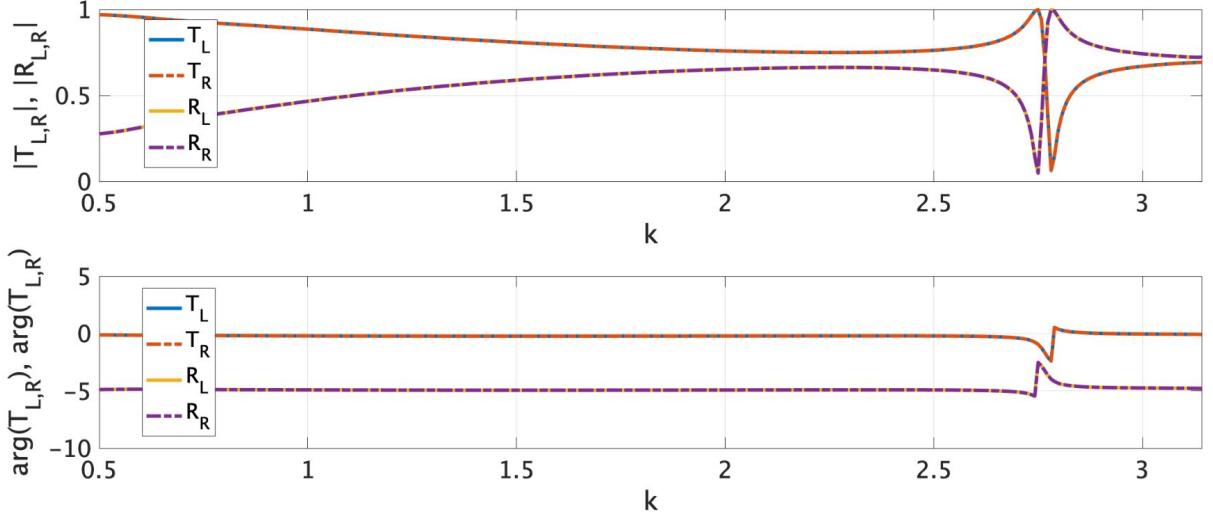


FIGURE 5.14 – Modulus and argument of the coefficients of the scattering matrix for $D = 0.1$, $\Gamma = 0$.

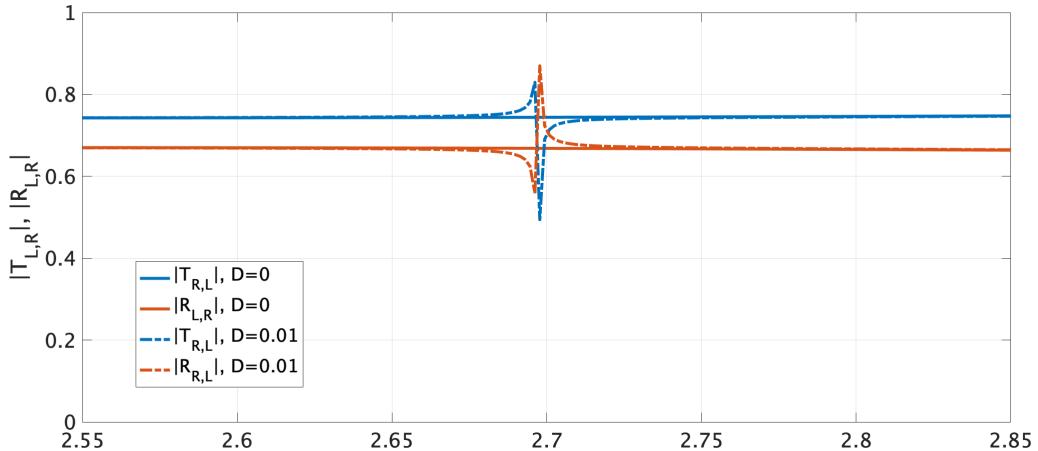


FIGURE 5.15 – Modulus of the transmission and reflection coefficients for $D = 0$ (plain lines) and $D = 0.01$ (dashed lines).

back to one.

5.5.4 Non-symmetric guide with flow

The system finally becomes non-reciprocal when the cylinder is shifted off the axis in the presence of flow. In such a case, no Fano resonance appears, but at the frequency of the quasi-trapped mode perfect transmission is achieved. The non-reciprocal

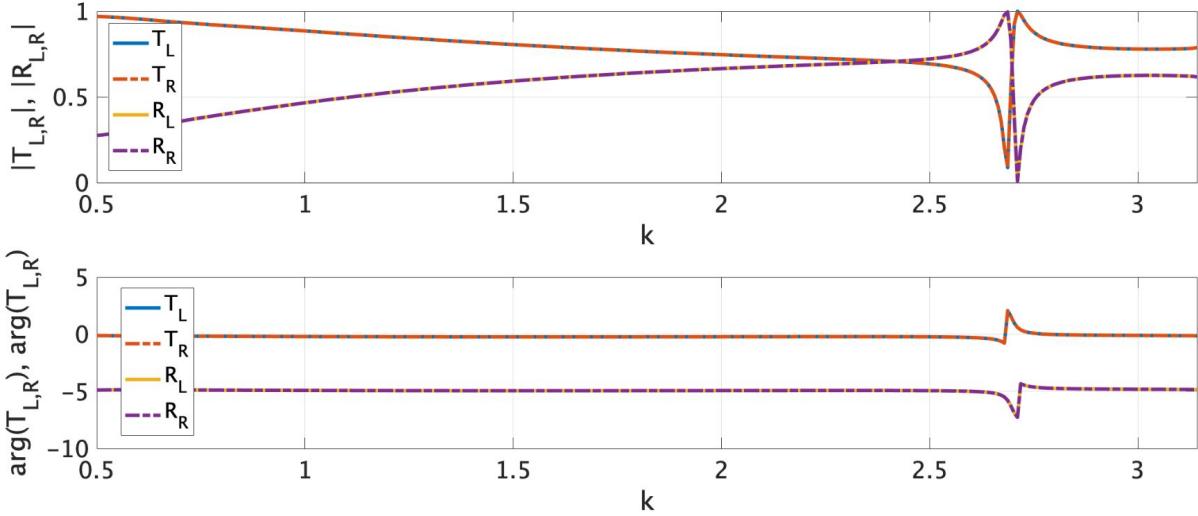


FIGURE 5.16 – Modulus and argument of the coefficients of the scattering matrix for $D = 0$, $\Gamma = 0.2$.

behavior of the system only appears on the arguments of the transmission coefficients. Indeed, it can be observed in Fig.5.17 that, for a range of frequencies around the frequency of the quasi-trapped mode, $\arg(T_L) \neq \arg(T_R)$. This phase difference, equal to 0 without

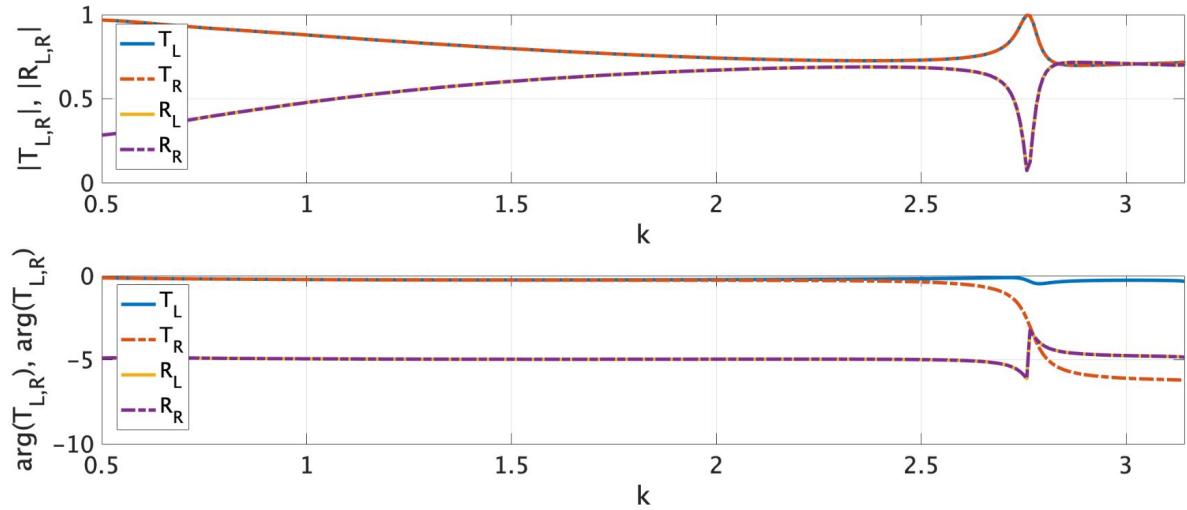


FIGURE 5.17 – Modulus and argument of the coefficients of the scattering matrix for $D = 0.1$, $\Gamma = 0.2$.

flow, decreases and spreads in frequency as Γ increases (see Fig. 5.18). This difference in phase can actually be justified by the symmetries of the system. Indeed, a \mathcal{PT} -symmetric

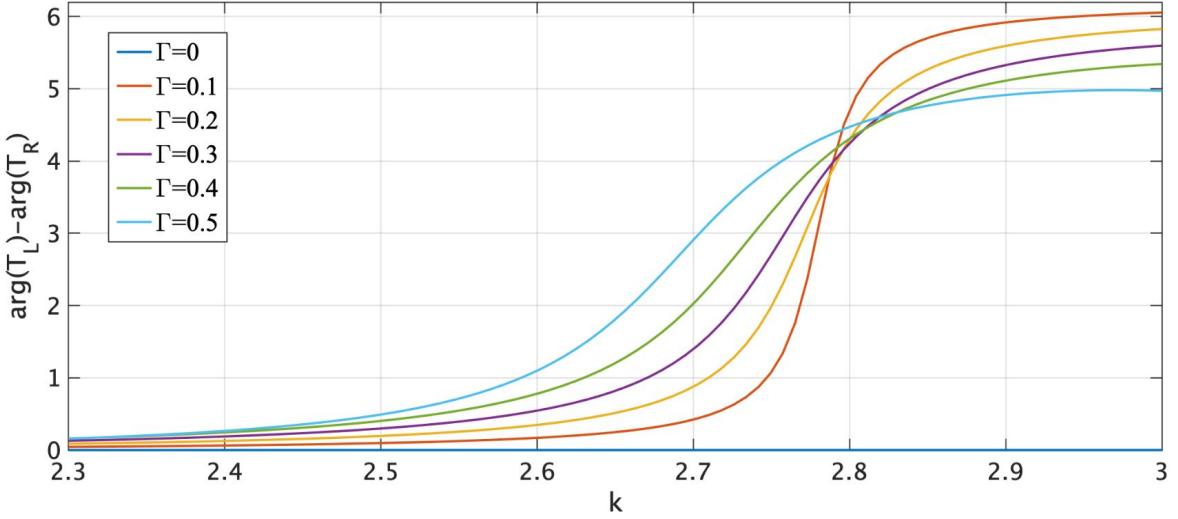


FIGURE 5.18 – Difference between the arguments of T_L and T_R for different values of Γ .

system (i.e. invariant by mirror symmetry $x/-x$ and phase conjugation) verifies [61] :

$$\sigma_x S^*(\omega^*) \sigma_x S(\omega) = \text{Id}, \quad (5.8)$$

with σ_x the first Pauli matrix. Then, for ω real :

$$\begin{cases} |T_L|^2 + R_L R_R^* = 1 \\ |T_R|^2 + R_L^* R_R = 1 \\ T_L^* R_R + R_R^* T_R = 0 \\ T_L R_L^* + R_L T_R^* = 0. \end{cases} \quad (5.9)$$

The two first equations lead to $\theta_{R_L} = \theta_{R_R}[\pi]$ and $|T_L| = |T_R|$. Moreover, the energy conservation imposes $|T_L| = |T_R|$, and therefore $|R_L| = |R_R|$. Consequently, $R_L = R_R = R$. Concerning the arguments of the transmission coefficients, the only relation we can write is :

$$e^{i(\theta_R - \theta_{T_L})} + e^{-i(\theta_R - \theta_{T_R})} = 0 \quad (5.10)$$

and nothing implies that these should be equal. This non-reciprocity is something that could be exploited in a periodic system, leading to interesting topological effects such as skin modes, as pointed in recent works [36].

5.6 Conclusion

We have seen here how trapped modes in a duct can be perturbed by the presence of flow as they start leaking energy towards infinity when the medium starts moving, therefore turning into quasi-trapped modes. Around the frequency of these modes, the scattering of the portion of guide containing the obstacle is also modified, leading to a non-reciprocal scattering. However, this non-reciprocity is only expressed on the phases of the transmission coefficients, leading to :

$$\begin{cases} R_L = R_R \\ |T_L| = |T_R|. \end{cases} \quad (5.11)$$

EFFET DE L'ÉCOULEMENT SUR UN SYSTÈME PÉRIODIQUE

Dans ce chapitre est étudié l'effet de l'écoulement dans un système périodique. Pour ce faire, le système étudié dans le chapitre précédent est considéré comme une cellule unitaire et répliqué selon une dimension. Le mode piégé ou quasi-piége de ce système provoque l'apparition de bandes interdites autour de sa fréquence. Afin de réaliser un grand nombre de calculs rapidement et d'étudier l'influence de plusieurs paramètres du système périodisé, un modèle-jouet est également étudié à l'aide d'une méthode de raccordement modal. Cette méthode permet notamment de construire un système dans lequel un mode d'interface est observé.

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6.1 Introduction : généralités sur les systèmes périodiques

On parle de système périodique pour un système composé de cellules unitaires identiques régulièrement agencées dans l'espace (Fig. 6.1).

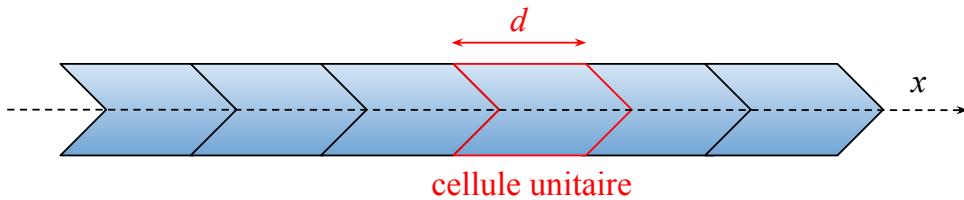


FIGURE 6.1 – Illustration d'un système périodique à une dimension de période spatiale d .

En supposant une périodicité en x de longueur d , un mode $\phi(x)$ du système vérifiera [6] :

$$\phi(x + d) = \phi(x)e^{j\alpha d}, \quad (6.1)$$

avec α la composante selon x du nombre d'onde. Ainsi, en définissant une fonction $\Psi(x)$ telle que $\Psi(x) = \Psi(x + d)$:

$$\phi(x) = e^{j\alpha x}\Psi(x). \quad (6.2)$$

Une solution $p(x)$ peut être décomposée en un mode se propageant vers la gauche et un mode se propageant vers la droite, ainsi $p(x) = \phi^+e^{j\alpha x} + \phi^-e^{-j\alpha x}$. En notant d'un indice L les amplitudes aller et retour à gauche d'une cellule donnée et d'un indice R celles à droites (voir Fig.6.2), il est possible de définir la matrice de transfert d'une cellule via :

$$\begin{pmatrix} \phi_R^+ \\ \phi_R^- \end{pmatrix} = \begin{bmatrix} A(k) & B(k) \\ C(k) & D(k) \end{bmatrix} \begin{pmatrix} \phi_L^+ \\ \phi_L^- \end{pmatrix} = \mathbf{M}_e(k) \begin{pmatrix} \phi_L^+ \\ \phi_L^- \end{pmatrix}. \quad (6.3)$$

Finalement, les ondes pouvant se propager dans le système ont un nombre d'onde $\alpha(k) =$

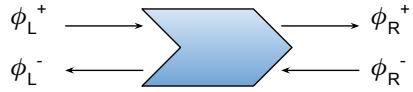


FIGURE 6.2 – Définition des amplitudes des ondes aller et retour de part et d'autre d'une cellule unitaire.

$q^\pm(k)$ tel que :

$$\begin{cases} e^{jq^+(k)d} = \lambda_1(k) \\ e^{jq^-(k)d} = \lambda_2(k), \end{cases} \quad (6.4)$$

où $\lambda_{1,2}$ sont les valeurs propres de M_e à une fréquence donnée.

Une onde de fréquence k pourra se propager dans le système si et seulement si q^\pm est réel, autrement dit si $|\lambda_{1,2}| = 1$. Il est ainsi possible de définir des bandes passantes et des bandes interdites, permettant ou non la propagation dans le système, comme illustré en Fig.6.3.

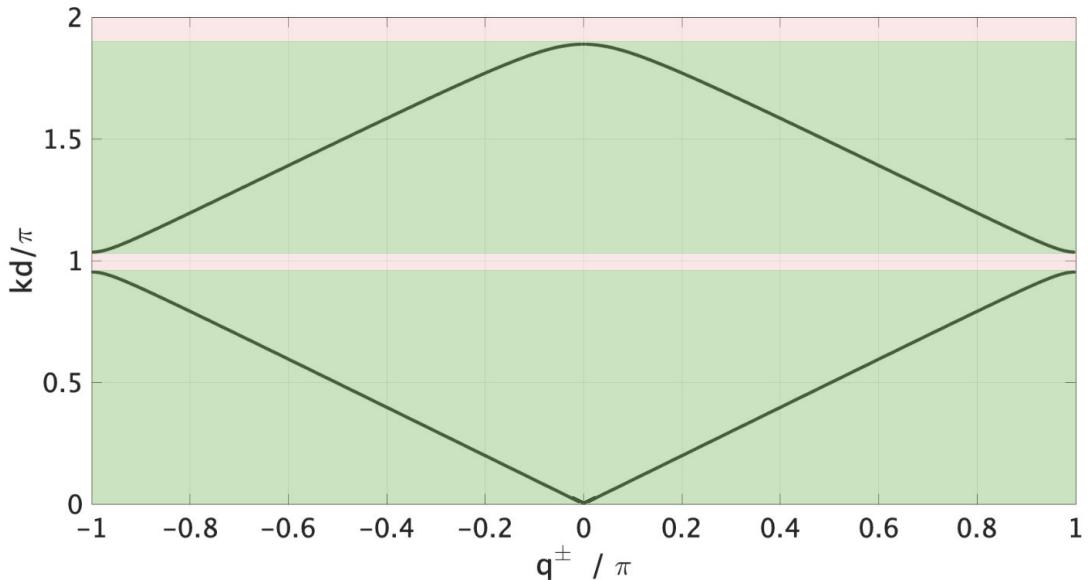


FIGURE 6.3 – Illustration de la première zone de Brillouin du diagramme de dispersion pour un système périodique admettant des bandes passantes (en vert) et des bandes interdites (en rouge).

6.2 Périodisation du système

On périodise le système étudié au chapitre précédent, ainsi on obtient un guide d’onde infini dans lequel des cylindres sont régulièrement espacés, tel que celui représenté en Fig. 6.4. Les cellules sont considérées comme étant assez espacées pour que les écoulements de deux cellules adjacentes n’interagissent pas.

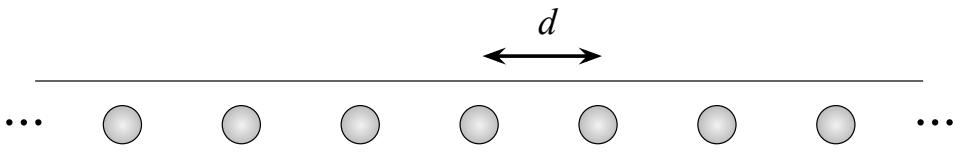


FIGURE 6.4 – Illustration du système périodisé

6.2.1 Définition d’une cellule unitaire

Les cellules unitaires sont constituées d’une portion de guide d’onde de longueur L , dans laquelle est situé un cylindre et dont la matrice de transmission / réflexion est calculée par une méthode éléments finis. On rappelle que l’écoulement autour du cylindre est contrôlé par l’amplitude du saut de potentiel de vitesse, notée Γ . De part et d’autre de cette portion de guide sont ajoutées des sections vides, de longueur l (Fig. 6.5). Ainsi, la cellule unitaire a pour longueur totale $d = L + 2l$. Le calcul par éléments finis du Chapitre 5 nous permet d’accéder aux coefficients de transmission et de réflexion de la cellule unitaire. On commence par passer d’une notation matrice de scattering (liant les ondes entrantes aux ondes sortantes) à une notation matrice de transfert (liant les ondes

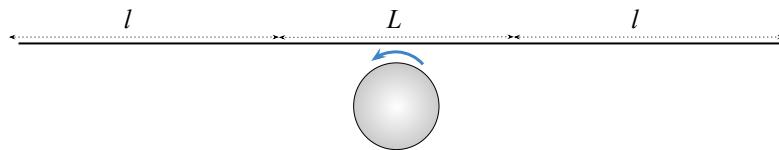


FIGURE 6.5 – Géométrie de la cellule unitaire.

à gauche de la cellule aux ondes à droite de la cellule) :

$$\begin{pmatrix} p_L^- \\ p_R^+ \end{pmatrix} = \begin{bmatrix} R_L & T_R \\ T_L & R_R \end{bmatrix} \begin{pmatrix} p_L^+ \\ p_R^- \end{pmatrix} \iff \begin{pmatrix} p_R^+ \\ p_L^- \end{pmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} p_L^+ \\ p_R^- \end{pmatrix}. \quad (6.5)$$

On identifie les coefficients A , B , C et D en fonction des coefficient $T_{L,R}$ et $R_{L,R}$. Finalement, la matrice de transfert de la portion de la cellule contenant l'obstacle s'écrit :

$$\mathbf{M} = \frac{1}{T_R} \begin{bmatrix} T_L T_R - R_L R_R & R_R \\ -R_L & 1 \end{bmatrix}. \quad (6.6)$$

Les portions vides du tube, de longueur l , ont quant à elle chacune pour matrice de transfert \mathbf{M}_v , telle que :

$$\mathbf{M}_v = \begin{bmatrix} e^{jkl} & 0 \\ 0 & e^{-jkl} \end{bmatrix}. \quad (6.7)$$

La matrice de transfert élémentaire de chaque cellule est donc donnée par :

$$\mathbf{M}_e = \mathbf{M}_v \mathbf{M} \mathbf{M}_v = \frac{1}{T_R} \begin{bmatrix} (T_L T_R - R_L R_R) e^{2jkl} & R_R \\ -R_L & e^{-2jkl} \end{bmatrix}. \quad (6.8)$$

6.2.2 Périodisation

À partir de la matrice de transfert de la cellule unitaire, on accède au diagramme en bandes, obtenu en identifiant les couples (k, q^\pm) qui vérifient :

$$q^\pm = \frac{-j}{d} \ln(\lambda_{1,2}). \quad (6.9)$$

où les $\lambda_{1,2}$ sont les valeurs propres de la matrice de transfert de la cellule unitaire. Dans les exemples ci-après, on s'intéresse au cas $R = 0.3$, $D = 0$ ou $D = 0.1$, $L = 5$, $l = 0$, $\Gamma = 0$ ou $\Gamma = 0.3$ ou $\Gamma = 0.5$. Dans le cas où $D = 0$ et $\Gamma = 0$, on obtient par exemple le diagramme de dispersion de la Fig.6.6. Comme on pouvait s'y attendre, des bandes interdites apparaissent pour $k = n\pi/d$, avec $n \in \mathbb{N}$. Ces fréquences correspondent aux fréquences de Bragg du système, pour lesquelles la longueur d'onde est un multiple de $2d$.

En décalant le cylindre de l'axe du guide d'une distance $D = 0.1$, la résonance de Fano due à la présence du mode piégé apparaît (Fig. 6.7), créant une nouvelle bande interdite. Cette bande interdite a une fréquence légèrement supérieure à celle du mode piégé.

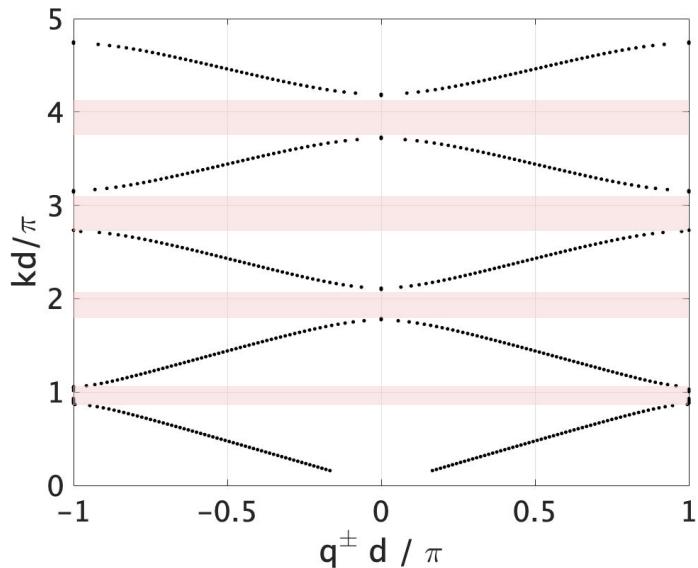


FIGURE 6.6 – Diagramme de dispersion pour un cylindre placé sur l’axe, sans écoulement, avec $l = 0$.

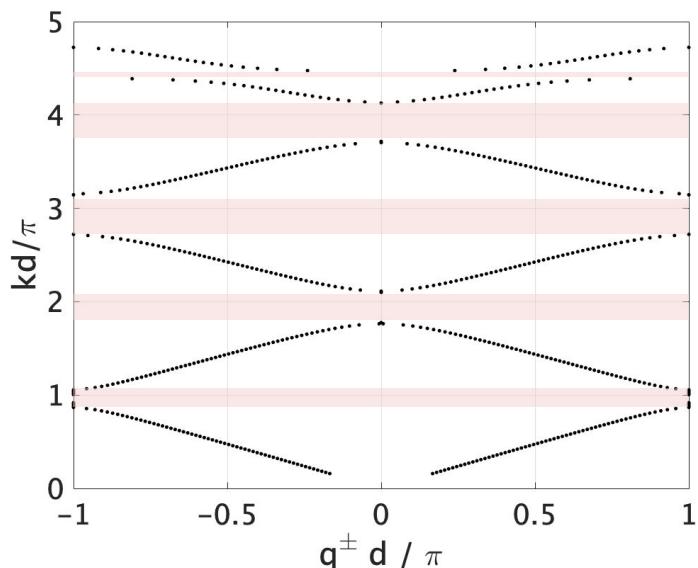


FIGURE 6.7 – Diagramme de dispersion pour un cylindre décalé de l’axe du guide d’une distance $D = 0.1$, sans écoulement, avec $l = 0$.

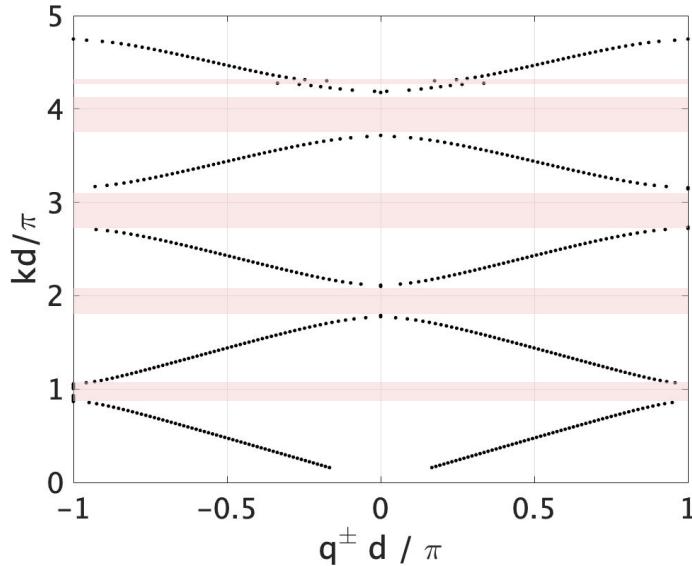


FIGURE 6.8 – Diagramme de dispersion pour un cylindre sur l'axe du guide, avec $\Gamma = 0.1$ et $l = 0$.

6.2.3 Effet de l'écoulement dans le système périodique

En ajoutant de l'écoulement au système, dans le cas où le guide est symétrique, là aussi une bande interdite apparaît à la fréquence du mode piégé. Cette bande interdite a une fréquence légèrement inférieure à celle du mode piégé. On constate particulièrement que l'effet de l'écoulement en basse fréquence est minime, et se concentre autour de la fréquence du mode piégé. En décalant le cylindre de l'axe du guide et en augmentant la valeur de Γ , la non-réciprocité du système devient plus évidente ($q^+ \neq q^-$). Là aussi, l'effet de l'écoulement en basse fréquence est minime.

En Fig. 6.9, un zoom est effectué autour de la fréquence du mode piégé. Une bande interdite très fine apparaît autour de cette fréquence. Le coût de calcul important pour obtenir cette résolution motivera par la suite l'emploi d'un modèle simplifié du système, résolu par raccordement modal.

6.3 Étude d'un modèle simplifié

Afin de pouvoir optimiser les temps de calcul et étudier efficacement les effets de l'arrangement des cellules dans le guide, un modèle simplifié est étudié par la suite. Celui-ci, schématisé en Fig. 6.10, substitut au cylindre une brique rectangulaire, séparant

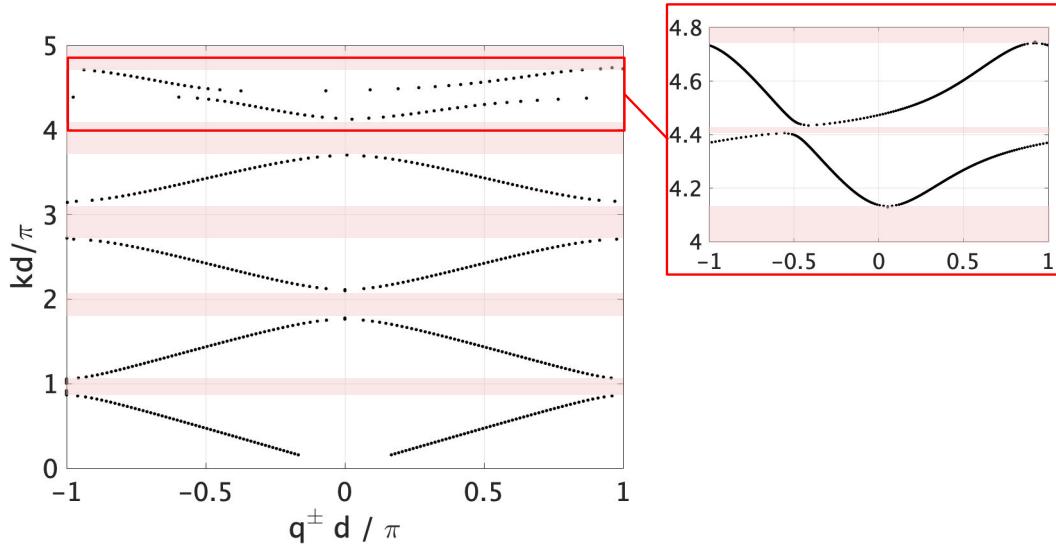


FIGURE 6.9 – Diagramme de dispersion pour un cylindre décalé de l’axe du guide d’une distance $D = 0.1$, avec $\Gamma = 0.2$ et $l = 0$.

le guide d’onde de hauteur totale $H = 1$ en deux portions de guide, de hauteurs H_1 et H_2 . Dans chacun de ces guides, un écoulement uniforme est défini, de nombre de Mach $M_{1,2}$, avec $H_1 M_1 + H_2 M_2 = 0$. L’écoulement étant défini comme nul dans le reste du conduit, la limite physique de ce modèle tient en la discontinuité d’écoulement moyen de part et d’autre de la brique.

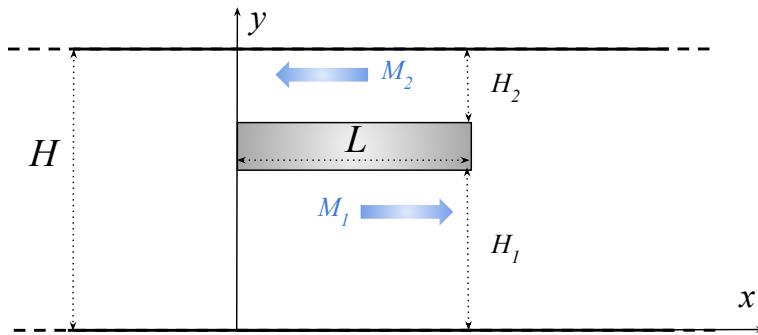


FIGURE 6.10 – Géométrie du modèle simplifié.

La description totale du modèle est donnée en annexe 6.5. En inversant l’Eq. 6.21, on peut accéder à la matrice \mathbf{S} liant les ondes entrantes aux ondes sortantes du modèle simplifié. Si les modes évanescents seront essentiels à la détermination du champs total, par la suite nous nous placerons toujours à $k < \pi$, c’est à dire sous la fréquence de coupure du premier mode transverse du guide d’onde. Ainsi, les coefficients $R_{L,R}$ et $R_{R,R}$ considérés

feront toujours référence à ceux liant les amplitudes des ondes aller et retour du mode plan uniquement, de part et d'autre de la brique.

6.3.1 Périodisation du toy model

On périodise la cellule unitaire en entourant chaque brique d'une section de guide vide de longueur l . On obtient donc (Fig. 6.11 a)) une cellule unitaire de longueur $d = l + L$. La cellule unitaire peut donc être décrite par sa matrice de transfert, $\mathbf{M}_{tot} = \mathbf{M}_t \mathbf{M} \mathbf{M}_t$, avec \mathbf{M}_t la matrice de transfert des sections de tube définie par :

$$\mathbf{M}_t = \begin{bmatrix} e^{jkl} & 0 \\ 0 & e^{jkl} \end{bmatrix}, \quad (6.10)$$

et \mathbf{M} la matrice de transfert de la partie "brique" dont les coefficients sont donnés en fonction de ceux de la matrice de scattering \mathbf{S} obtenue précédemment par :

$$\mathbf{M} = \frac{1}{T_R} \begin{bmatrix} -|\mathbf{S}| & R_R \\ -R_L & 1 \end{bmatrix}. \quad (6.11)$$

La solution étant d -périodique, les valeurs propres λ_i de la matrice M_{tot} sont liées aux modes propres du système via :

$$\begin{cases} \lambda_1 = e^{jq^+d} \\ \lambda_2 = e^{jq^-d}. \end{cases} \quad (6.12)$$

En isolant les cas où les q^\pm sont réels, on a accès au diagramme de dispersion du système. En doublant la taille de la cellule (Fig. 6.11 b)), on replie le diagramme de dispersion, en créant des points de Dirac en $q = \pm\pi$ (Fig. 6.12). La matrice de transfert de la cellule unitaire est alors donnée par $\mathbf{M}_{tot} = \mathbf{M}_t \mathbf{M} \mathbf{M}_t \mathbf{M} \mathbf{M}_t$.

On décale ensuite à l'intérieur de chaque cellule les deux briques de manière symétrique, soit en les écartant, soit en les rapprochant l'une de l'autre (Fig. 6.11 c) et d)). On obtient donc en plus les paramètres L_1 et L_2 (tels que $L_1 + L_2 = 2L$), ainsi qu'à l'intérieur de la cellule l_1 et l_2 , tous représentés en Fig. 6.13. La matrice de transfert de la cellule s'écrit alors $\mathbf{M}_{tot} = \mathbf{M}_1 \mathbf{M} \mathbf{M}_2 \mathbf{M} \mathbf{M}_1$, avec $M_{1,2}$ les matrices de transfert des sections de longueur $l_{1,2}$. Dans ce cas, on est capable d'ouvrir un bandgap au niveau des points de Dirac précédemment créés (voir Fig. 6.14).

En particulier, on peut remarquer que ce diagramme de dispersion sera le même pour une

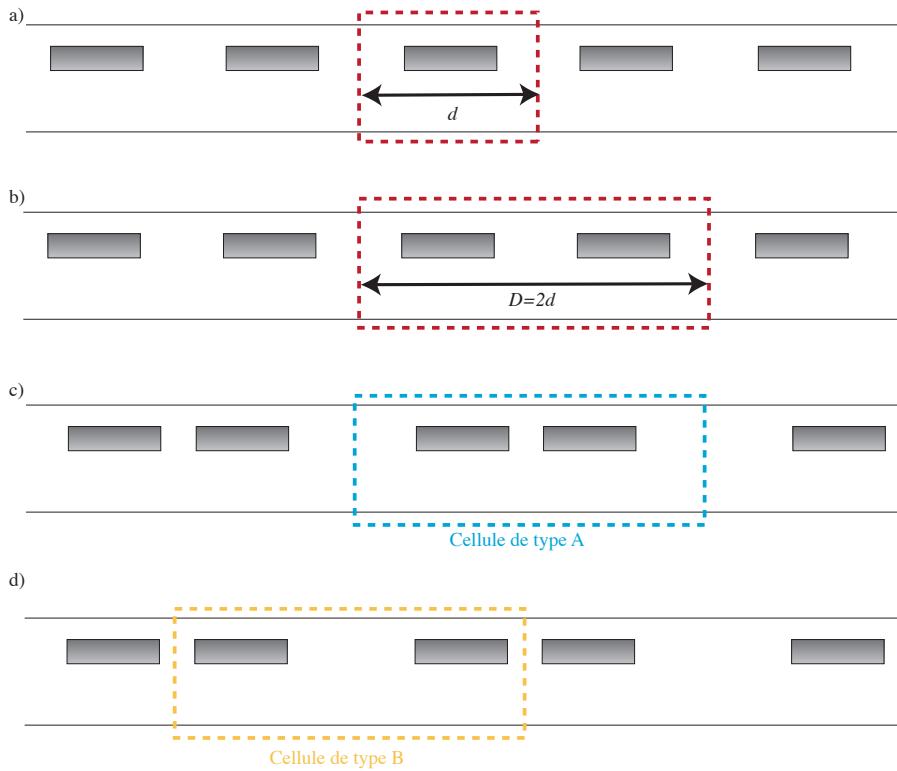


FIGURE 6.11 – Différents cas étudiés.

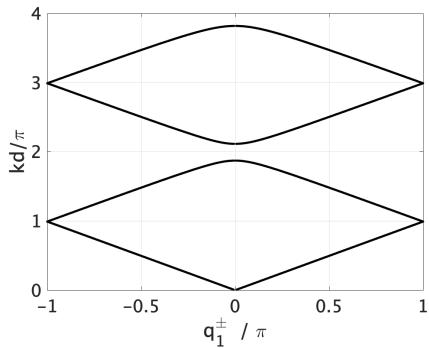


FIGURE 6.12 – Diagramme de dispersion pour le cas b) de la Fig. 3, avec $H_1 = 0.5$, $H_2 = 0.25$ et $L = 0.3$

cellule de type A (briques rapprochées) ou de type B (briques écartées).

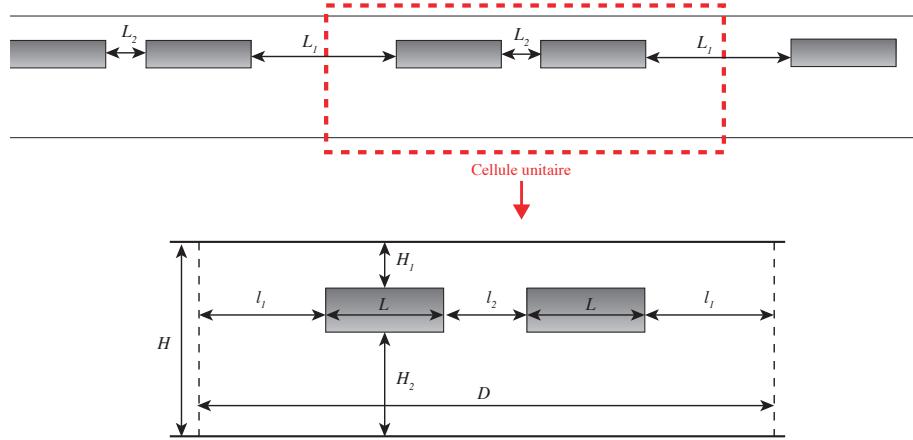


FIGURE 6.13 – Définition des différentes grandeurs du problème. En supposant $L_1 > L_2$, la cellule de type A est définie par $l_1 = L_1/2$ et $l_2 = L_2$; la cellule de type B par $l_1 = L_2/2$, $l_1 = L_1$.

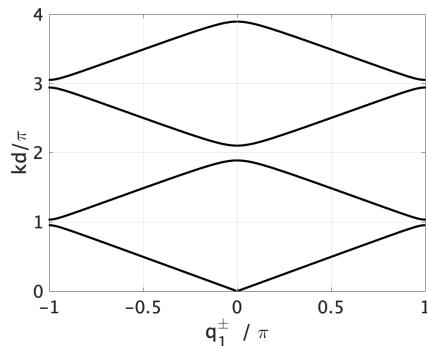


FIGURE 6.14 – Diagramme de dispersion pour le cas b) de la Fig. 3, avec $H_1 = 0.5$, $H_2 = 0.25$ et $L_1 = 0.1$ et $L_2 = 0.5$

6.3.2 Perspectives : création d'interface

Cette partie présente de travaux en cours. On crée une interface entre N cellules de type A et N cellules de type B. Cette configuration particulière permet une transmission particulière associée à un mode d'interface [67]. On compare ensuite la transmission par un réseau formé de $2N$ cellules A (ou B), comme en Fig. 6.15 a) et b) à la transmission par un réseau avec interface (Fig. 6.15 c)). Les résultats, tracés en Fig. 6.16 montrent l'apparition du mode d'interface dans les bandgaps qui correspondent à l'ouverture des points de Dirac. À cette fréquence particulière, on a un pic de transmission.

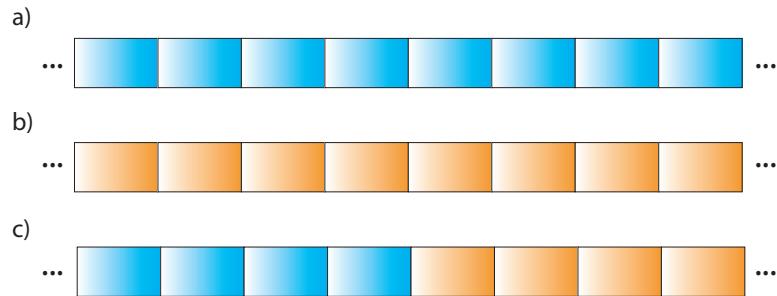


FIGURE 6.15 – Création d’interface entre cellules de type A (en bleu) et cellules de type B (en orange).

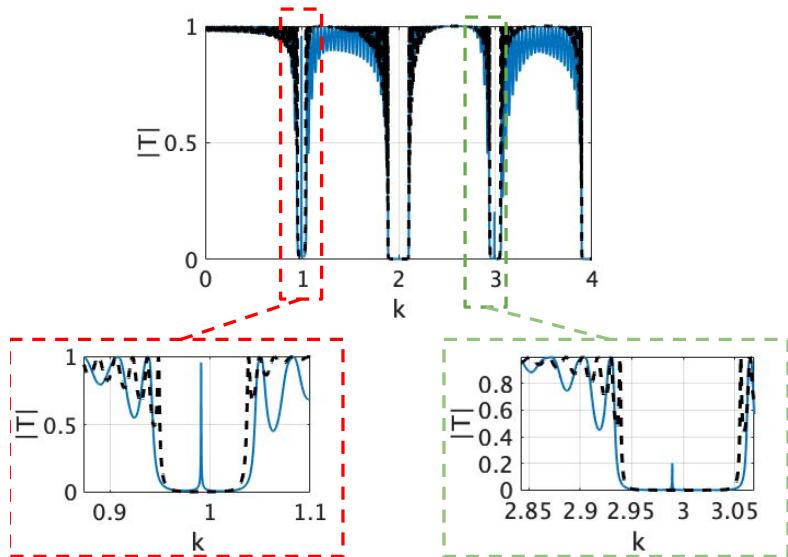


FIGURE 6.16 – Coefficient de transmission pour $2N = 50$ cellules de type A (en noir) et pour 25 cellules de type A connectées à 25 cellules de type B (en bleu).

La présence de ce mode d’interface se traduit par une concentration de l’énergie autour de l’interface. En Fig.6.17 est représentée l’amplitude de pression le long d’un arrangement de 25 cellules de type A et 25 cellules de type B excité à gauche par une onde incidente d’amplitude unitaire à la fréquence du premier mode d’interface. On observe que non seulement l’amplitude à droite est également égale à 1 (on a transmission parfaite), mais en plus l’amplitude au niveau de l’interface est plus de six fois supérieure à celle injectée.

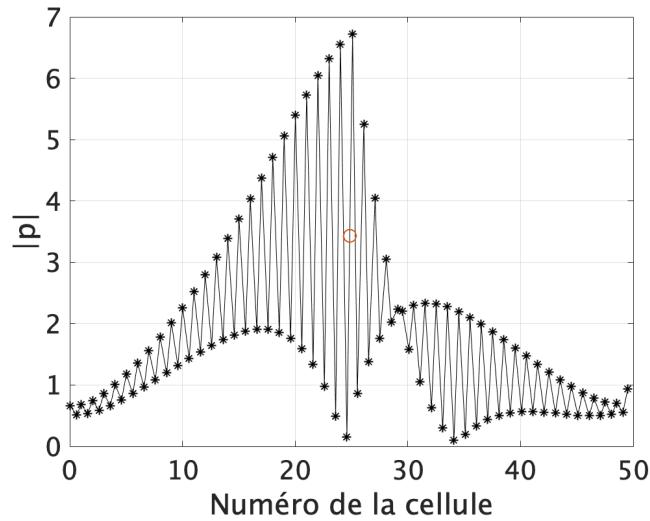


FIGURE 6.17 – Amplitude de la pression à gauche de chaque cellule. L'interface est notée par un point rouge pour une onde incidente à gauche d'amplitude unitaire à la fréquence du premier mode d'interface de la Fig.6.16.

6.3.3 Effet de l'écoulement

On ajoute ensuite progressivement de l'écoulement. Dans un premier temps on compare les diagrammes de dispersion en fonction de M (Fig.6.18). Comme dans le

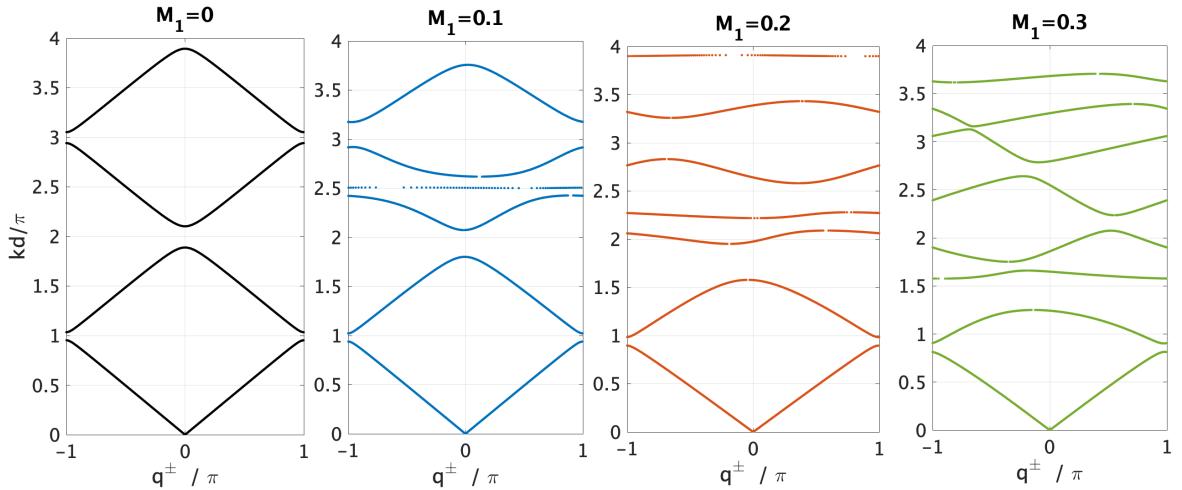


FIGURE 6.18 – Diagramme en bandes, pour $H_1 = 0.$, $H_2 = 0.25$, $L_1 = 0.5$, $L_2 = 0.1$, en fonction de M .

problème général avec cylindre, le diagramme est bien plus perturbé en hautes fréquences.

La première bande voit sa pente légèrement modifiée mais garde globalement la même allure. En revanche les bandes supérieures sont fortement perturbées, avec l’apparition de plusieurs nouvelles bandes (et coupures, par conséquent). Ce phénomène se relie bien à la transmission dans le système à interface entre 25 cellules de type A et 25 cellules de type B (Fig 6.19). Finalement, le mode d’interface est protégé par le peu de perturbation due à l’écoulement autour de la première bande et se retrouve même avec des écoulements forts. L’effet de l’écoulement est en effet concentré en hautes fréquences et autour de la résonance. On remarque toutefois que la fréquence de ce mode varie un peu avec la vitesse

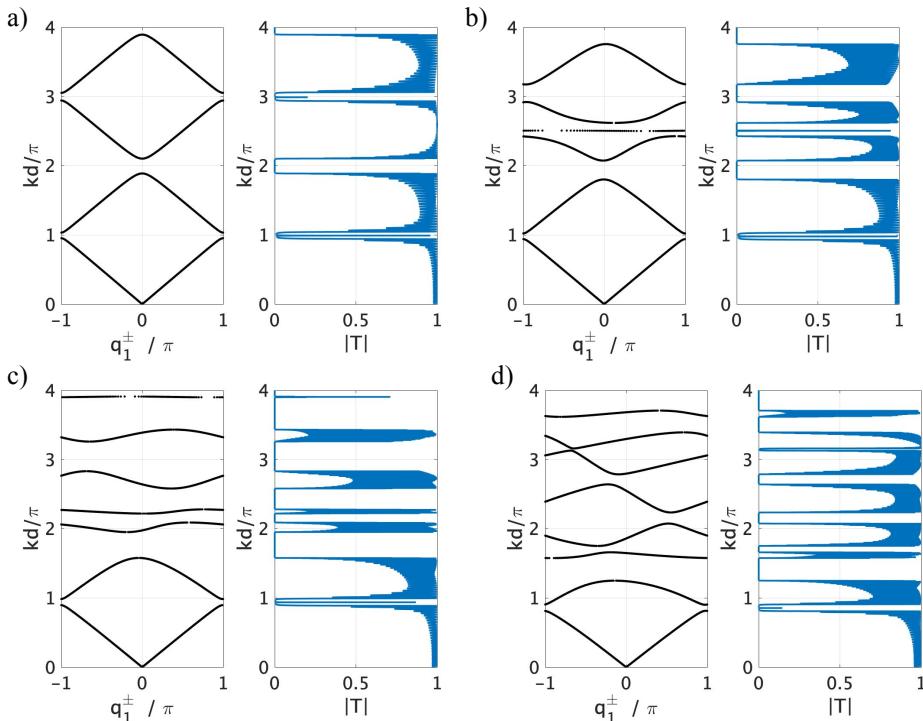


FIGURE 6.19 – Diagramme en bandes et coefficient de transmission du système avec interface pour $H_1 = 0.$, $H_2 = 0.25$, $L_1 = 0.5$, $L_2 = 0.1$, et M_1 = a) 0, b) 0.1, c) 0.2 et d) 0.3.

de l’écoulement. On trace en Fig. 6.20 la fréquence de ce mode en fonction de M_1 pour des vitesses d’écoulement M_1 allant jusqu’à 0.49 (limite d’écoulement subsonique dans le conduit 2). On voit bien apparaître une dépendance à M_1 de sa fréquence. A partir de $M = 0.35$, on voit apparaître un second mode d’interface à une fréquence supérieure. Pour mieux visualiser l’évolution de $|T|$ en fonction de M_1 , on trace en Fig. sa valeur en fonction de la fréquence et de l’écoulement dans le conduit supérieur, dans le cas où 10 cellules de type A sont connectées à 10 cellules de type B, mais aussi pour 20 cellules

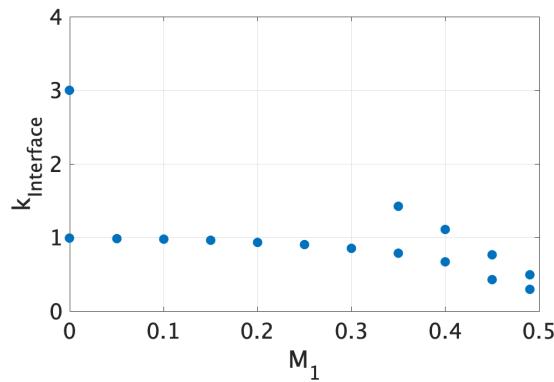


FIGURE 6.20 – Fréquence du ou des modes d'interface en fonction de M_1 .

de type A seulement. Le mode d'interface dans le troisième bandgap disparaît alors très clairement après une certaine valeur de M_1 . L'effet semble être inverse sur le mode piégé (autour de $kd/\pi = 2.5$ sans écoulement) : celui-ci s'élargit jusqu'à rejoindre une bande passante. Globalement, on peut faire le même constat que sur la Fig. 6.19, qui est que la présence d'écoulement complique beaucoup la dépendance fréquentielle en affinant les bandes passantes et interdites et donc en les multipliant. Cependant, on voit aussi que la présence de nouveaux bandgaps donne naissance à d'autres modes d'interface, ce qui explique aussi la "réapparition" du second mode pointée en Fig. 6.20.

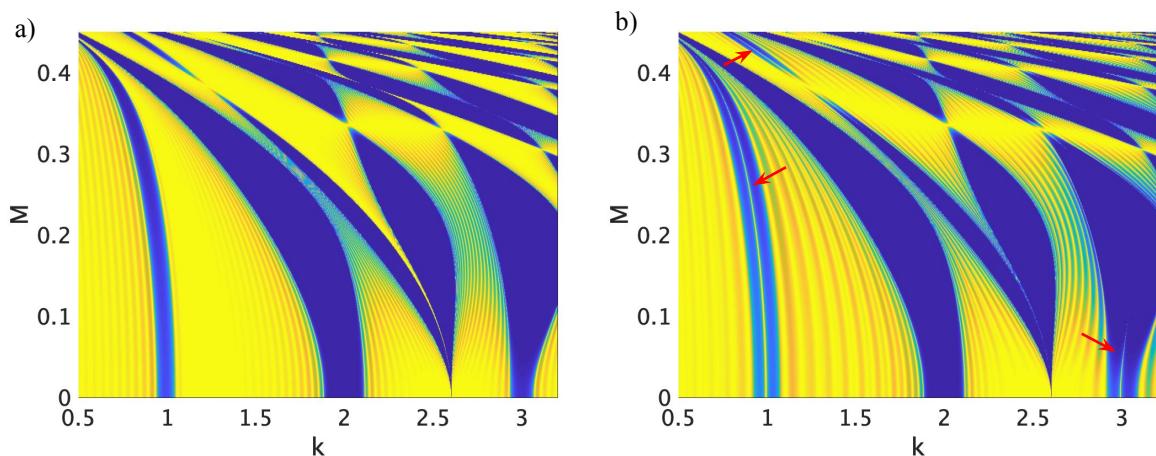


FIGURE 6.21 – $|T_L|$ en fonction de k et M_1 pour a) 20 cellules de type A, b) 10 cellules de type A connectées à 10 cellules de type B. Les flèches rouges pointent les modes d'interface. Les zones jaunes correspondent à $|T_L| = 1$ (bandes passantes) et les zones bleues à $|T_L| = 0$ (bandes interdites).

6.3.4 Robustesse du mode d’interface face à l’introduction de désordre

On introduit sur chaque cellule une variation aléatoire de M_1 . On conserve la relation $M_2 = -H_1/H_2 M_1$, afin que le débit moyen reste nul sur une section du guide. On note la nouvelle valeur de M_1 \tilde{M}_1 , de telle sorte que :

$$\tilde{M}_1 = M_1(1 + \xi), \quad (6.13)$$

où ξ est un réel aléatoire dans l’intervalle $[-\Xi, \Xi]$, $\Xi \in [0, 1]$. Dans la suite, on s’intéresse à la valeur de $|P_{int}|$, la pression à l’interface, en fonction de Ξ et de la fréquence, notamment autour de la fréquence du mode d’interface quand l’écoulement est le même sur toutes les cellules.

À $M_1 = 0.1$, on zoomé autour du premier mode d’interface et on s’intéresse à son évolution en fonction de Ξ et de k . On constate avant tout un étalement fréquentiel du phénomène

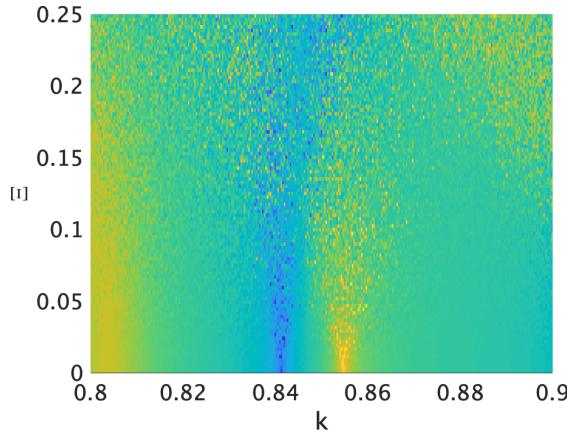


FIGURE 6.22 – $|T|$ en fonction de k et Ξ pour $H_1 = 0.5$, $H_2 = 0.25$, $M_1 = 0.3$.

lorsque l’on introduit un désordre sur M . La robustesse de ce mode n’est par ailleurs plus vérifiée à partir d’un certain taux de désordre.

6.4 Conclusion et perspectives

Dans ce chapitre, la périodisation du système faisant apparaître un mode piégé a permis d’étudier d’autres effets de l’écoulement. La présence du mode piégé résulte en effet en l’ouverture d’une bande interdite autour de la fréquence de résonance.

L'écoulement, qui on l'a vu au chapitre précédent modifie la transmission et la réflexion d'une cellule unitaire, vient modifier la fréquence et la largeur de cette bande interdite. En annexe 6.6, on note aussi que l'asymétrie sur les phases des coefficients de transmission de la cellule unitaire se répercute sur le système périodisé.

Pour mettre en lumière la notion de mode d'interface, le développement d'un modèle simplifié a permis la réalisation d'un grand nombre de simulations. Cet effet de la topologie du système se révèle être sensible à la présence d'écoulement dans les cellules, si bien que l'écoulement est capable de faire apparaître ou disparaître le phénomène selon son amplitude.

6.5 Annexe : description mathématique du système simplifié

Les amplitudes des ondes à gauche de la brique ($x = 0$) sont notées avec un indice L , celles à droite de la brique ($x = L$) sont notées avec un indice R et celles des conduits inférieur et supérieur respectivement avec un indice 1 et 2.

6.5.1 Continuité des potentiels de vitesse

On considère N modes dans le conduit principal (à gauche et à droite), P modes dans le conduit supérieur et Q modes dans le conduit inférieur. On a donc les potentiels suivants :

$$\begin{cases} \phi_L = \sum_{n=0}^N (A_{Ln}^+ e^{i\alpha_n x} + A_{Ln}^- e^{-i\alpha_n x}) f_n(y) \\ \phi_R = \sum_{n=0}^N (A_{Rn}^+ e^{i\alpha_n x} + A_{Rn}^- e^{-i\alpha_n x}) f_n(y) \\ \phi_1 = \sum_{p=0}^P (A_{1p}^+ e^{i\alpha_{1p}^+ x} + A_{1p}^- e^{-i\alpha_{1p}^- x}) g_p(y) \\ \phi_2 = \sum_{q=0}^Q (A_{2q}^+ e^{i\alpha_{2q}^+ x} + A_{2q}^- e^{-i\alpha_{2q}^- x}) h_q(y), \end{cases} \quad (6.14)$$

avec $f_n(y) = B_n \cos(\beta_n y)$, où $\beta_n = n\pi/H$, $g_p = B_{1p} \cos(\beta_{1p} y)$, où $\beta_{1p} = p\pi/H_1$ et $h_q = B_{2q} \cos(\beta_{2q} (y - H))$, où $\beta_{2q} = q\pi/H_2$; ainsi que $\alpha_n^2 = k^2 - \beta^2$ et $\alpha_{in}^\pm = (-kM_i \pm \sqrt{k^2 + (M_i^2 - 1)\beta_{in}^2})/(1 - M_i^2)$ ($i = 1, 2$). Les modes transverses sont normalisés, de telle sorte que :

$$\int_0^H f_n f_m dy = \delta_n^m, \quad (6.15)$$

(idem pour les g_p et les h_q , respectivement sur H_1 et H_2). Pour vérifier ça, on a : $B_n = \sqrt{(2 - \delta_0^n)/H}$. (idem pour les B_{1p} et les B_{2q} , avec respectivement H_1 et H_2).

En $x = 0$ et en $x = L$, on a continuité des potentiels. Ainsi :

$$\begin{cases} \phi_L(0, y) = \phi_1(0, y) \text{ pour } y \in [0, H_1], \\ \phi_L(0, y) = \phi_2(0, y) \text{ pour } y \in [H - H_2, H], \\ \phi_R(L, y) = \phi_1(L, y) \text{ pour } y \in [0, H_1], \\ \phi_R(L, y) = \phi_2(L, y) \text{ pour } y \in [H - H_2, H]. \end{cases} \quad (6.16)$$

En projetant ces relations, respectivement sur les g_p et sur les h_q , il vient :

$$\begin{cases} \int_0^{H_1} \phi_L(0, y) g_p dy = \int_0^{H_1} \phi_1(0, y) g_p dy \\ \int_{H-H_2}^H \phi_L(0, y) h_q dy = \int_{H-H_2}^H \phi_2(0, y) h_q dy \\ \int_0^{H_1} \phi_R(L, y) g_p dy = \int_0^{H_1} \phi_1(L, y) g_p dy \\ \int_{H-H_2}^H \phi_R(L, y) h_q dy = \int_{H-H_2}^H \phi_2(L, y) h_q dy. \end{cases} \quad (6.17)$$

Et en utilisant la normalisation des modes :

$$\begin{cases} \sum_{n=0}^N (A_{Ln}^+ + A_{Ln}^-) \int_0^{H_1} f_n g_p dy = A_{1p}^+ + A_{1p}^-, \\ \sum_{n=0}^N (A_{Ln}^+ + A_{Ln}^-) \int_{H-H_2}^H f_n h_q dy = A_{2q}^+ + A_{2q}^-, \\ \sum_{n=0}^N (A_{Rn}^+ e^{i\alpha_n L} + A_{Rn}^- e^{-i\alpha_n L}) \int_0^{H_1} f_n g_p dy = A_{1p}^+ e^{i\alpha_{1p}^+ L} + A_{1p}^- e^{-i\alpha_{1p}^- L}, \\ \sum_{n=0}^N (A_{Rn}^+ e^{i\alpha_n L} + A_{Rn}^- e^{-i\alpha_n L}) \int_{H-H_2}^H f_n h_q dy = A_{2q}^+ e^{i\alpha_{2q}^+ L} + A_{2q}^- e^{-i\alpha_{2q}^- L}. \end{cases} \quad (6.18)$$

Pour alléger les notations, on notera par la suite $\xi_1 = \int_0^{H_1} f_n g_p dy$ et $\xi_2 = \int_{H-H_2}^H f_n h_q dy$.

6.5.2 Continuité des vitesses normales

On a également continuité des $\partial_x \phi$ en $x = 0$ et $x = L$, qui après projection sur les f_n s’écrit :

$$\left\{ \begin{array}{l} \alpha_n(A_{Ln}^+ - A_{Ln}^-) = \sum_{p=0}^P (\alpha_{1p}^+ A_{1p}^+ - \alpha_{1p}^- A_{1p}^-) \xi_1 + \sum_{q=0}^Q (\alpha_{2q}^+ A_{2q}^+ - \alpha_{2q}^- A_{2q}^-) \xi_2, \\ \alpha_n(A_{Rn}^+ e^{i\alpha_n L} - A_{Rn}^- e^{-i\alpha_n L}) = \sum_{p=0}^P (\alpha_{1p}^+ A_{1p}^+ e^{i\alpha_{1p}^+ L} - \alpha_{1p}^- A_{1p}^- e^{i\alpha_{1p}^- L}) \xi_1 \\ \quad + \sum_{q=0}^Q (\alpha_{2q}^+ A_{2q}^+ e^{i\alpha_{2q}^+ L} - \alpha_{2q}^- A_{2q}^- e^{-i\alpha_{2q}^- L}) \xi_2. \end{array} \right. \quad (6.19)$$

6.5.3 Système final

On a donc un système de $2N + 2P + 2Q$ équations, dans lequel on isole les amplitudes connues (celles des ondes entrantes, A_L^+ et A_R^-) :

$$\left\{ \begin{array}{l} \sum_{n=0}^N A_{Ln}^+ \xi_1 = A_{1p}^+ + A_{1p}^- - \sum_{n=0}^N A_{Ln}^- \xi_1, \\ \sum_{n=0}^N A_{Ln}^+ \xi_2 = A_{2q}^+ + A_{2q}^- - \sum_{n=0}^N A_{Ln}^- \xi_2, \\ \sum_{n=0}^N A_{Rn}^- e^{-i\alpha_n L} \xi_1 = A_{1p}^+ e^{i\alpha_{1p}^+ L} + A_{1p}^- e^{-i\alpha_{1p}^- L} - \sum_{n=0}^N A_{Rn}^+ e^{i\alpha_n L} \xi_1, \\ \sum_{n=0}^N A_{Rn}^- e^{-i\alpha_n L} \xi_2 = A_{2q}^+ e^{i\alpha_{2q}^+ L} + A_{2q}^- e^{-i\alpha_{2q}^- L} - \sum_{n=0}^N A_{Rn}^+ e^{i\alpha_n L} \xi_2, \\ \alpha_n A_{Ln}^+ = \sum_{p=0}^P (\alpha_{1p}^+ A_{1p}^+ - \alpha_{1p}^- A_{1p}^-) \xi_1 + \sum_{q=0}^Q (\alpha_{2q}^+ A_{2q}^+ - \alpha_{2q}^- A_{2q}^-) \xi_2 + \alpha_n A_{Ln}^-, \\ -\alpha_n A_{Rn}^- e^{-i\alpha_n L} = \sum_{p=0}^P (\alpha_{1p}^+ A_{1p}^+ e^{i\alpha_{1p}^+ L} - \alpha_{1p}^- A_{1p}^- e^{i\alpha_{1p}^- L}) \xi_1 + \sum_{q=0}^Q (\alpha_{2q}^+ A_{2q}^+ e^{i\alpha_{2q}^+ L} - \alpha_{2q}^- A_{2q}^- e^{-i\alpha_{2q}^- L}) \xi_2 \\ \quad - \alpha_n A_{Rn}^+ e^{i\alpha_n L}, \end{array} \right. \quad (6.20)$$

que l'on ré-écrit sous forme matricielle :

$$\begin{bmatrix} -\Xi_1 & 0_{P \times N} & \text{Id}_{P \times P} & \text{Id}_{P \times P} & 0_{P \times Q} & 0_{P \times Q} \\ -\Xi_2 & 0_{Q \times N} & 0_{Q \times P} & 0_{Q \times P} & \text{Id}_{Q \times Q} & \text{Id}_{Q \times Q} \\ 0_{P \times N} & -\Xi_1 \mathsf{P}^+ & \mathsf{P}_1^+ & \mathsf{P}_1^- & 0_{P \times Q} & 0_{P \times Q} \\ 0_{Q \times N} & -\Xi_2 \mathsf{P}^+ & 0_{Q \times P} & 0_{Q \times P} & \mathsf{P}_2^+ & \mathsf{P}_2^- \\ \mathsf{K} & 0_{N \times N} & \Xi_1^T \mathsf{K}_1^+ & -\Xi_1^T \mathsf{K}_1^- & \Xi_2^T \mathsf{K}_2^+ & -\Xi_2^T \mathsf{K}_2^- \\ 0_{N \times N} & -\mathsf{K} \mathsf{P}^+ & \Xi_1^T \mathsf{K}_1^+ \mathsf{P}_1^+ & -\Xi_1^T \mathsf{K}_1^- \mathsf{P}_1^- & \Xi_2^T \mathsf{K}_2^+ \mathsf{P}_2^+ & -\Xi_2^T \mathsf{K}_2^- \mathsf{P}_2^- \end{bmatrix} \begin{pmatrix} \mathbf{A}_L^- \\ \mathbf{A}_R^+ \\ \mathbf{A}_1^+ \\ \mathbf{A}_1^- \\ \mathbf{A}_2^+ \\ \mathbf{A}_2^- \end{pmatrix} = \begin{bmatrix} \Xi_1 & 0_{P \times N} \\ \Xi_2 & 0_{Q \times N} \\ 0_{P \times N} & \Xi_1 \mathsf{P}^- \\ 0_{Q \times N} & \Xi_2 \mathsf{P}^- \\ \mathsf{K} & 0_{N \times N} \\ 0_{N \times N} & -\mathsf{P}^- \mathsf{K} \end{bmatrix} \begin{pmatrix} \mathbf{A}_L^+ \\ \mathbf{A}_R^- \end{pmatrix}. \quad (6.21)$$

Avec :

- Ξ_1 la matrice de taille $P \times N$, telle que $(\Xi_1)_{pm} = \xi_1 = \int_0^{H_1} f_n g_p dy$;

- Ξ_2 la matrice de taille $Q \times N$, telle que $(\Xi_2)_{qm} = \xi_2 = \int_{H-H_2}^H f_n h_q dy$;
- P^\pm la matrice diagonale de taille $N \times N$, telle que $(\mathsf{P}^\pm)_{nn} = e^{\pm i\alpha_n L}$,
- P_1^\pm la matrice diagonale de taille $P \times P$, telle que $(\mathsf{P}_1^\pm)_{pp} = e^{\pm i\alpha_{1p}^\pm L}$,
- P_2^\pm la matrice diagonale de taille $Q \times Q$, telle que $(\mathsf{P}_2^\pm)_{qq} = e^{\pm i\alpha_{2q}^\pm L}$,
- K , K_1^\pm et K_2^\pm les matrice diagonales de taille respective $N \times N$, $P \times P$ et $Q \times Q$ contenant respectivement les α_n , les α_{1p}^\pm et les α_{2q}^\pm .

On calcule finalement les ξ_1 et ξ_2 afin de les implémenter numériquement. On a :

$$\begin{cases} \xi_1 = \int_0^{H_1} f_n g_p dy = \frac{B_n B_{1p}}{2} \left[\frac{1}{\gamma_n + \gamma_{1p}} \sin((\gamma_n + \gamma_{1p}) H_1) + \frac{1}{\gamma_n - \gamma_{1p}} \sin((\gamma_n - \gamma_{1p}) H_1) \right] \text{ pour } \gamma_n \neq \gamma_{1p} \\ \xi_1 = B_n B_{1p} \frac{H_1}{2} \text{ pour } \gamma_n = \gamma_{1p} \text{ avec } \{n, p\} \neq \{0, 0\} \\ \xi_1 = B_n B_{1p} H_1 \text{ pour } \{n, p\} = \{0, 0\} \\ \xi_2 = \int_{H-H_2}^H f_n h_q dy = -B_n B_{2q} \frac{\gamma_n}{\gamma_n^2 - \gamma_{2q}^2} \sin(\gamma_n(H - H_2)) \cos(p\pi) \text{ pour } \gamma_n \neq \gamma_{2q} \end{cases} \quad (6.22)$$

6.5.4 Sélection des modes évanescents décroissants :

Pour éviter des modes évanescents croissants, on sélectionne uniquement les modes évanescents sortants à gauche et à droite. Dans les conduits 1 et 2, on sélectionne les modes i décroissants, et on note :

$$\phi_{(1,2)i} = A_{1,2}^+ \exp^{ik_{(1,2)i}x} + A_{1,2}^- \exp^{-ik_{(1,2)i}(x-L)}; \quad (6.23)$$

6.6 Annexe : effet du nombre de cellules

On peut observer l’effet du nombre de cellules dans le système périodique sur la transmission d’une onde incidente. Pour ce faire, on enchaîne N cellules décrites en Fig. 6.5. À la manière du chapitre précédent, on s’intéresse à l’asymétrie de transmission en traçant en Fig. 6.23 la différence des phases des coefficients de transmission T_L et T_R . On voit apparaître une dépendance directe de cette différence au nombre N de cellules enchaînées, telle que $\arg(T_L) - \arg(T_R) = N[\arg(T_{L,0}) - \arg(T_{R,0})]$ où $T_{L,0}$ et $T_{R,0}$ sont les coefficients de transmission à gauche et à droite de la cellule unitaire.

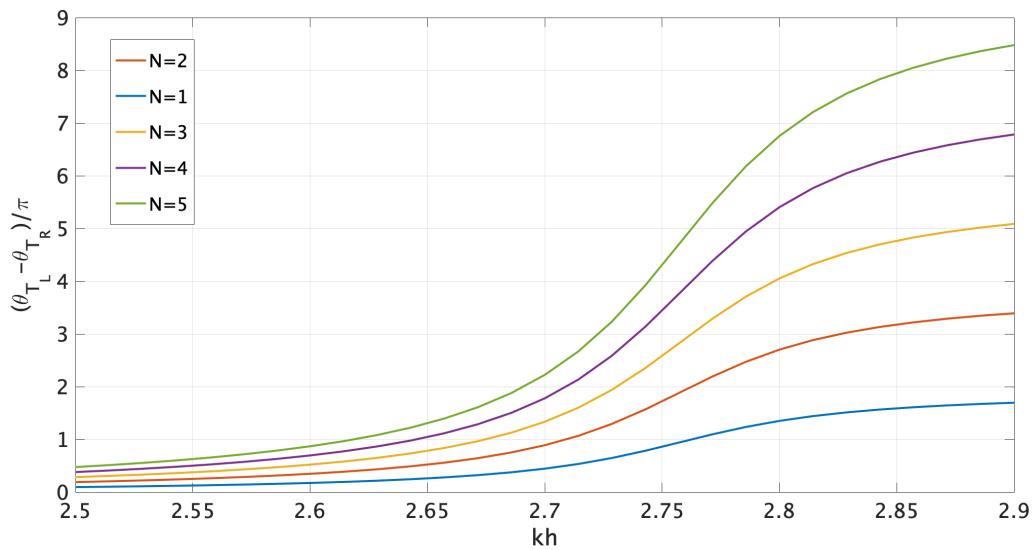


FIGURE 6.23 – Différence de phase entre les coefficients de transmission à gauche (T_L) et à droite (T_R) pour N cellules unitaires contenant un cylindre de rayon $R = 0.3$, pour $L = 5$, $l = 0$ et $\Gamma = 0.1$.

CONCLUSION ET PERSPECTIVES

Dans ce manuscrit, ce sont plusieurs applications du caractère asymétrique d'un écoulement non-uniforme qui ont été proposées.

Les chapitres 2, 3 et 4 interrogent la conservation de l'énergie à travers une couche de cisaillement, tout en mettant en lumière des phénomènes de transmission ou de réflexion remarquables dus à la présence de l'écoulement. Les chapitres 2 et 3 ont permis de conclure sur la conservation de l'énergie à travers une couche de cisaillement. Dans le chapitre 4, la présence de couches critiques fait apparaître des singularités dans le cas où le profil d'écoulement n'est pas linéaire qui pourraient indiquer que l'énergie n'est plus conservée.

Dans les chapitres 5 et 6, ce sont cette fois-ci des propriétés de résonance d'un système qui sont modifiées par l'écoulement, menant à des effets de transmission/réflexion non-réciroques autour de ces résonances. Nous avons également montré que l'ajout d'écoulement transformait un mode piégé en un mode quasi-piége. La périodisation du système donne lieu à des bandes passantes ou interdites, modifiées autour de la résonance par la présence de l'écoulement.

Les perspectives d'évolution de ces recherches concernent principalement les effets de topologie mis en évidence dans le dernier chapitre. En effet, nous avons montré ici que la périodisation de cellule présentant des effets non-réciroques permet l'apparition de modes liés à la topologie du système. Cette propriété pourrait être étendue à l'étude de modes de peau (*skin modes*) [65]. L'obtention de tels modes suppose que le système soit non conservatif. Ici, l'introduction de pertes pourrait par exemple être réalisée grâce à un film résistif de type wiremesh, comme proposé en Fig. 6.24. Le wiremesh placé en $x = x_0$ impose un saut de pression tel que $[p]_{x_0^-}^{x_0^+} = -\gamma u(x_0)$ [17]. Ainsi, on identifie la

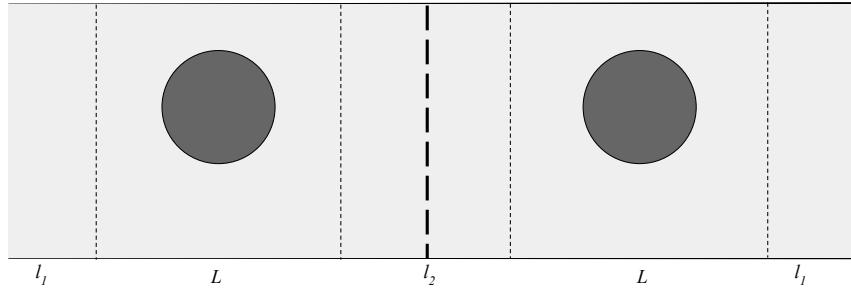


FIGURE 6.24 – Géométrie de la cellule avec pertes.

matrice de transfert de la portion de tube contenant le wiremesh comme étant :

$$\mathbf{M}_l = \begin{bmatrix} (1 - \gamma/2)e^{jkl_2} & 0 \\ 0 & (1 + \gamma/2)e^{-jkl_2} \end{bmatrix}. \quad (6.24)$$

On a finalement une cellule unitaire dont la matrice de transfert \mathbf{M}_e est donnée par $\mathbf{M}_e = \mathbf{M}_v \mathbf{M} \mathbf{M}_l \mathbf{M} \mathbf{M}_v$, avec \mathbf{M}_v la matrice de transfert des portions de tube vides de part et d'autre, et \mathbf{M} la matrice de transfert (obtenue par éléments finis) des portions de tube contenant un cylindre. La périodisation d'un tel système pourra mener à l'apparition de modes de peau, dont toute l'énergie sera concentrée sur le bord du système.

Une autre perspective imaginable, serait d'étendre ce réseau périodique en deux dimensions (Fig. 6.25). Cependant cette périodisation pourrait poser des soucis de mécanique des fluides, notamment vis à vis de la stabilité de l'écoulement.

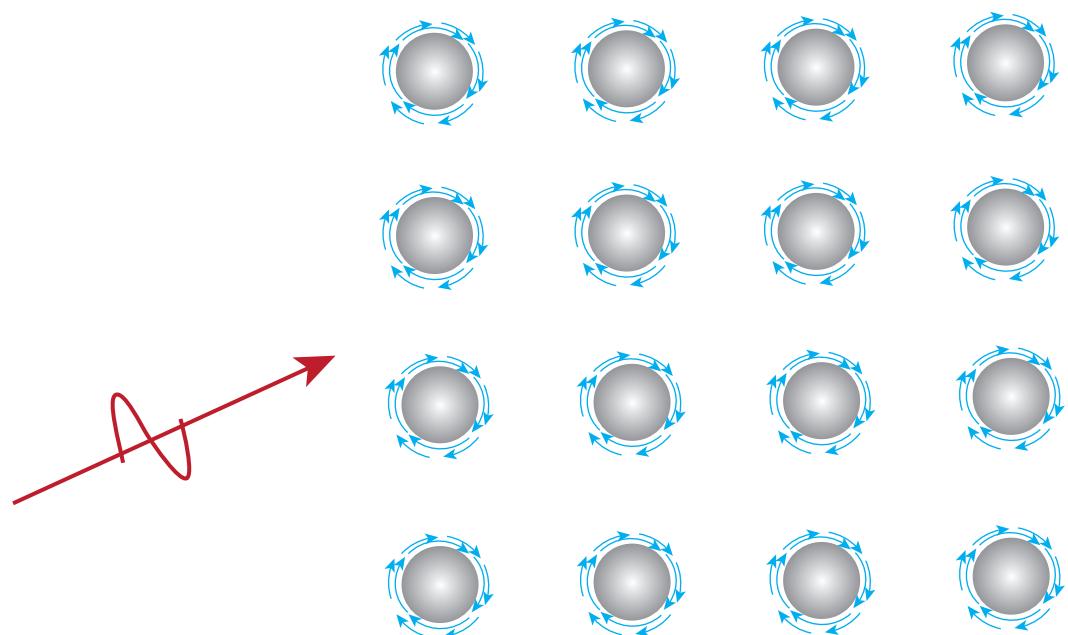


FIGURE 6.25 – Exemple d'une périodisation en deux dimensions.

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Titre : Effets d'écoulements non-uniformes sur la transmission et la réflexion d'ondes acoustiques

Mot clés : aéroacoustique ; non-réciprocité ; écoulement cisaillé ; métasurfaces ; modes piégés ; couches critiques

Résumé : Les interactions entre écoulements fluides et ondes acoustiques sont un sujet largement étudié. Or, ces interactions donnent naturellement lieu à des phénomènes non-réciroques. Dans cette thèse de doctorat, nous nous proposons d'exploiter cette propriété dans plusieurs systèmes donnant lieu à des transmissions et réflexions exotiques. Les cas particuliers d'un écoulement linéairement cisaillé entre deux milieux en mouvement uniforme, d'une couche de cisaillement au-dessus d'une paroi admettante et d'un cylindre en rotation entraînant le fluide autour de lui feront l'objet d'études approfondies. À travers des simulations numériques, des études analytiques et des développements asymptotiques l'effet de l'écoulement sera discuté. Celui-ci est en

effet susceptible d'agir sur la conservation de l'énergie acoustique dans un système donné, mais aussi sur les caractéristiques (amplitude et direction) du champs acoustique réfléchi. L'ajout d'écoulement jouera en particulier sur plusieurs phénomènes physiques ou applications propres aux systèmes considérés tels que la réfraction négative, l'absorption parfaite, le décalage de Goos-Hänchen, la manipulation de front d'onde, ou encore la présence de modes piégés. Pour chacun de ces phénomènes, les caractéristiques de l'écoulement (nombre de Mach moyen ou épaisseur de couche de cisaillement par exemple) sont variées afin de pouvoir conclure sur leur rôle dans une éventuelle modification, amplification, voire annulation du phénomène.

Title: Effects of non-uniform flows on the scattering of acoustic waves

Keywords: aeroacoustics ; non-reciprocity ; shear flow ; metasurfaces ; trapped modes ; critical layers

Abstract: The interaction between fluid flows and acoustic waves is a widely studied subject. These interactions naturally give rise to non-reciprocal phenomena. In this PhD thesis, we propose to exploit this property in several systems where exotic transmissions and reflections occur. The particular cases of a linearly sheared flow between two uniformly moving media, of a shear layer above an admitting wall and of a rotating cylinder driving the fluid around it will be studied in depth. Through numerical simulations, analytical studies and asymptotic developments, the effect of the flow will be discussed. The flow is indeed likely to

affect the conservation of acoustic energy in a given system, but also the characteristics (amplitude and direction) of the reflected acoustic field. The addition of flow will affect several physical phenomena or applications specific to the considered systems such as negative refraction, perfect absorption, Goos-Hänchen shift, wavefront manipulation, or the presence of trapped modes. For each of these phenomena, the flow characteristics (e.g. average Mach number or shear layer thickness) are varied in order to conclude on their role in a possible modification, amplification or even cancellation of the phenomenon.