Optimization Methods
Lab 13 Session

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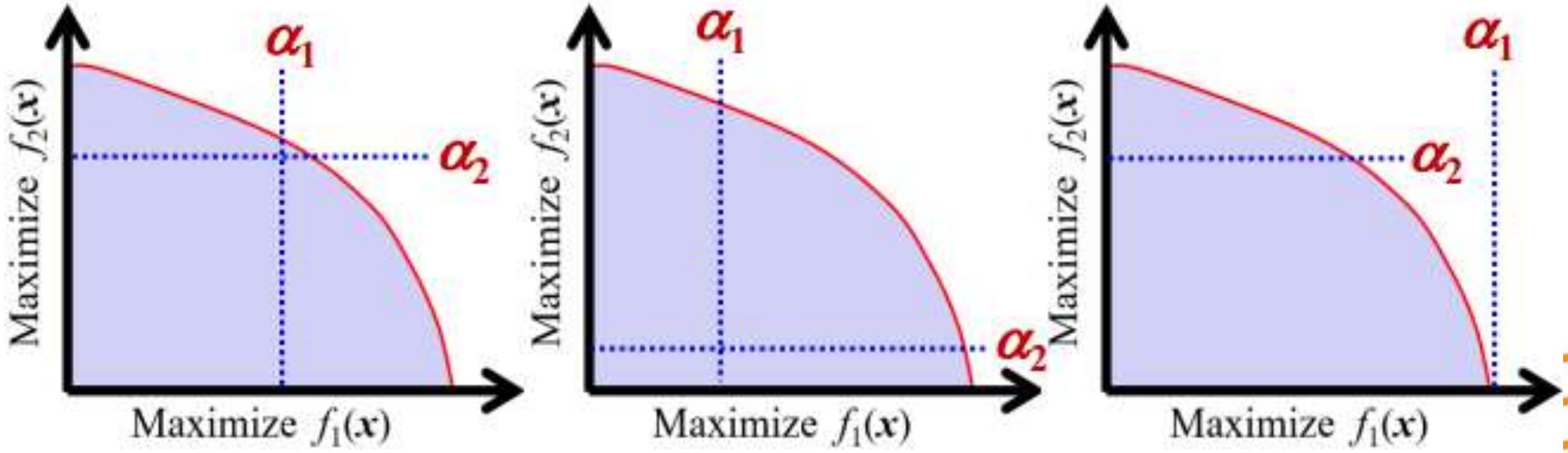
Task1

Lab Session Task 1:

Original Two-Objective Problem: Maximize $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$

Let us assume that we have the following inequality conditions from the decision maker: $f_1(\boldsymbol{x}) \geq \alpha_1$ and $f_2(\boldsymbol{x}) \geq \alpha_2$.

Since the decision maker does not know the true Pareto front, these inequalities can be infeasible. Please design an algorithm to find the best solution for the decision maker.



1. **Initialization**:
   1. Define objective functions $f1(x)$ and $f2(x)$.
   2. Obtain the inequality conditions provided by the decision-maker, such as $f1(x) \geq a$ and $f2(x) \geq b$, where a and b are thresholds set by the decision-maker.
2. **Setting Initial Parameters**:
   1. Choose an initial solution $\boldsymbol{x_0}$.
   2. Define an initial step size Δ for adjusting decision variables.
   3. Define a tolerance ϵ to determine satisfaction of the inequality conditions.
3. **Define Objective Function**:
   1. Construct a combined objective function that penalizes when the decision-maker's inequality conditions are not satisfied.

$$\text{Objective} = \max(f1(x) + f2(x) - \lambda \cdot (\max(0, a - f1(x)) + \max(0, b - f2(\mathrm{x}))))$$

4. **Iterative Search**:
   1. Use genetic algorithms to iteratively adjust the solution **x** to optimize the combined objective function.
   2. In each iteration, check if the current solution satisfies the inequality conditions. If satisfied, record the current solution as a potential optimal solution.
5. **Termination Criteria**:
   1. Terminate the search when the combined objective function no longer significantly improves or when the maximum number of iterations is reached.
   2. Return the recorded potential optimal solution.

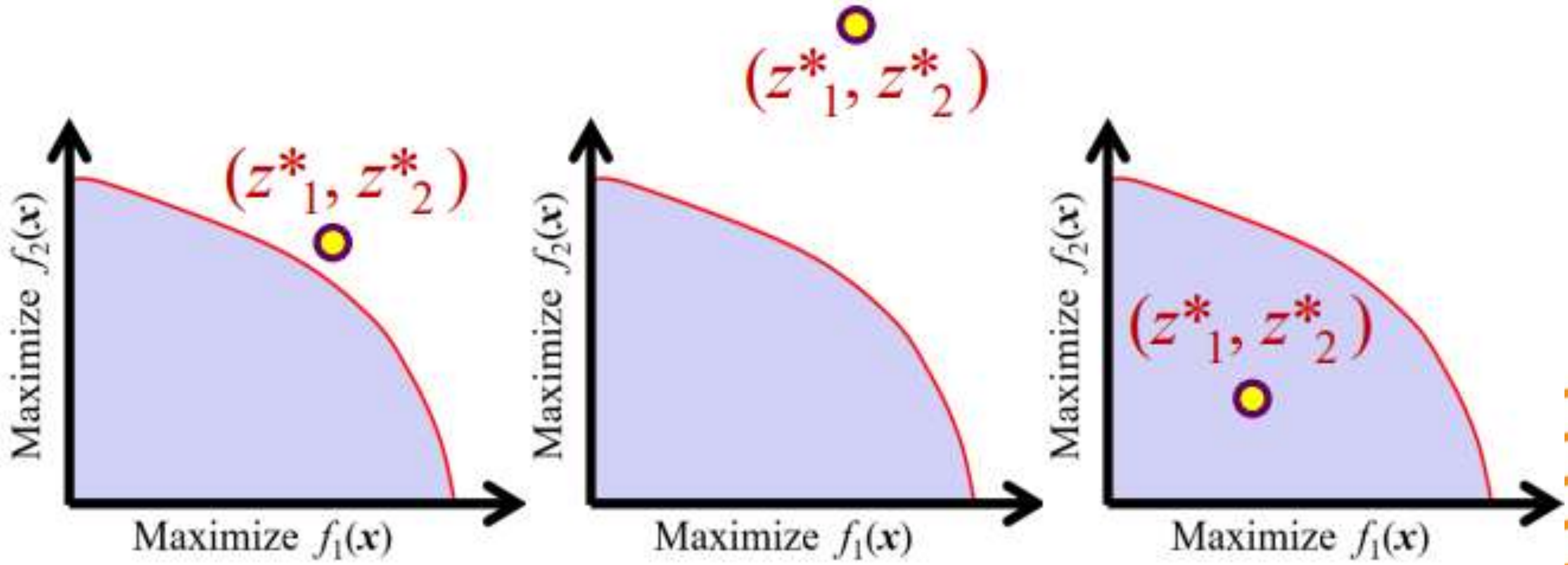
Task2

Lab Session Task 2:

Original Two-Objective Problem: Maximize $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$

Let us assume that we have the following target (ideal) point from the decision maker: $(f_1(\boldsymbol{x}), f_2(\mathbf{x})) = (z^*_1, z^*_2)$.

Since the decision maker does not know the true Pareto front, this target point can be inside the feasible region. Please design an algorithm to find the best solution for the decision maker.



1. Intialization

    Define objective functions $f1(x)$ and $f2(x)$.

2. Setting initial parameter
    1. Choose an initial solution $\boldsymbol{x_0}$.
    2. Define a tolerance ϵ to determine satisfaction of the inequality

3. Define object function

$$Objective = \min((f_1(x) - z_1^*)^2 + (f_2(x) - z_2^*)^2)$$

4. Iterative Search:

1. Use genetic algorithms to iteratively adjust the solution **x** to optimize the combined objective function.
2. In each iteration, check if the current solution satisfies the inequality conditions. If satisfied, record the current solution as a potential optimal solution.

5. Termination Criteria:

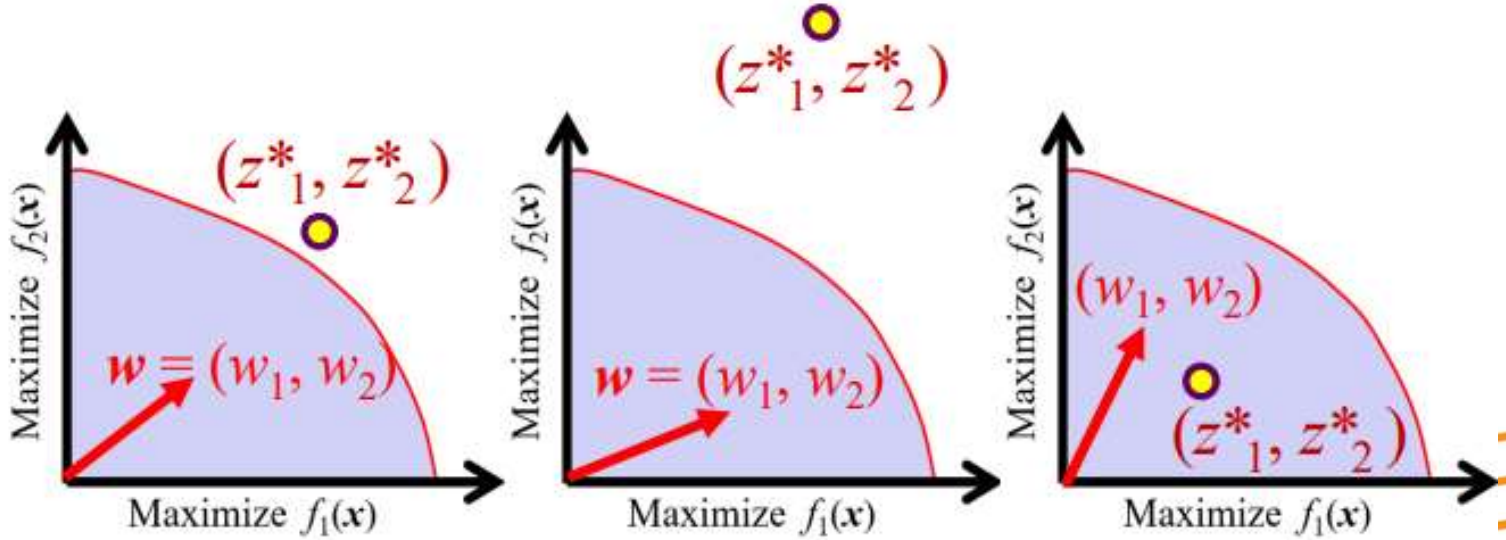
1. Terminate the search when the combined objective function no longer significantly improves or when the maximum number of iterations is reached.
2. Return the recorded potential optimal solution.

Task3

Lab Session Task 3:

Original Two-Objective Problem: Maximize $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$

Let us assume that we have the target (ideal) point $\boldsymbol{z}^* = (z^*_1, z^*_2)$ and the weight vector $\boldsymbol{w} = (w_1, w_2)$ from the decision maker. Using them, please design a function to find a final solution for the decision maker. The weighted sum $f(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}))$ is a simple example (which ignores the target point).



1. Intialization

    Define objective functions $f1(x)$ and $f2(x)$.

2. Setting initial parameter
    1. Choose an initial solution $\boldsymbol{x_0}$.
    2. Define a tolerance ϵ to determine satisfaction of the inequality

3. Define object function

$$Objective = \min(w_1(f_1(x) - z_1^*)^2 + w_2(f_2(x) - z_2^*)^2)$$

4. **Iterative Search**:
   1. Use genetic algorithms to iteratively adjust the solution **x** to optimize the combined objective function.
   2. In each iteration, check if the current solution satisfies the inequality conditions. If satisfied, record the current solution as a potential optimal solution.
5. **Termination Criteria**:
   1. Terminate the search when the combined objective function no longer significantly improves or when the maximum number of iterations is reached.
   2. Return the recorded potential optimal solution.