Optimization Methods
Lab 10 Session

By Shi Jimao

Task1

The main idea of my algorithm is to use a mate-surrogate model which is mainly based on past empirical probability estimation and evolutionary algorithm.

I define a series mate-surrogate models H, and each dimension has a mate-surrogate model H_j.

Each model maintains two series of probability values $PS_{i,j}$ and $PL_{i,j}$. The followings are the definition of them.

$$PS_{i,j} = P\left(f\left(x_{i,j}^{o}\right) < f(x_{i,j}^{p}) \middle| x_{i,j}^{o} < x_{i,j}^{p}\right)$$
$$PL_{i,j} = P\left(f\left(x_{i,j}^{o}\right) < f(x_{i,j}^{p}) \middle| x_{i,j}^{o} > x_{i,j}^{p}\right)$$

Surrogate process

For generating new solutions, the strategy is in the following:

For i=1 to n

$$x_{i,j}^{o} = x_{1,j}^{p} + \sigma_{i,j} \cdot \mathrm{N}(0,1)$$

N(0,1) is Gaussian mutation operator

For i=n+1 to λ

$$x_{i,j}^{o} = x_{1,j}^{p} + \sigma_{i,j} \cdot C(0,1)$$

C(0,1) is Cauchy mutation operator

The purpose of combining these two mutation operators is to balance the search efficiency and the effectiveness.

To determine whether to keep each offspring solution, I use $PS_{i,j}$ and $PL_{i,j}$ to decide.

If $x_{i,j}^{o} < x_{1,j}^{p}$ and $PS_{i,j}$ <random(0,1) $x_{i,j}^{o} = x_{i,j}^{p}$

Else if $x_{i,j}^{o} > x_{1,j}^{p}$ and $PL_{i,j}$ <random(0,1) $x_{i,j}^{o} = x_{i,j}^{p}$

Solution updating

If $F\left(x_1^p\right) > minF(x_i^o) \qquad where\ 1 \leq i \leq \lambda$

$$x_1^p = \arg\min_{x_i^o} F(x_i^o)$$

Model and operator updating

B is a boolean function that returns 1 is the statement is true else return 0.

Each $\sigma_{i,j}$, $PS_{i,j}$, $PL_{i,j}$ is adapted for every iteration again in terms of the 1/5 successful rule.

$$\sigma_{i,j} = \sigma_{i,j} \cdot \exp^{\frac{1}{\sqrt{2}}}\{B(x_{i,j}^o \neq x_{i,j}^p) \cdot B(F\left(x_1^p\right) \geq F(x_i^o) - \frac{1}{5})\}$$

$$PS_{i,j} = PS_{i,j} \cdot \exp^{\frac{1}{\sqrt{2}}}\{B(x_{i,j}^o < x_{i,j}^p) \cdot B(F\left(x_1^p\right) \geq F(x_i^o) - \frac{1}{5})\}$$

$$PL_{i,j} = PL_{i,j} \cdot \exp^{\frac{1}{\sqrt{2}}}\{B(x_{i,j}^p < x_{i,j}^o) \cdot B(F\left(x_1^p\right) \geq F(x_i^o) - \frac{1}{5})\}$$

Complete pseudocode

For a problem with D dimension(D dimension, λ, n) do iteration as the following until it satisfy our goal.

For j = 1 to D

  for i = 1 to λ

    $PS_{i,j} = 1.0$, $PL_{i,j} = 1.0$, $\sigma_{i,j}$ =1.0

  Initialize $x_{1,j}^{p}$ randomly

For j = 1 to D

  for i = 1 to n

    $x_{i,j}^{o} = x_{1,j}^{p} + \sigma_{i,j} \cdot \mathrm{N}(0,1)$

  for i = n+1 to λ

    $x_{i,j}^{o} = x_{1,j}^{p} + \sigma_{i,j} \cdot C(0,1)$

  for i = 1 to λ

    If $x_{i,j}^{o} < x_{1,j}^{p}$ and $PS_{i,j}$ <random(0,1) $x_{i,j}^{o}= x_{i,j}^{p}$

    Else if $x_{i,j}^{o} > x_{1,j}^{p}$ and $PL_{i,j}$ <random(0,1) $x_{i,j}^{o}= x_{i,j}^{p}$

surrogate model

If $F\left(x_1^p\right) > minF(x_i^o)$ $where\ 1 \le i \le \lambda$

  $x_1^p = \arg\min_{x_i^o} F(x_i^o)$

For j = 1 to D

  for i = 1 to λ

    $\sigma_{i,j}= \sigma_{i,j} \cdot \exp^{\frac{1}{\sqrt{2}}}\left\{B\left(x_{i,j}^{o} \neq x_{i,j}^{p}\right) \cdot B\left(F\left(x_1^p\right) \geq F(x_i^o) - \frac{1}{5}\right)\right\}$

    $PS_{i,j} = PS_{i,j} \cdot \exp^{\frac{1}{\sqrt{2}}}\{B(x_{i,j}^{o} < x_{i,j}^{p}) \cdot B(F\left(x_1^p\right) \geq F(x_i^o) - \frac{1}{5})\}$

    $PL_{i,j} = PL_{i,j} \cdot \exp^{\frac{1}{\sqrt{2}}}\{B(x_{i,j}^{p} < x_{i,j}^{o}) \cdot B(F\left(x_1^p\right) \geq F(x_i^o) - \frac{1}{5})\}$

Analysis

- This algorithm is more suitable for high dimension optimization problem.
- It can also get a good result of low dimension while it may cause longer time.