Optimization Methods

Lab 3 Session

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（1）Adjacent Two-City Change

Since we exchange two adjacent points, for n cities, we just consider each point exchange with its left point. Therefore, the result is n

（2）Arbitrary Two-City Change

Since we exchange two arbitrary points, for n cities, each point has n-1 choice to exchange. Considering repeated situation, we should divide it by 2.

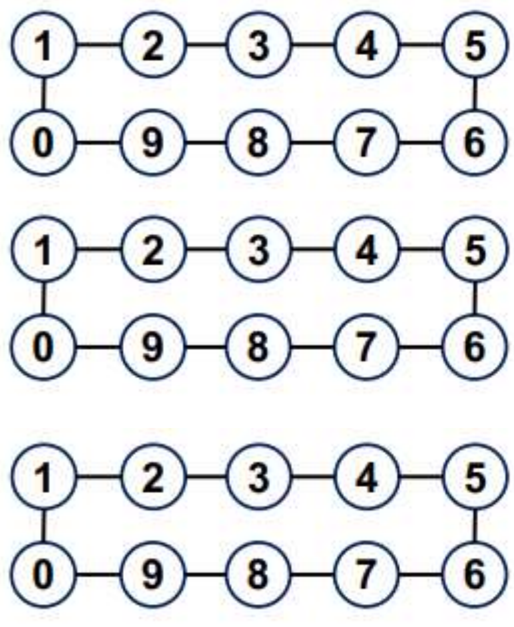
So the final result is $\frac{n(n-1)}{2}$

（3） Insertion(Shift)

Since we insert one point from n points, each point has n-2 position to insert.

So the number is n(n-2).

However there exist repeated situation.



For example, situation 1 and 2 are same with different insertion strategy. We can find for each adjacent point has such a situation. For n points, there exist n choice that will lead to repetition.

So we should minus n from n(n-2). Then the final result will be n(n-2)-n=n(n-3)

（4）Inversion(Arbitrary Two-Edge Change)

Since we invert an arbitrary sequence, we can consider this problem as we choose two edge, and then make them across with each other.

There is n edge, so the number is $\frac{n*(n-1)}{2*1} = \frac{n*(n-1)}{2}$.

However, for three adjacent points, their 2 edges can't cross. So we should minus it by (n-1).

The final result is $\frac{n*(n-1)}{2} - n = \frac{n*(n-3)}{2}$

（5）Arbitrary Three-City Change

Repetition only occurs if and only if one of 3 selected points doesn't change it location. For example, 0-1-2-3-4-5-6-7-8-9, if we select 1-2-3 and change it into 2-1-3, it is similar with that we select 0-1-2 and change it into 0-2-1.

Therefore, we can divide this problem into 2 parts.

1. All points of which we picked change its location. That's will be $2 * \frac{n*(n-1)*(n-2)}{3*2*1} = \frac{n*(n-1)*(n-2)}{3}$.
2. Only two points change their location, which is $\frac{n*(n-1)}{2*1}$.

Then we can get the final result:

$$\frac{n*(n-1)*(n-2)}{3} + \frac{n*(n-1)}{2} = \frac{2n^3 - 3n^2 + n}{6}$$