Optimization Methods

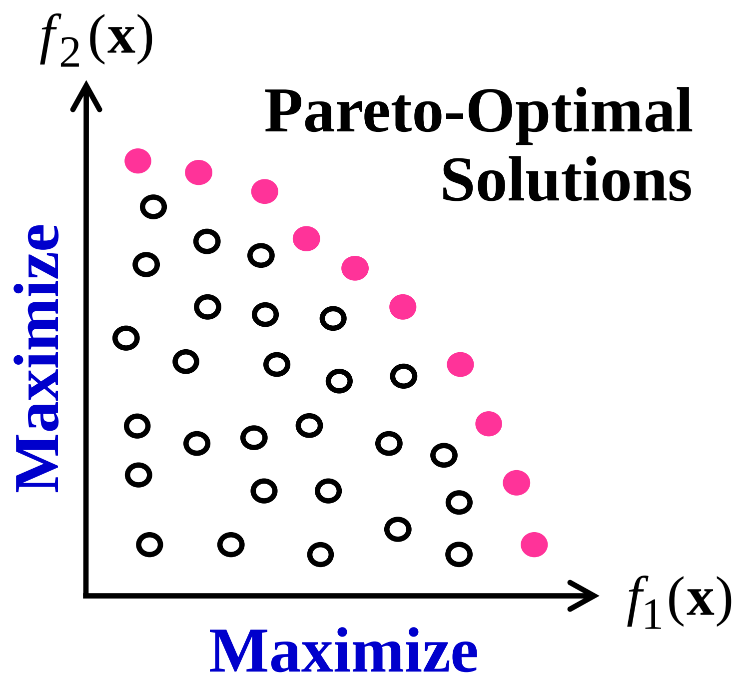
1. Introduction.
2. Greedy algorithms for combinatorial optimization.
3. LS and neighborhood structures for combinatorial optimization.
4. Variable neighborhood search, neighborhood descent, SA, TS, EC.
5. Branch and bound algorithms, and subset selection algorithms.
6. Linear programming problem formulations and applications.
7. Linear programming algorithms.
8. Integer linear programming algorithms.
9. Unconstrained nonlinear optimization and gradient descent.
10. Newton's methods and Levenberg-Marquardt modification.
11. Quasi-Newton methods and conjugate direction methods.
12. Nonlinear optimization with equality constraints.
13. Nonlinear optimization with inequality constraints.
14. Problem formulation and concepts in multi-objective optimization.
15. Search for single final solution in multi-objective optimization.
16: Search for multiple solutions in multi-objective optimization.

Two Approaches to Multi-Objective Optimization Problems

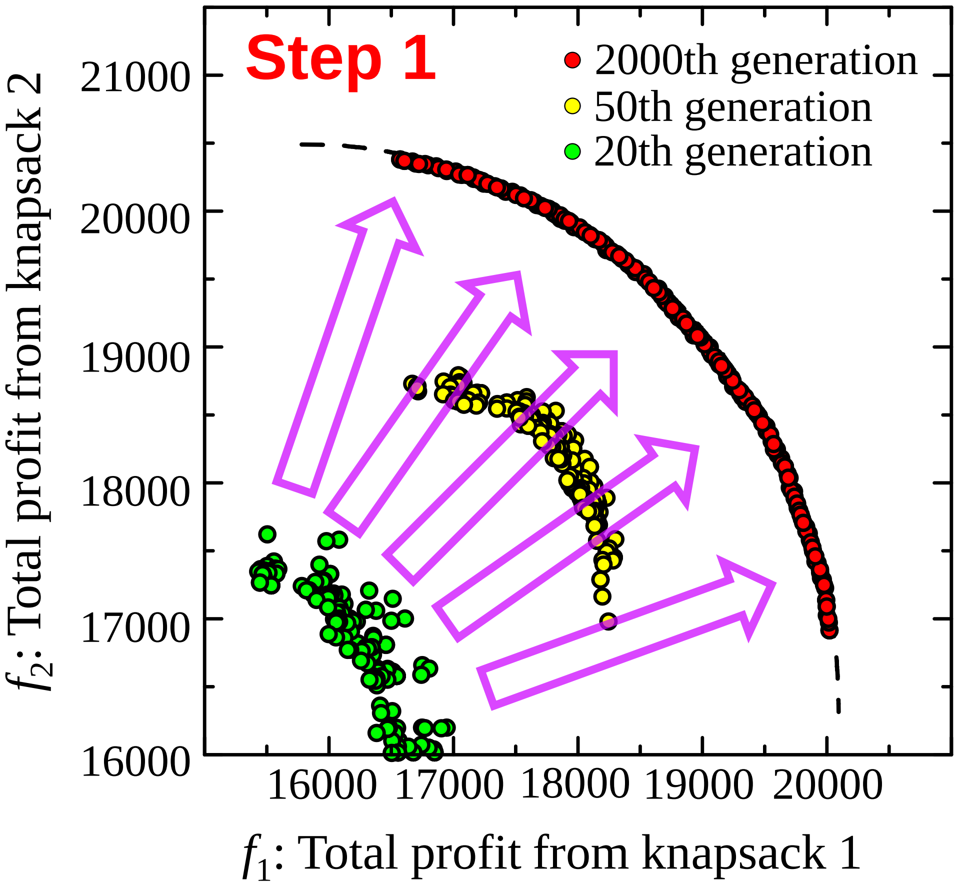
1. Use of Additional Information after Optimization

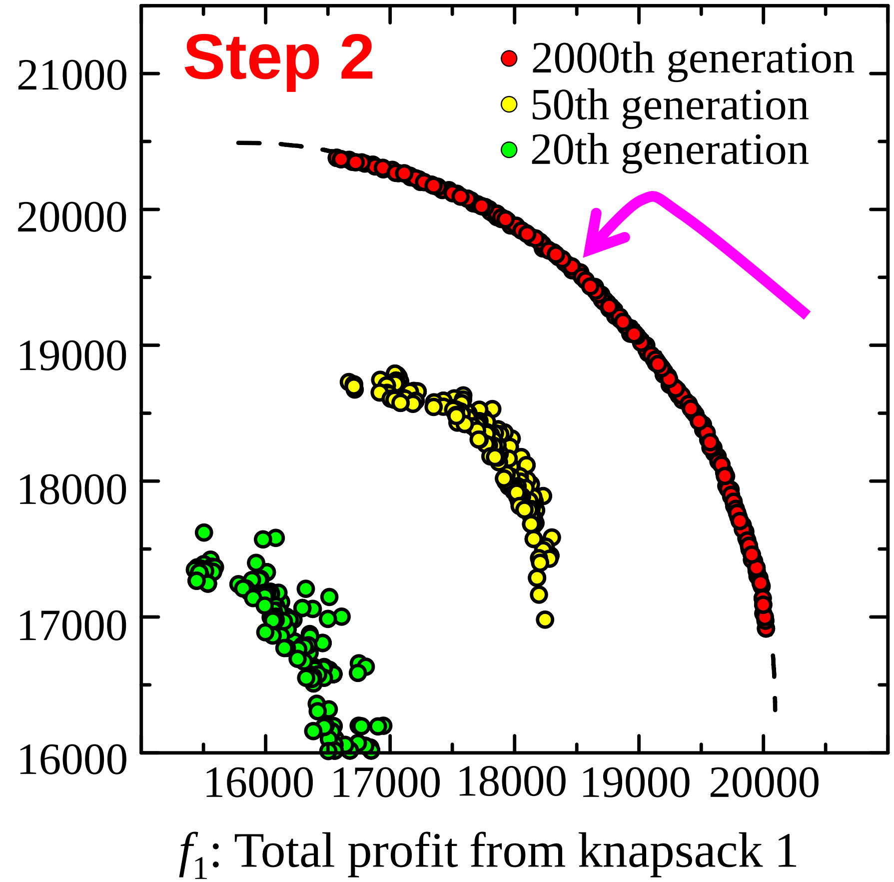
First, a number of Pareto optimal solutions are found, and presented to the decision maker. Then, a final solution is selected by the decision maker.

EMO (Evolutionary Multi-Objective Optimization) Approach



Basic Idea of Decision Making in EMO
(Evolutionary Multiobjective Optimization)





Step 1: Search for non-dominated solutions along the Pareto front.

Step 2: Selection of a single solution from the obtained solutions by the decision maker.

Two Approaches to Multi-Objective Optimization Problems

1. Use of Additional Information after Optimization

First, a number of Pareto optimal solutions are found, and presented to the Decision Maker. Then, a final solution is selected by the Decision Maker.

EMO (Evolutionary Multi-Objective Optimization) Approach

2. Use of Additional Information before Optimization

First, multiple objectives are combined into a single objective function using additional information from the decision maker. Then, the objective function is optimized to find a single final solution.

Traditional Approach   Example: Weighted sum approach:

$$f(\boldsymbol{x}) = w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$$

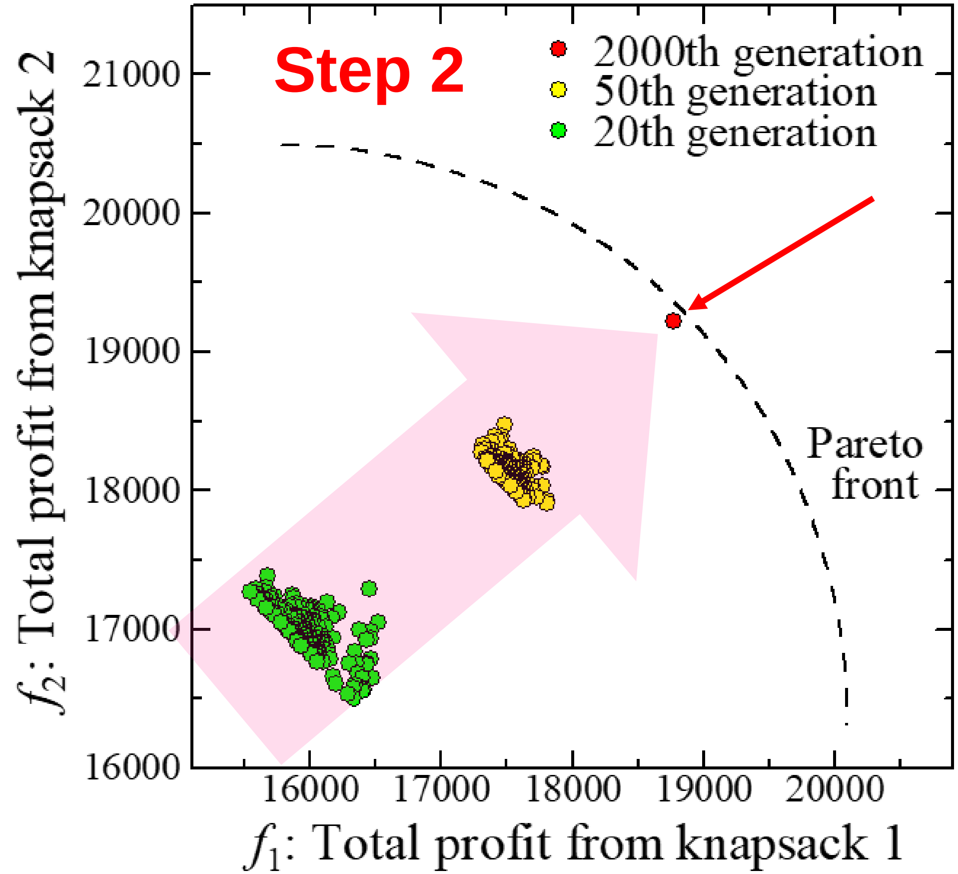
Basic Idea of the Traditional Approach
(MCDM: Multi-Criteria Decision Making)

Step 1

Maximize $f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x})$

Maximize

$f(\boldsymbol{x}) = 0.55\, f_1(\boldsymbol{x}) + 0.45\, f_2(\boldsymbol{x})$



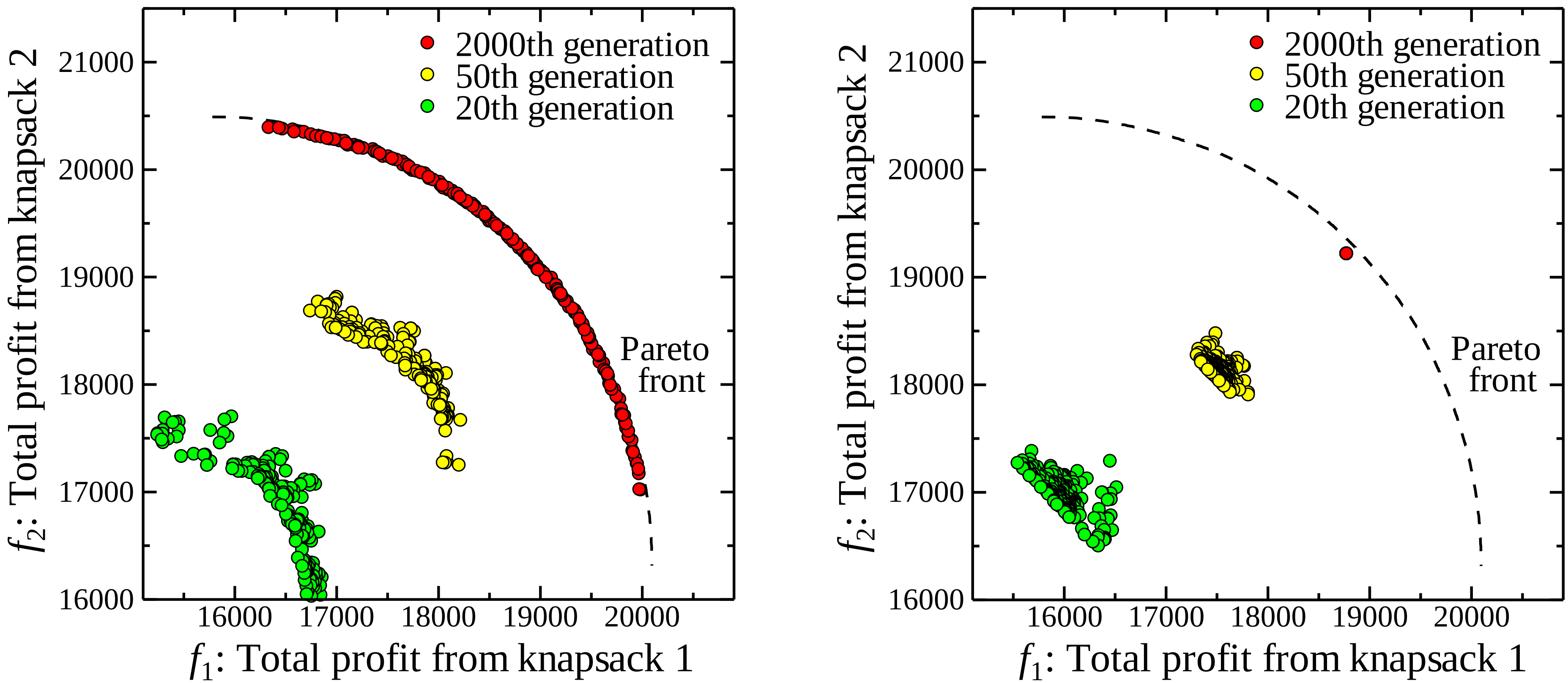
Step 1: Formulation of a single-objective optimization problem.

Step 2: Search for the optimal solution of the formulated problem.

Comparison of the Two Approaches

Two-Objective Optimization and Weighted Sum

Maximize { $f_1(x)$, $f_2(x)$ } Maximize $w_1 f_1(x) + w_2 f_2(x)$



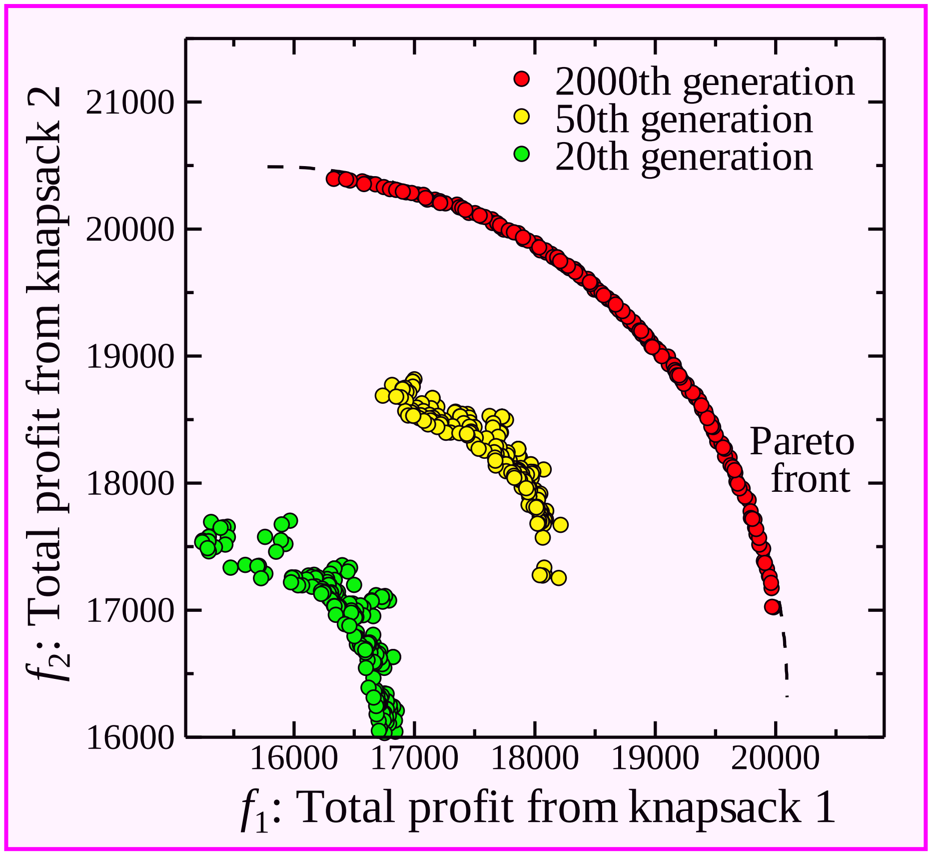
EMO Approach **Weighted Sum Approach**

Experimental results of a single run of each approach

Comparison of the Two Approaches

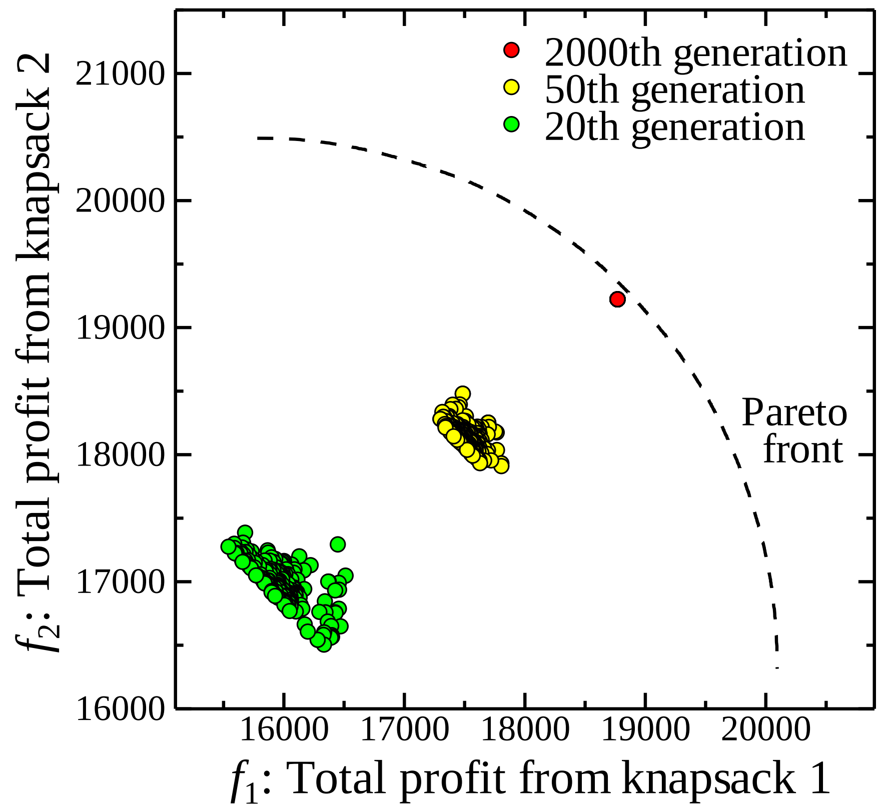
Two-Objective Optimization and Weighted Sum

Maximize { $f_1(x)$, $f_2(x)$ }



EMO Approach

Maximize $w_1 f_1(x) + w_2 f_2(x)$



Weighted Sum Approach

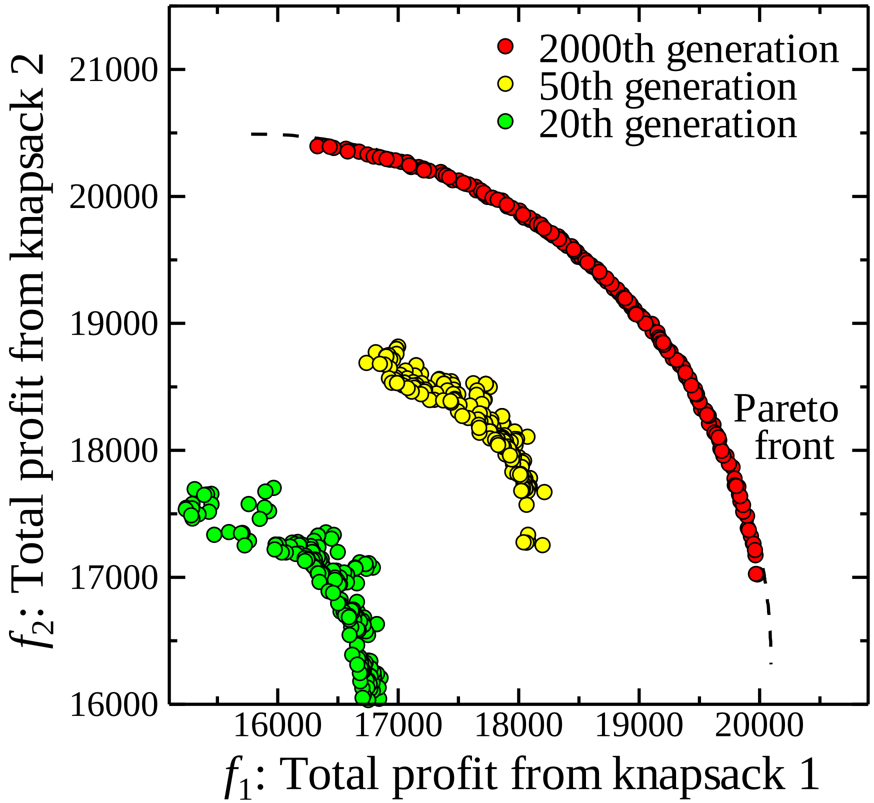
A large number of solutions are obtained along the Pareto front in EMO.

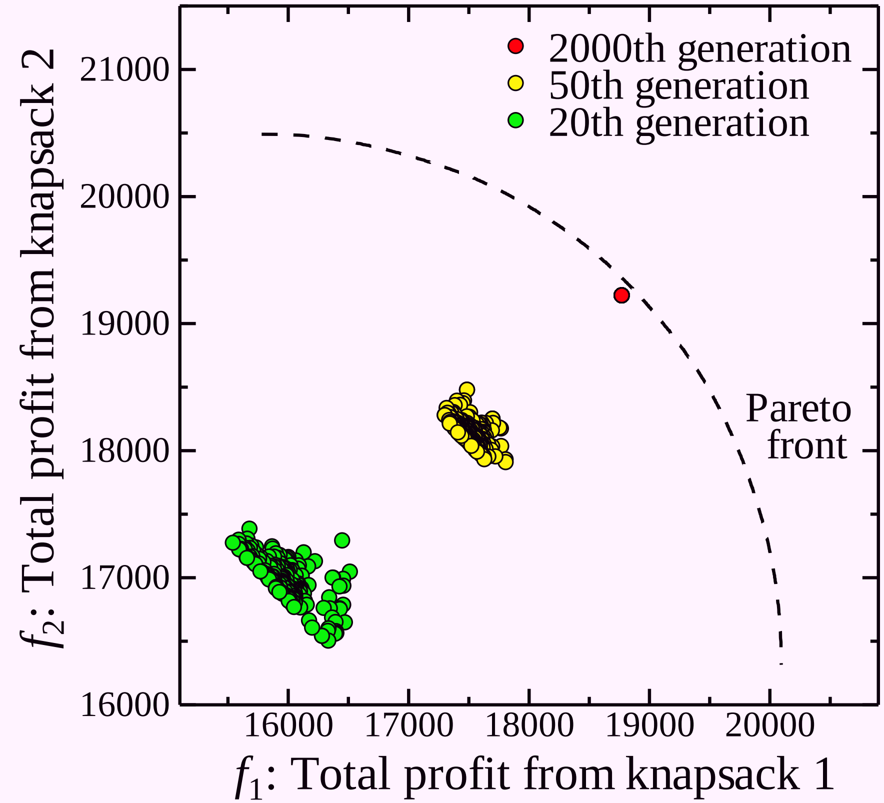
Comparison of the Two Approaches

Two-Objective Optimization and Weighted Sum

Maximize { $f_1(x)$, $f_2(x)$ }

Maximize $w_1 f_1(x) + w_2 f_2(x)$





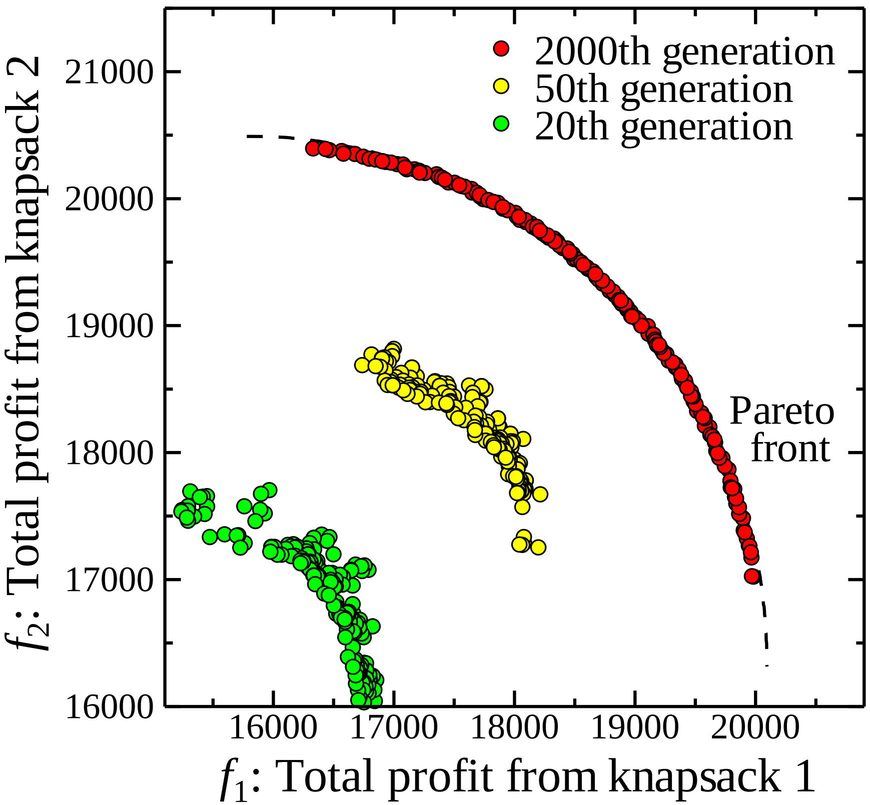
EMO Approach

Weighted Sum Approach

Only a single best solution with respect to the weighted sum is obtained.

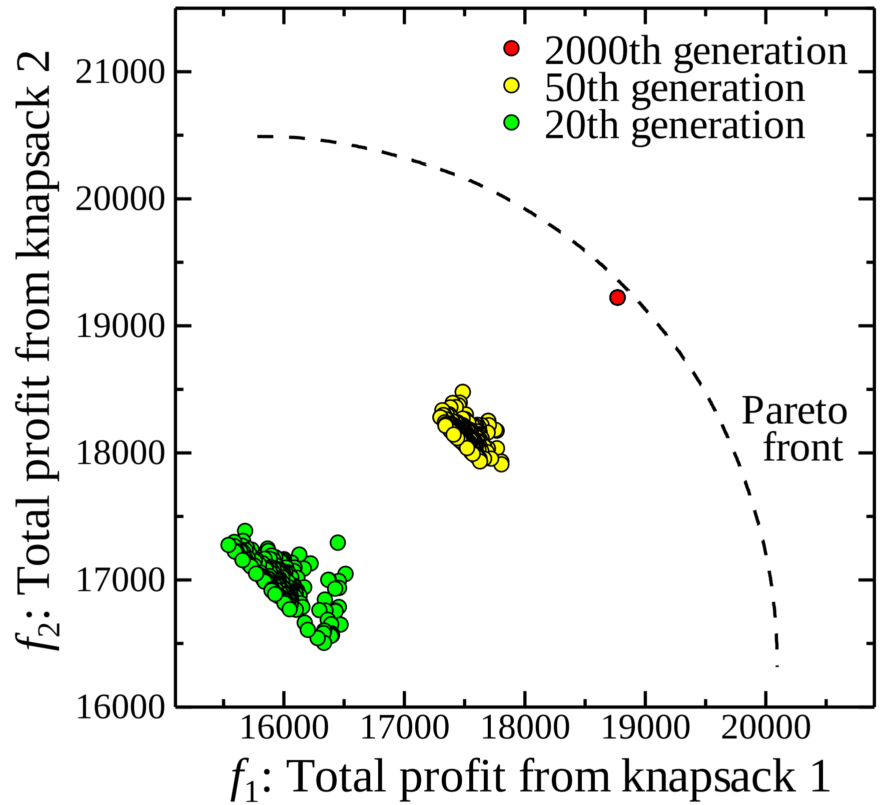
Advantages of EMO Approach

The Pareto front is shown to the decision maker as a result of a single run of an EMO algorithm.



EMO Approach

Maximize $\{ f_1(\boldsymbol{x}),\ f_2(\boldsymbol{x}) \}$

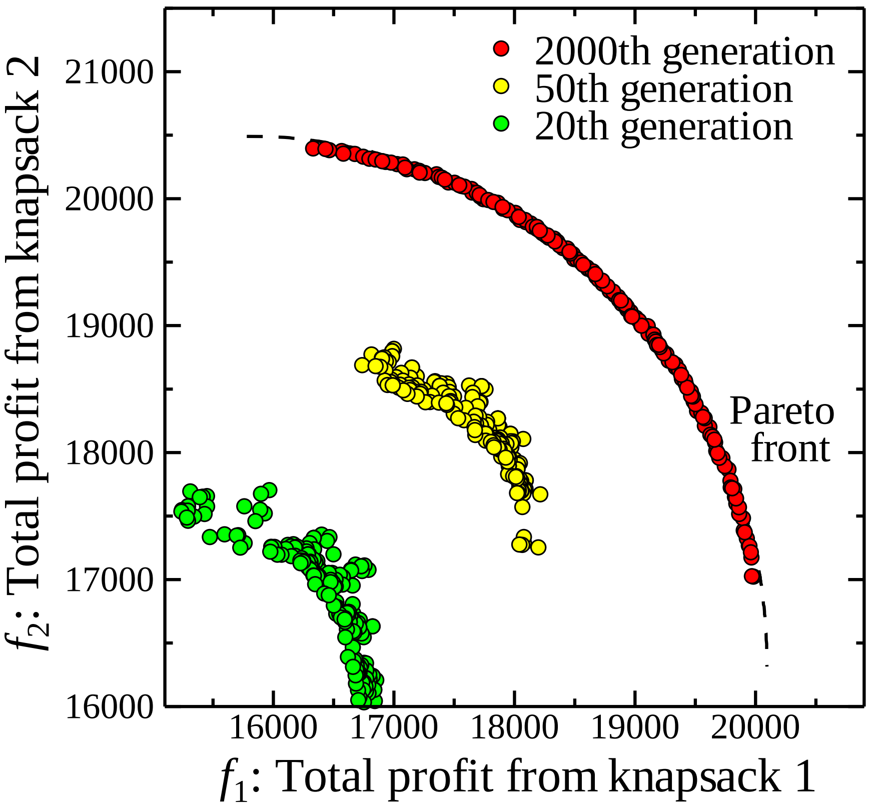


Weighted Sum Approach

Maximize $w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$

Advantages of EMO Approach

In some cases (not always), the EMO solutions have the better convergence than the single-objective solution



EMO Approach

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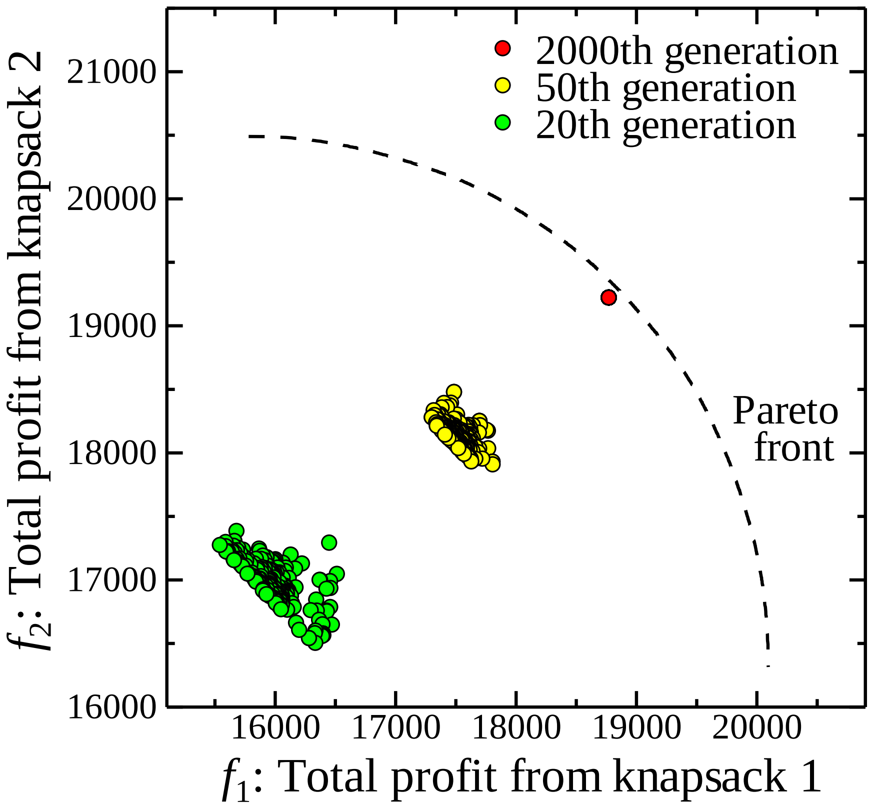
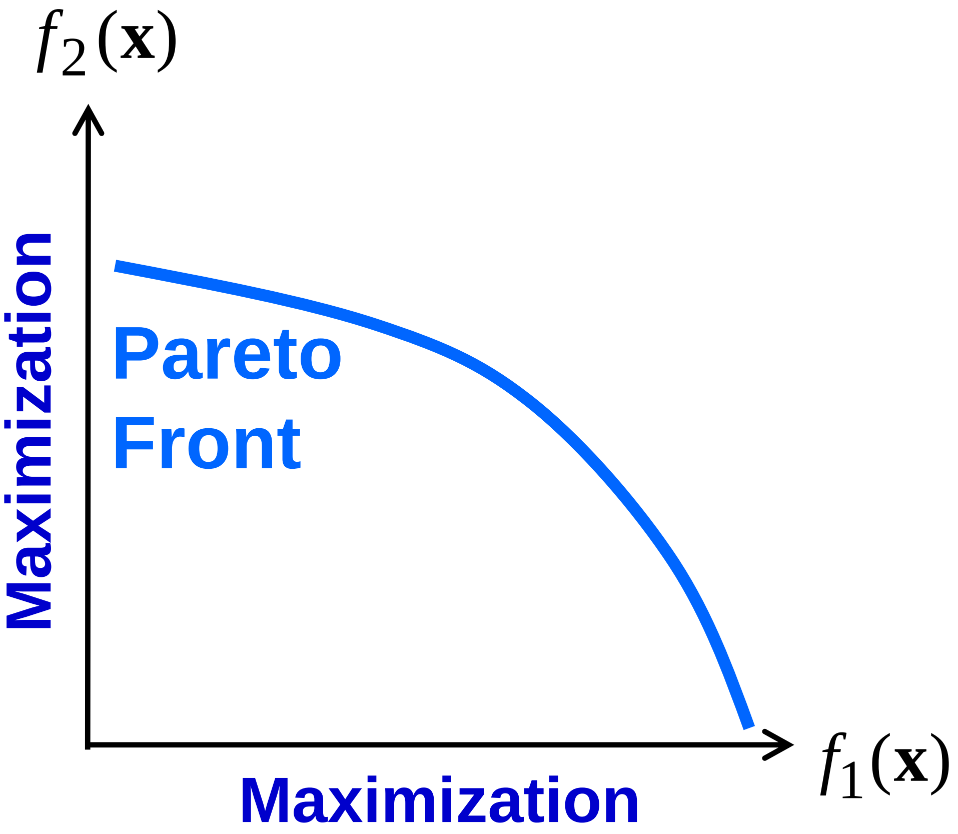


Weighted Sum Approach

Maximize $w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$

The 2nd Approach: Weighted Sum

Maximize $f(\boldsymbol{x}) = w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$



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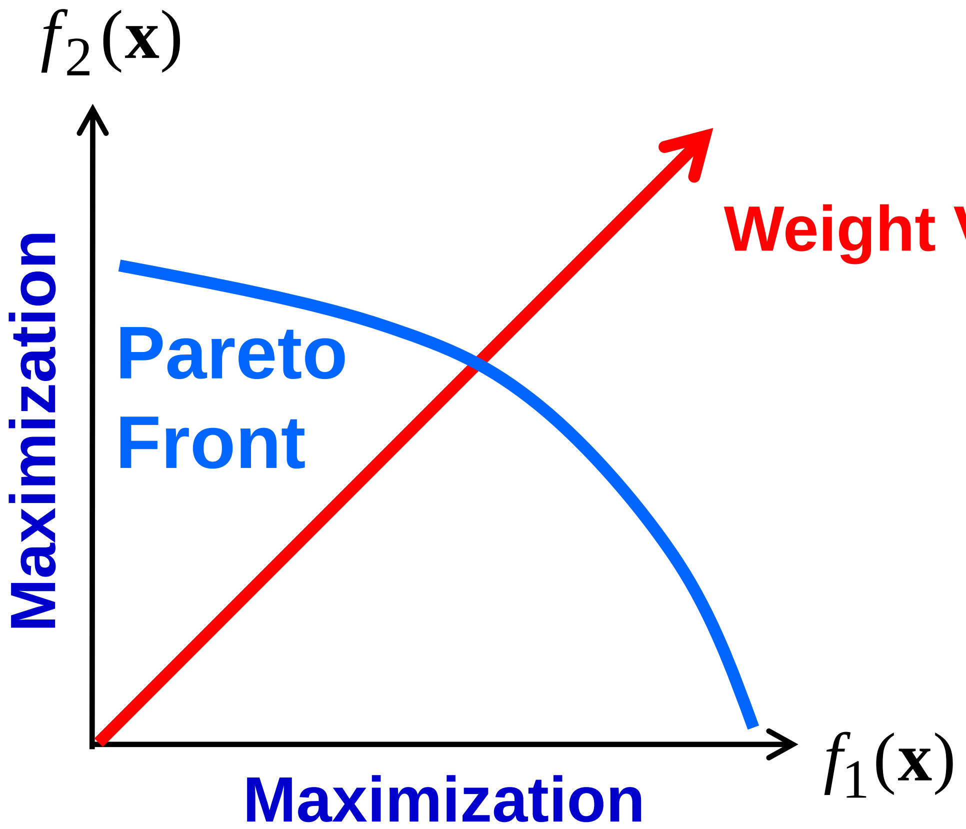

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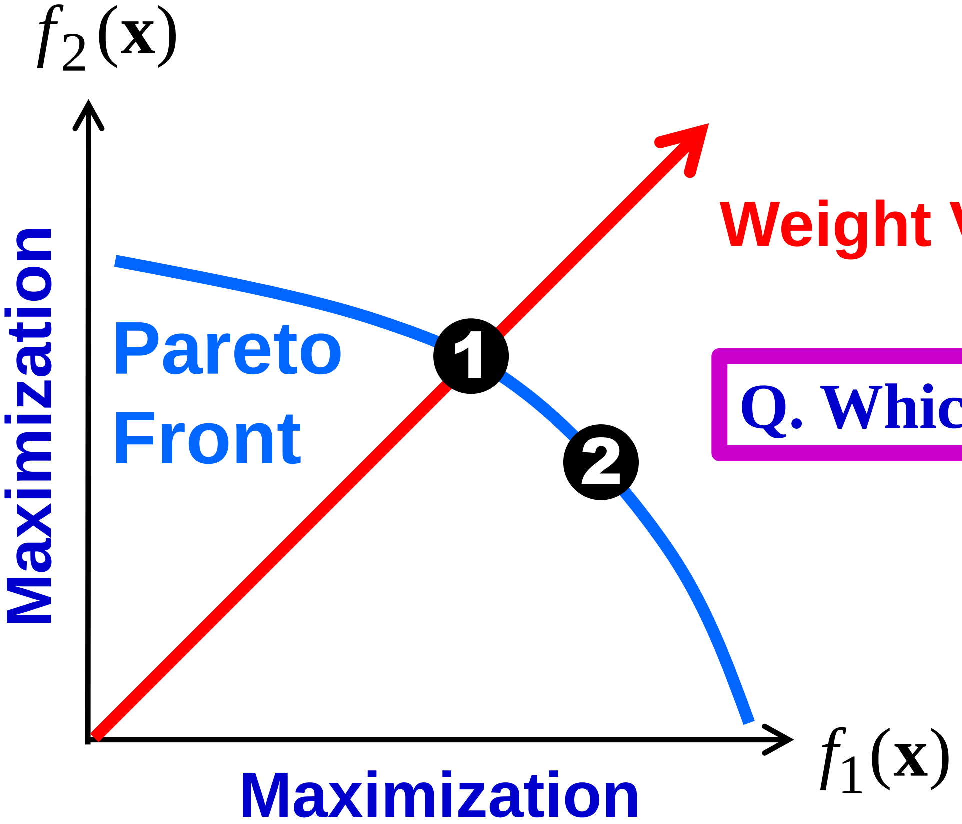
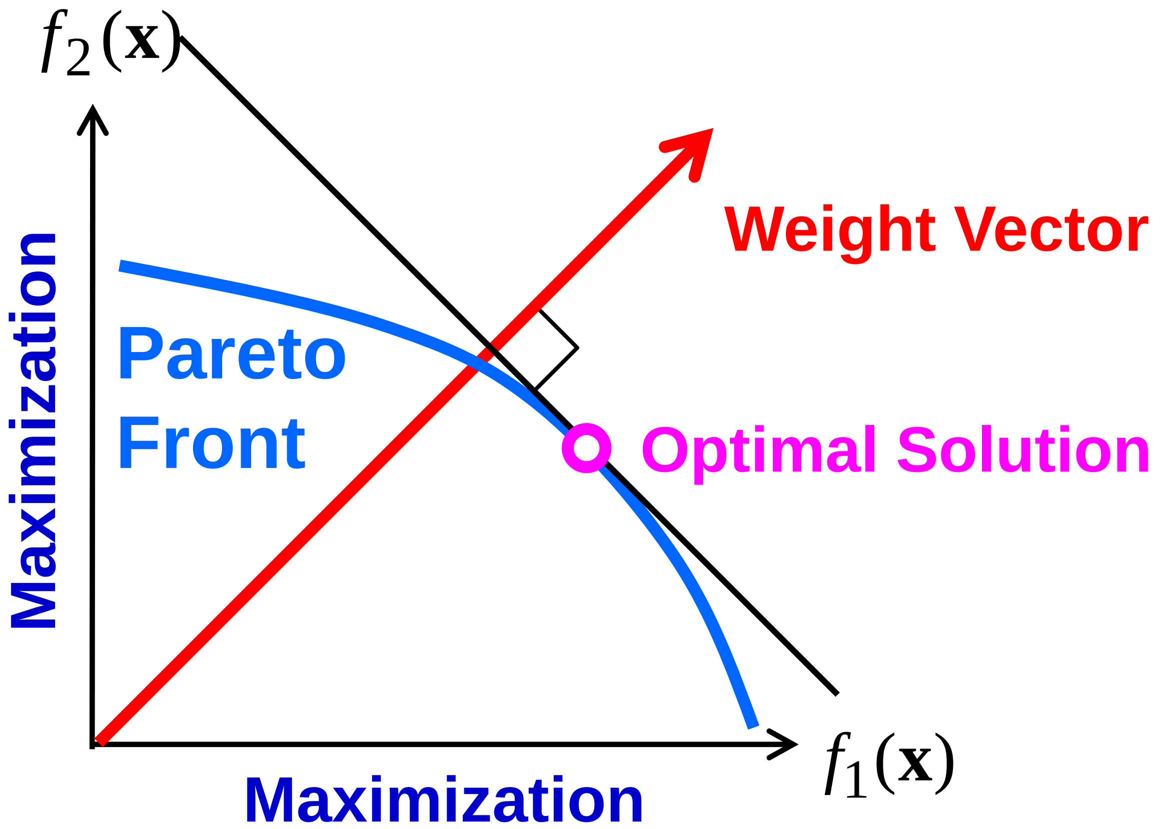


Q. Which solution will be obtained ?

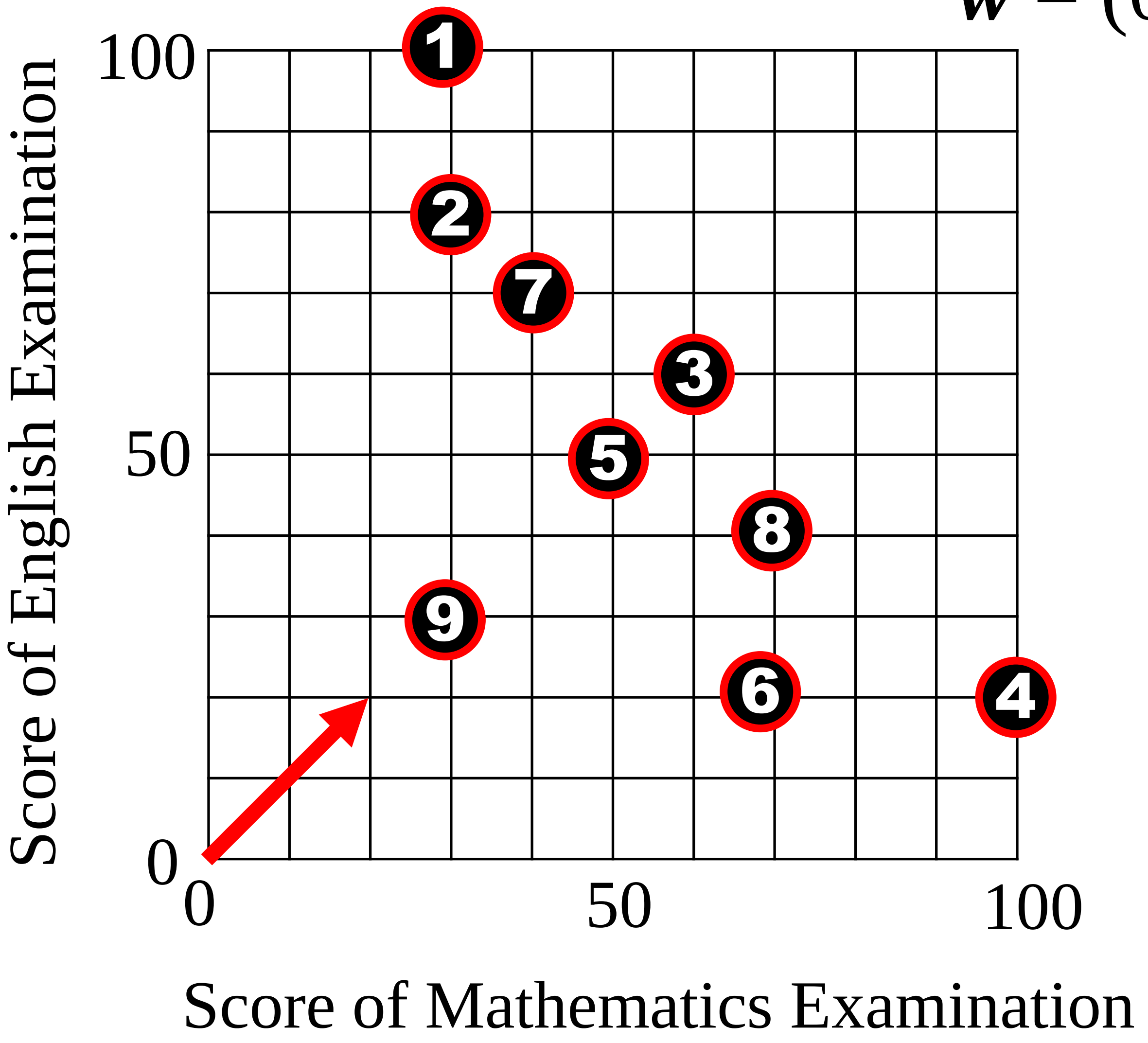
The 2nd Approach: Weighted Sum

Maximize $f(\boldsymbol{x}) = w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$



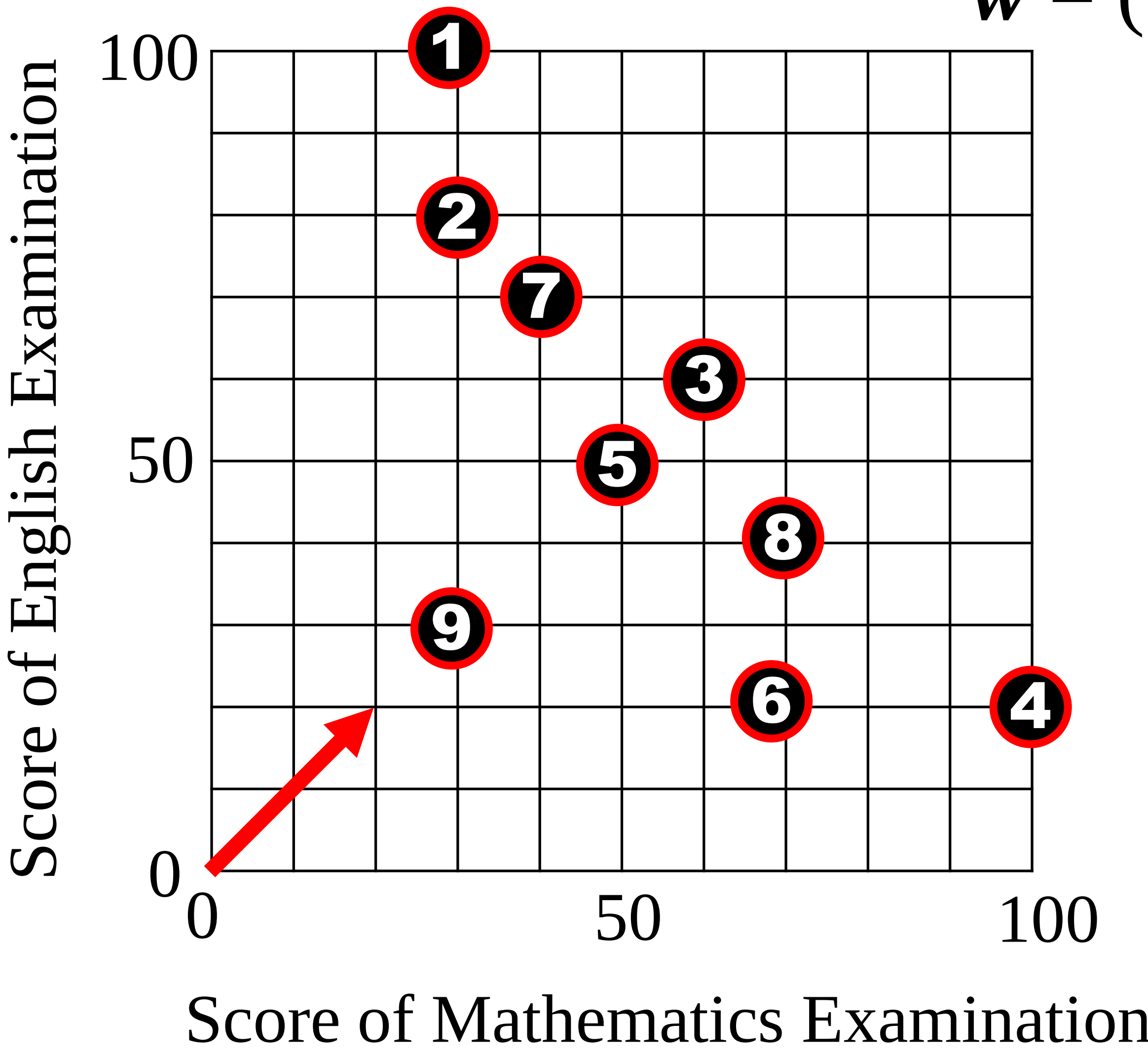
Q. which solution will be obtained ?

$\boldsymbol{w} = (0.5, 0.5)$



Q. which is the best student ?

$\boldsymbol{w} = (0.5, 0.5)$



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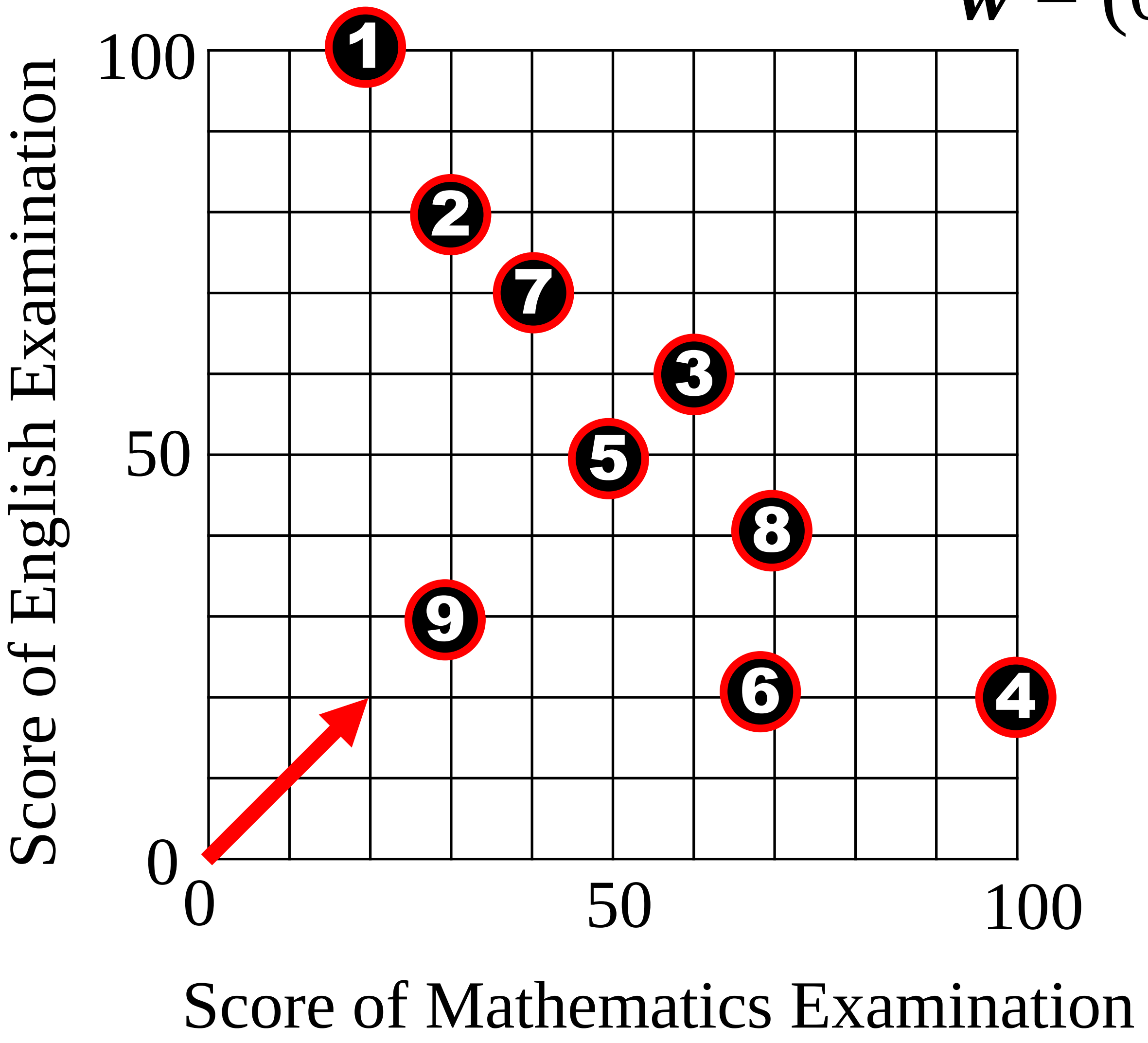
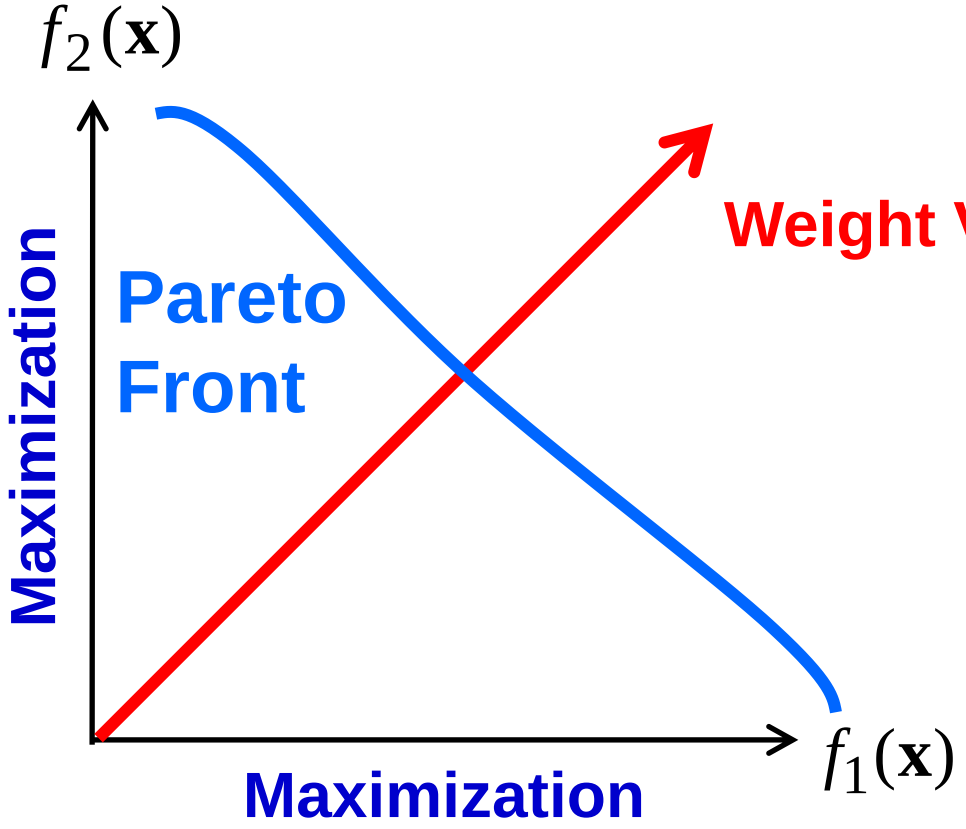


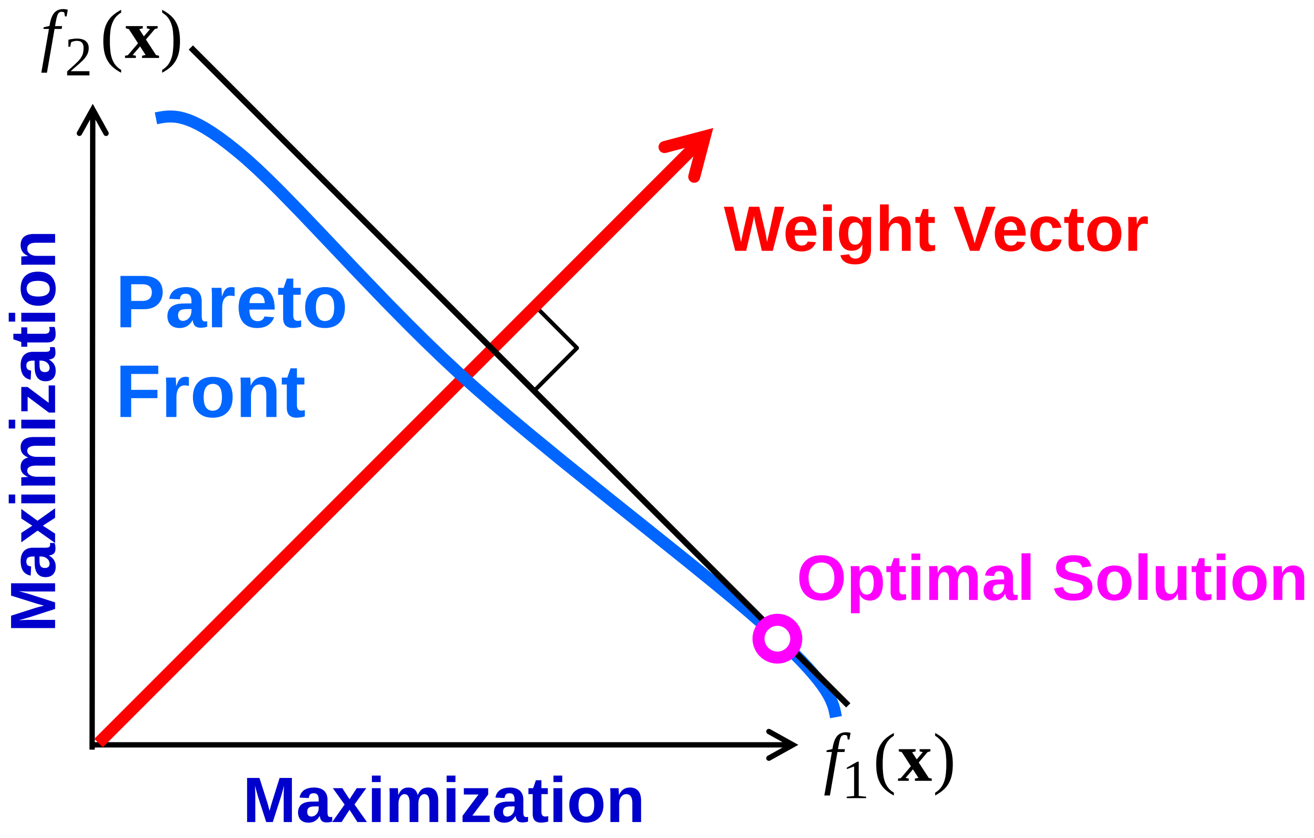
The 2nd Approach: Weighted Sum

Maximize $f(\boldsymbol{x}) = w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$



The 2nd Approach: Weighted Sum

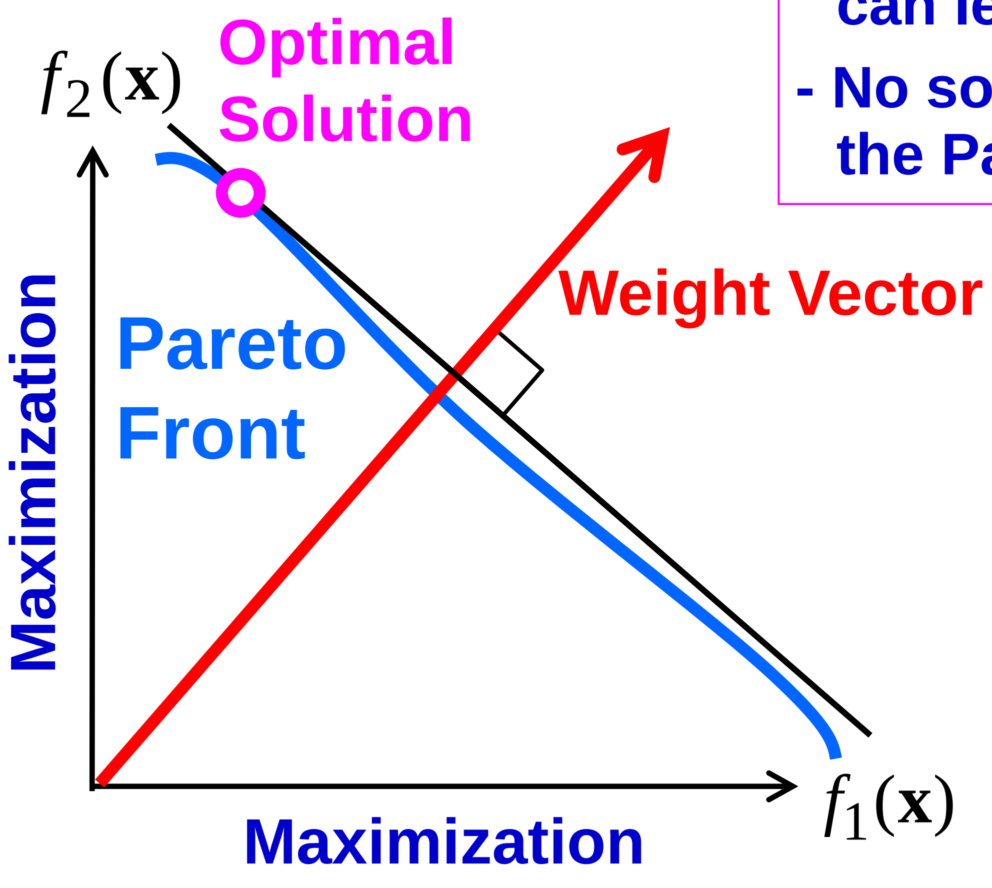
$$\textbf{Maximize} \quad f(\boldsymbol{x}) = w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$$



The 2nd Approach: Weighted Sum

Maximize $f(\boldsymbol{x}) = w_1 f_1(\boldsymbol{x}) + w_2 f_2(\boldsymbol{x})$

- Slight change in the weight vector can lead to a large jump.
- No solution around the center of the Pareto front can be found.



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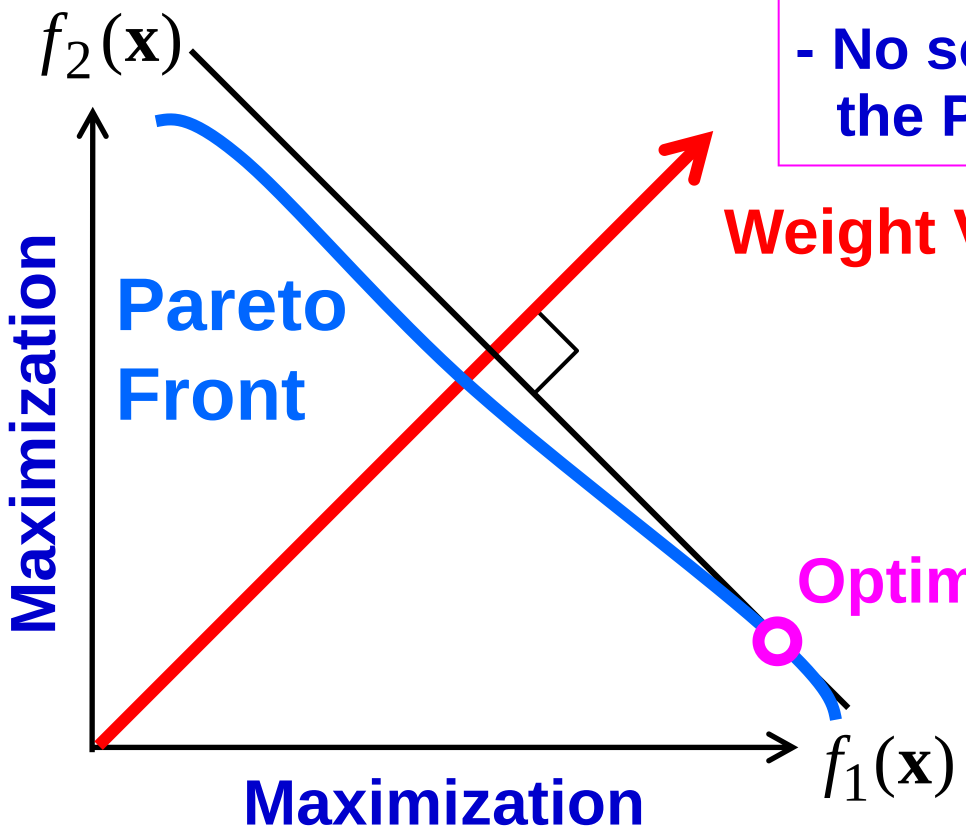


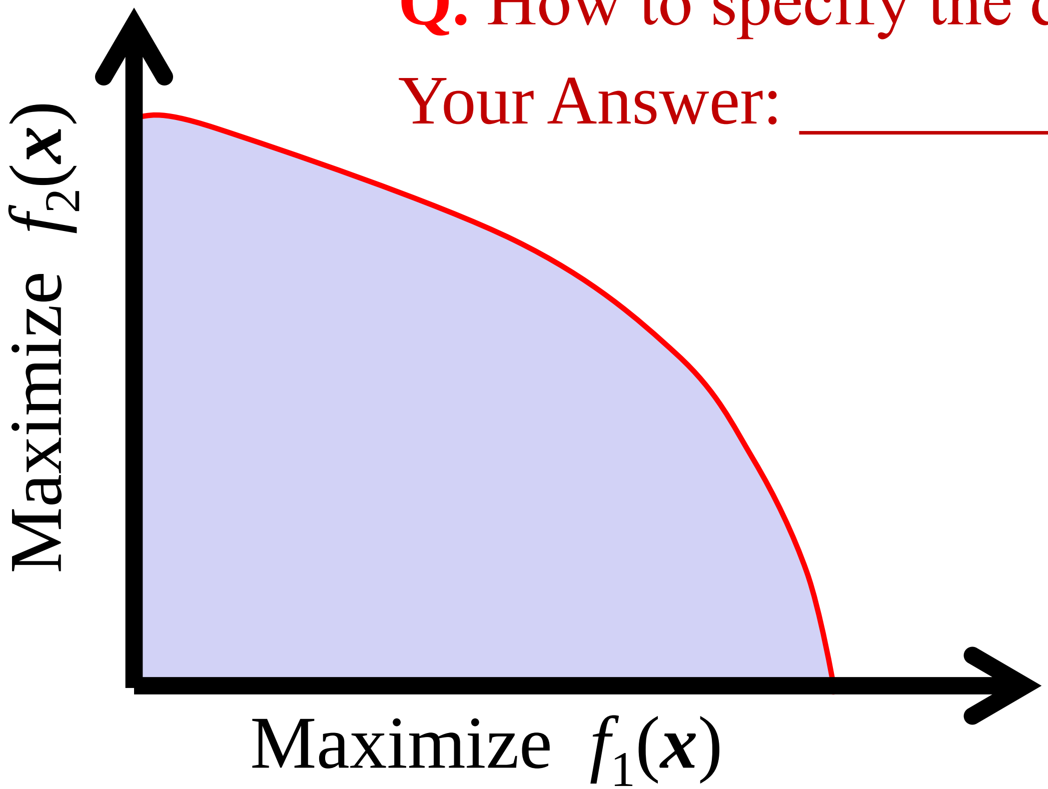
Use of Additional Information before Optimization

First, multiple objectives are combined into a single objective function using additional information from the decision maker. Then, the objective function is optimized to find a single final solution.

Original Two-Objective Problem: Maximize $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$

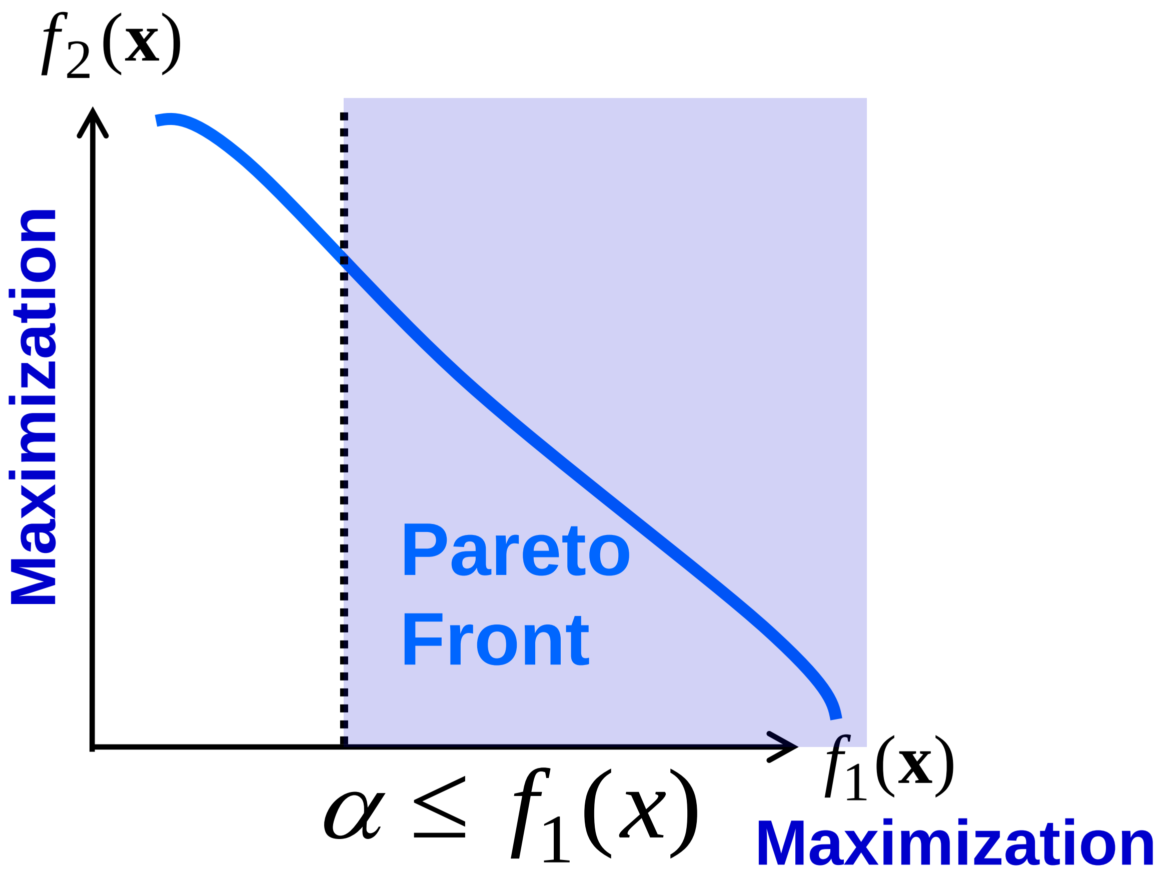
Q. How to specify the decision maker's preference ?

Your Answer: ______________________________.



The 2nd Approach:
Use of Constraint Condition

Maximize $f(\boldsymbol{x}) = f_2(\boldsymbol{x})$ **subject to** $f_1(x) \geq a$



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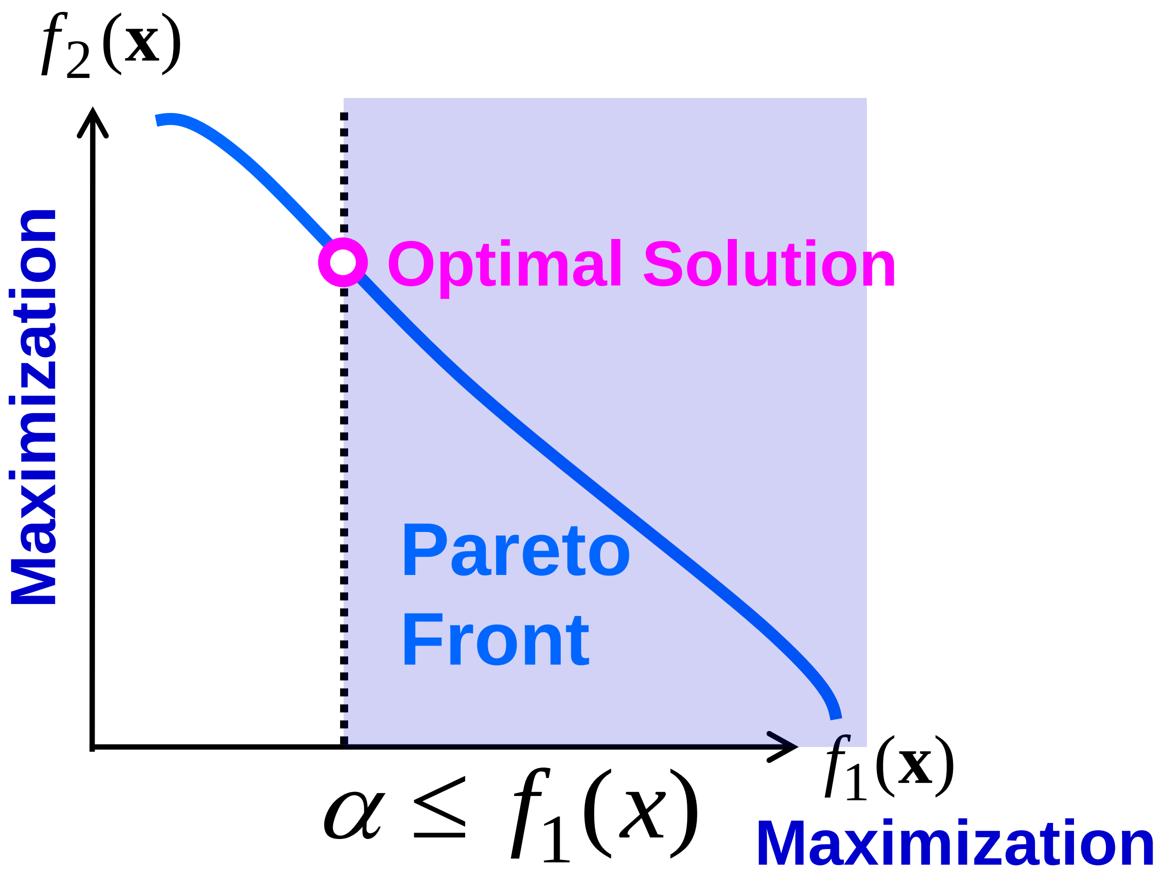
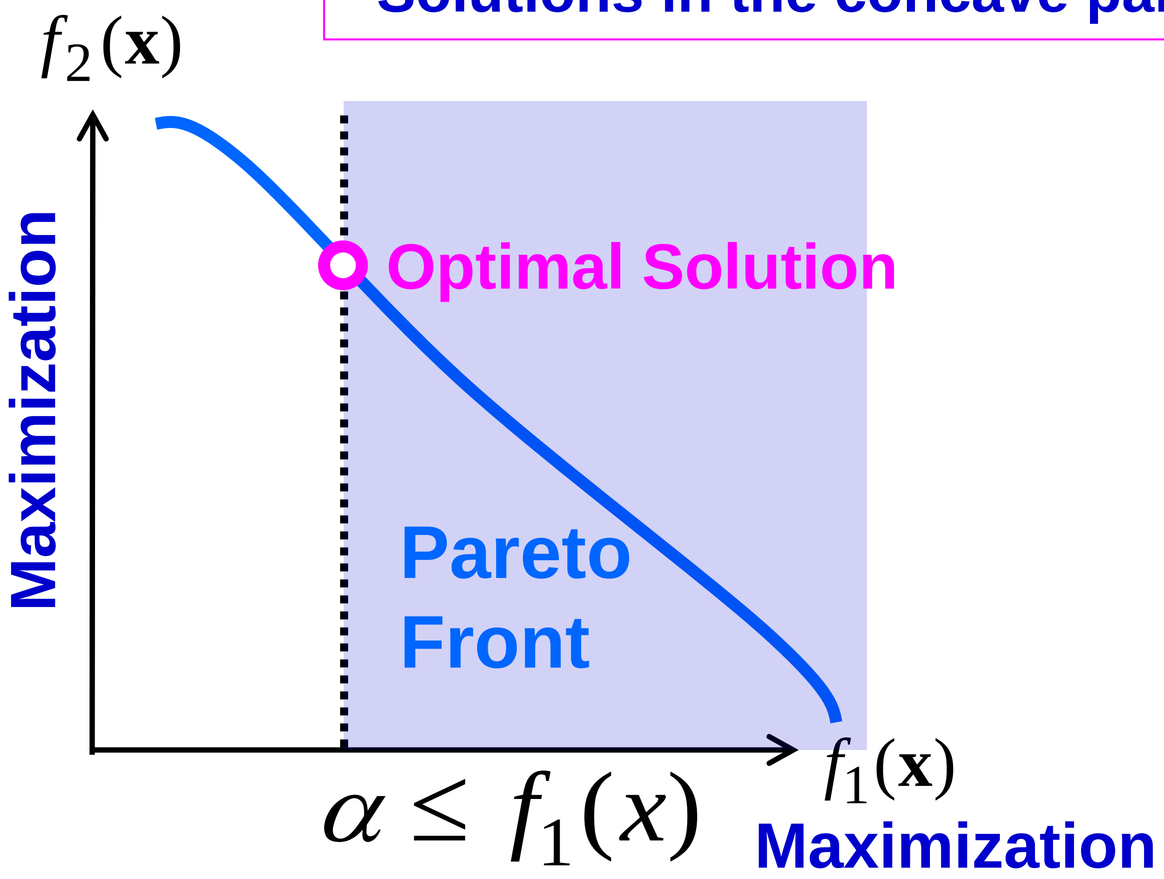


The 2nd Approach:
Use of Constraint Condition

Maximize $f(\boldsymbol{x}) = f_2(\boldsymbol{x})$ **subject to** $f_1(x) \geq a$

- Solutions in the concave part can be found.



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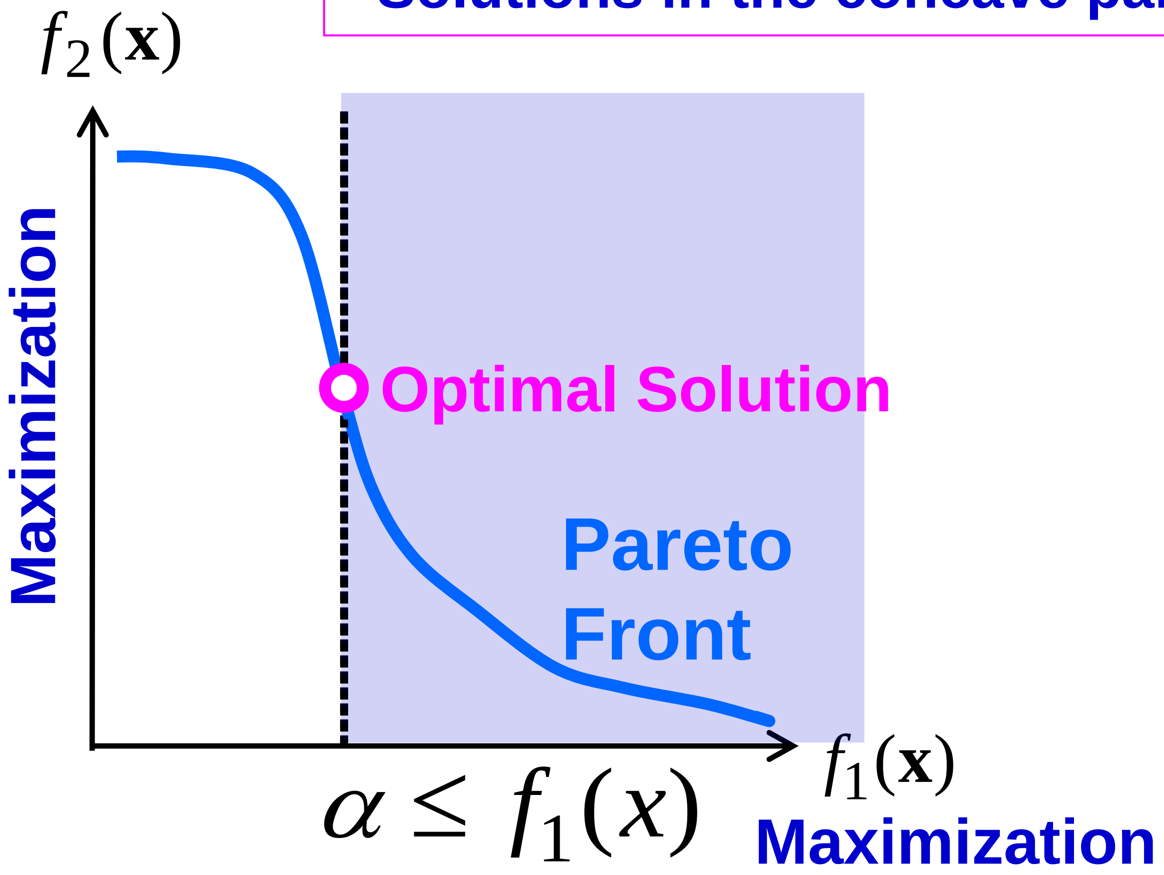
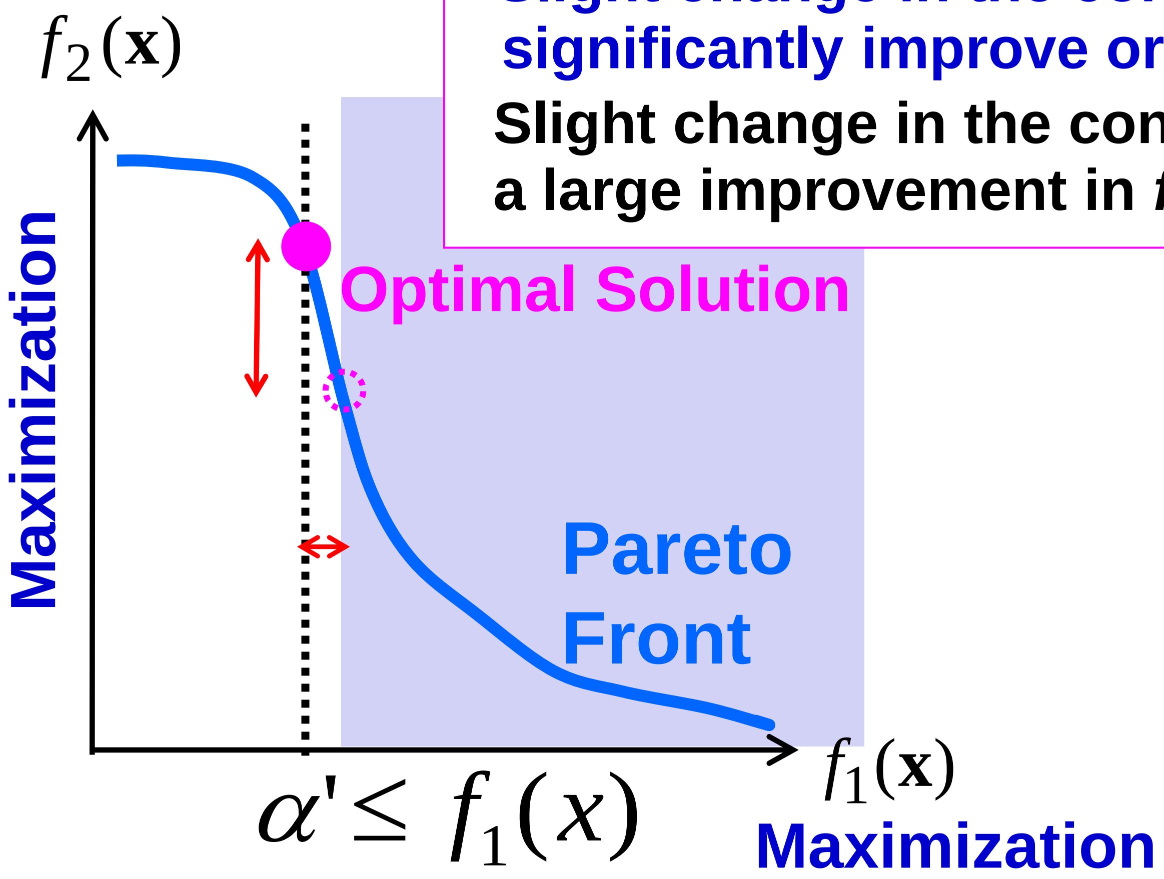


The 2nd Approach:
Use of Constraint Condition

Maximize $f(\boldsymbol{x}) = f_2(\boldsymbol{x})$ **subject to** $f_1(x) \geq a$

- Slight change in the constraint condition can significantly improve or degrade the solution:

Slight change in the constraint leads to a large improvement in f_2(x) in this example.



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Maximize $f(\boldsymbol{x}) = f_2(\boldsymbol{x})$ **subject to** $f_1(x) \geq a$

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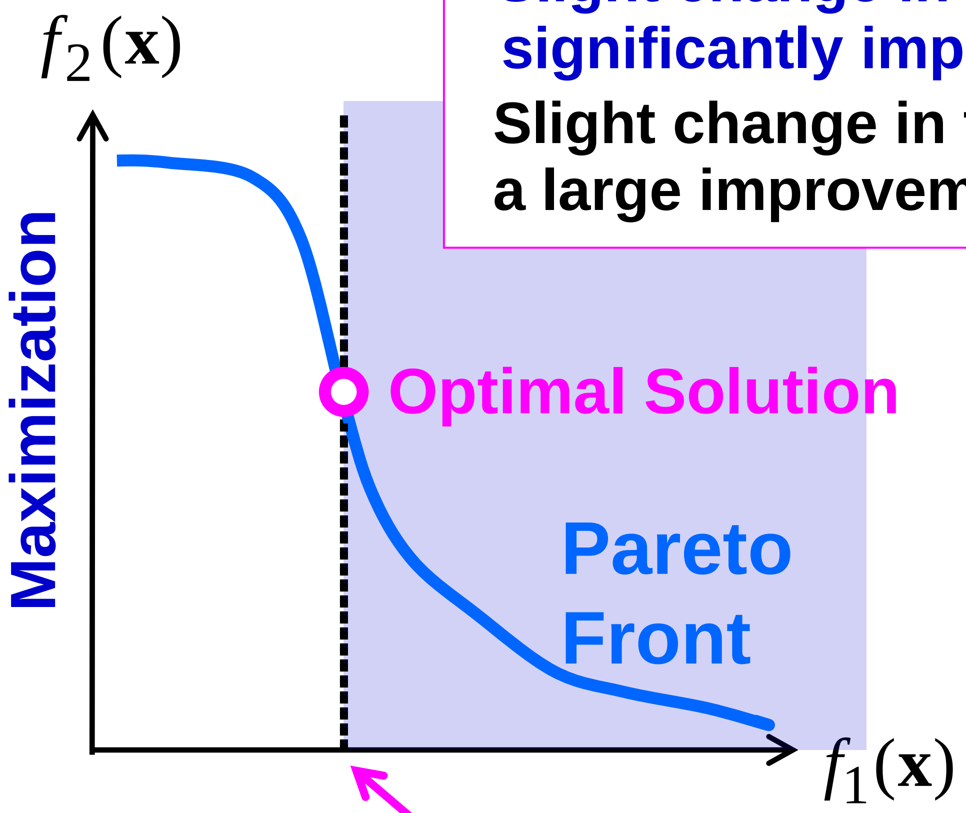
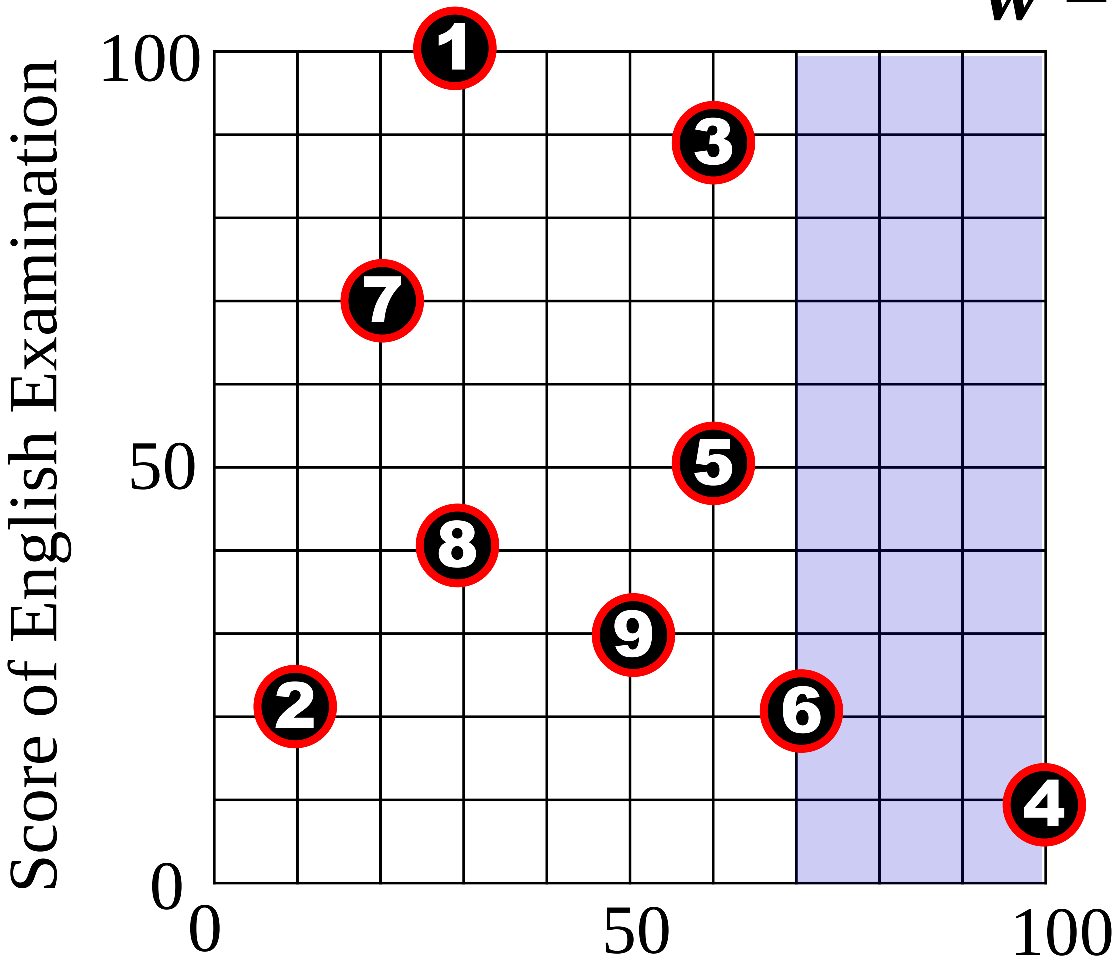


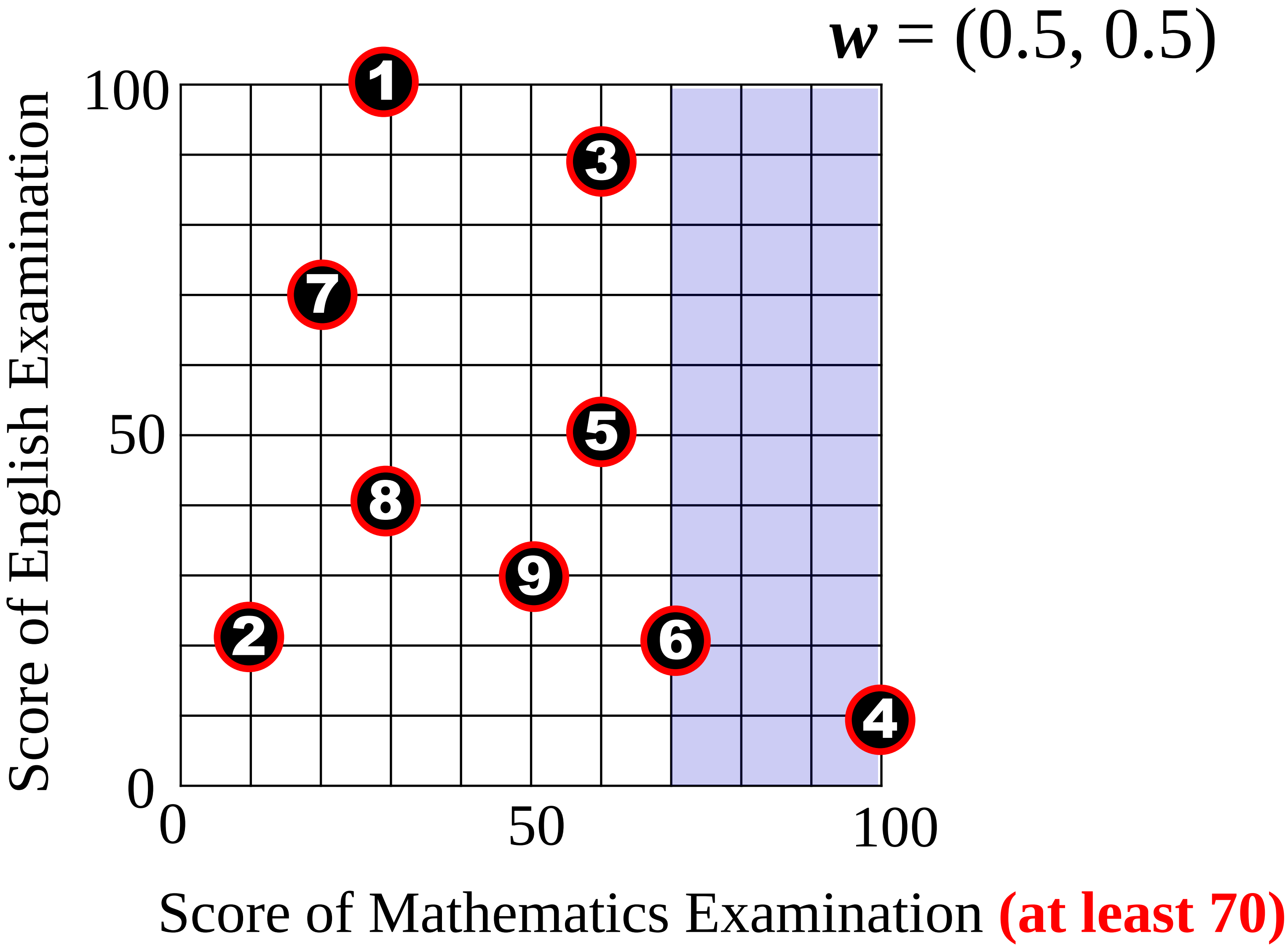
This is not a good specification.

Q. which solution will be obtained ?

$\boldsymbol{w} = (0.5, 0.5)$

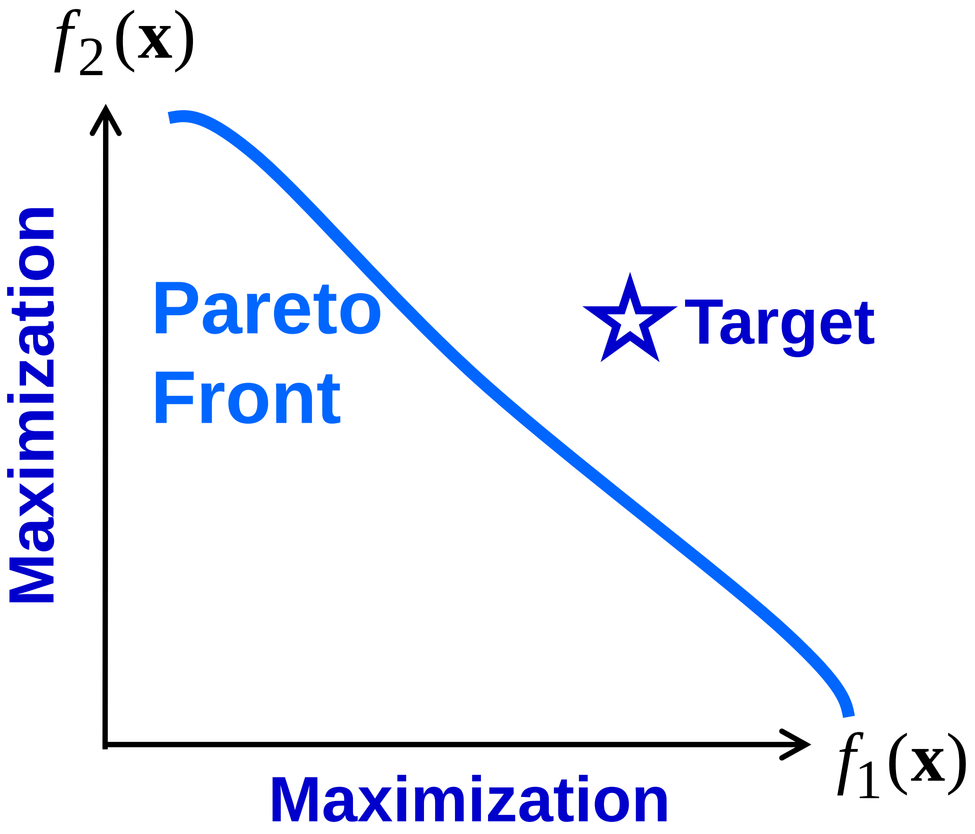


Q. which is the best student ?



The 2nd Approach:
Use of a Target Solution

Minimize the distance from the target



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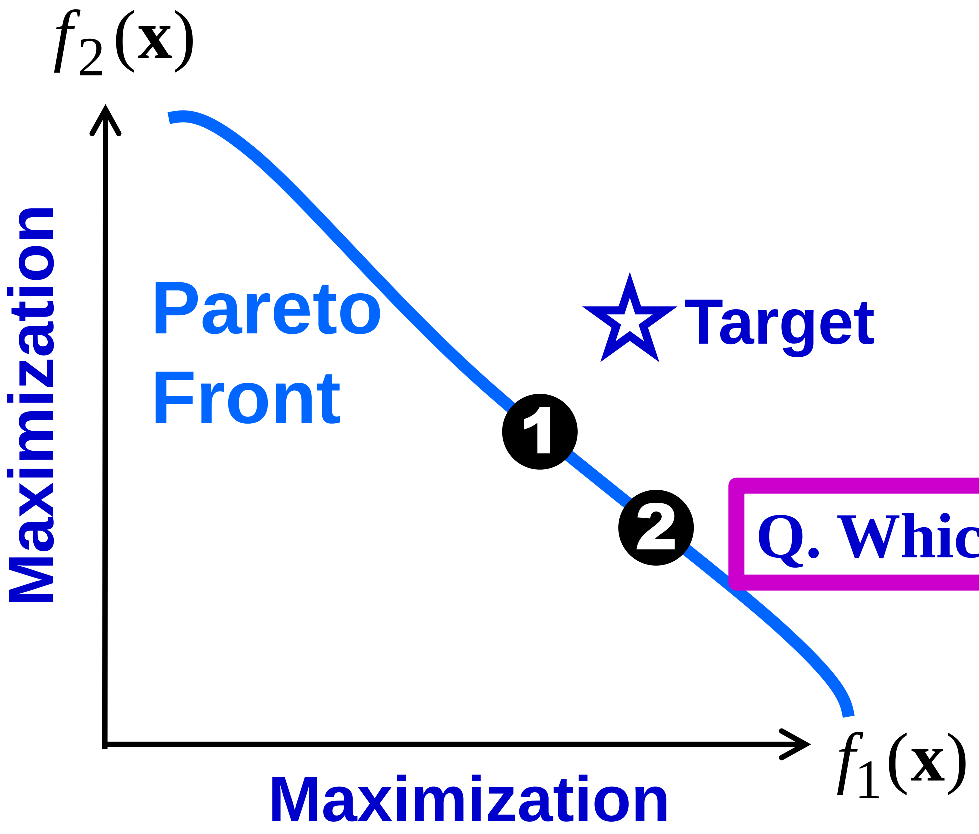

Q. Which solution will be obtained ?

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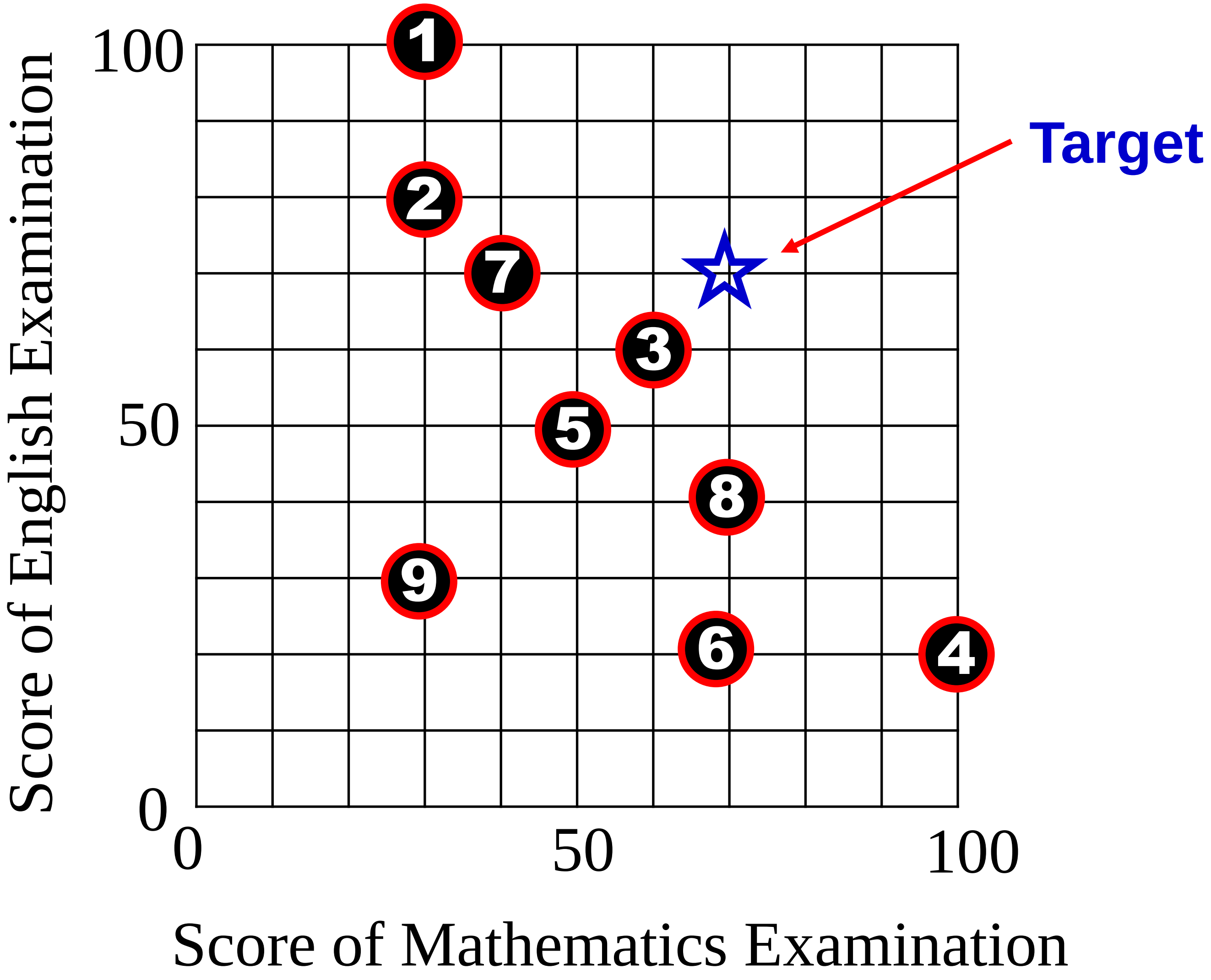

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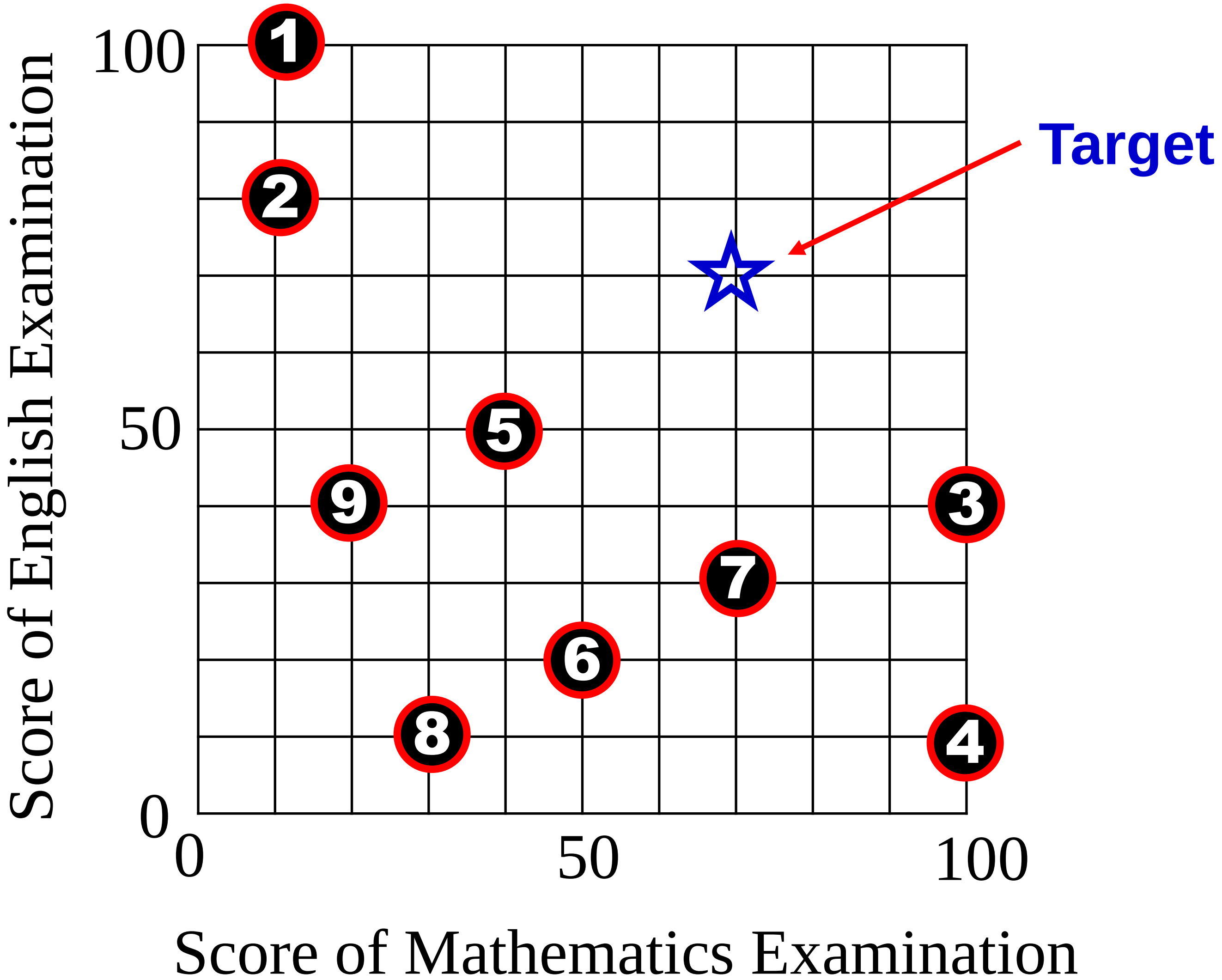

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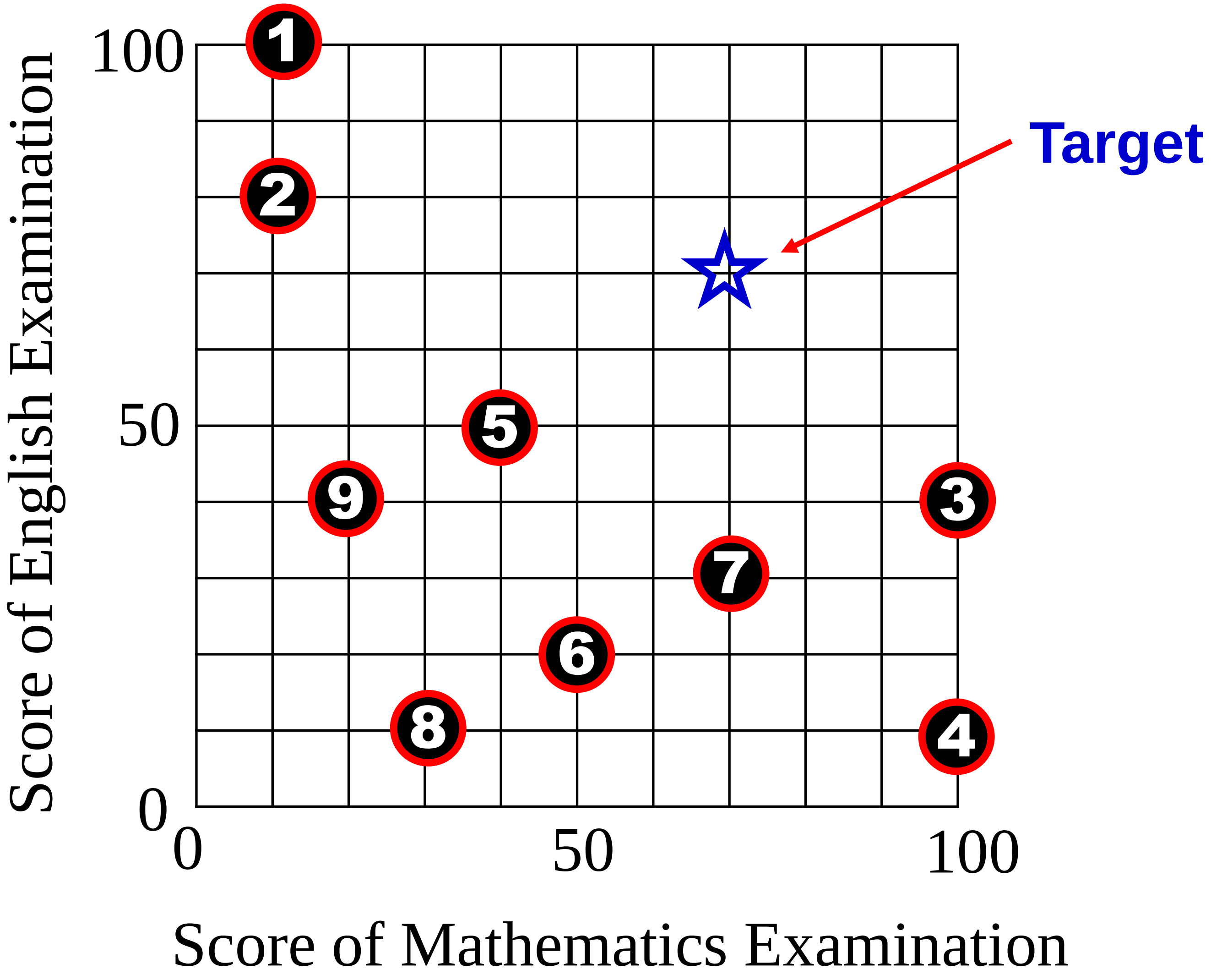
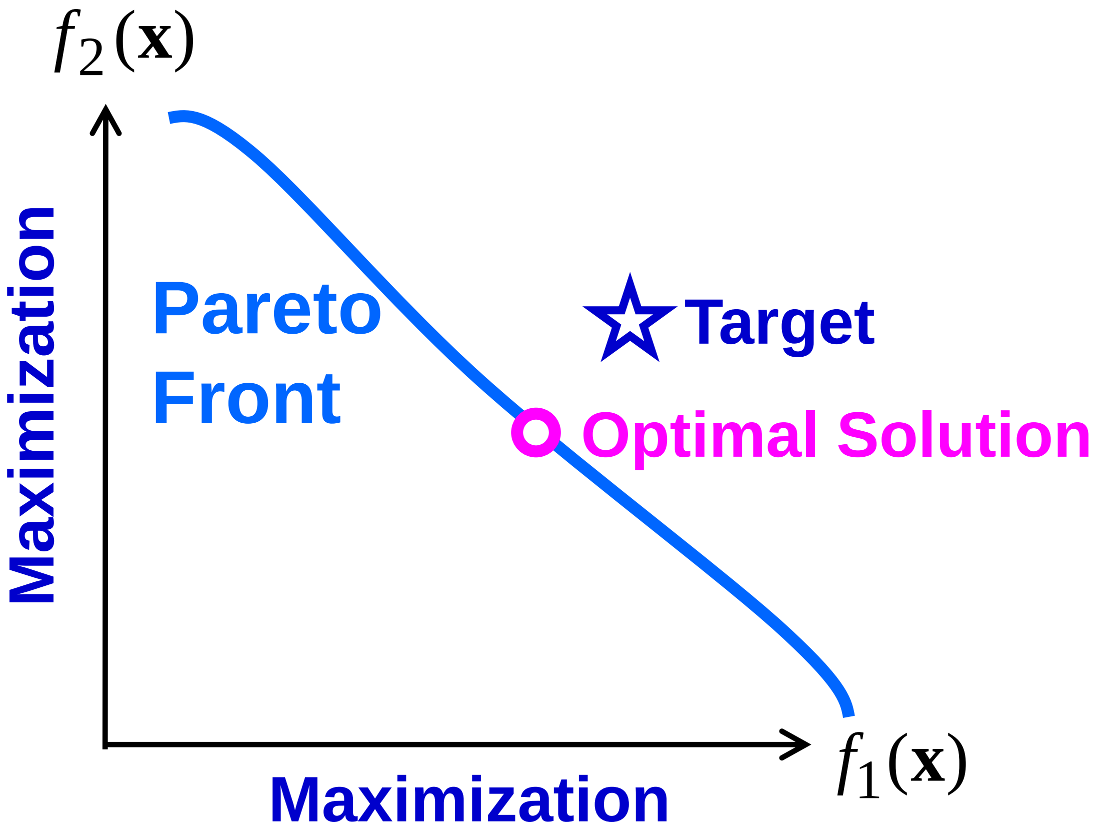


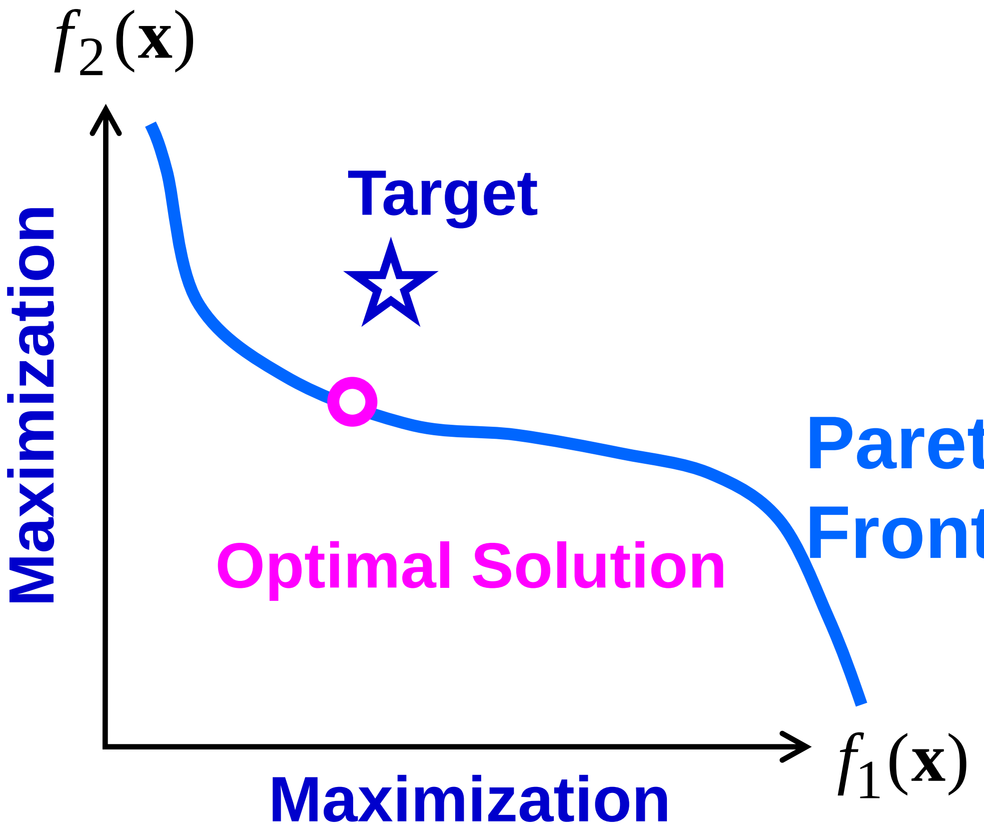
The 2nd Approach:
Use of a Target Solution

Minimize the distance from the target



The 2nd Approach:
Use of a Target Solution

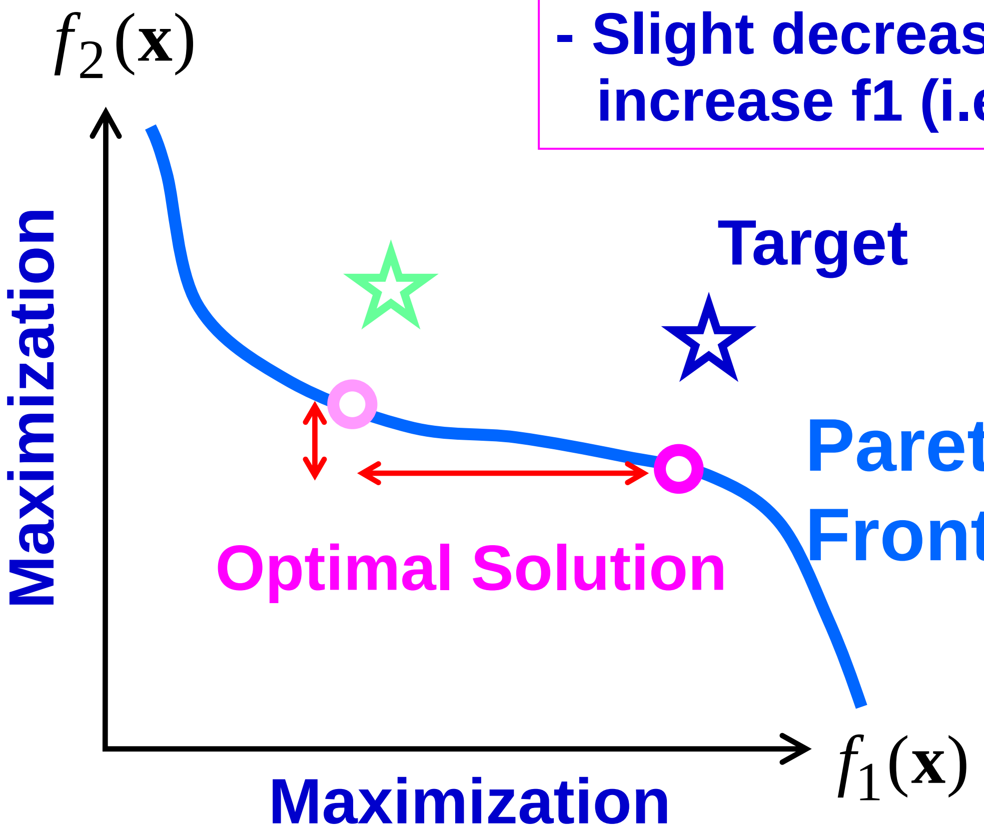
Minimize the distance from the target



The 2nd Approach:
Use of a Target Solution

Minimize the distance from the target

- Slight decrease in f2 can significantly increase f1 (i.e., improve f1).



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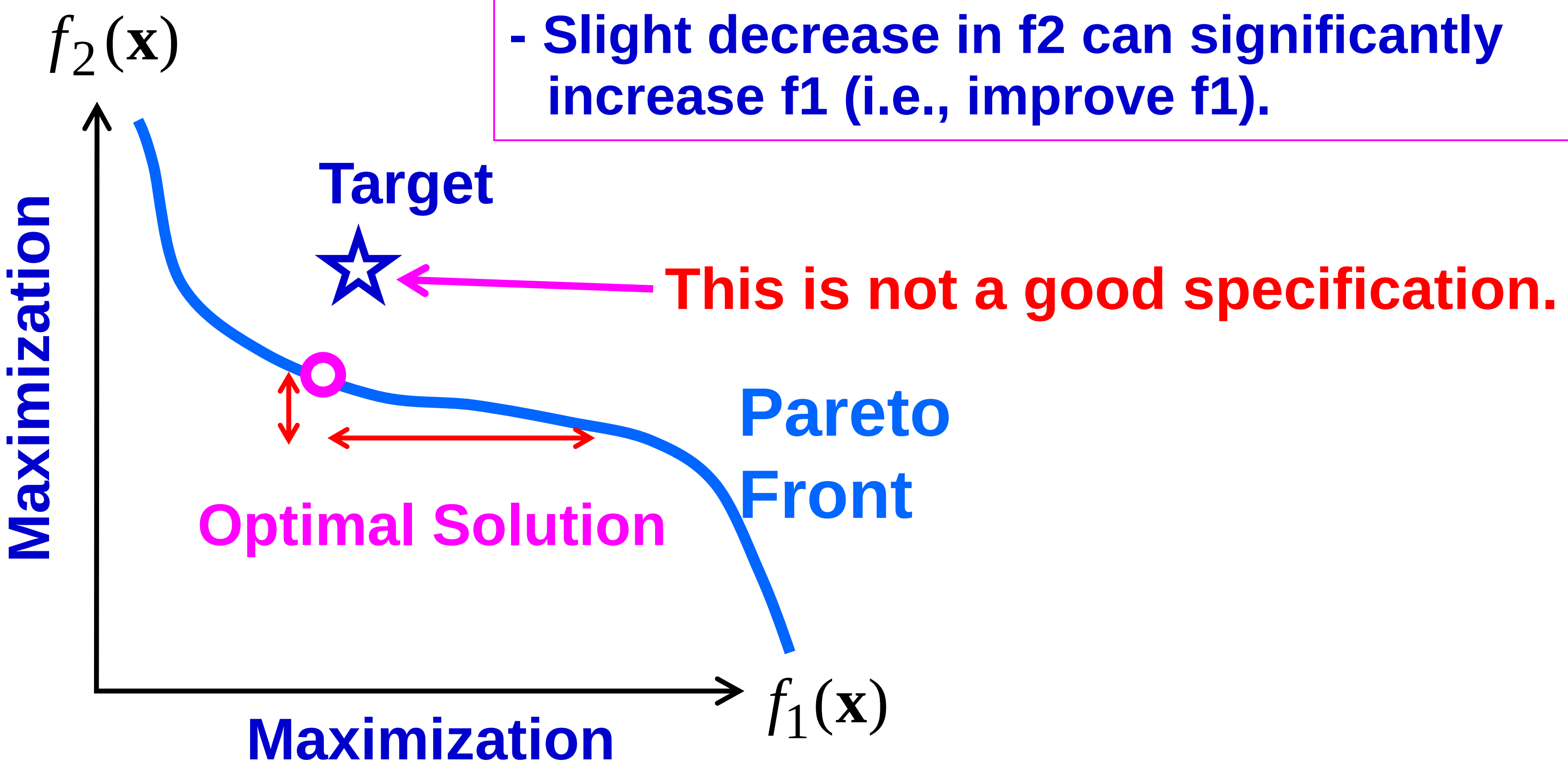

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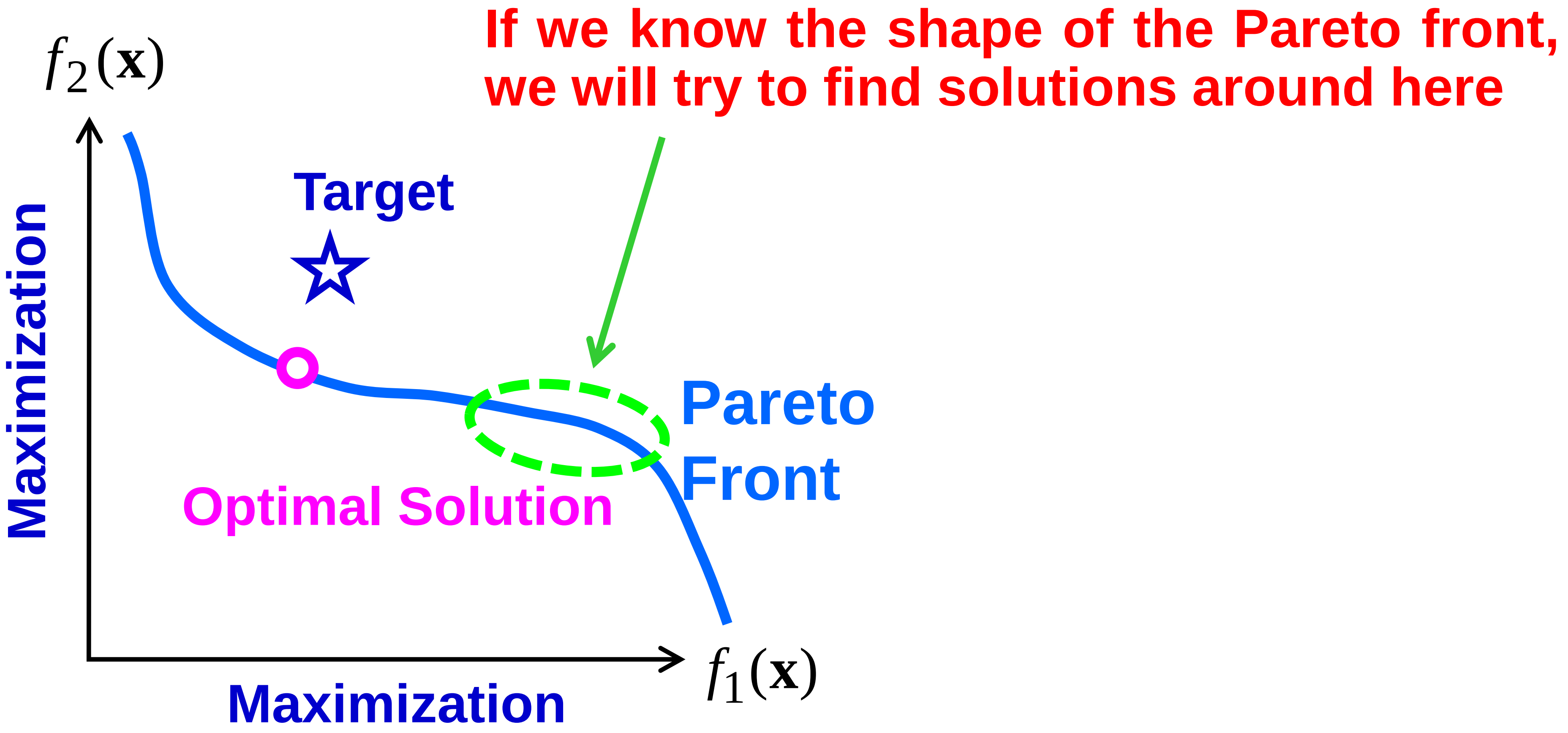

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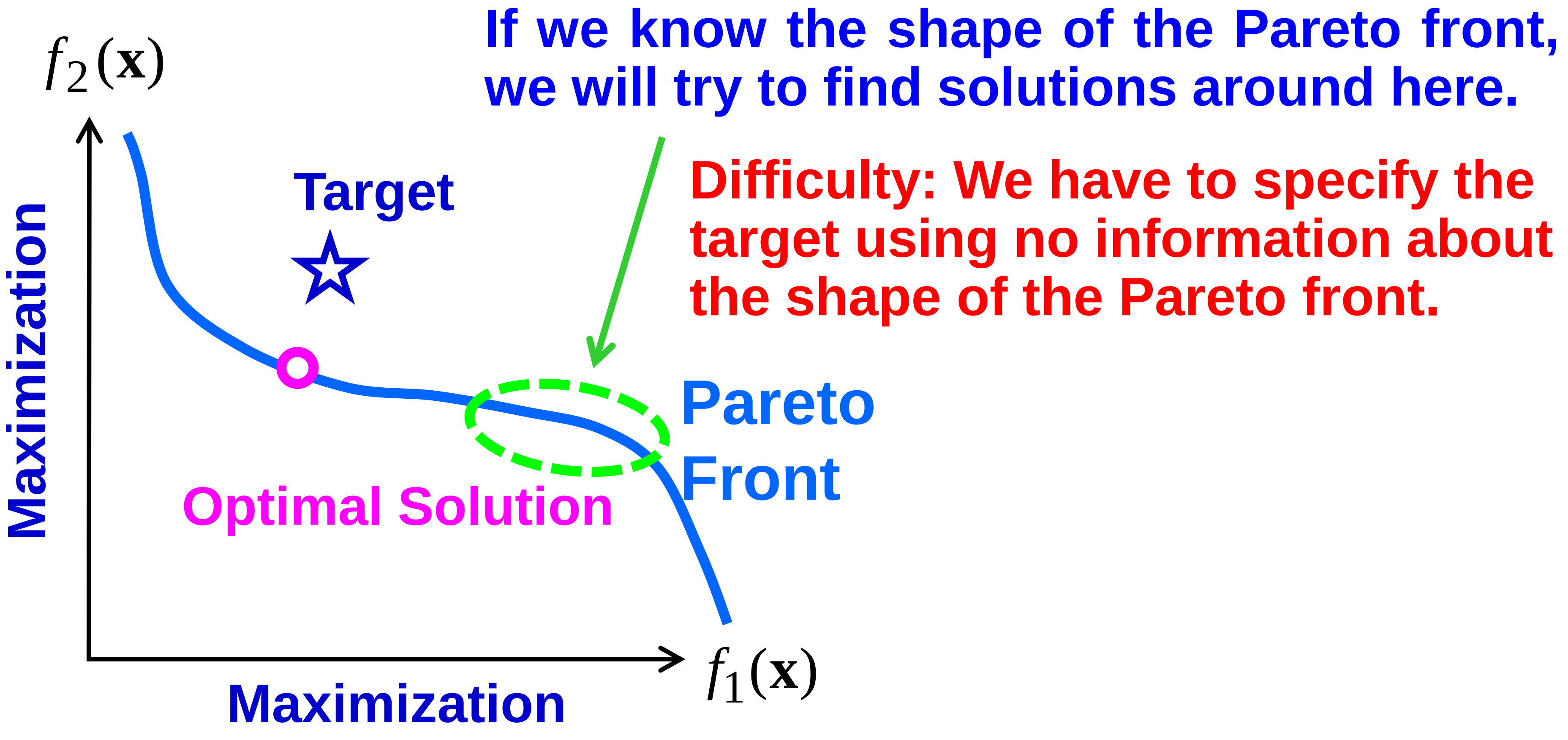
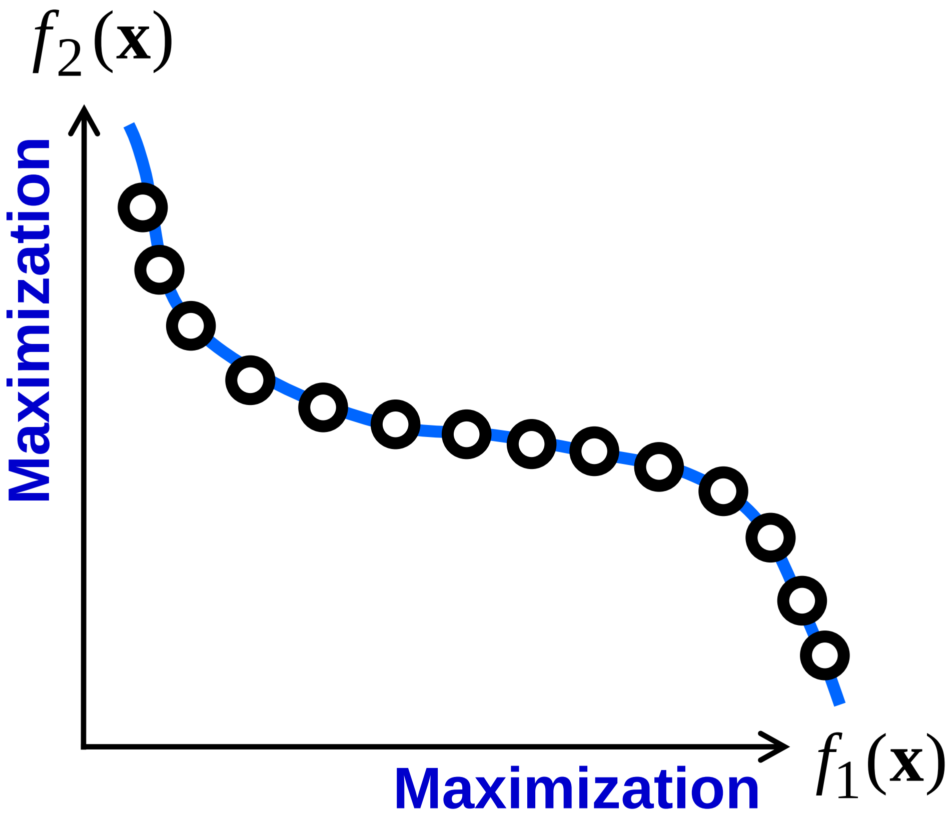
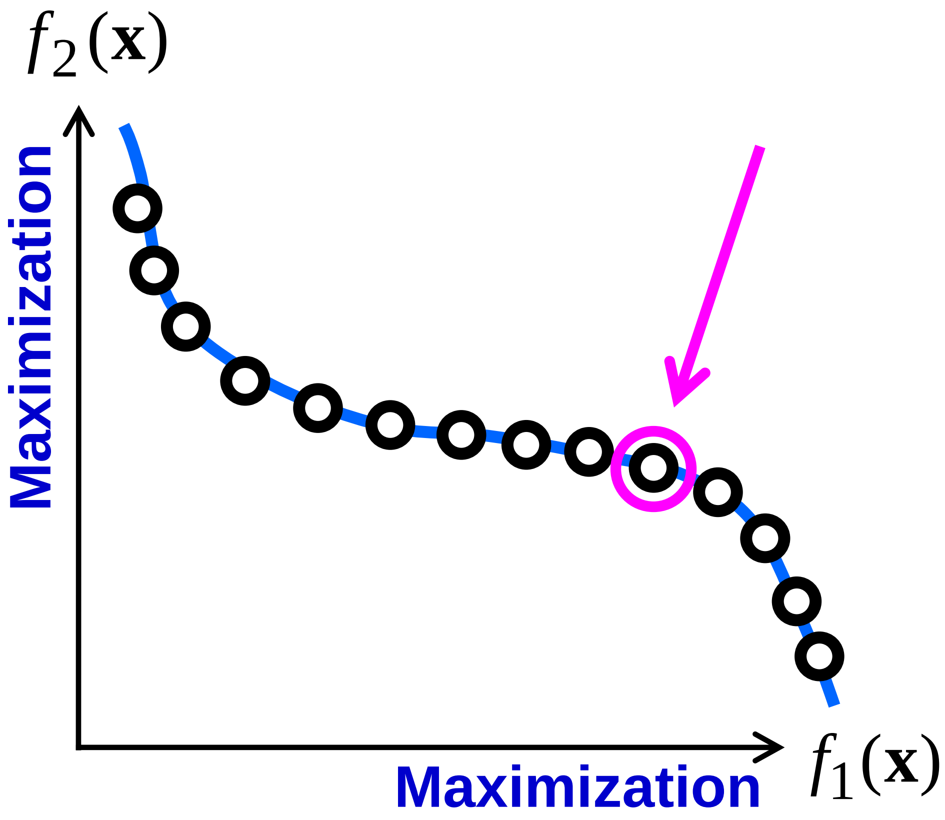


Advantages of EMO Approach:

Many solutions are shown to the decision maker.



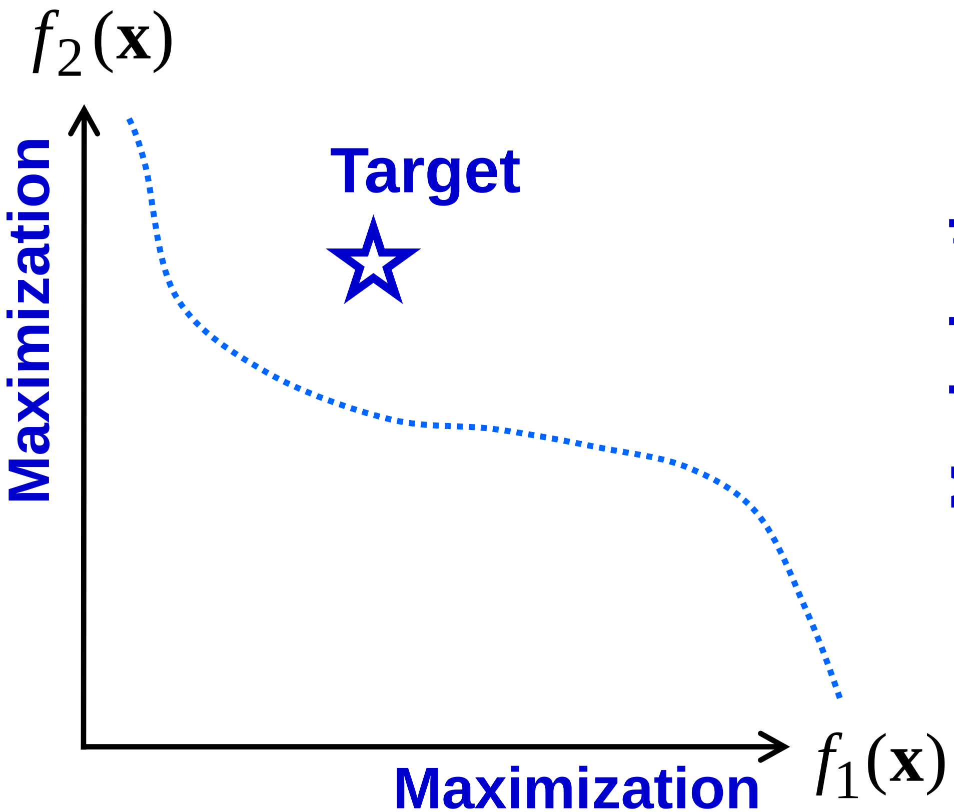


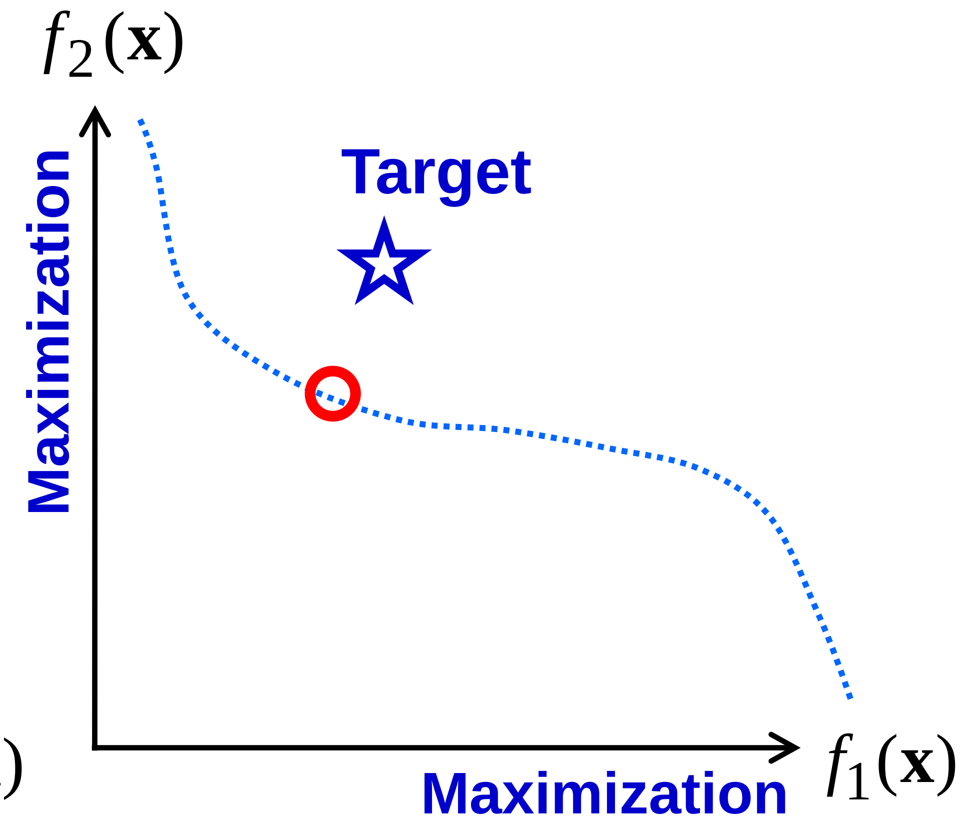
Step 1: Search for Pareto optimal solutions by an EMO algorithm.

Step 2: Choice of a single final solution.

Target Solution-based Method:

No information is available when a target is specified.



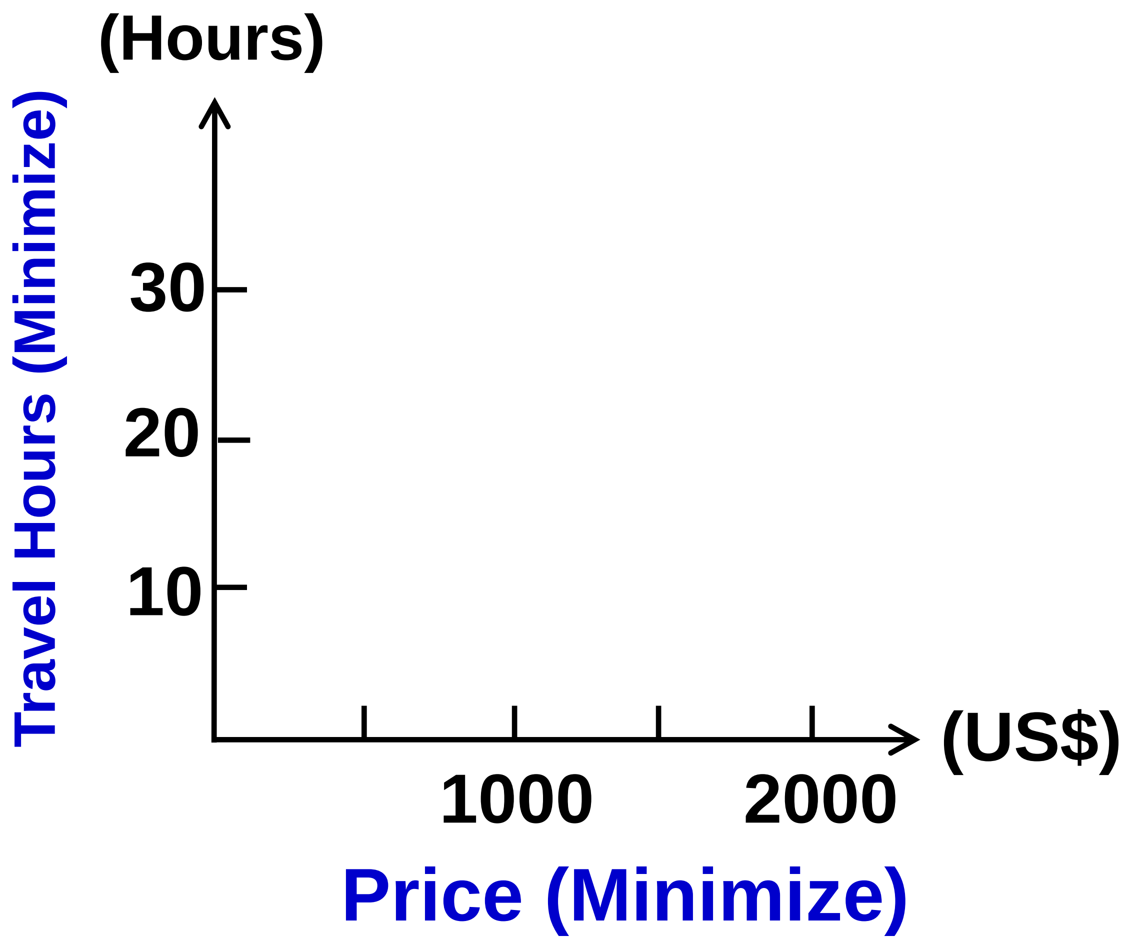


Step 1: Specification of a target solution.

Step 2: Search for the best solution.

Examples: Air Ticket to New York

If we know the problem very well, it may be easy to give an appropriate target.



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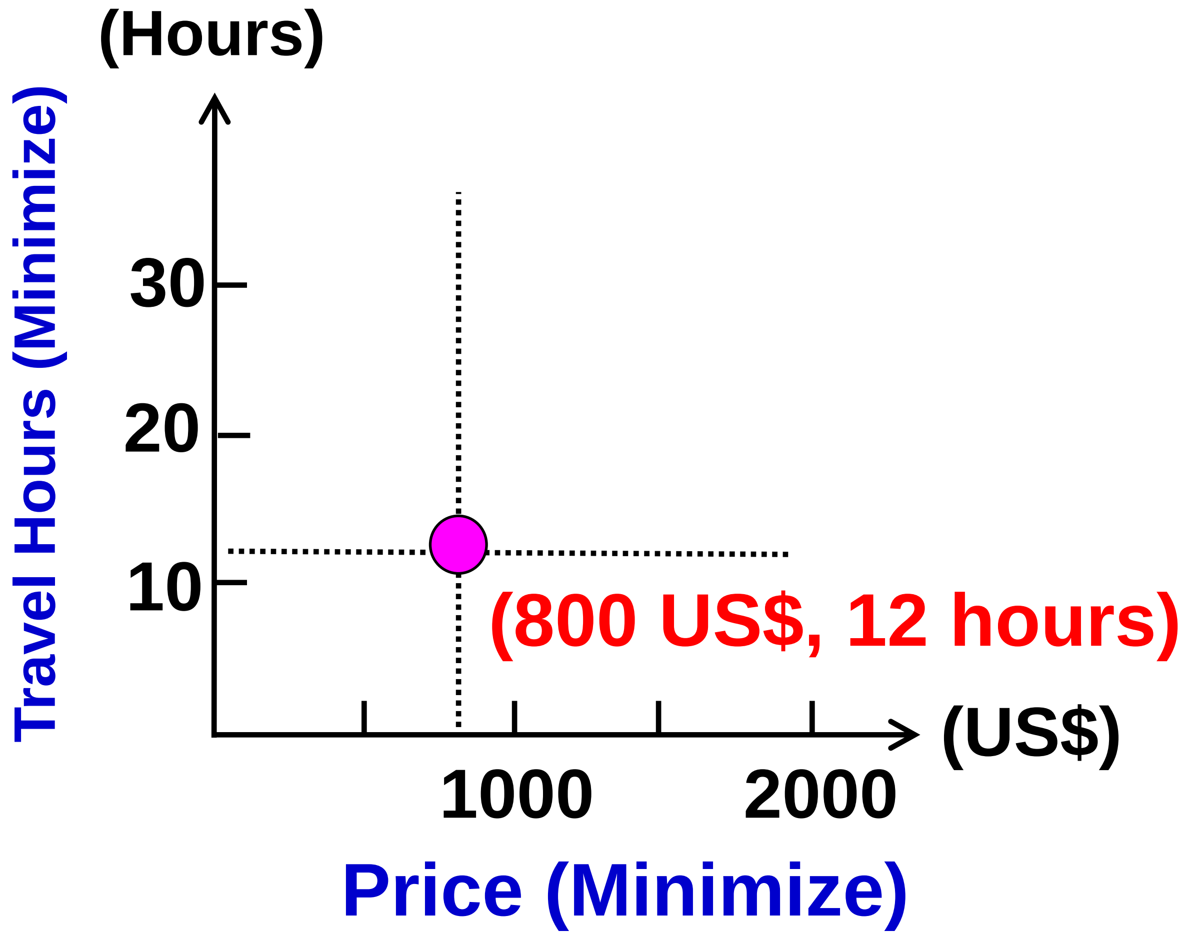

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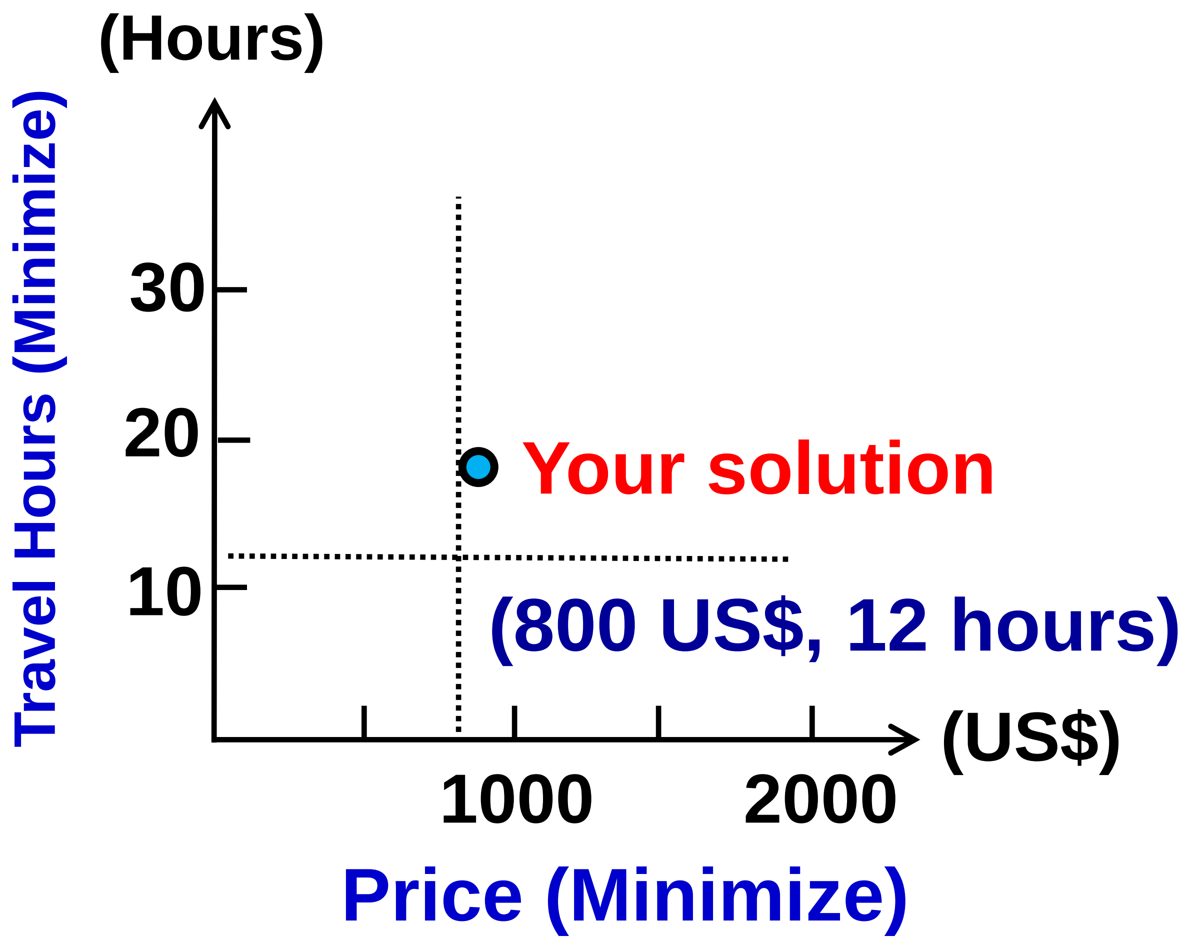

Examples: Air Ticket to New York

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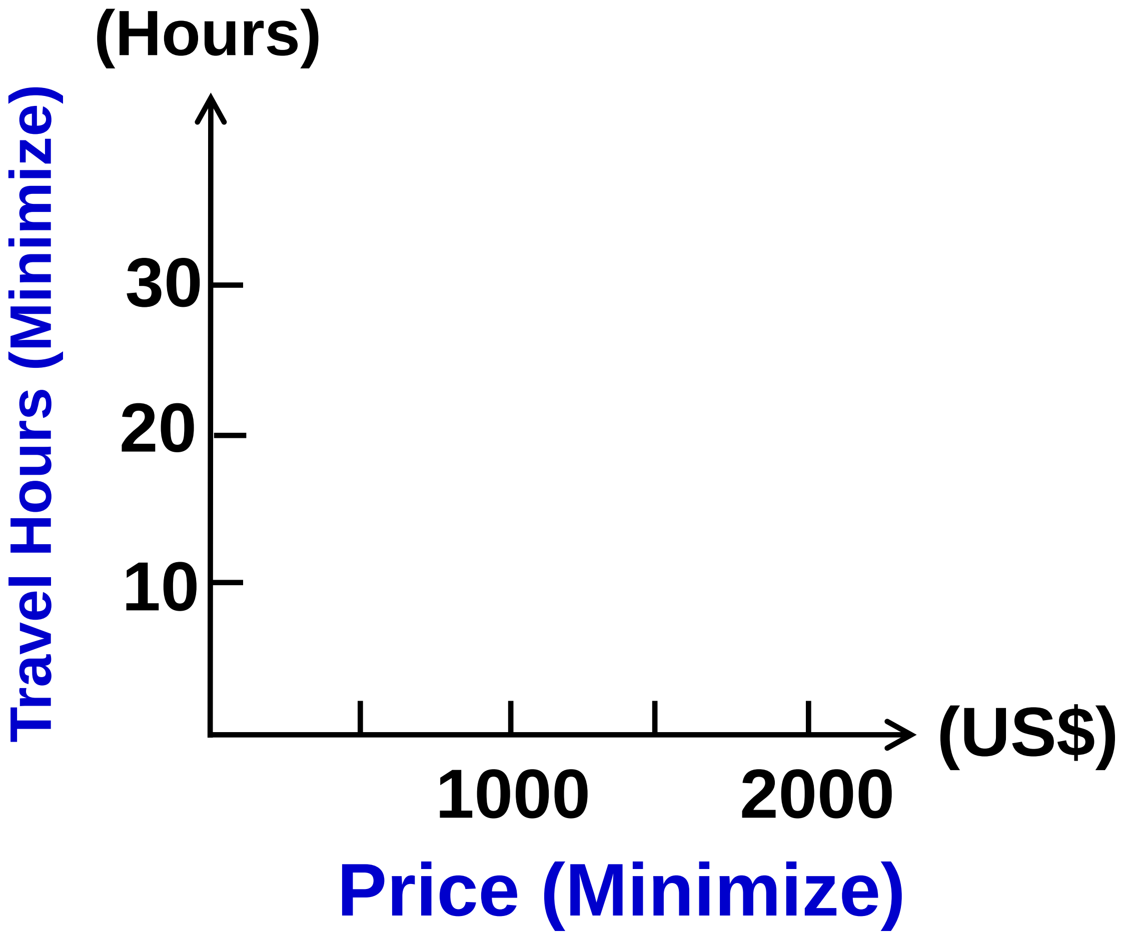
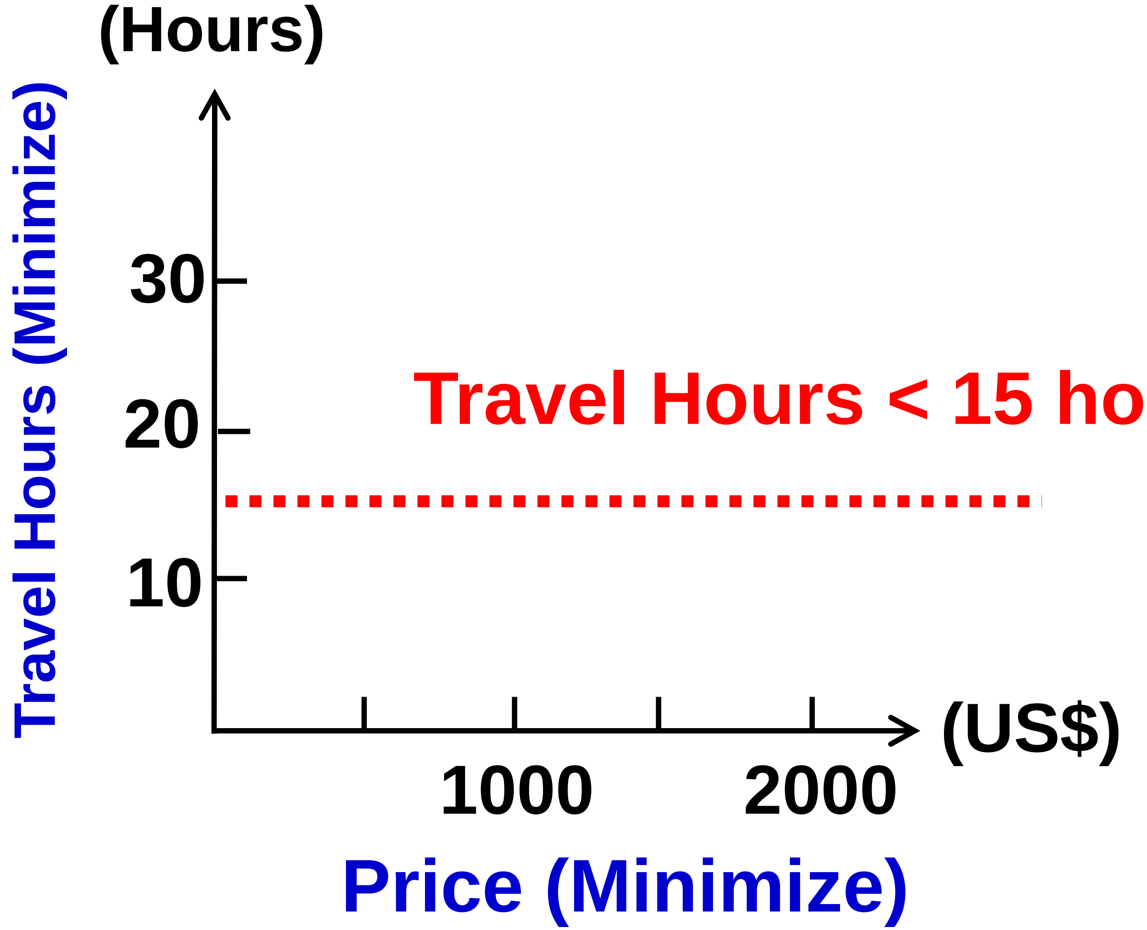


Examples: Air Ticket to New York

If we know the problem very well, it may be easy to give an appropriate constraint condition.



Examples: Air Ticket to New York

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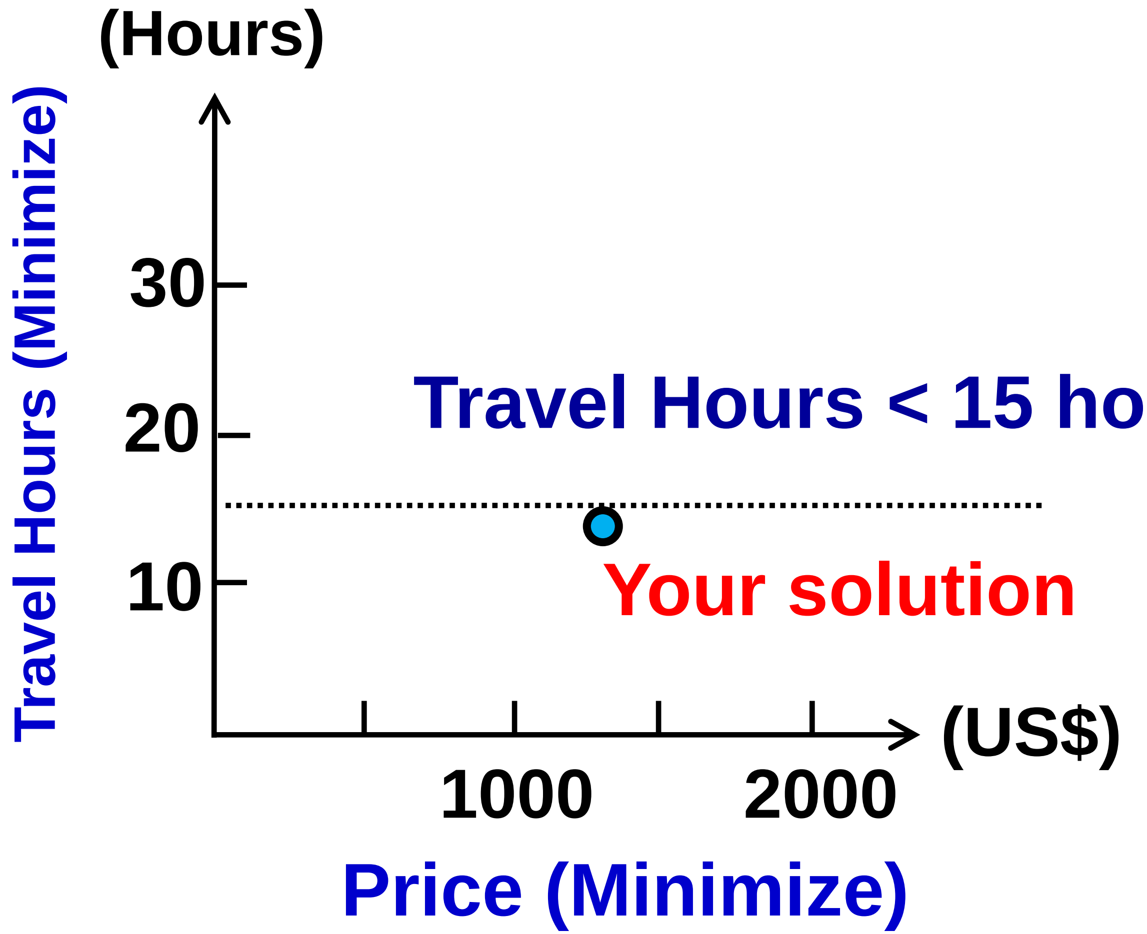
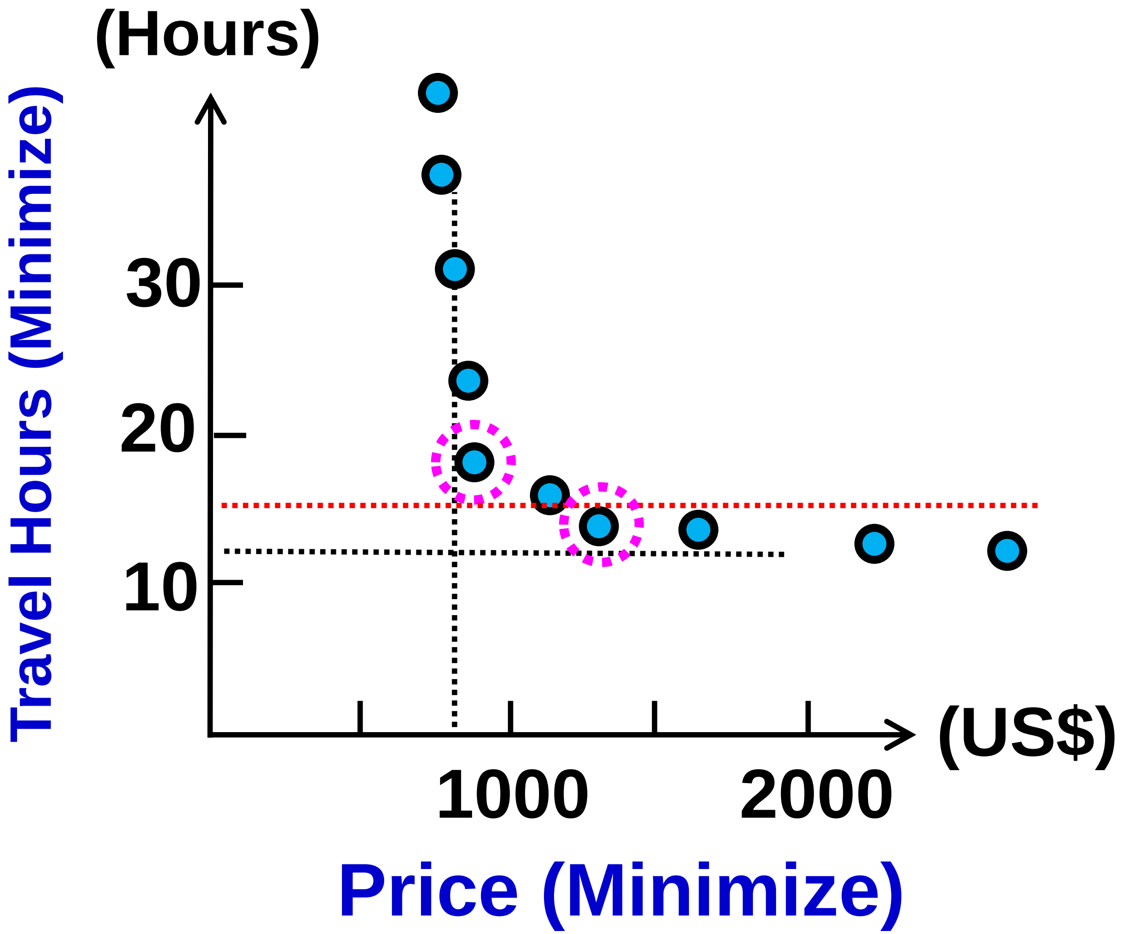


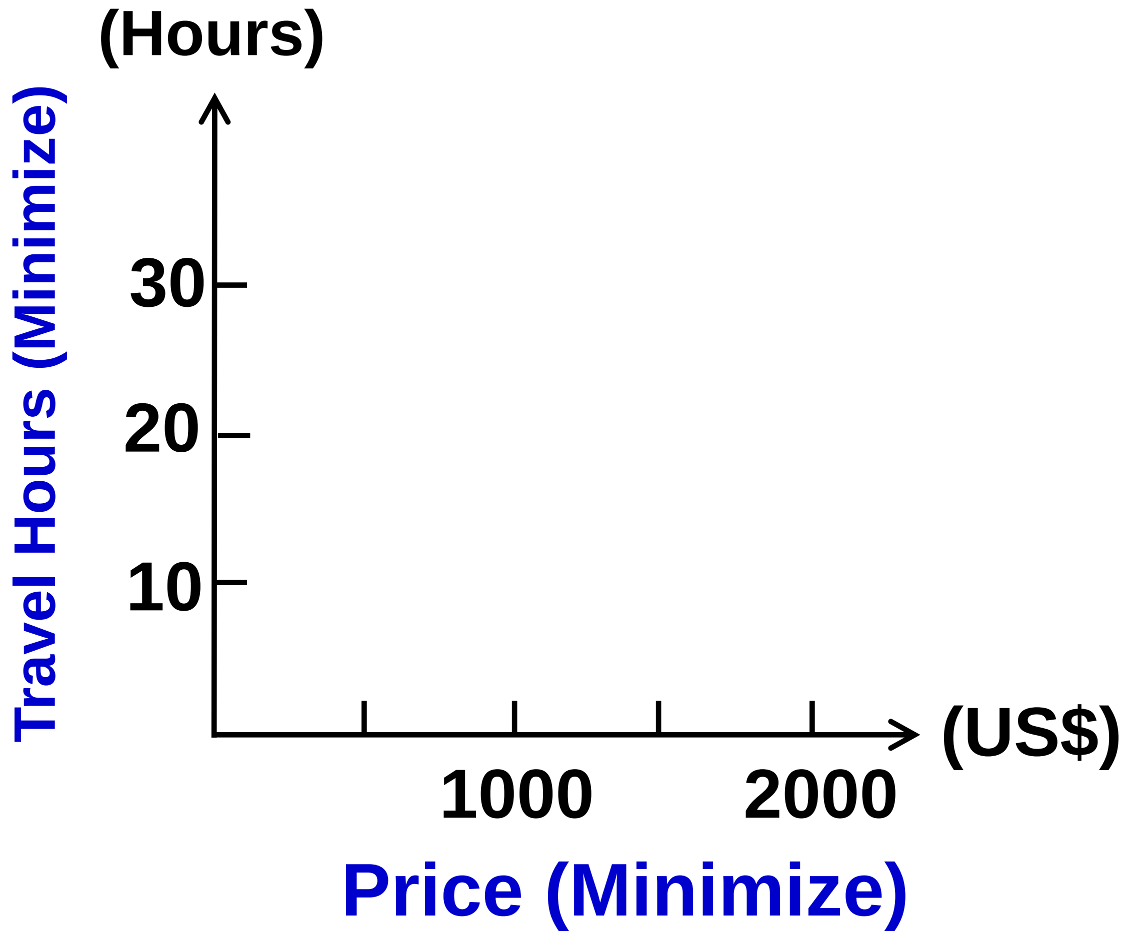
Examples: Air Ticket to New York

You may want to see other solutions.
EMO approach can show you all of them.



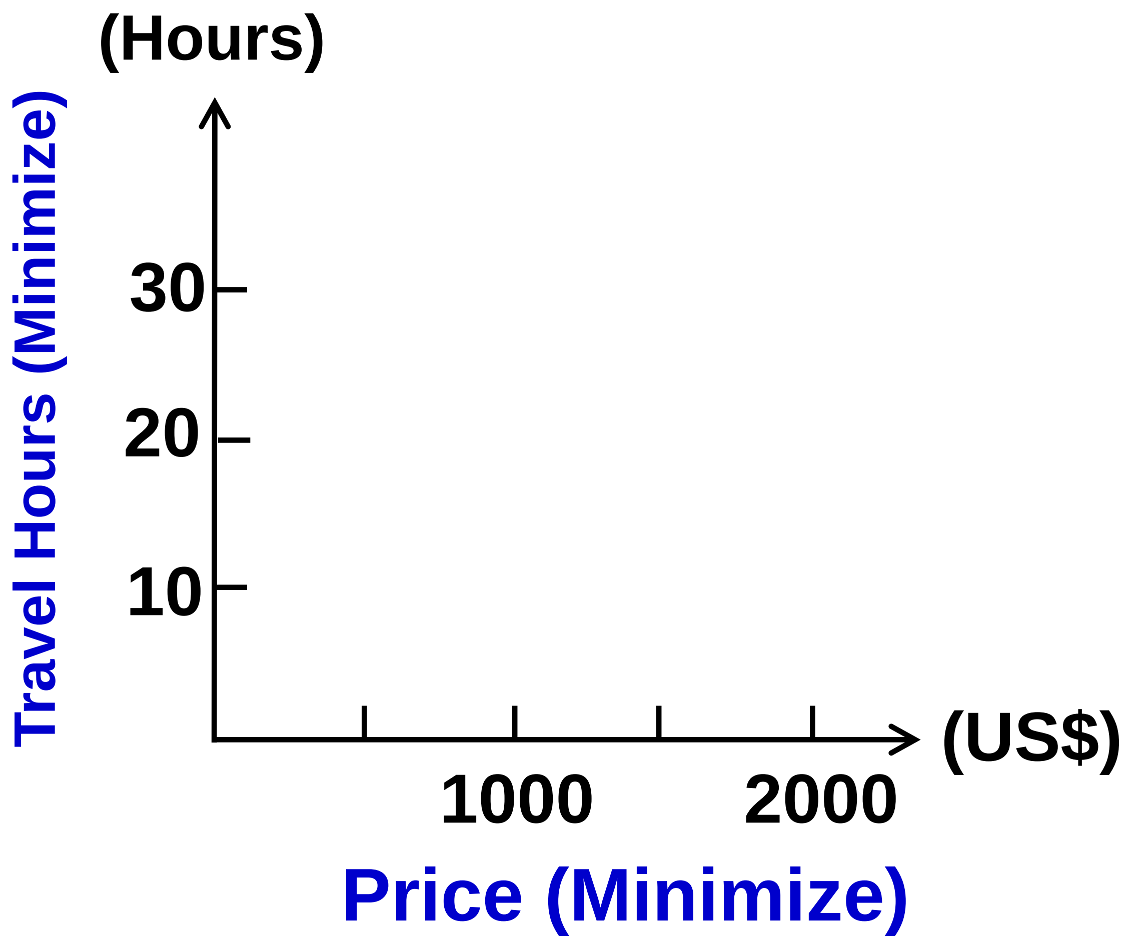
Examples: Air Ticket to Cape Town

If we do not know the problem very well, it may be difficult to give an appropriate target.



Examples: Air Ticket to Cape Town

If we do not know the problem very well, it may be difficult to give an appropriate constraint.



Examples: Air Ticket to Cape Town

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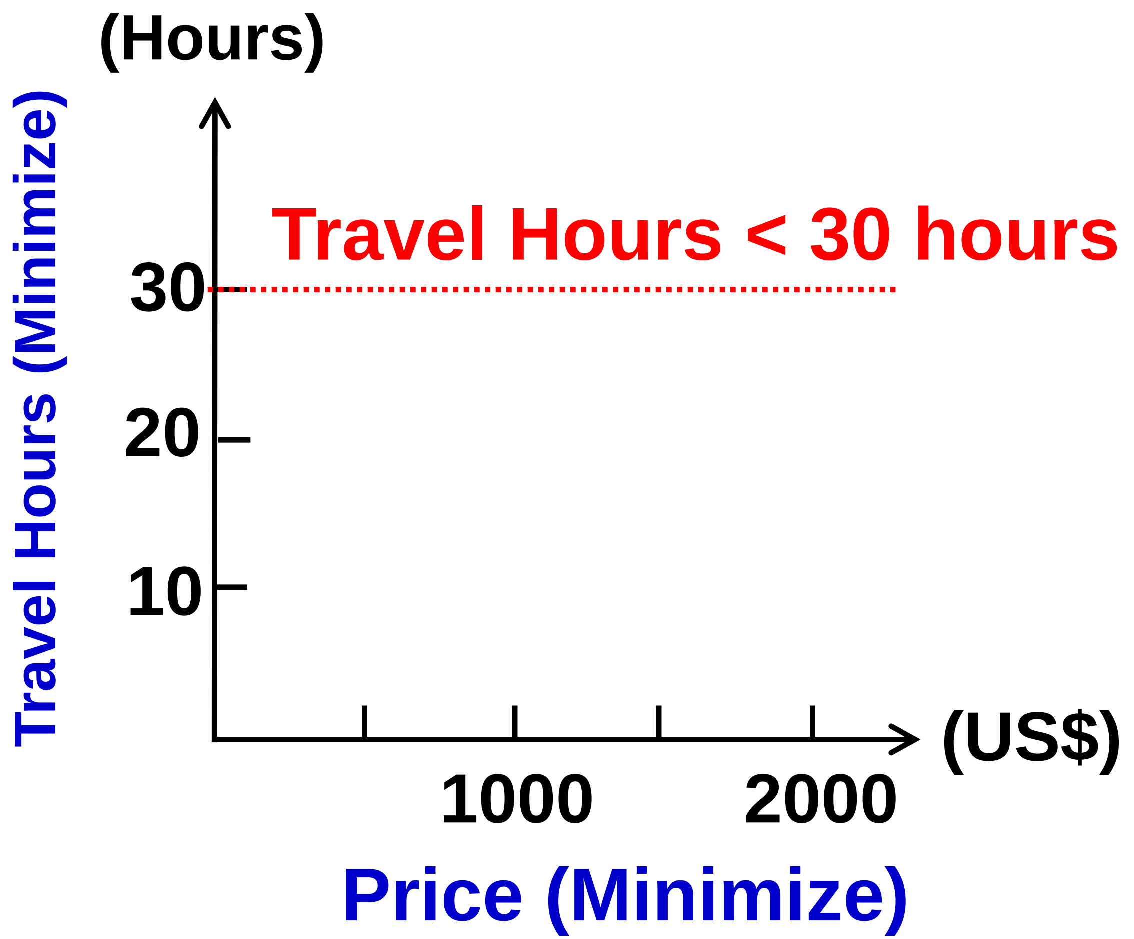

Examples: Air Ticket to Cape Town

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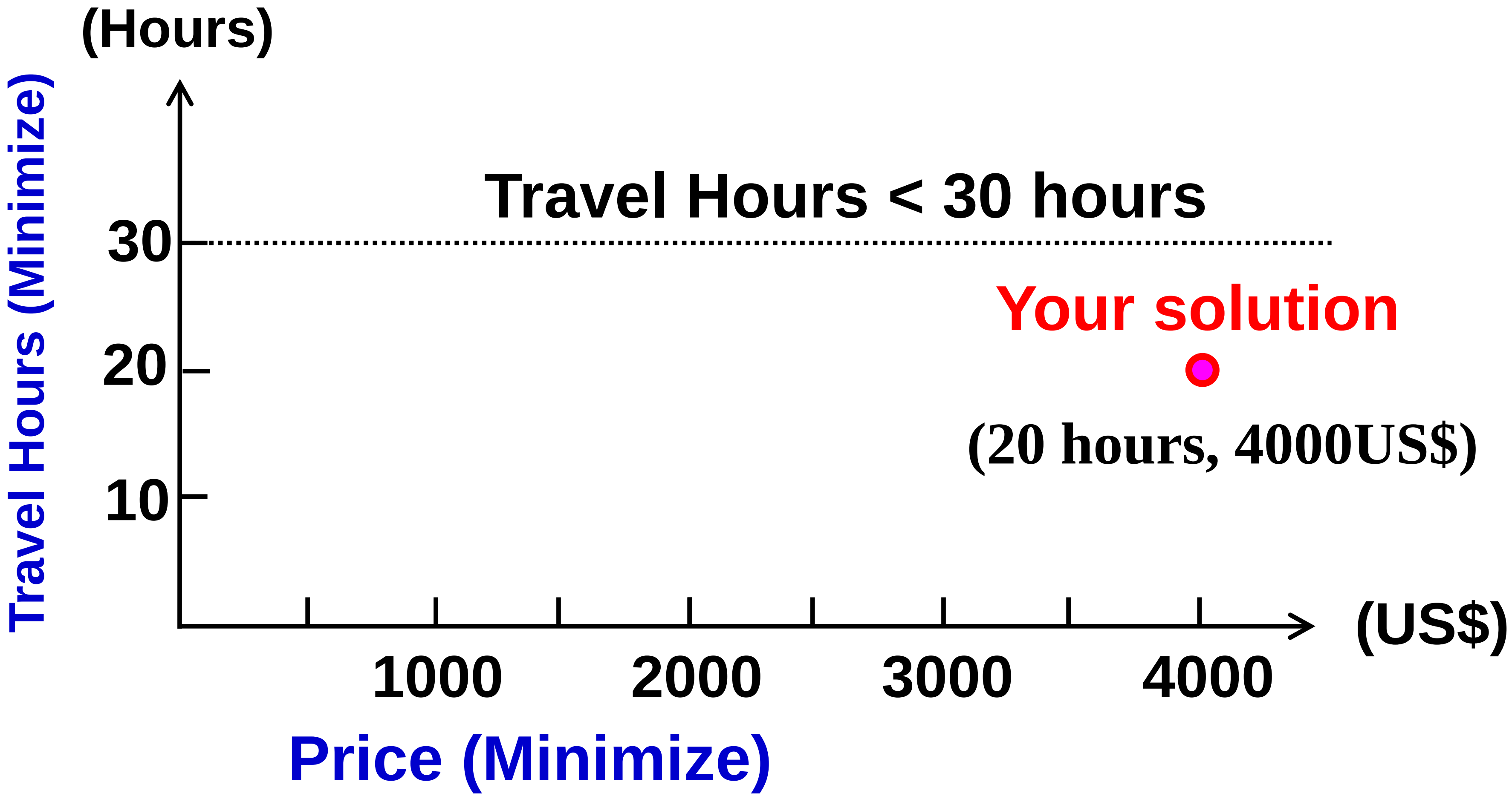

Examples: Air Ticket to Cape Town

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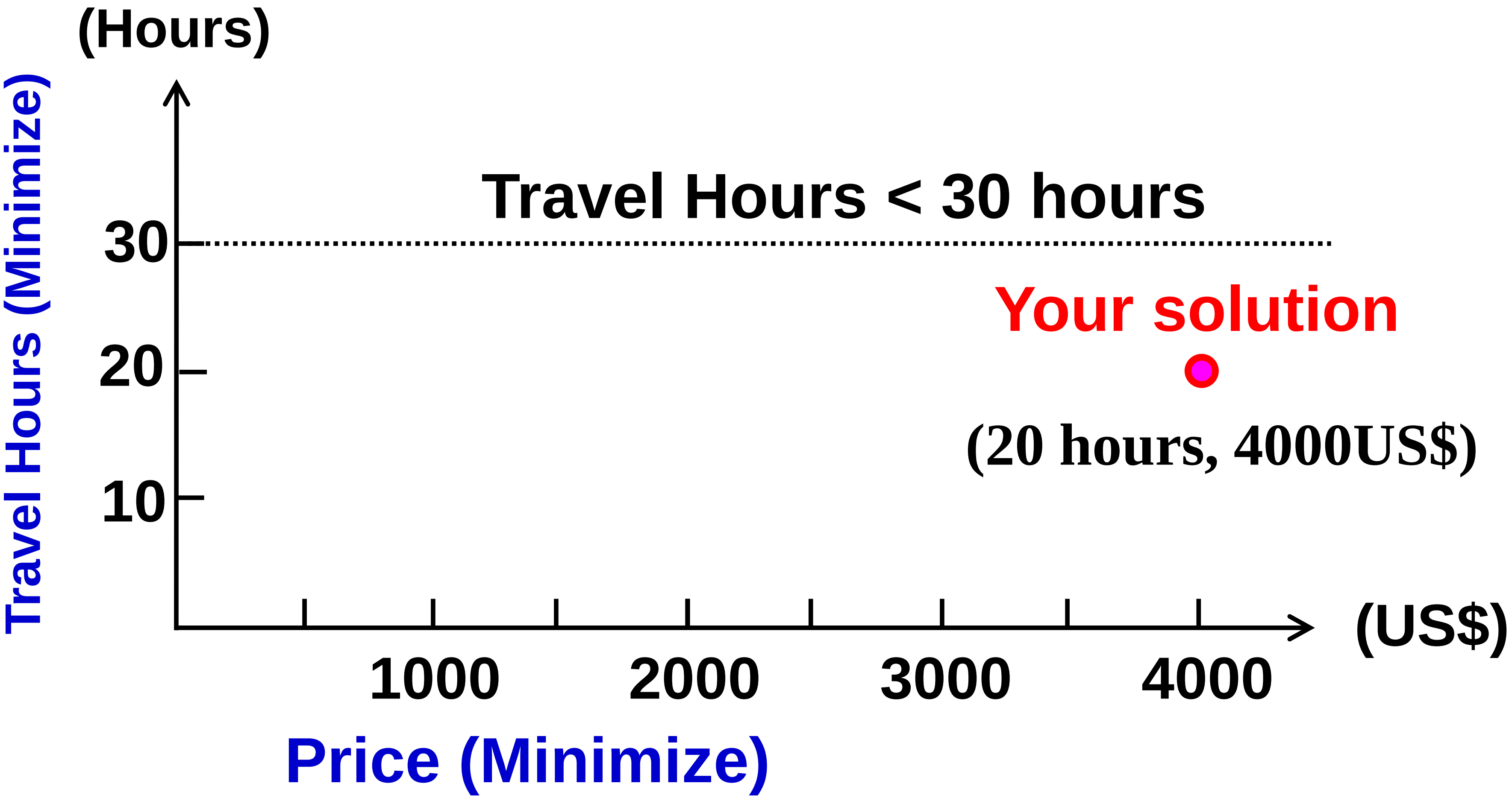

Examples: Air Ticket to Cape Town

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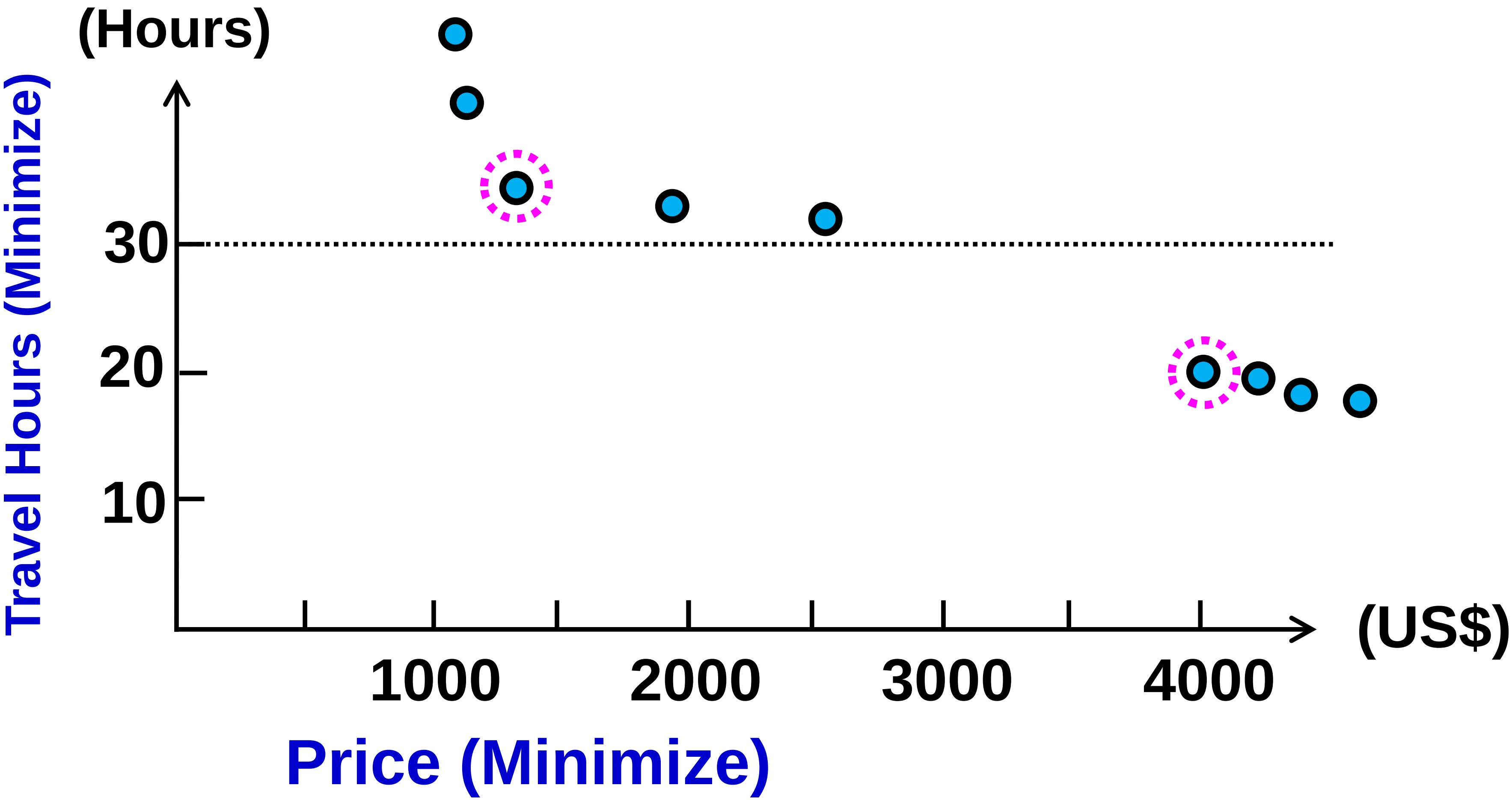
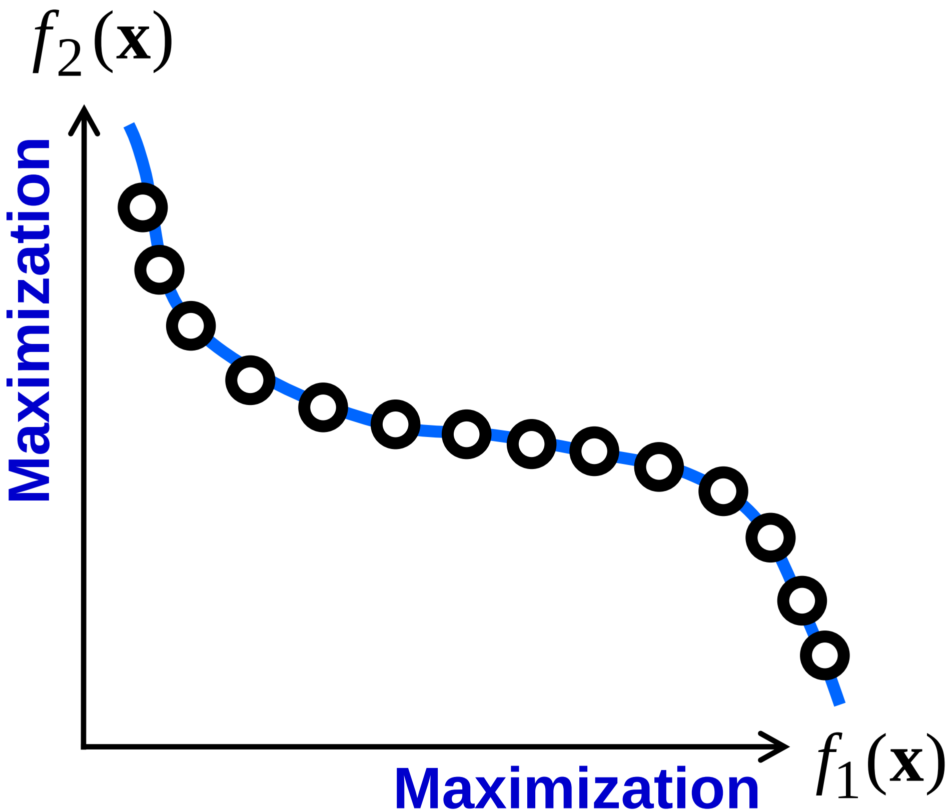
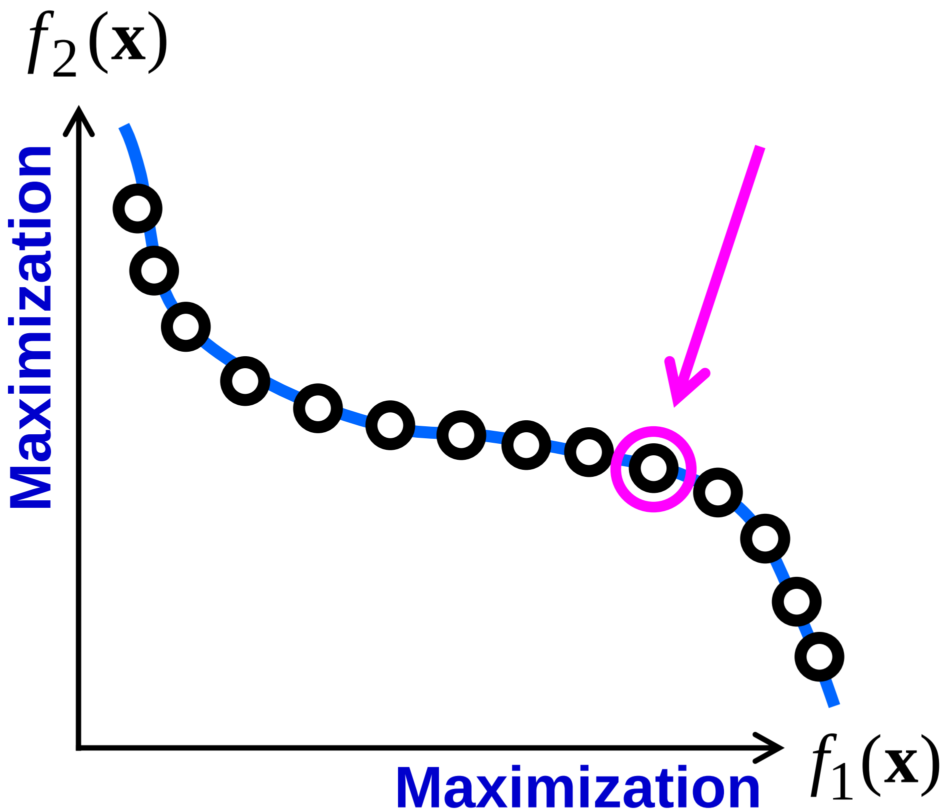


Advantages of EMO Approach:

Many solutions are shown to the decision maker.





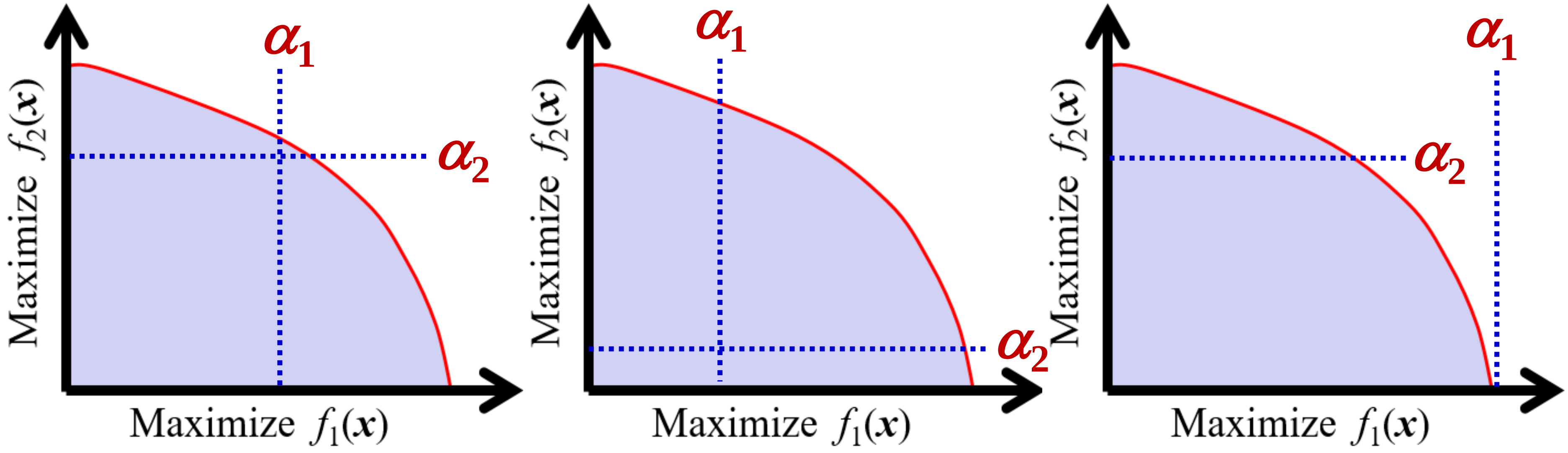
Step 1: Search for Pareto optimal solutions by an EMO algorithm.

Step 2: Choice of a single final solution.

Lab Session Task 1:

Original Two-Objective Problem: Maximize $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$

Let us assume that we have the following inequality conditions from the decision maker: $f_1(\boldsymbol{x}) \geq \alpha_1$ and $f_2(\boldsymbol{x}) \geq \alpha_2$.
Since the decision maker does not know the true Pareto front, these inequalities can be infeasible. Please design an algorithm to find the best solution for the decision maker.

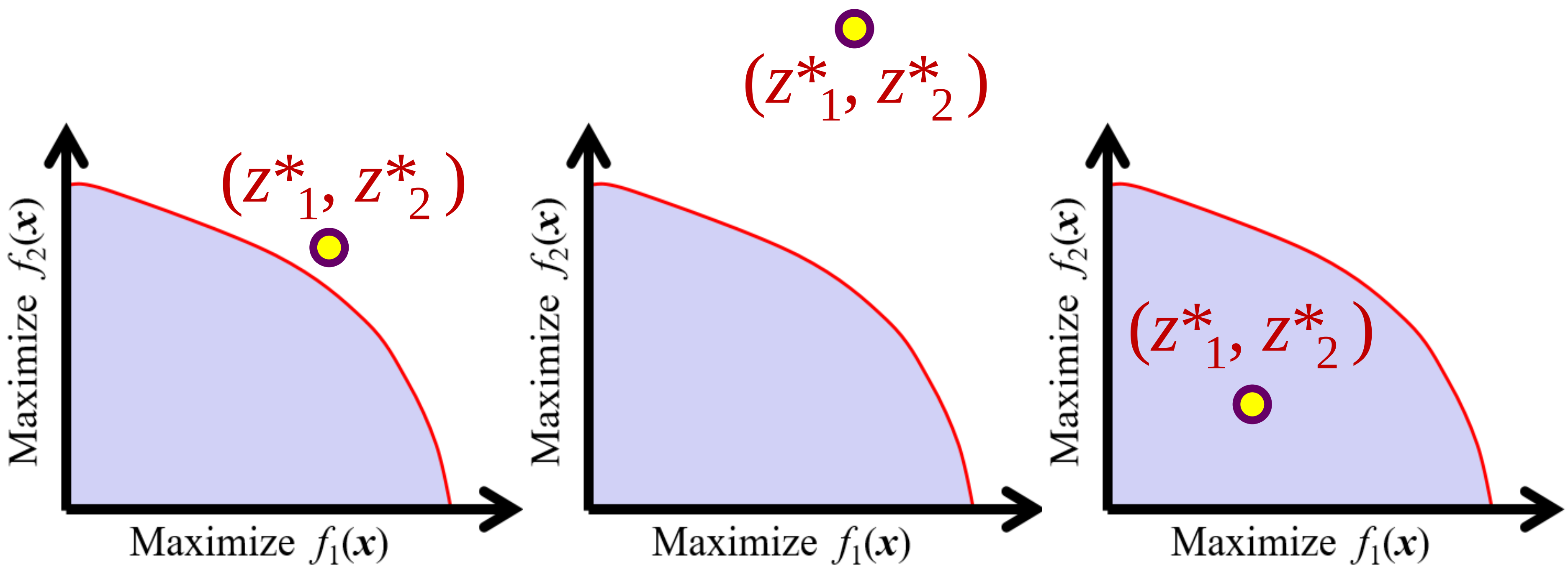


Lab Session Task 2:

Original Two-Objective Problem: Maximize $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$

Let us assume that we have the following target (ideal) point from the decision maker: $(f_1(\mathbf{x}), f_2(\mathbf{x})) = (z^*_1, z^*_2)$.

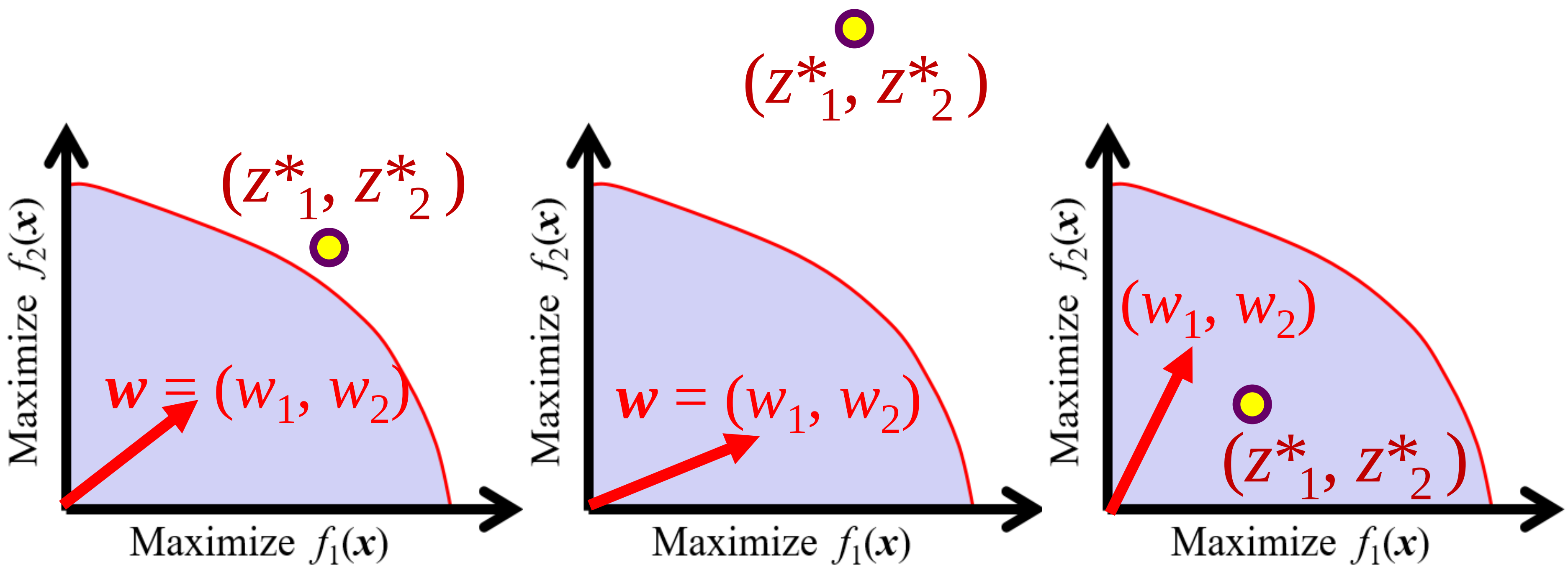
Since the decision maker does not know the true Pareto front, this target point can be inside the feasible region. Please design an algorithm to find the best solution for the decision maker.



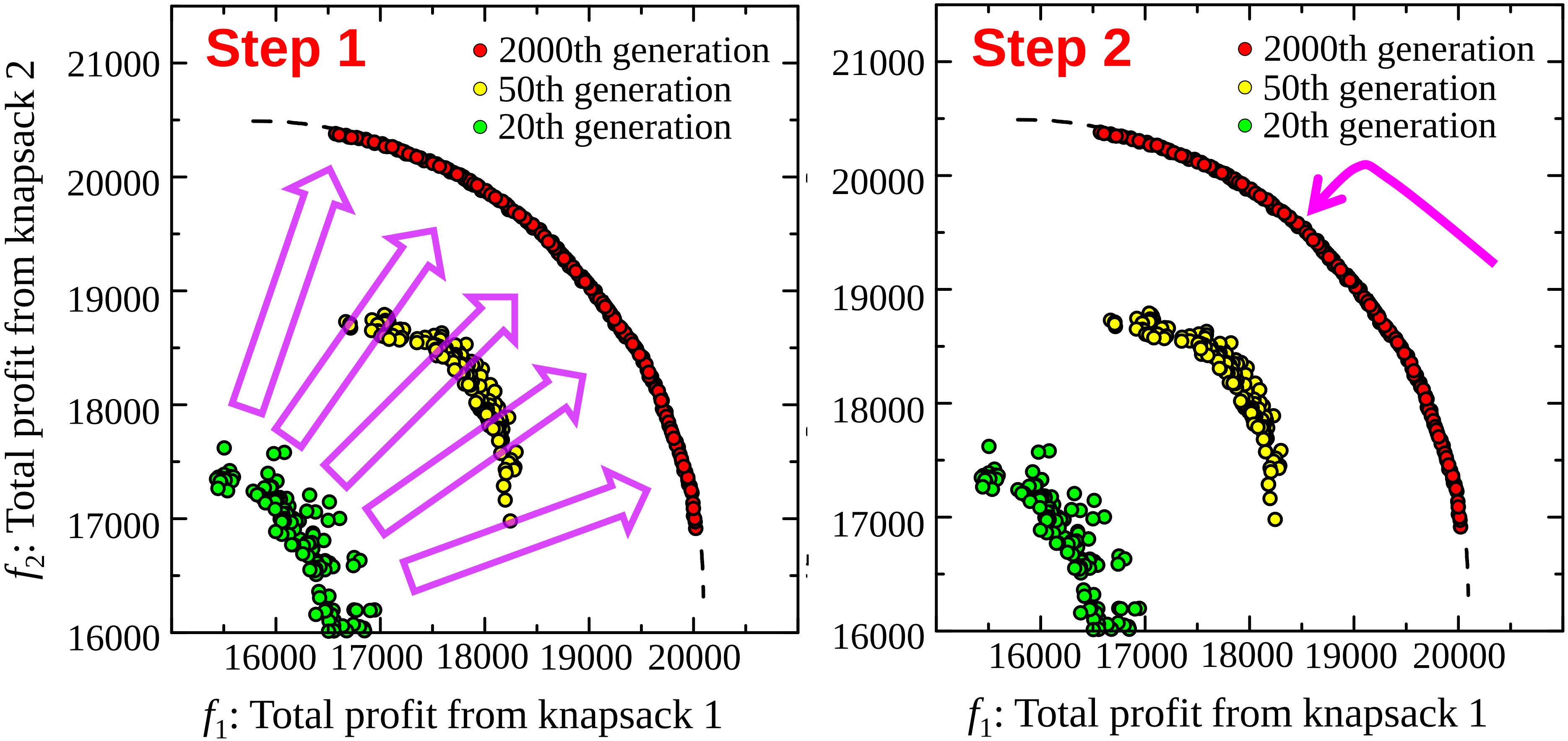
Lab Session Task 3:

Original Two-Objective Problem: Maximize $f_1(\boldsymbol{x})$ and $f_2(\boldsymbol{x})$

Let us assume that we have the target (ideal) point $\boldsymbol{z}^* = (z^*_1, z^*_2)$ and the weight vector $\boldsymbol{w} = (w_1, w_2)$ from the decision maker. Using them, please design a function to find a final solution for the decision maker. The weighted sum $f(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x})$) is a simple example (which ignores the target point).



Basic Idea of Decision Making in EMO
(Evolutionary Multiobjective Optimization)

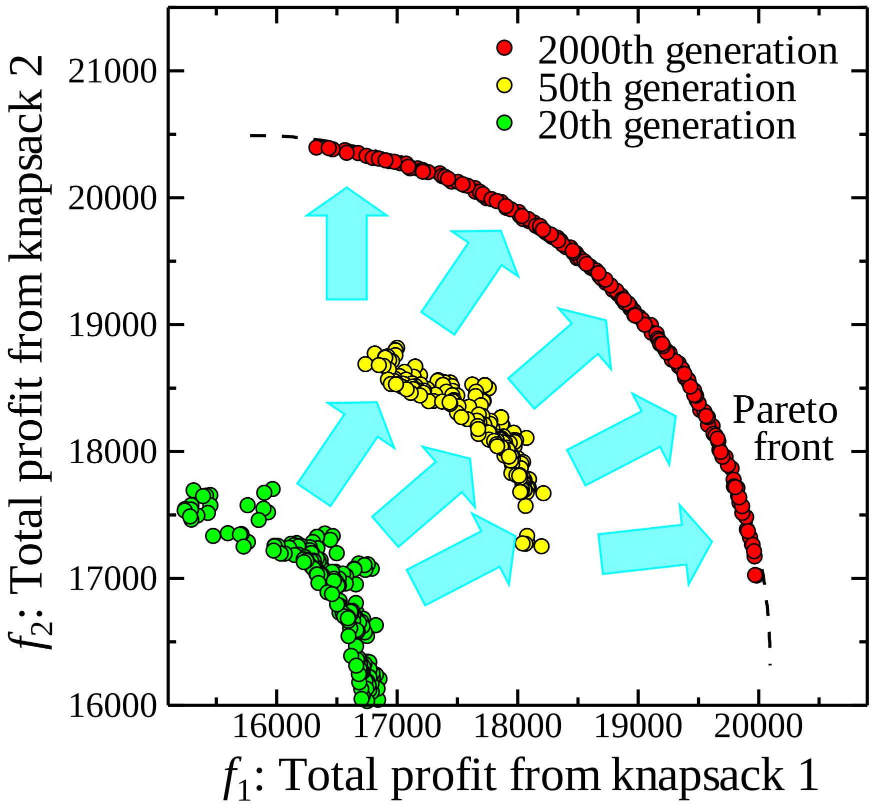


Step 1: Search for non-dominated solutions along the Pareto front.

Step 2: Selection of a single solution from the obtained solutions by the decision maker.

EMO (Evolutionary Multi-Objective Optimization)
= Evolutionary Search for Pareto Optimal Solutions

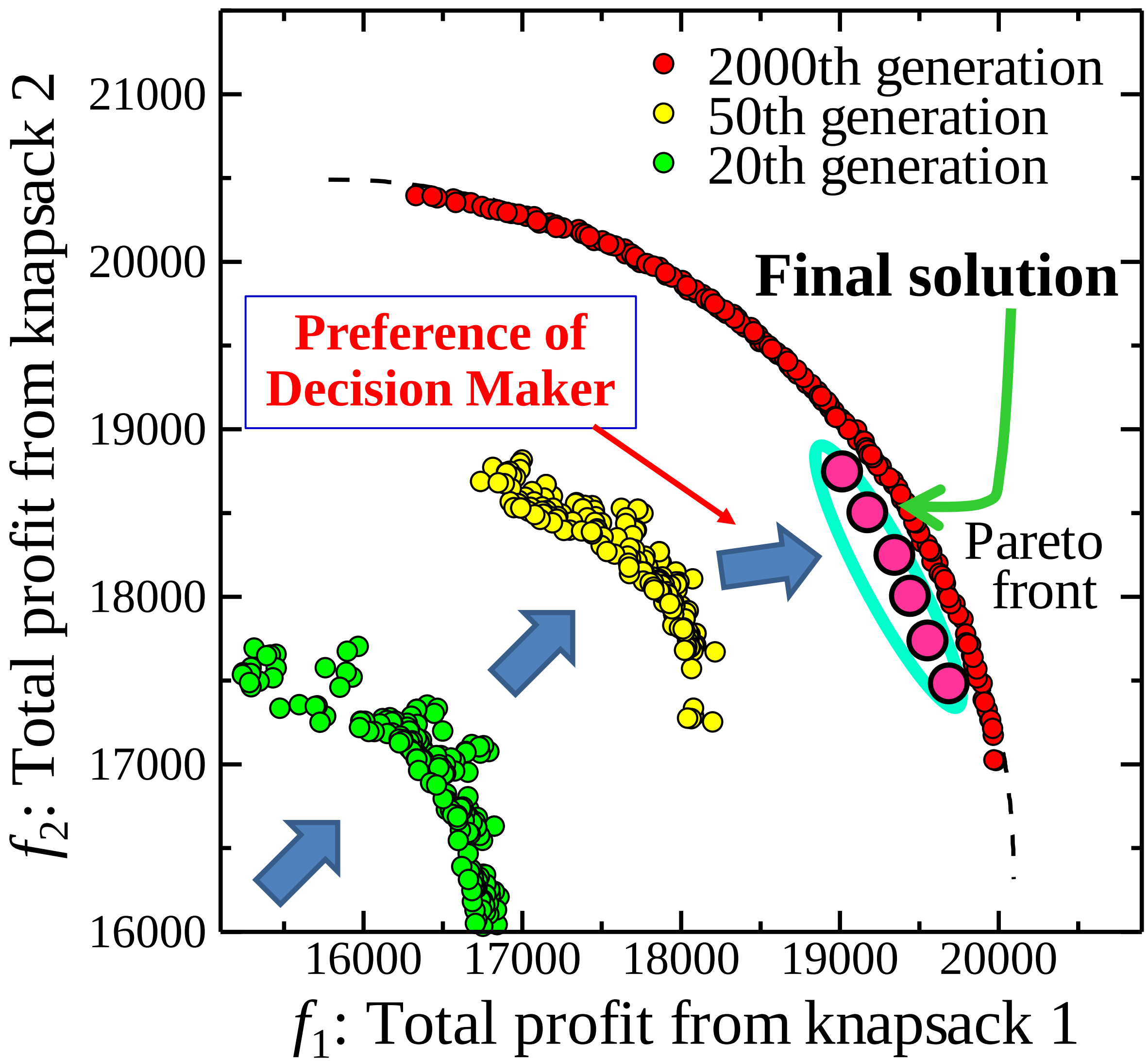
Important Issue in EMO-Approach: How to search for a well-distributed solution set along the Pareto front.



Desired search behavior of EMO algorithms

Combination of Two Approach. Interactive EMO Algorithms

If the decision maker's preference is available in the middle of evolution, it is a good idea to focus on the preferred region.



Many EMO algorithms and test problems are available through the Internet:

jMetal (for Java users)

J.J. Durillo, and A. J. Nebro, “jMetal: A Java framework for multi-objective optimization,” Advances in Engineering Software (2011).

PlatEMO (for MATLAB users)

Y. Tian, R. Cheng, X. Zhang, and Y. Jin, “PlatEMO: A MATLAB platform for evolutionary multi-objective optimization,” *IEEE Computational Intelligence Magazine* (2017)

Pymoo (for Python users)

J.Blank and K. Deb, “Pymoo: Multi-objective optimization in Python,” *IEEE Access* (2020).