Optimization Methods
Lab 15 Session

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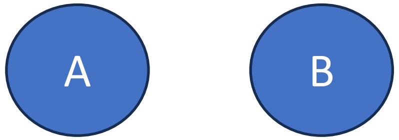
Task1

Let’s analyse the first turn, the people who leave must satisfy only 2 colors are same of all people.



Suppose there are only 2 people are blue among all people. A knows that there is only one blue guy. Since at least one participant has the same color as his/her color since all of them were true logicians, he can know he is blue, similar with B. Therefore, at the first turn, A and B will leave.

Let’s analyse the second turn, the people who leave must satisfy only 3 colors are same of all people.

Suppose there are only 3 people are green among all people. According to the previous analysis, we can know they won’t leave at the first turn. In each person’s view, they can see only two same guys are left after the first turn, so he/she knows there must another one is green, but he/she can just see 2 green guys, so he knows that himself is green. So these 3 guys C,D,E will left at the second turn.

Similarly, we can get a conclusion that at i turn, color with i+1 person will leave.

Let’s come back to this problem.

1. 4 leave → 2 different color (eg. 2 green, 2 blue)
2. All red guys leave → 3 red guys totally
3. Nobody left → there don’t exist 4 persons with same color
4. At least one left → the color with 5 person left(5*k k>=1)
5. Two different color left and other person → 6*(2+m)=12 people leave(m >= 0)
6. There are (31-4-3-12-5*k-6*m)= 12 → k must is 1, since if k = 2, in this terms there just left 2 guys which leads to a conflict. Similar with m, it must equal to 0. Therefore, in 6 terms, there are only 7 people left and they will leave in this terms.

So, we just need to ring 6 times.

Task2

Distribution of Solutions

(i) **Solutions well distributed over the entire Pareto front:**

•This implies that the solutions are spread uniformly across the Pareto front, covering the entire front without any gaps. This distribution ensures that the hypervolume is maximized because it explores all possible combinations of objective values within the defined front.

(ii) **Solutions on the boundary of the Pareto front:**

•Here, most solutions lie on the boundary (extremes) of the Pareto front. This can also maximize the hypervolume because the boundary solutions are typically the most extreme (and hence the furthest away) from the reference point, contributing significantly to the hypervolume measure.

Both distributions can maximize the hypervolume for the following reasons:

1.Well Distributed Solutions:

1. When solutions are well distributed, the hypervolume is maximized because every possible combination of f1,f2,f3 values is considered within the Pareto front constraints. This leads to a comprehensive coverage of the objective space, ensuring that no potential volume is left out. Therefore, each small volume element contributes to the overall hypervolume.

2.Boundary Solutions:

1. In contrast, solutions on the boundary, especially at the extreme points, have the advantage of being farthest from the reference point. These extreme points can cover a significant volume individually. When these boundary points are chosen strategically, they can create a large hypervolume by encompassing a substantial part of the objective space. Essentially, the distance of these points from the reference point (1000, 1000, 1000) is maximized, which in turn maximizes the hypervolume.