Optimization Methods
Lab 12 Session

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(1) We randomly generate two points A and B in $[0, 1]^2$. Calculate the probability of A being dominated by B.

mathematically calculate the solution

Two points $A = (a_1, a_2)$ and $B = (b_1, b_2)$ are randomly generated in $[0, 1]^2$. Point A is dominated by point B if $b_1 \geq a_1$ and $b_2 \geq a_2$.

The probability that $b_1 \geq a_1$ is $\frac{1}{2}$ since a_1 and b_1 are uniformly distributed and independent. Similarly, the probability that $b_2 \geq a_2$ is also $\frac{1}{2}$.

Since these events are independent:

$$P(A \text{ is dominated by } B) = P(b_1 \geq a_1) \times P(b_2 \geq a_2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

numerically estimate the solution

0.248585(1000 times)

(2) We randomly generate two points A and B in $[0, 1]^4$. Calculate the probability of A being dominated by B.

mathematically calculate the solution

Using the same reasoning as in problem 1, for d-dimensional space, the probability of one coordinate being dominated is $\frac{1}{2}$. Hence, for four dimensions:

$$P(A\ is\ dominated\ by\ B) = \left(\frac{1}{2}\right)^4$$

numerically estimate the solution

0.062072(1000 times)

(3) We randomly generate two points A and B in $[0, 1]^{10}$. Calculate the probability of A being dominated by B.

mathematically calculate the solution

Similarly, for ten dimensions:

$$P(A\ is\ dominated\ by\ B) = \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$$

numerically estimate the solution

0.000967(1000 times)

(4) We randomly generate 200 points in $[0, 1]^2$. Calculate the expected number of non-dominated solutions among them.

mathematically calculate the solution

$$200\int_0^1\int_0^1[1-(1-x)(1-y)]^{199}\mathrm{dxdy}$$

$$=200\int_0^1\int_0^1[x+y-\mathrm{xy}]^{199}\mathrm{dxdy}$$

$$=200\int_0^1\left[\frac{(x+y-\mathrm{xy})^{200}}{200(1-y)}\right]^{-1}\mathrm{dxdy}$$

$$=\int_0^1(1-y)^{199}\mathrm{dxdy}$$

$$=\int_0^1 1+y+y^2+\cdots+y^{199}\mathrm{dy}$$

$$=1+\frac{1}{2}+\cdots+\frac{1}{200}$$

$$=5.87$$

numerically estimate the solution

5.74(1000 times)

(5) We randomly generate 2000 points in $[0, 1]^2$. Calculate the expected number of non-dominated solutions among them.

mathematically calculate the solution

Similar with (4), we can get

$$2000\int_0^1\int_0^1 [1-(1-x)(1-y)]^{1999}\mathrm{dxdy}$$

$$=1+\frac{1}{2}+\cdots+\frac{1}{2000}$$

$$=8.18$$

numerically estimate the solution

8.174(1000 times)

(6) We randomly generate 200 points in $[0, 1]^{10}$. Calculate the expected number of non-dominated solutions among them.

mathematically calculate the solution

$$200\int_0^1 \int_0^1 \left[1-\prod_{i=1}^{10}(1-x_i)\right]^{199} \mathrm{dx}_1 \mathrm{dx}_2 \ldots \mathrm{dx}_{10}$$

numerically estimate the solution

180.315(1000 times)

(7) We randomly generate 2000 points in $[0, 1]^{10}$. Calculate the expected number of non-dominated solutions among them.

mathematically calculate the solution

$$2000\int_0^1 \int_0^1 \left[1-\prod_{i=1}^{10}(1-x_i)\right]^{1999} \mathrm{d}x_1 \mathrm{d}x_2 \ldots \mathrm{d}x_{10}$$

numerically estimate the solution

1381.012(1000 times)