Optimization Methods
Lab 9 Session

By Shi Jimao

Task1

Problem 1: Minimize $x_1 + x_2$

subject to $x_1 + x_2 \geq 1$

$\mathbf{0 \leq x \leq 1}$

Since $x_1 + x_2 \geq 1$, to minimize $x_1 + x_2$, it is simple to find know that the minimum is 1.

$$x_1 = t, x_2 = 1 - t \ \ where\ 0 \leq t \leq 1$$

Task1

Problem 2: Minimize $x_1 + x_2 + x_3 + x_4$

$$
\begin{aligned}
\text{subject to } & x_1 + x_2 = 1 \\
& x_2 + x_3 = 1 \\
& x_3 + x_4 = 1 \\
& x_1 + x_4 = 1 \\
& \mathbf{0 \le x \le 1}
\end{aligned}
$$

If we add the first and third constraint, we can directly get that $x_1 + x_2 + x_3 + x_4 = 1$, which means that the minimum value is 1

Task2

In this task , I use these 3 LP solvers: **GLPK (GNU Linear Programming Kit), Gurobi, PuLP**, and get the result for two problems shown in the following tables.

Problem1

	x1	x2	Object function
GLPK	1.0	0.0	1.0
Gurobi	1.0	0.0	1.0
PuLP	1.0	0.0	1.0

Problem2

	x1	x2	x3	x4	Object function
GLPK	0.0	1.0	0.0	1.0	2.0
Gurobi	1.0	0.0	1.0	0.0	2.0
PuLP	0.0	1.0	0.0	1.0	2.0

Task3

Task 3: Compare the following four methods on your 100-item knapsack problem (used in the lab session on Simulated Annealing) with respect to the solution quality (i.e., the objective function value), and the total computation time.

(i) A heuristic method used to generate an initial solution for SA.

(ii) Simulated algorithm with your parameter setting

(iii) Use of an LP solver (i.e., use of the LP relaxation problem), and create a feasible solution from the LP solutions.

(iv) Use of an integer LP (ILP) solver

Approach to change LP to integer problem:
Set boundary = 1;
Set element = (element<boundary)
While constraint is satisfied:
    Set element = (element<boundary)
    boundary-=0.01

	(i)	(ii)	(iii)	(iv)
Running time	0.06ms	200ms	1.14ms	431ms
Object value	1667	2953	2987	3093

Task4

200-item

	(i)	(ii)	(iii)	(iv)
Running time	0.43ms	413ms	2.14ms	631ms
Object value	4190	6248	6260	6341

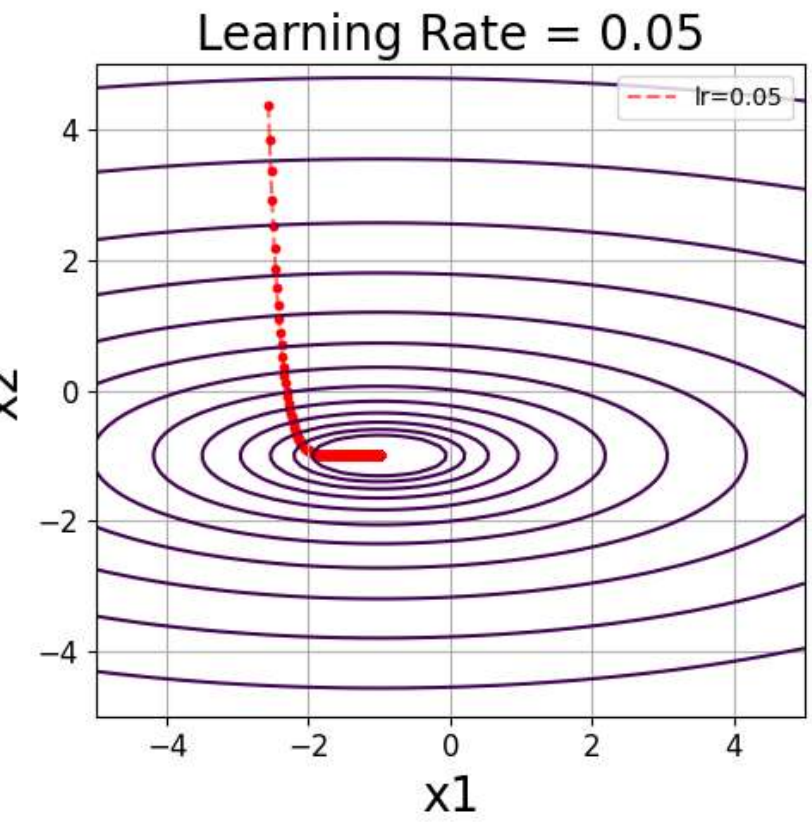
400-item

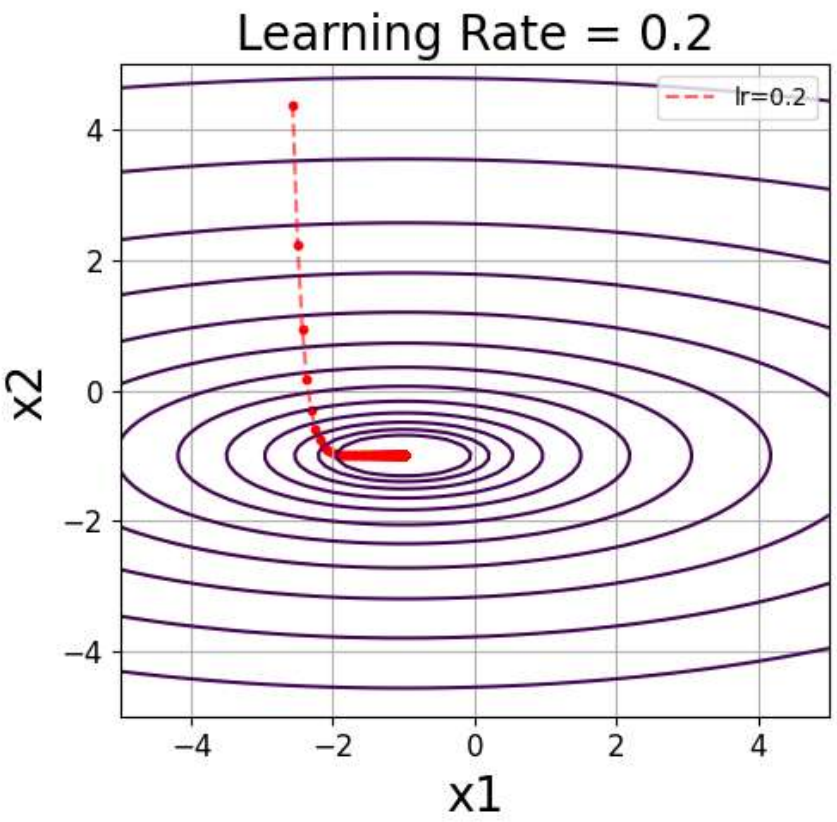
	(i)	(ii)	(iii)	(iv)
Running time	0.89ms	723ms	3.41ms	90067ms
Object value	9841	12918	12929	12936

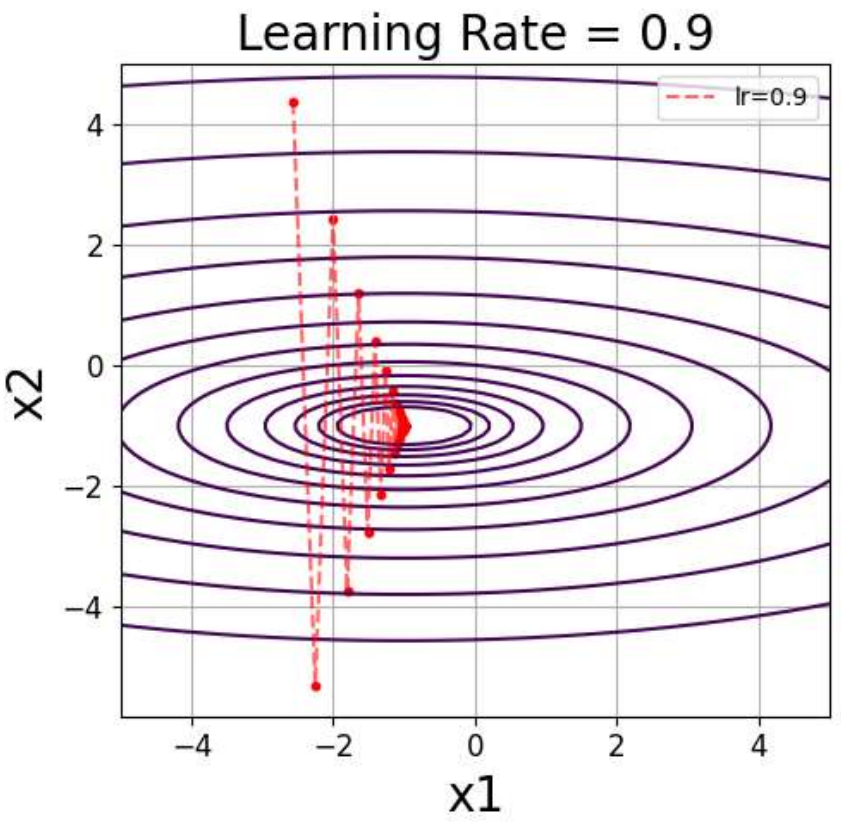
For using an integer LP (ILP) solver, the running time increase very fast when the scale of the problem grows, while using an LP solver increase nearly linearly.

Task5

I set 3 learning rate: 0.05,0.2 and 0.9







When learning rate is too small, it will take a long time to search at beginning. And if learning rate is too large, it will cause shaking. Therefore, select learning rate with a median value like 0.2 is reasonable .