Adding of two velocities.

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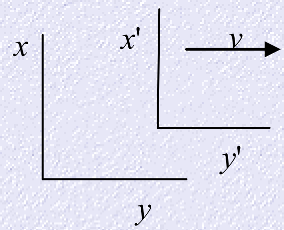
Introduction.

A frame is moving with a velocity *v* to the right with a respect to a frame at rest. In this moving frame an object is moving with the velocity *w* to the right. In this article we will show how these velocities add.

Moving frame and frame at rest.



The second frame is moving to the right with a velocity *v* m/sec.

According to the Lorentz transformation formulae the relation between the space and time, in the frame at rest and moving frame, is:

$$x' = \frac{x - v.t}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad t' = \frac{t - \frac{v.x}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The velocity *w* in the moving frame is:

$$\frac{x'}{t'} = \frac{x - v.t}{t - \frac{v.x}{c^2}} = \frac{\frac{x}{t} - v}{1 - \frac{v}{c^2}.\frac{x}{t}} = w$$

Hence:

$$\frac{x}{t} - v = w - w.\frac{v}{c^2}.\frac{x}{t} \Rightarrow \frac{x}{t}.\left(1 + \frac{w.v}{c^2}\right) = w + v \Rightarrow \frac{x}{t} = \frac{w + v}{1 + \frac{w.v}{c^2}}$$

Thus adding the velocity *v,* of the moving frame with respect to the frame at rest, and the velocity *w* of the object with respect to the moving frame, results in the velocity of the moving object with respect to the frame at rest:

$$\frac{x}{t} = \frac{w + v}{1 + \frac{w.v}{c^2}}$$

Thus if w and v are small the resulting velocity is $w+v$, which is well known in the Newtonian calculation. If w and/or v are equal to the velocity of light c then the resulting velocity is C.