Derivation of the Lorentz Transformation.

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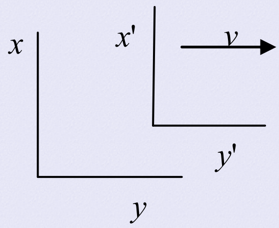
Introduction.

In this article the Lorentz transformations, providing the important formulae for the Relativity Theory, are derived.

For this purpose two coordinate systems are considered. One is in rest while the other is moving with a velocity v m/sec to the right. The most important assumption here is that the velocity of light c is the same in both coordinate systems

Derivation.



The second coordinate system is moving to the right with a velocity v m/sec.

The origins of both systems on time zero coincide. The speed of light is the same in both systems so a light flash on moment zero in the origin of both systems will result in:

$$x(t) = ct \quad \text{and} \quad x' = ct'$$

Transformation from the first system to the second (moving to the right):

$$x' = \gamma(x - vt)$$

Transformation from the second system to the first system (in rest):

$$x = \gamma(x' + vt')$$

Here the factor γ has to be found.

Thus:

$$ct' = \gamma(x - vt)$$
$$ct = \gamma(x' + vt')$$

Multiplification of these two equations:

$$ct'.ct = \gamma^2(x - vt)(x' + vt') = \gamma^2(ct - vt)(ct' + vt')$$

$$=> c^2tt' = \gamma^2 t(c - v)t'(c + v) = \gamma^2 tt'(c - v)(c + v)$$

$$=> \gamma^2 = \frac{c^2}{(c-v)(c+v)} = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$=> \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

a)

Hence :

$$=> x^{'} = \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad => x = \frac{x^{'} + vt^{'}}{\sqrt{1-\frac{v^2}{c^2}}}$$

b)

$$x = \gamma(x^{'} + vt^{'}) => t^{'} = \frac{\frac{x}{\gamma} - x^{'}}{v}$$

and $x^{'} = \gamma(x - vt)$ result in

$$t^{'} = \frac{\left\{\frac{x}{\gamma} - \gamma(x - vt)\right\}}{v} = \frac{\gamma\left(\frac{x}{\gamma^2} - x + vt\right)}{v} = \frac{\gamma\left\{x\left(1 - \frac{v^2}{c^2}\right) - x + vt\right\}}{v}$$

$$t^{'} = \frac{\gamma\left(vt - \frac{v^2}{c^2}x\right)}{v} = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$=> t^{'} = \frac{t - \frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{and} \quad t = \frac{t^{'} + \frac{v}{c^2}x^{'}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$c)$

The formulae a), b) and c) are the Lorentz transformation formulae.