# Elaboration on why the velocity of light is always the same independent of a particular coordinate system.

## Introduction

The writer of this article had some queries on the thesis of Albert Einstein that the velocity of light is always the same in each uniform moving system; especially the case of a rocket, fast moving from us or towards us. The question is: why does a traveler in a rocket experience the velocity of light still as the standard Light Velocity C in all cases considered below?

## Considered Cases

1. A light pulse is emitted from the Earth towards a rocket that is moving away from the Earth.
From the Earth perspective the difference of the velocities of the light pulse and the rocket is small so it would take quite some time before the pulse travels from the incoming site of the rocket to the end of the rocket. One would expect that the velocity of light, in the rocket, would be perceived, by the traveler, as very low.

2. A light pulse is emitted from the Earth towards a rocket that is approaching the Earth.
In this case one would expect that the velocity of light, in the rocket, is very high. Together with the fact that the length of the rocket has decreased; the light would reach the far end of the rocket very fast.

3. A light pulse is emitted from the Earth to one side of the rocket to the other side, so perpendicular on the moving direction of the rocket. It is assumed that the rocket is just passing by the Earth. From the perspective of the Earth, the light travels in a straight line. However when it hits the other side, from the perspective of the rocket, it appears more to the left, in the opposite direction, of the, to the right, moving rocket.
Because the perpendicular case is considered, the speed of the rocket in this direction is zero with respect to the Earth, thus there is no contraction of the width of the rocket. The time in the rocket has still the well known time dilation related to the velocity of the rocket in the length direction. The clock in the rocket is slower, so one would expect that the inhabitant of the rocket experiences a greater velocity of the light.

4. This is the alternative of case 3. Now it is considered that the light pulse is emitted inside the rocket from one side to the other side, so perpendicular on the moving direction of the rocket. It is

assumed that the rocket is just passing by the Earth. From the perspective of the rocket, the light travels in a straight line. However when it hits the other side, with respect to the Earth, it appears more to the right, in the moving direction, of the rocket.
Because the perpendicular case is considered, the speed of the rocket in this direction is zero with respect to the Earth, thus there is no contraction of the width of the rocket. The time in the rocket has still the well known time dilation related to the velocity of the rocket in the length direction.

## Elaboration

*Case 1*:
From Earth's perspective the length contraction of the rocket is:

$$L = l\sqrt{1-\frac{v^2}{c^2}} = \frac{l}{\gamma} \ \ where \ \ \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$x_1^{'}$ : is the location of the entrance of the rocket
$x_2^{'}$ : is the location the end of the rocket
$t_1^{'}$ : is the time the light pulse enters the rocket
$t_2^{'}$ : is the time the light pulse reaches the end of the rocket

The velocity of the light pulse in the rocket is $v_l = \frac{x_2^{'}-x_1^{'}}{t_2^{'}-t_1^{'}} = \frac{l}{t_2^{'}-t_1^{'}}$ . Now we have to show that $v_l \equiv \boldsymbol{C}$ being the standard Light Velocity.

From Earth point of view we can derive two equations for the distance the light travels from the entrance of the rocket to the end of the rocket:

$$c(t_2-t_1) = \ l\ \sqrt{1-\frac{v^2}{c^2}} + \ v(t_2-t_1) \ \rightarrow (t_2-t_1) = \frac{l\ \sqrt{1-\frac{v^2}{c^2}}}{c-v}$$

$$c(t_2-t_1) = \ (x_2-x_1)$$

According to the Lorentz formula

$$t^{'} = \frac{t-\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \ \rightarrow \ t_2^{'}-t_1^{'} = \frac{(t_2-t_1)-\frac{v}{c^2}(x_2-x_1)}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{(t_2 - t_1) - \frac{v}{c^2} c(t_2 - t_1)}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(t_2 - t_1)\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\frac{l\sqrt{1 - \frac{v^2}{c^2}}}{c - v}\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}} = l\frac{1 - \frac{v}{c}}{c - v} = \frac{l}{c}$$

Now $v_l$ the speed of light in the rocket, is: $v_l = \frac{l}{t_2' - t_1'} = \frac{l}{\frac{l}{c}} = \boldsymbol{C}$. Thus the velocity

of the light pulse in the rocket is equal to the standard Light Velocity.

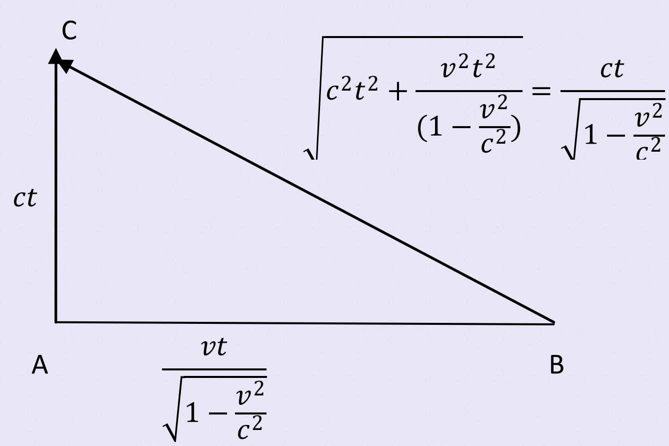
*Case 2*:

In this case the speed of the rocket is towards the Earth so $v = -v$.

If one substitutes $-v$ in the formulae of case 1 then one can see that this results to the same conclusion as in case 1.

*Case 3*:

The light pulse is emitted from the Earth to one side of the rocket but when the pulse reaches the other side in a straight line the rocket has moved. So from the Earth it appears that the point where the pulse hits the other side has moved as well. The distance between the points of emitting and hitting, from Earth's perspective, is now longer than the distance D between the two sides.



A rocket is moving to the right with a velocity $v$

Assume the light pulse hits the rocket on $x = 0, t = 0$ thus $x^{'} = 0,\ t^{'} = 0$

The light pulse hits the other side of the rocket at time $t$ and travels a distance AC which equals $ct$.

When the pulse hits the other side the situation at C is:

$$x = 0$$

$$t^{'} = \frac{t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$x^{'} = \frac{vt}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

This last equation is the distance between A and B. Thus from AB and AC the hypotenuse BC can be calculated.

$$BC = \sqrt{c^2t^2 + \frac{v^2t^2}{(1 - \frac{v^2}{c^2})}} = \frac{ct}{\sqrt{1 - \frac{v^2}{c^2}}}$$

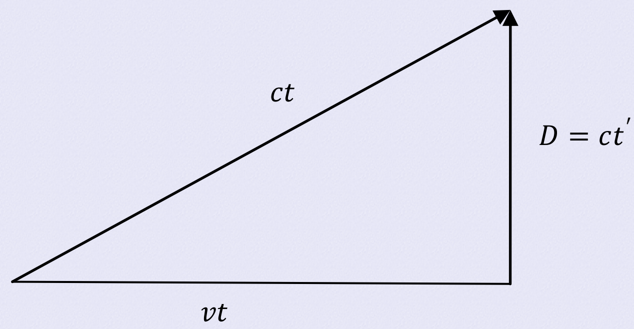
This the distance the light pulse travels with respect to the coordinate system of the rocket.

$$\frac{BC}{t^{'}} = \frac{\frac{ct}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}}{\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}} = c$$

So the speed of the light in the rocket is equal to the standard Light Velocity **C**.

*Case 4*:

The light pulse is emitted from one side of the rocket but when the pulse reaches the other side in a straight line the rocket has moved. So from the Earth it appears that the point where the pulse hits the other side has moved as well. The distance between the points of emitting and hitting, from Earth's perspective, is now longer than the distance D between the two sides.




$$c^2t'^2 + v^2t^2 = c^2t^2 \rightarrow c^2t'^2 = c^2t^2 - v^2t^2$$

$$t'^2 = \frac{c^2\,t^2 - v^2t^2}{c^2} \rightarrow t'^2 = t^2\left(1 - \frac{v^2}{c^2}\right) \rightarrow t' = t\,\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

This is in accordance with the time dilation formula.

## Conclusion

Thus the consideration of all these cases above proves that the velocity of light is always equal to the standard Light Velocity **C** independent of a particular coordinate system.