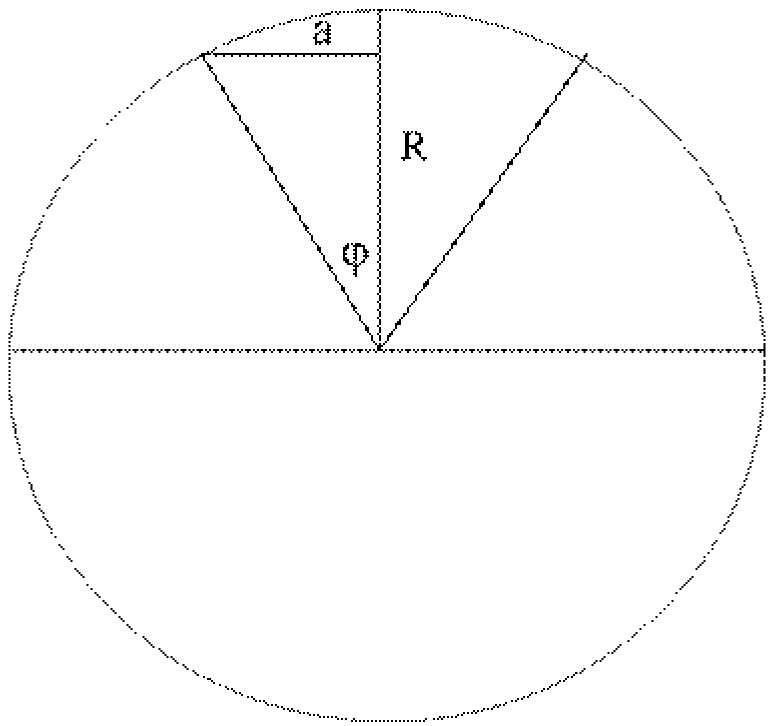
Calculation of curved surface

Calculate the size of a curved surface. A "flat" surface is considered from the perspective of a 2 dimensional person, i.e. on earth. How can a person on earth find out that his surface is curved? When the surface is really flat the surface is πr^2. In case the surface is curved the surface area could be considered as a segment of a bol/sphere.

Let us calculate the segment of a sphere:



$a = R sin\theta$.

Thus calculation of the segment:

A= $\int_0^{\emptyset} 2\pi R \sin\emptyset . R d\theta = 2\pi R^2 (1 - \cos\emptyset)$ is size of segment.

Half the diameter of the segment is: $\emptyset R$

Hence the size of the segment from the perspective of the flat person:

$\pi(\emptyset R)^2$. When the flat person measures the size of the sphere he will find the real size of the segment. The radius is defined as the surface divided by π and taking the square root.

Thus dividing the size by π and taking the square root he will find his real radius.

Thus $\sqrt{2R^2(1-\cos\emptyset)} = R\sqrt{2(1-\cos\emptyset)}$ is real radius from perspective of flat person. Measuring his own radius he will find $\emptyset R$.

Hence the ratio between the real radius and the measured radius is:

real/measured = $R\sqrt{2(1-\cos\emptyset)} / \emptyset R = \sqrt{\frac{2(1-\cos\emptyset)}{\theta^2}}$

Applying Taylor on $\cos\emptyset$ results in 1 - $\frac{\emptyset^2}{2!} + \frac{\emptyset^4}{4!} - \frac{\emptyset^6}{6!} + \frac{\emptyset^8}{8!} \ldots\ldots$ ==>

$$\frac{2(1-\cos\emptyset)}{\theta^2} = 1 - \frac{2\emptyset^2}{4!} + \frac{2\emptyset^4}{6!} - \frac{2\emptyset^6}{8!} + \frac{2\emptyset^8}{10!} \ldots\ldots$$

Real Radius/Measured Radius = $\sqrt{\frac{2(1-\cos\emptyset)}{\theta^2}} = \sqrt{1 - \frac{2\emptyset^2}{4!} + \frac{2\emptyset^4}{6!} - \frac{2\emptyset^6}{8!} + \frac{2\emptyset^8}{10!} \ldots\ldots}$ = R_r/R_m

Thus R_r is smaller than the R_m. As an example, in case of $\emptyset = \pi/2$: R_r/R_m=0.90. Hence the ratios between the surfaces is 0.81

For the earth is this when Rm=10,000 km and 0.99 (or 1%) when Rm=3120km