### Via Schwarzschild approach

A summary of the results derived in the Schwarzschild equation in chapter [Trajectories of massive particles-Second Derivation](#_Trajectories_of_massive) and **Error! Reference source not found.**The semi-major axis is:

The parameter *e* measures the eccentricity of the orbit. The perihelion is and the aphelion is

So for a circle *e*=0 and r=a.

When then

This is the aphelion.

Find the angle between *v* and the *vtan* (perpendicular with *r,* to find the angular momentum):

Gives

Velocity along the earth surface:

Or:

Instantaneous velocity as function of

From previous chapter

Earth

d

Ø

r=R

α

h

Bullet

trajectory

α

Ø

h

r

R

1/2d

From (2d) we will derive *e*:

For the starting point, the intersection of Earth and trajectory *r=R*. (*R* is here the radius of the Earth) and

The given velocity at the *r=R* point is Thus for a velocity there are two solutions of *e*.

Here is *h* the highest point of the bullet trajectory

Here *d* is the distance on Earth, *v* is the starting velocity of the bullet and *R* is the Earth radius. As seen above

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From (2a)

From (2c)

Together with (2a)

From (2b)

Fill inn

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Result of the example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Conservative | | Schwarzschild | |
| Horizontal distance (m) | 10 | 10 |  |  |
| Horizontal velocity (m/sec) | 5 | 500 |  |  |
| Total velocity (m/sec) | 11.06 | 500 | 11.06 | 500 |
| time | 2 | 2\*10-4 |  |  |
| Height (m) | 4.93 | 4.93\*10-4 | 4.93 | 4.91\*10-4 |

Note: is hard to solve. By iterative processing this can be approximated or by “seek goal” in the program Excel.

Derivation of the circumference of an ellipse

For the **circumference of an ellipse** there is no simple closed solution.

There are approximations, for instance the Ramanujan approximation:

Detailed results of calculations on the example mentioned above, showing that there are two solutions per initial velocity.

The starting points are the velocity of the bullet and the distance to be covered.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| velocity(m/s) | **11** | | **500** | |
| d(m) | **10** | **10** | **10** | **10** |
| epsilon | 5E-03 | 1E-03 | 5E-07 | 3E+00 |
| e(centricity) | -1.00000E+00 | -9.99998E-01 | -9.96014E-01 | -1.00000E+00 |
| a(km) | 3178 | 3178 | 3185 | 3185 |
| h(m) | 4.9522 | 1.2624 | 4.91E-04 | 12693.05 |
| alpha(deg) | -63.3 | -26.8 | -0.01 | 89.98 |
| L (ang. mom.) | 3.E+07 | 6.E+07 | 3.E+09 | 1.E+06 |
| cos(alpha) | 0.449 | 0.892 | 1.000 | 0.000 |
| cos(alpha+phi) | 0.449 | 0.892 | 1.000 | 0.000 |
| vx0(m/s) | 4.96 | 9.87 | 500.00 | 0.21 |
| vy0(m/s) | -9.89 | -4.99 | -0.10 | 500.00 |
| Circ.(km) | 12662 | 12663 | 12894 | 12686 |