

Final Year B. Tech., Sem VII 2022-23

Cryptography And Network Security

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Assignment No. 10

1. Aim:

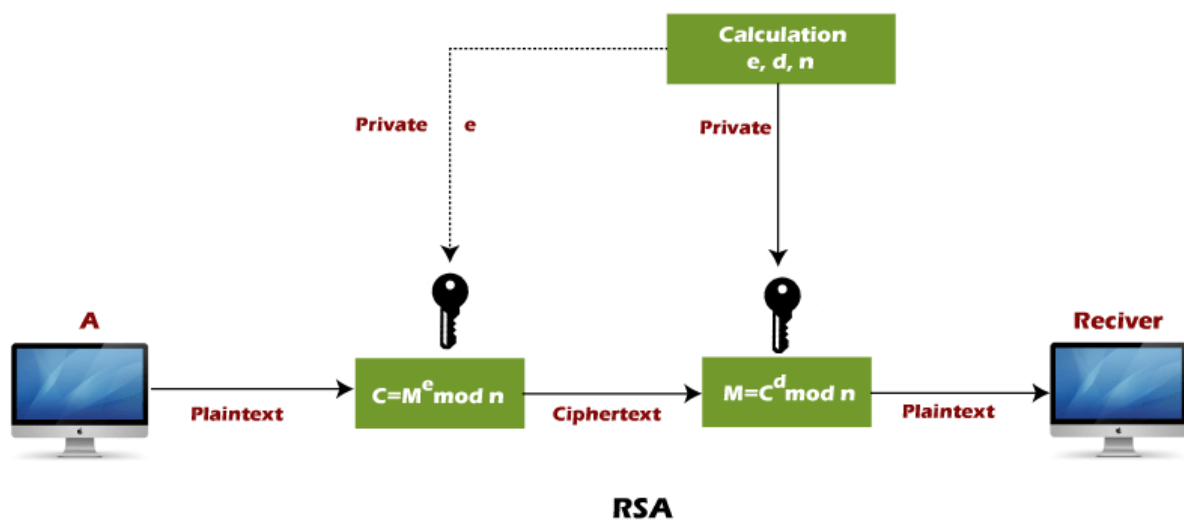
Implementation of RSA Algorithm

2. Theory:

RSA encryption algorithm is a type of public-key encryption algorithm.

RSA encryption algorithm:

RSA is the most common public-key algorithm, named after its inventors Rivest, Shamir, and Adelman (RSA).



RSA algorithm uses the following procedure to generate public and private keys:

- Select two large prime numbers, p and q .
- Multiply these numbers to find $n = p \times q$, where n is called the modulus for encryption and decryption.

- Choose a number e less than n , such that n is relatively prime to $(p - 1) \times (q - 1)$. It means that e and $(p - 1) \times (q - 1)$ have no common factor except 1. Choose " e " such that $1 < e < \phi(n)$, e is prime to $\phi(n)$,
 $\gcd(e, \phi(n)) = 1$
- If $n = p \times q$, then the public key is $\langle e, n \rangle$. A plaintext message m is encrypted using public key $\langle e, n \rangle$. To find ciphertext from the plain text following formula is used to get ciphertext C .

$$C = m^e \bmod n$$

Here, m must be less than n . A larger message ($>n$) is treated as a concatenation of messages, each of which is encrypted separately.
- To determine the private key, we use the following formula to calculate the d such that:

$$D_e \bmod \{(p - 1) \times (q - 1)\} = 1$$

Or

$$D_e \bmod \phi(n) = 1$$
- The private key is $\langle d, n \rangle$. A ciphertext message c is decrypted using private key $\langle d, n \rangle$. To calculate plain text m from the ciphertext c following formula is used to get plain text m .

$$m = c^d \bmod n$$

Let's take some example of RSA encryption algorithm:

This example shows how we can encrypt plaintext 9 using the RSA public-key encryption algorithm. This example uses prime numbers 7 and 11 to generate the public and private keys.

Explanation:

Step 1: Select two large prime numbers, p , and q .

$$p = 7$$

$$q = 11$$

Step 2: Multiply these numbers to find $n = p \times q$, where n is called the modulus for encryption and decryption.

First, we calculate

$$n = p \times q$$

$$n = 7 \times 11$$

$$n = 77$$

Step 3: Choose a number e less than n , such that n is relatively prime to $(p - 1) \times (q - 1)$. It means that e and $(p - 1) \times (q - 1)$ have no common factor except 1. Choose " e " such that $1 < e < \phi(n)$, e is prime to $\phi(n)$, $\gcd(e, \phi(n)) = 1$.

Second, we calculate

$$\phi(n) = (p - 1) \times (q - 1)$$

$$\phi(n) = (7 - 1) \times (11 - 1)$$

$$\phi(n) = 6 \times 10$$

$$\phi(n) = 60$$

Let us now choose relative prime e of 60 as 7.

Thus, the public key is $\langle e, n \rangle = (7, 77)$

Step 4: A plaintext message m is encrypted using public key $\langle e, n \rangle$. To find ciphertext from the plain text following formula is used to get ciphertext C .

To find ciphertext from the plain text following formula is used to get ciphertext C .

$$C = m^e \bmod n$$

$$C = 9^7 \bmod 77$$

$$C = 37$$

Step 5: The private key is $\langle d, n \rangle$. To determine the private key, we use the following formula d such that:

$$D_e \bmod \{(p - 1) \times (q - 1)\} = 1$$

$$7d \bmod 60 = 1, \text{ which gives } d = 43$$

The private key is $\langle d, n \rangle = (43, 77)$

Step 6: A ciphertext message c is decrypted using private key $\langle d, n \rangle$. To calculate plain text m from the ciphertext c following formula is used to get plain text m .

$$m = c^d \bmod n$$

$$m = 37^{43} \bmod 77$$

$$m = 9$$

In this example, Plain text = 9 and the ciphertext = 37

3. Code:

```
#include <bits/stdc++.h>

using namespace std;

void file()
{
    #ifndef ONLINE_JUDGE
    freopen("input.txt", "r", stdin);
    freopen("output.txt", "w", stdout);
    #endif
}

// Function for extended Euclidean Algorithm

int ansS, ansT;

int findGcdExtended(int r1, int r2, int s1, int s2, int t1, int t2)
{
    // Base Case
    if (r2 == 0)
    {
        ansS = s1;
        ansT = t1;
        return r1;
    }

    int q = r1 / r2;
    int r = r1 % r2;
    int s = s1 - q * s2;
    int t = t1 - q * t2;
```

```
cout << q << " " << r1 << " " << r2 << " " << r << " " << s1 << " " << s2 << " " << s << " "
<< t1 << " " << t2 << " " << t << endl;
```

```
return findGcdExtended(r2, r, s2, s, t2, t);
```

```
}
```

```
int modInverse(int A, int M)
```

```
{
```

```
int x, y;
```

```
int g = findGcdExtended(A, M, 1, 0, 0, 1);
```

```
if (g != 1) {
```

```
cout << "Inverse doesn't exist";
```

```
return 0;
```

```
}
```

```
else {
```

```
// m is added to handle negative x
```

```
int res = (ansS % M + M) % M;
```

```
cout << "Inverse is: " << res << endl;
```

```
return res;
```

```
}
```

```
}
```

```
long long powM(long long a, long long b, long long n)
```

```
{
```

```
if (b == 1)
```

```
return a % n;
```

```
long long x = powM(a, b / 2, n);
```

```
x = (x * x) % n;
```

```

if (b % 2)

x = (x * a) % n;

return x;

}

int findGCD(int num1, int num2)

{

if (num1 == 0)

return num2;

return findGCD(num2 % num1, num1);

}


// Code to demonstrate RSA algorithm

int main()

{

file();

// Two random prime numbers

long long p, q, e, msg;

//17 31 7 2

cin >> p >> q >> e >> msg;

cout<<"Two prime Numbers are: "<<p<<" "<<q<<endl;

// First part of public key:

long long n = p * q;

cout << "Product of two prime number n is " << n << endl;

// Finding other part of public key.

// e stands for encrypt

cout << "Taken e is " << e << endl;

long long phi = (p - 1) * (q - 1);

```

```
cout << "phi is " << phi << endl;

while (e < phi) {

    // e must be co-prime to phi and

    // smaller than phi.

    if (findGCD(e, phi) == 1)

        break;

    else

        e++;

}

cout << "Final e value is " << e << endl;

// Private key (d stands for decrypt)

long long d = modInverse(e, phi);

cout << "d is " << d << endl;

cout << "\nSo now our public key is " << "<" << e << "," << n << ">" << endl;

cout << "\nSo now our private key is " << "<" << d << "," << n << ">" << endl << endl;

// Message to be encrypted

cout << "Message data is " << msg << endl;

// Encryption  $c = (msg^e) \% n$ 

long long c = powM(msg, e, n);

cout << "Encrypted Message is " << c << endl;

// Decryption  $m = (c^d) \% n$ 

long long m = powM(c, d, n);

cout << "Original Message is " << m << endl;

return 0;

}
```

4. Output:

```

input.txt  X
input.txt
1  17 31 7 2

output.txt  X
output.txt
1  Two prime Numbers are: 17 31
2  Product of two prime number n is 527
3  Taken e is 7
4  phi is 480
5  Final e value is 7
6  0 7 480 7 1 0 1 0 1 0
7  68 480 7 4 0 1 -68 1 0 1
8  1 7 4 3 1 -68 69 0 1 -1
9  1 4 3 1 -68 69 -137 1 -1 2
10 3 3 1 0 69 -137 480 -1 2 -7
11 Inverse is: 343
12 d is 343
13
14 So now our public key is <7,527>
15
16 So now our private key is <343,527>
17
18 Message data is 2
19 Encrypted Message is 128
20 Original Message is 2
21

```

5. Conclusion:

Successfully implemented RSA Algorithm.