Probability Theory

Practice Set

Continuous Random Variables and its distributions

1. Let X be a random variable with probability density function.

$$f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

(a) find value of c.

(b) what CDF of X

(c) Calculate $P(0.3 \le X \le 0.9)$, $P(w \le 0.5)$, $P(w = 0.5)$

$F_x(0.5)$

2. A random variable V, has cumulative distribution function

$F(v) = 1 - e^{-0.5v}$, $v \ge 0$ (a) find PDF

(b) Calculate $P(V > 2)$, $P(V < 1)$, $P(V = 0.8)$

3. Find the expected value and variance of the random variable X, that has PDF

$$f(x) = \frac{1}{18}(x^2 - 2x) \quad , \quad 2 \le x \le 5$$

4. A continuous random variable Y, has probability density function

$$f(y) = \frac{3}{y^4} \quad , \quad y \ge 1$$

(a) Calculate $E[y]$, $E[y^2]$, $E[5y-2]$, $Var[y]$, $Var[5y-2]$

$E\left[\frac{5y+1}{6}\right]$

5. The random variable X has PDF as

$$f(x) = \frac{3}{4}(1-x^2), \quad -1 \le x \le 1$$

Does X has symmetrical distribution?

6. The random V has the PDF

$$f(v) = \frac{1}{2}v \quad 0 \le v \le 2.$$

(a) Calculate the 4^{th} moment of V.

(b) Third central moment of V

(c) Second order moment of V about 1.

7. If $X \sim Unif(-2, 2)$.

(a) find $P(X<1)$

(b) $P(|X-1| \ge 1/2)$

8. If X is uniformly distributed in $[-\alpha, \alpha]$ with $\alpha > 0$ then determine α such that $P(X>1) = 1/3$.

9. A bus arrives every 10 minutes at a bus stop. Assuming waiting time X for bus is uniformly distributed. find the probability that a person has to wait for the bus for more than 7 minutes.

10. Claim amounts for an insurer are exponentially distributed with parameter 0.005 calculate the Probability that a claim exceeds 210.

11 Write CDF for $X \sim exp(0.5)$. use it to Calculate

(a) $P(X<3)$

(b) $P(2<X<8)$

(c) $E[3x+2]$

12. If random variable X represents the life-time of a battery with mean 500 hours. Calculate the Probability that the battery lasts

(a) more than 700 hours

(b) between 400 to 600 hours.

13. If $X \sim exp(\lambda)$ and $P(X>40)=0.7$

(a) Calculate λ

(b) Calculate $P\left(\frac{X>105}{X>80}\right)$

14 use table and find probabilities

(a) $P(z<1.23)$

(b) $P(z>2.17)$

(c) $P(z>-0.08)$

(d) $P(z<-1.50)$

(e) $P(-2.92<z<-1.67)$

(f) $P(-0.92<z<0.83)$

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(15) Calculate the probabilities using interpolation

(a) $P(Z < 1.048)$

(b) $P(-0.704 < Z < 0.897)$

(16) If $X \sim N(100, 16)$. Calculate

(a) $P(X > 110)$

(b) $P(95 < X < 107)$

(17) Suppose that X is normal random variable with mean 5. If $P\{X > 9\} = 0.2$. approximately what is $Var(X)$.

18. Suppose that X is a random variable with mean and variance both equal to 20. What can be $P(0 < X < 40)$?

(19) From past experience a professor knows that the test score of a student taking her final exam is a random variable with mean 75.

(a) Give an upper bound for the probability that a student's test score will exceed 85.

(b) In addition, that the professor knows that the variance of a student's test score is equal to 25. What can be the probability that a student will score between 65 and 85.

20. Let $X_1, X_2 \dots X_{20}$ be independent poisson random variables with mean 1. Find approximate probability $P\{\sum_{1}^{20} x_i > 15\}$

21. Find a lower bound on $P[-3<X<3]$ where $E[X]=0$ and $\sigma^2=1$.

22. The probability of any given policy in a portfolio of term assurance policies lapsing before it expires is considered to be 0.15. For a group of 100 such policies. Calculate the approximate probability that more than 20 will lapse before they expire)
(Discuss about its actual distribution and your assumption to calculate approximate probability)

23. The number of claims arising in a month under a home insurance policy ~~follow~~ having mean 0.075. Calculate the approximate probability that at least 50 claims in total arise in a month under a group of 500 independent such policies.
(Discuss about its actual distribution and your assumption to calculate approximate probability).