(1) (a;

Indian Institute of Information Technology Chittoor, SriCity

Discrete Mathematics - Mid Semester 1 examination

Total marks: 50; Time: 1 hour 30 minutes

$2c = c + c =$

$n = 2c \quad c \in I$

$2c = k+5 + k-5$

$2c = 2k$

$k = c$ const unique

Instructions

- All the questions are to be attempted.
- All the answers should be written in formal way.
- All questions have the equal weightage.
- This is a closed book, closed notes exam.
- Calculators (non-programming) may be allowed in the exam hall.

1. Examine the validity of the following argument, using mathematical logic:
   (a) The IIITS students can be of three kinds: either BTech or MS or PhD students.
   (b) For all MS students the primary goal is research.
   (c) For all PhD students the primary goal is research.
   (d) For any IIITS student, if the primary goal is not research then he/she is a BTech student.
2. Prove that $((p \to r) \land (q \to r)) \leftrightarrow ((p \lor q) \to r)$ is a tautology.
3. You and your brother are playing a game. You toss a coin. Whoever wins, will get a chocolate. If you get a chocolate, then you will gift it to your father. If your brother gets a chocolate, then he will gift it to your mother. Using Mathematical logic, prove that either of your parents will get a chocolate.
4. (3+2 marks) Prove that (not using truth table) $\neg(p \leftrightarrow q)$ and $(\neg p \leftrightarrow q)$ are logically equivalent. Find the Prenix Normal Form of the statement: $\neg \exists x P(x) \to \forall x Q(x)$.
5. Obtain PDNF of the given expression: $((p \lor q) \land (q \to r)) \leftrightarrow r$.
6. (4+1 marks) Prove that, if n is an even integer, then there is a unique integer k such that n can be represented as a sum of k-5 and k+5. Prove further that the integer 5 can be replaced by any integer t.
7. (2+3 marks) Prove or disprove the following statements:
   (a) For any two positive real numbers x and y, $\lceil x \rceil + \lfloor y \rfloor = \lceil x+y \rceil$.
   (b) For any two positive real numbers x and y, $\lceil x \rceil + \lfloor y \rfloor \geq \lceil x+y \rceil$.

   Ceiling func. $5.4 = 6$ $\quad \neg(\neg p) \lor q$
8. (3+2 marks) Prove the following:
   (a) For each integer x, either $x^2+1>3$ or $x<2$.
   (b) Negative of any irrational number is irrational.

   $(\neg p \to q) \land (q \to \neg p)$

   $(p \lor q) \land (\neg q \lor \neg p)$
9. (3+2 marks)
   (a) Prove that the integer xy is an odd integer iff x and y both are odd integers.
   (b) Find the mistake in the following proof:
   (5+5)(5-5) = 5*(5-5)*
   => 5+5 = 5 *[Right cancellation law for integers]*
   =>10 = 5

   $x \to irr$

   $-x = r = \frac{p}{q} \neq 0$

   $x + (-x)$

   $x + \frac{p}{q} = \frac{xq+p}{q \neq 0}$
10. (4+1 marks)
    (a) Prove that, for any three sets A, B and C, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
    (b) Find $\prod_{i=1}^{\infty} A_i$, where $A_i = (-i, i)$ for all integer i.

5.8 $\quad$ 7.1 $\quad$ 5.8 7.1 12.9 $\quad$ 13

6+8 IIITS → BVMS V PhD

(14) MS → R $\quad$ ¬R → ¬MS

PhD → R $\quad$ ¬R → ¬PhD

I: $p(x)$, $p(x) \to q(x)$, $q(x) \to r(x)$, $r(x) \to f$

Bro: $p(y)$, $p(y) \to q(y)$, $q(y) \to r(y)$, $r(y) \to m$

x: Me, y: Bro, p → win, q → get C, r → gift C

$p \land (\neg q \lor \neg p) \lor q \land (\neg q \lor \neg p)$

$(p \land \neg q) \lor (p \land \neg p) \lor (q \land \neg q) \lor (q \land \neg p)$

F $\quad$ F

$(p \land \neg q) \lor (q \land \neg p) \quad \neg(\neg p \lor q) \lor (\neg p \land q)$

$p : r$, $q : r$, $p \to q$, $\neg q \to \neg p$

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8 Pages

Answer Booklet

☑ Mid_Sem / ☐ End_Sem Examination (✓whichever applicable)

Programme...CSE.......Roll No. 201601061.......Subject and Code....DM......

Date ...08.09.2016...... No. of Addl. Sheets....+1.......... Invigilator's Signature........

Room No./Seat No. 305, A1

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total
Maximum Marks											
Marks Obtained	2	5	3.5	3	0	5	2	5	3	1	29.5

Signature of the Faculty for the course (after evaluation) :

Please start writing here

(1.) Let x be students

$P(x)$: Students study at IIITS

$Q(x)$: Students study BTech course

$R(x)$: Students study MS course

$S(x)$: Students study PhD course

$T(x)$: Main goal of student is research.

Consider case $T,T,F \to T \Delta T \Delta F$ $= F \Delta F = F$

why XOR?

2.

(a) $P(x) \to Q(x) \Delta R(x) \Delta S(x)$ is true Δ: X-OR

(b) $R(x) \to T(x) \equiv \neg T(x) \to \neg R(x)$ is true ; $x \in P(x)$

(c) $S(x) \to T(x) \equiv \neg T(x) \to \neg S(x)$ is true ; $x \in P(x)$

For ~~all~~ $\neg T(x)$ to be true

$\Rightarrow$ $\neg R(x)$ is true

$\neg S(x)$ is true

Incomplete Proof.

For $P(x) \to Q(x) \Delta R(x) \Delta S(x)$ to be true when $R(x)$ is false and $S(x)$ is false

$Q(x)$ must be true (Must be a B.Tech Student)

Hence it proves that for any IIITS student, if primary goal is not research, he/she is a BTech Student.

→ q and q and conclusion ... argument form, which
ed the fallacy of affirming the conclusion.

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(1)

T: true
F: false

(2) $((p \to r) \wedge (q \to r)) \leftrightarrow ((p \vee q) \to r)$

p	q	r	$(p \to r)$	$(q \to r)$	$(p \to r) \wedge (q \to r)$	$(p \vee q)$	$(p \vee q) \to r$	$((p \to r) \wedge (q \to r)) \leftrightarrow ((p \vee q) \to r)$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F	T
T	F	T	T	T	T	T	T	T
T	F	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F	T
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	T

5

Since all final outcomes of the truth table are True (T) hence $((p \to r) \wedge (q \to r)) \leftrightarrow ((p \vee q) \to r)$ is a tautology.

(3) ~~Let p: I win~~
~~q: I gift chocolate to my father~~
~~r:~~

~~Considering p is true.~~
p : ~~I win, I get a chocolate~~
~~p→q :~~

(3) Let x = I (me) y = My brother

$P(a)$: a wins
$Q(a)$: a gets a chocolate
$R(a)$: a gifts the chocolate to
m: mother gets chocolate , f: father gets chocolate

In a coin toss, there can be only two cases, either I win, or my brother wins.

Case 1: I win

$P(x)$ is true.

$P(x) \rightarrow Q(x)$ is true.

$Q(x) \rightarrow R(x)$ is true. $P(x) \rightarrow R(x)$ (Hypothetical syllogism)

According to question:

When I win, I gift the chocolate to my father

$R(x) \rightarrow f$; $P(x) \rightarrow f$ (Hypothetical syllogism)

Result 1: Father gets chocolate.

Case 2: Brother wins

$P(y)$ is true

$P(y) \rightarrow Q(y)$ is true

$Q(y) \rightarrow R(y)$ is true $P(y) \rightarrow R(y)$ (Hypothetical syllogism)

According to question:

When my brother wins, he gifts the chocolate to my mother.

$R(y) \rightarrow m$

$P(y) \rightarrow m$ (Hypothetical syllogism)

3.5

Result 2: Mother gets chocolate.

Seeing both results we can easily conclude that whichever case may occur (whether I win or my brother wins), either parent would get a chocolate.

Case 1 Δ Case 2 (Both cannot win simultaneously)

$(P(x) \rightarrow f) \Delta (P(y) \rightarrow m)$ Δ: X-OR

Hence proved.

You had to prove $f \Delta m$.

Incomplete proof.

(4.) $\neg(p \leftrightarrow q)$ and $(\neg p \leftrightarrow q)$

$\neg(p \leftrightarrow q)$: LHS

$\equiv \neg((p \to q) \wedge (q \to p))$ $\quad \because p \leftrightarrow q = (p \to q) \wedge (q \to p)$

$\equiv \neg[(\neg p \vee q) \wedge (\neg q \vee p)]$ $\quad \because a \to b \equiv \neg a \vee b$

$\equiv [\neg(\neg p \vee q)] \vee [\neg(\neg q \vee p)]$ $\quad \because \neg(a \wedge b) \equiv \neg a \vee \neg b$

$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$ $\quad \because \neg(\neg a) \equiv a$

$\equiv [p \vee (q \wedge \neg p)] \wedge [\neg q \vee (q \wedge \neg p)]$ $\quad$ Distributive law $a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$

$\equiv [(p \vee q) \wedge (p \vee \neg p)] \wedge [(\neg q \vee q) \wedge (\neg q \vee \neg p)]$

$\equiv ((p \vee q) \wedge T) \wedge (T \wedge (\neg q \vee \neg p))$ $\quad \because a \wedge \neg a = T$

$\equiv (p \vee q) \wedge (\neg q \vee \neg p)$ — ① $\quad \because a \wedge T = a$

$(\neg p \leftrightarrow q)$: RHS

~~$\equiv (\neg(\neg p) \vee q) \wedge$~~

$\equiv (\neg p \to q) \wedge (q \to \neg p)$ $\quad \because (a \leftrightarrow b) \equiv (a \to b) \wedge (b \to a)$

$\equiv (\neg(\neg p) \vee q) \wedge (\neg q \vee \neg p)$ $\quad \because (a \to b) \equiv (\neg a \vee b)$

$\equiv (p \vee q) \wedge (\neg q \vee \neg p)$ — ② $\quad \because \neg(\neg a) \equiv a$

Since ① = ②

Hence proved that $\neg(p \leftrightarrow q)$ and $(\neg p \leftrightarrow q)$ are logically equivalent.

3

$\exists x P(x) \to \forall x Q(x)$

$\equiv [\neg(\exists x P(x))] \vee [\forall x Q(x)]$ $\quad \because (a \to b) \equiv \neg a \vee b$

$\equiv [\forall x \neg P(x)] \vee [\forall x Q(x)]$ $\quad \because \neg \exists a (P(a)) \equiv \forall a \neg P(a)$

$\equiv \forall x [\neg P(x) \vee Q(x)]$

(5.) $((p \vee q) \wedge (q \rightarrow r)) \leftrightarrow r$

$\equiv [(p \vee q) \wedge (\neg q \vee r)] \leftrightarrow r$ $\quad \because (a \rightarrow b) \equiv \neg a \vee b$

$\cancel{(p \vee q) \wedge (}$

$[p \wedge (\neg q \vee r) \vee (q \wedge (\neg q \vee r))] \leftrightarrow r$

$(p \wedge \neg q) \vee (p \wedge r) \vee \cancel{(q \wedge \neg q)} \vee (q \wedge r)$

$[(p \wedge \neg q) \vee (p \wedge r) \vee (q \wedge r)] \leftrightarrow (q \wedge r)$

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(6.) By the direct method:

Let $n = 2c$ where $c \in I$

Hence n is an even number

$n = 2c = c + c$

Adding and subtracting 5 from RHS

$n = c + (c+5) - 5$ (Commutative law)

$= (c+5) + (c-5)$ (Associative law)

$\equiv (k+5) + (k-5)$

where $\boxed{k = c}$

Hence there exists a unique integer k for every c such that $n = k-5+k+5 = 2k$ (even)

As both $+5$ and -5 are being added in $2k$; $+5-5=0$ so here any integer t could be taken because $+t-t=0$ for all integers

Hence $n = (k-t) + (k+t)$

where a unique integer k exists for every n and t can be any integer.

(7). (a.) $\lceil x \rceil + \lceil y \rceil = \lceil x+y \rceil$

This is not ~~to~~ true:

Let $x = 5.8$

$y = 7.1$

$\lceil x \rceil = 6$

$\lceil y \rceil = 8$

$\lceil x \rceil + \lceil y \rceil = 14$

$\lceil x+y \rceil = \lceil 12.9 \rceil = 13$

As $14 \neq 13$; it is not true.

(b) For any two positive real nos x and y:

$\lceil x \rceil \geq x$

$\lceil y \rceil \geq y$

$\lceil x \rceil + \lceil y \rceil \geq x + y$

For $\lceil x \rceil = x$; $\lceil y \rceil = y$

$\lceil x \rceil + \lceil y \rceil = x + y = \lceil x + y \rceil$

For $\lceil x \rceil > x$; $\lceil y \rceil > y$.

$\lceil x \rceil + \lceil y \rceil > x + y$

Hence $\Rightarrow$ $\lceil x \rceil + \lceil y \rceil \geq \lceil x + y \rceil$: Proved.

(8)(a.) $x^2 + 1 > 3$ or $x < 2$

For $x < 2$

$(-\infty, 1]$ condition satisfies.

3

For $x \geq 2$

$[2, \infty)$:

$P(2)$: $4 + 1 = 5 > 3$

$P(3)$: $9 + 1 = 10 > 3$

As $2 < 3 < 4 \ldots < \infty$; the condition is always true

Hence for each integer: either

$x^2 + 1 > 3$ or $x < 2$; $x \in I$ is true.

(b) By using contra-positive method:

we take two rational numbers

$\frac{p}{q}$ and $\frac{r}{s}$; $q \neq 0$; $s \neq 0$

$\frac{p}{q} - \frac{r}{s} = 0$

Date

Roo

Maxi
Ma
Ma
Obta

Signa

Plea

$$\frac{ps - qr}{qs} = 0 \qquad qs \neq 0$$

$$ps - qr = 0$$

$$ps = qr$$

$$\frac{p}{q} = \frac{r}{s} \Rightarrow$$ Nos are equal in magnitude

2

3) As $\frac{p}{q} = -\frac{r}{s}$

i) ~~Negative of any~~ ?

ii) For any rational no; its negative is rational

i) $\Rightarrow$ Negative of any irrational no is irrational.

(9) (a.) Let $x = 2a+1$; $a \in I$

$y = 2b+1$; $b \in I$

$$xy = (2a+1)(2b+1)$$
$$= 4ab + 2a + 2b + 1$$
$$= 2(2ab + a + b) + 1$$
$$= 2k' + 1 = \text{odd} \qquad k' = 2ab + a + b$$

1

Hence when x & y are odd, xy is odd.

(b.) $(5+5) * (5-5) = 5 * (5-5)$

$\Rightarrow 5+5 = 5$ By Right Cancellation

Here we cannot apply right cancellation because $5-5=0$ and we cannot divide by 0 both sides

Hence the proof is invalid.

2