(9) $X \sim$ uniformly distributed continuous r.v.

X - waiting time

$X \sim \text{unif}(0, 10)$

Bus arrives every 10 minutes

$f(x) = \frac{1}{10-0} = 1/10$

$0 \le x \le 10$

$P(X>7) = ?$

$$P(X>7) = \int_7^{10} \frac{1}{10}\, dx$$

$$= \frac{1}{10}(x)\Big|_7^{10}$$

$$= 3/10$$

(10) $X \sim \exp(0.005)$ $\lambda = 0.005$

PDF of exp distribution

$f(x) = \lambda e^{-\lambda x}$ for $x \ge$

$E(x) = 1/\lambda$

$Var(x) = 1/\lambda^2$

$$P(X>210) = \int_{210}^{\infty} f(x)\, dx$$

$$= \int_{210}^{\infty} (0.005)\, e^{-0.005x}\, dx$$

$$= \frac{0.005\, e^{-0.005}}{-0.005}\Big|_{210}^{\infty} = e^{-1.05} = 0.3499$$

(11) $X \sim \exp(0.5)$, $\lambda = 0.5$

PDF of exp distribution

$f(x) = \lambda e^{-\lambda x}$ $x > 0$

CDF $F_X(x) = 1 - e^{-\lambda x}$

$E[X] = 1/\lambda = 1/0.5 = 2$

$$P(X \cancel{\ge} 3) = F_X(3)$$

$$= 1 - e^{-0.5 \times 3}$$

$$= 1 - e^{-1.5}$$

$$P(2 < X < 8) = P(X<8) - P(X<2)$$

$$= F_X(8) - F_X(2)$$

Answer $= e^{-1} - e^{-4}$

$$E[3x+2] = 3E[x]+2 = 3\times 2 + 2 = 8$$

(12) X - represents life-time of battery

$X \sim exp(\lambda)$

$$\boxed{PDF \text{ of } exp(\lambda) = \lambda e^{-\lambda x}}$$

$E[X] = 500$ given

$\frac{1}{\lambda} = 500 \Rightarrow \lambda = 1/500 = 0.002$

$$P(X > 700) = \int_{700}^{\infty} \lambda e^{-\lambda x} dx = \cancel{0.24660}$$

$$= \int_{700}^{\infty} 0.002 e^{-0.002x} dx = e^{-1.4}$$

$|||^{ly}$

(Ans)

$$P(400 < X < 600) = e^{-0.8} - e^{-1.2}$$

$$\rightarrow \int_{400}^{600} \lambda e^{-\lambda x} dx$$

(13) $X \sim exp(\lambda)$ $P(X > 40) = 0.7$ $\lambda = ??$

$P(X > 40) = 0.7$

$$\int_{40}^{\infty} \lambda e^{-\lambda x} dx = 0.7$$

$$\left. \frac{\lambda e^{-\lambda x}}{-\lambda} \right|_{40}^{\infty} = 0.7$$

$$e^{-40\lambda} = 0.7 \Rightarrow \boxed{\lambda = -1/_{40} \ln 0.7}$$

$$P\left(\frac{X>105}{X>80}\right) = \frac{P(X>105 \text{ and } X>80)}{P(X>80)} = \frac{P(X>105)}{P(X>80)} = \frac{\int_{105}^{\infty} \lambda e^{-\lambda x} dx}{\int_{80}^{\infty} \lambda e^{-\lambda x} dx}$$

$$= \underline{\underline{0.8}}$$

14) (a) $P(Z<1.23) = 0.8907$ (directly read from table)

(b) $P(Z>2.17) = 1 - P(Z<2.17)$

$= 1 - 0.9850$ → from table

$= 0.0150$

(c) $P(Z>-0.08) = P(Z<0.08)$

$= 0.5319$

(d) $P(Z<-1.50) = P(Z>1.50) = 1 - P(Z<1.50)$

$= 1 - 0.9332$

$= 0.06681$

(e) $P(-2.92<Z<-1.67)$

$= P(Z<-1.67) - P(Z<-2.92)$

$= P(Z>1.67) - P(Z>2.92) = 1 - P(Z<1.67) - (1 - P(Z<2.92))$

$= P(Z<2.92) - P(Z<1.67)$

$= 0.9982 - 0.9525$

$=$ ________

(f) $P(-0.92<Z<0.83) = P(Z<0.83) - P(Z<-0.92)$

$= P(Z<0.83) - P(Z>0.92)$

$= P(Z<0.83) - (1 - P(Z<0.92))$

$= 0.7967 - 1 + 0.8212$

$= \underline{\underline{0.6179}}$

(15) (a) $P(z<1.048)$

1.048 in between 1.04 and 1.05

$\downarrow$ 0.8508 $\qquad \downarrow$ 0.8531

$$= \underset{\text{starting probability}}{0.8508} + \frac{1.048-1.04}{1.05-1.04}\underset{\text{diff of 2 probabilities}}{(0.8531-0.8508)}$$

$= \underline{\underline{0.8527}}$

(b) $P(-0.704 < z < 0.897)$

$P(z<0.897) - P(z<-0.704)$

$P(z<0.897) - (1 - P(z<0.704))$

Use Interpolation like 15(a)

$= \underline{\underline{0.5744}}$

(16) $X \sim N(100, 16)$.

Here X is Normally distributed which should be standardized to read value from table

$\mu = 100$

$\sigma^2 = 16$

$$\boxed{Z = \frac{X-\mu}{\sigma}}$$

$$P(X>110) = P\left(\frac{X-\mu}{\sigma} > \frac{110-\mu}{\sigma}\right)$$

$$= P\left(\frac{X-100}{\sqrt{16}} > \frac{110-100}{\sqrt{16}}\right)$$

$$= P\left(Z > 10/4\right) = P(z>2.5)$$

$$= 1 - P(z<2.5)$$

$$= 1 - 0.9938 = 0.0062$$

(b) $P(95 < X < 107)$

$= P(X < 107) - P(X < 95)$

$= P\left(\frac{X-100}{\sqrt{16}} < \frac{107-100}{\sqrt{16}}\right) - P\left(\frac{X-100}{\sqrt{16}} < \frac{95-100}{\sqrt{16}}\right)$

$= P(Z < 7/4) - P(Z < -5/4)$

$= P(Z < 1.75) - (1 - P(Z < 1.25))$

get values from table

$= \underline{\underline{0.8543}}$

(17) $X \sim N(5, \sigma^2)$

$P(X>9) = 0.2 \quad Var(X) = ?$

$P\left(\frac{X-5}{\sigma} > \frac{9-5}{\sigma}\right) = 0.2$

$P\left(Z > \frac{4}{\sigma}\right) = 0.2$

$1 - P\left(Z < \frac{4}{\sigma}\right) = 0.2$

$P\left(Z < \frac{4}{\sigma}\right) = 0.8$

$\frac{4}{\sigma} \approx 0.85 \rightarrow$ from table

$\sigma = \frac{4}{0.85} =$

$\sigma = \underline{\underline{22.14}}$

(18) $E[X] = 20, \; Var[X] = 20$

$P(0 < X < 40)$ Here donot know It's distribution we can find approximate probability by using inequalities

Since Mean and Variance are known we can use Chebyshev inequality

$P(0<X<40) = P(0-20 < X-20 < 40-20)$

$= P(-20 < X-20 < 20)$

$= P(|X-20| < 20)$

By Chebyshev Inequality we know

$P(|X-\mu| \geq k) \leq \frac{\sigma^2}{k^2}$. i.e $P(|X-20| \geq 20) \leq \frac{20}{20^2} = 1/20$

$\therefore P(|X-20| \geq k) \leq 1/20$

$P(0 < X < 40) = P(|X-20| \leq 20) = 1 - P(|X-20| \geq 20)$

$\geq 1 - 1/20$

$\geq 19/20$

$\therefore P(0 < X < 40) \geq 19/20$

(19) $E[X] = 75$ given. Let X — test score of student, distribution of X is unknown

(a) $P(X > 85)$

By Markov inequality

$$P(X \geq a) \leq \frac{E[X]}{a}$$

$$P(X > 85) < \frac{75}{85}$$

$P(X > 85) < 15/17$

(b) $E[X] = 75$ and $Var(X) = 25$

$P(65 < X < 85) = P(65-75 < X-75 < 85-75)$

$= P(-10 < X-75 < 10)$

$= P(|X-75| < 10)$

we know by Chebyshev's Inequality

$$P(|X-75| \geq 10) \leq \frac{\sigma^2}{k^2} = \frac{25}{10^2} = 0.25$$

$\therefore P(65 < X < 85) = P(|X-75| < 10) = 1 - P(|X-75| \geq 10)$

$\geq 1 - 0.25 = 0.75$

(20) $X_1, X_2 \ldots X_{20} \sim poi$

Mean $= 1$

$\therefore \lambda = 1$

$P\left\{ \sum_{1}^{20} x_i > 15 \right\}$

By CLT

$\sum x_i \sim N(n\lambda, n\lambda) \sim N(20, 20)$

$P\left\{ \sum_{1}^{20} x_i > 15 \right\}$ Continuity Correction

$= P\left\{ \sum_{1}^{20} x_i > 15.5 \right\}$

$= P\left\{ Z > \frac{15.5 - 20}{\sqrt{20}} \right\}$

$P\{Z > -1.01\}$ → taking upto 2 decimals

$P\{Z < 1.01\} = \underline{\underline{0.8438}}$

2) $E[X] = 0$, $\sigma^2 = 1$

$$P[-3 < X < 3] = P[|X - 0| < 3]$$

We know that

$$P[|X - 0| \geq 3] \leq \frac{\sigma^2}{k^2} = \frac{1}{9}$$

$$\therefore P[|X - 0| < 3] = 1 - P[|X| \geq 3]$$

$$\geq 1 - 1/9 = 8/9$$

$\therefore$ The lower bound is $8/9$.

(22) $X \sim Bin(100, 0.15)$

$n = 100,\ p = 0.15$

$\downarrow$

p is not too small

So cannot approximate to poisson

Here $np = 100 \times 0.15$

$npq = 100 \times 0.15 \times 0.85 = 12.75$

Satisfies $np \geq 5$ and $np(1-p) > 5$

So central limit theorem can be used

$\therefore\ X \overset{\cdot}{\sim} N(np, np(1-p))$

$X \overset{\cdot}{\sim} N(15, 12.75)$

$P(X > 20) = \cancel{P\left(\frac{X-15}{\sqrt{12.75}}\right)}$ using continuity correction

$= P(X > 20.5) = P\left(\frac{X-15}{\sqrt{12.75}} > \frac{20.5-15}{\sqrt{12.75}}\right) = P(Z > 1.54)$

read from table

$= \underline{\qquad\qquad}$

(23) $X \sim Poi(0.075)$ $\quad n = 500$

n is large so can be approximated by CLT

$X \overset{\cdot}{\sim} N(n\lambda, n\lambda) \Rightarrow X \overset{\cdot}{\sim} N(37.5, 37.5)$

$P(X \geq 50) \xrightarrow{\text{Continuity Correction}} P(X > 49.5) = P\left(\frac{X-37.5}{\sqrt{37.5}} > \frac{39.5-37.5}{\sqrt{37.5}}\right)$

$= P(Z > 1.960) = 0.025$ //

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