

DMPT
Assignment Solutions

1. A: Set of Integers divisible by 2

$$|A| = \left\lfloor \frac{250}{2} \right\rfloor = 125$$

B: divisible by 3

$$|B| = \left\lfloor \frac{250}{3} \right\rfloor = 83$$

C: divisible by 7

$$|C| = \left\lfloor \frac{250}{7} \right\rfloor = 35$$

$|A \cap B|$: divisible by 2 and 3 i.e. divisible by 6

$$|A \cap B| = \left\lfloor \frac{250}{6} \right\rfloor = 41$$

$$|A \cap C| = \text{divisible by 2 and 7 i.e. by 14} \\ = \left\lfloor \frac{250}{14} \right\rfloor = 17$$

$$|B \cap C| = \left\lfloor \frac{250}{21} \right\rfloor \text{ divisible by 3 and 7 i.e. by 21} \\ = 11$$

$$|A \cap B \cap C| = \text{divisible by 2, 3, 7 i.e. 42} \\ \left\lfloor \frac{250}{42} \right\rfloor = 5$$

By inclusive exclusive principle

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ = 125 + 83 + 35 - 41 - 17 - 11 + 5 \\ = \underline{\underline{179}}$$

② (a) at least 8 characters = $P_8 + P_9 + P_{10} + P_{11} + P_{12}$
but not more than 12

Password can be \rightarrow lower case (or) upper case (or) digit
any letter any letter

$$= 26 + 26 + 10 + 6$$

(or) Special Char

$= 68 \rightarrow$ Total 68 characters are available

$P_8 \rightarrow$
68 68 68 68 68 68 68 68 \rightarrow repetition is allowed order matters
 $(68)^8$

$$P_8 + P_9 + P_{10} + P_{11} + P_{12}$$

$$= 68^8 + 68^9 + 68^{10} + 68^{11} + 68^{12}$$

$$= 9920671339261 \times 10^{21} \dots = 9.920671339 \times 10^{21}$$

$$\approx \underline{\underline{9.9 \times 10^{21}}}$$

(b) No. of ways to have one occurrence or at least of the special characters

$=$ Total permutation $-$ No of ways of occurrence of no special character

$$P_8 = 68^8 - 62^8$$

$$P_9 = 68^9 - 62^9$$

$$P_{10} = 68^{10} - 62^{10}$$

$$P_{11} = 68^{11} - 62^{11}$$

$$P_{12} = 68^{12} - 62^{12}$$

$$P = P_8 + P_9 + P_{10} + P_{11} + P_{12}$$

$$= \underline{\underline{6.64151496 \times 10^{21}}}$$

3) (a) Answer - 22 (use tree diagram)

(b) $4 \times 3 + 5 \times 2 = \underline{\underline{22}}$

(a) By pigeon hole principle,
He has to take out 3 socks to be sure that
he had at least two socks of same color.

(b) 14

(5) By pigeon hole principle

$$\left\lceil \frac{61327}{30} \right\rceil = 2045. \quad \text{One dictionary should have at least 2045 pages.}$$

(6) (a) 150

(b) 25

(c) 6

(d) 2

7) 9

8) $20 + 5 - 1 \underset{20}{C} = 24 \underset{20}{C} \quad (or) \quad 24 \underset{4}{C}$

⑨ 5 Cards to 6 players deck of 48

- group of 5 each
- 1 $48C_5$
 - 2 $43C_5$
 - 3 $38C_5$
 - 4 $33C_5$
 - 5 $28C_5$
 - 6 $23C_5$

$$\frac{48!}{(5!)^6 \times 18!}$$

permutation with indistinguishable objects

(or)

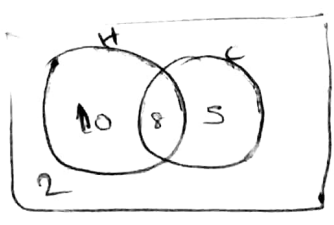
$$48C_5 \times 43C_5 \times 38C_5 \times 33C_5 \times 28C_5 \times 23C_5$$

remains 18

⑩ indistinguishable objects to distinguishable bins
in combination with repetition

$$8+10-1C_{10} = 17C_{10} \text{ (or) } 17C_7$$

⑪ (a) 8



(b) 20.

