

Probability Theory

Discrete Random Variables

Practice set

1. A fair coin is tossed three times. Let X be the random variable representing the total number of heads. Find

- (i) Probability Mass function
- (ii) cumulative distribution function
- (iii) $E[X]$, $\text{Var}[X]$

2. A random sample of four policy holders is taken from a group of eight, comprising 3 men and five women. Determine

- (i) probability mass function
- (ii) Calculate $F_X(2)$

3. The random variable W has a PMF

| | | | |
|----------|-----|-----|-----|
| w | 2 | 4 | 5 |
| $P(W=w)$ | 0.2 | 0.5 | 0.3 |

- (i) Determine CDF for W . Sketch graph of CDF
- (ii) Find $F_W(1)$, $F_W(4.5)$, $F_W(10)$

4. Let X represent the difference between the number of head and the number of tails obtained when coin is tossed n times. What are possible values of X ?

5. Suppose that $P(X=0) = 1 - P(X=1)$. If $E[X] = 3 \text{Var}[X]$ find $P(X=0)$

6. If CDF of random variable X is given by

$$F_X(x) = \begin{cases} 0 & x < 0.1 \\ \frac{3}{8} & 0.1 \leq x < 0.15 \\ \frac{5}{8} & 0.15 \leq x < 0.2 \\ 1 & x \geq 0.2 \end{cases}$$

Find

(i) $P(X=0.15)$

(ii) $P(X > 0.1)$

(iii) $P(X < 0.2)$

(iv) $P(X = 0.18)$

(v) $F_X(0.18)$

7. A random variable X has mean 6. Find

(i) $E[5X+7]$ (ii) $E\left[\frac{3X+1}{8}\right]$ (iii) $E[9-4Z]$

8. Two coins are to be flipped. The first coin will land on heads with probability 0.6. The second coin with probability 0.7. Assume that the results of the flips are independent. and let X be the total number of heads that result.

(i) Find $P(X=1)$

(ii) Find PHF

(iii) $E[X]$

9. A Random Variable Z has mean 6 and Standard deviation 2. Find

(i) $\text{Var}[5Z+1]$ (ii) $\text{Var}[9-4Z]$ (iii) $E(Z^2)$

10. Suppose that the Random Variable X is equal to the number of hits obtained by a certain baseball player in his next 3 at bats. If $P(X=1) = 0.3$, $P(X=2) = 0.2$, $P(X=0) = 3P(X=3)$ find $E[X]$, $\text{Var}(X)$, $\text{SD}(X)$

(ii) Calculate third order central moment