1) Express the following statements using $C(x)$, $D(x)$, quantifiers and logical connectives.

a) If any student of UG1 is good in DM then he/she is good in C as well.

$$\forall x \, (D(x) \rightarrow C(x))$$

b) There may be some students in UG1 who are good in C but not in DM.

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$$\exists x \, (C(x) \wedge \neg D(x))$$

2) Prove the following statements are equivalent i) using truth table ii) without using truth table.

a) $(p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

p	q	$\neg p$	$\neg q$	$p \rightarrow q$	$\neg p \rightarrow \neg q$	$(p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

$(p \wedge q) \vee (\neg p \wedge \neg q)$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

(2)

$\therefore$ Both the statements are equivalent

$(p \to q) \wedge (\neg p \to \neg q)$

$\equiv (\neg p \vee q) \wedge (p \wedge \neg q)$

$\equiv (((\neg p \vee q) \wedge p) \vee ((\neg p \vee q) \wedge \neg q))$

$\equiv ((\neg p \wedge p) \vee (q \wedge p)) \vee ((\neg p \wedge \neg q) \vee (q \wedge \neg q))$

$\equiv (F \vee (q \wedge p)) \vee ((\neg p \wedge \neg q) \vee F)$

$\equiv (q \wedge p) \vee (\neg p \wedge \neg q)$

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\equiv (p \to q) \wedge (\neg p \to \neg q)$ are equivalent

Properties ???

I.S

b) $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

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$(p \vee q) \rightarrow r$

p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

2

$$(p \rightarrow r) \wedge (q \rightarrow r)$$

$$\equiv (\neg p \vee r) \wedge (\neg q \vee r)$$

Properties???

$$\equiv (\neg p \wedge \neg q) \vee r$$

1.5

$$\equiv \neg (p \vee q) \vee r$$

$$\equiv (p \vee q) \rightarrow r$$

3) $((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$

$\neg ((p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r)) \vee r$ [$p \rightarrow r \equiv \neg p \vee q$]

$\neg ((p \vee q) \wedge ((\neg p \wedge \neg q) \vee r)) \vee r$ [Distributive law]

$\neg ((p \vee q) \wedge (\neg (p \vee q) \vee r)) \vee r$

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$\neg((p \vee q) \wedge \neg(p \vee q)) \vee ((p \vee q) \wedge r)) \vee r$

$\neg(F \vee ((p \vee q) \wedge r)) \vee r$

[Distributive law]

$\neg((p \vee q) \wedge r) \vee r$

2

$(\neg p \wedge \neg q) \vee \neg r \vee r$

$(\neg p \wedge \neg q) \vee (\neg r \vee r)$ Associative law

$(\neg p \vee \neg q) \vee T$

T

4) Negation of the following expression

$\exists y (\forall x \exists z \; A(x,y,z) \vee \exists x \forall z \; B(x,y,z))$

$\forall y (\exists x \forall z \neg A(x,y,z) \wedge \forall x \; \exists z \neg B(x,y,z))$

5) Determine whether the following argument is valid or not.

x be a student

$D(x)$: x is good at DM

$C(x)$: x is good at CP

$E(x)$: x has good logical ability

$F(x)$: x can get a good job

$\forall x \; (D(x) \rightarrow E(x))$

$\forall x \; (C(x) \rightarrow E(x))$

$\forall x\,(E(x) \to F(x))$ — (2)

From (1) & (2) By Hypothetical ~~Syllogism~~

$\forall x\,(C(x) \to F(x))$ $[p \to q \equiv (\neg p \vee q)]$

$\forall x\,(\neg C(x) \vee F(x))$

But $\exists x\,(\neg C(x) \wedge F(x))$

$\therefore$ The argument is ~~in~~valid

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6) Find out the mistake in the following

The wrong statement is c because $P(C) \wedge Q(C) \to P(C)$ by simplification but not $P(C) \vee Q(C) \to P(C)$

The wrong statement is e because $P(C) \wedge Q(C) \to P(C)$ by simplification but not $P(C) \vee Q(C) \to Q(C)$

2

g is also wrong as $\forall x\, P(x) \vee \forall x\, Q(x)$ is not true by conjuction but not $\forall x\, P(x) \vee \forall x\, Q(x)$

7) Prove that sum of any 2 rational numbers is rational

Let us assume two rational numbers as

2

$a = p/q$ and $b = r/s$

$a+b = p/q + r/s$

$= \dfrac{ps + rq}{qs}$ $(qs \neq 0)$

$ps + rq = n_1$

$qs = n_2$

$\therefore a+b = \frac{n_1}{n_2}$ is in the form of a rational no.

9) Contrapositive approach: 5n^2+2 is odd no $\therefore$ the n is odd number

Contrapositive $p \to q$

$\neg q \to \neg p$

say n is even number

$n = 2k$

2

$(5(2k)^2+2$

$(10k)^2+2$

$100k^2+2$

$2(50k^2+1)$

$2(k'+1)$ $\therefore 50k^2 = k'$

$\therefore$ 5n^2+2 is even

8) a) K: The knife is in the store room

b) S: We saw it when we cleared the store room

4, 1. $K \to S$

a) b: Murder was committed at the basement

b) a: Murder was committed inside the apartment

2. $b \vee a$

a) b: Murder was committed at the basement

b) d: Knife is in the yellow dust bin.

3. $b \to d$

a) $\neg s$: We did not see a knife when we cleared the store room

4. $\neg s$

a) o: Murder was committed outside the building.

b) u: We are unable to find the knife.

5. $o \to u$

a) a: Murder was committed inside the apartment

b) k: The knife is in the store room

6. $a \to k$

from 1 and 4 $k \to s$

$\neg s$ [Using Modus tollens]

7. $\neg k$

from 7 and 6

$a \to k$ [Using Modus Tollens]

8. $\neg a$

5

from 2 and 8

$b \vee a$ [Disjunctive Syllogism]

9. b

from 3 and 9

$b \to d$ [ ]

10. d

1) So knife is in the yellow dust bin

2) Then murder was committed at the basement.

10) Direct Approach.

Given $x+y$ is even

$$x+y = 2k$$

Let x be even $\therefore x = 2p$

$$2p + y = 2k$$

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$$y = 2(k-p) \quad (\text{for } k>p)$$

$f = k-p$

$\therefore y$ is even as it is of the form $2f$.

Let x be odd $\therefore x = 2p+1$

$$2p+1+y = 2k$$

$$y = 2(k-p)-1 \quad (\text{for } k>p)$$

$$y = 2n-1$$

$n = k-p$

$\therefore x$ and y must be both odd, or even...