Discrete Maths - Assignment 2

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(1) $(p \to q) \wedge (\neg p \to \neg q)$

PDNF:

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ $\quad \because (a \to b) \equiv \neg a \vee b$

$\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \wedge (p \vee \neg q))$ $\quad$ (Distributive law)

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$

$\equiv [F \vee (\neg p \wedge \neg q)] \vee [(q \wedge p) \wedge F]$ $\quad \because \neg a \wedge a = F$

$\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$ $\quad \because F \vee a = a$

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PCNF:

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ $\quad \because (a \to b) \equiv \neg a \vee b$

(2) $\exists y (\forall x \exists z\, A(x,y,z) \to \exists x \forall z\, B(x,y,z))$

$\equiv \exists y [\neg (\forall x \exists z\, A(x,y,z)) \vee (\exists x \forall z\, B(x,y,z))]$ $\quad \because (a \to b) \equiv \neg a \vee b$

$\equiv \exists y [\exists x \forall z \neg (A)(x,y,z) \vee \exists x \forall z\, B(x,y,z)]$

$\equiv \exists y \exists x \forall z (\neg A(x,y,z) \vee B(x,y,z))$

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(3.) Prove that the difference between rational and irrational nos is always irrational.

By using contradiction method:

Considering the difference to be ~~a~~ rational no; p = irrational

Let a/b be a rational no and x/y ; $b \neq 0$; $y \neq 0$

$$\frac{a}{b} - p = \frac{x}{y}$$

$$p = \frac{a}{b} - \frac{x}{y} = \frac{ay - bx}{by} \quad \text{where } by \neq 0$$

= rational no.

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But this contradicts our previous assumption of p being irrational and ~~the~~ hence out assumption that the difference is rational is wrong. This proves by the contradiction method that difference between any rational and irrational no is always irrational.